Ch: 8 Potential Energy & Conservation of Energy

* Forces in Nature:

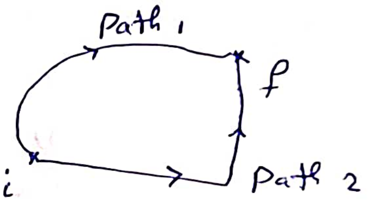
① Gravitational Force $= mg$
② spring Force $= -kx$
} → Conservative Forces ⇒ potential Energy

③ Normal Force N
④ frictional Force f_s, f_k
⑤ Drag Force $(D = \frac{1}{2} C \rho A v^2)$
} → Non-Conservative Forces

* properties of Conservative Forces:

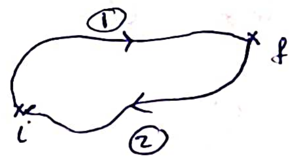
1- work done by $F_{(conservative)}$ is path independent.

$$W_1 = W_2$$



2- work done by F_{cons} around a closed path equal zero.

$$W_{i \to i} = 0$$



3- work done against F_{cons} don't lost, But stored as Energy called potential Energy (U)

$$W_{app\, i \to f} = \Delta U = U_f - U_i$$

4- Work done by $F_{cons} = -\Delta U$

$$W_F = -W_{app} = -\Delta U = -(U_f - U_i)$$
$$= U_i - U_f$$

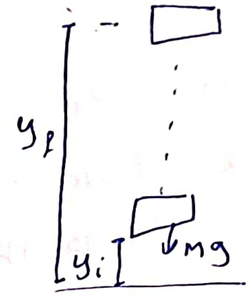
* Gravitational Potential Energy:

$$W_f = -\Delta U = -(U_f - U_i)$$

$$\int_{r_i}^{r_f} \bar{F}_{cons} \cdot d\bar{r} = -\Delta U \Rightarrow \boxed{\Delta U = -\int_{r_i}^{r_f} \vec{F}_{cons} \cdot d\bar{r}}$$

$$\int_{r_i}^{r_f} \bar{F} \cdot dr = \int_{x_i}^{x_f} F_x \, dx + \int_{y_i}^{y_f} F_y \, dy + \int_{z_i}^{z_f} F_z \, dz$$

For Gravitational Force:



$$U_f - U_i = -\int_{r_i}^{r_f} F_g \cdot d\bar{r}$$

$$= -\int_{y_i}^{y_f} (-mg) dy = mgy \Big|_{y_i}^{y_f}$$

$$U_f - U_i = mgy_f - mgy_i \quad , \text{ let } y_i = 0$$

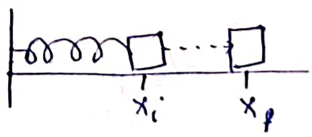
$$U_i = 0$$

(reference point)

$$U_f = mgy$$

$$\boxed{U_g = mgy}$$

* spring potential Energy.



$$U_f - U_i = -\int_{r_i}^{r_f} \vec{F}_s \cdot d\bar{r}$$

$$= -\int_{x_i}^{x_f} (-kx) dx = k \frac{x^2}{2} \Big|_{x_i}^{x_f}$$

$$= \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2 \quad , \text{ let } x_i = 0$$

$$U_i = 0$$

$$\boxed{U_s = \frac{1}{2} k x^2}$$

* Mechanical Energy = Kinetic Energy + Potential Energy

$$E_{mech} = K.E + P.E$$

$$\boxed{E_{mech} = \frac{1}{2} m v^2 + U}$$

* Conservation of Mechanical Energy:

If the only Force acting on the system is Conservative force $\Rightarrow$ E_{mech} is Conserved.

$$W_{cons} = -\Delta U \quad , \quad W_{net} = \Delta K$$

$$\Delta K = -\Delta U \quad \Rightarrow \quad \Delta K + \Delta U = 0$$

$$\underset{\text{التغير}}{\Delta}(K + U) = 0$$

$$K + U = \text{Const}$$

$$\Rightarrow \boxed{(K + U)_i = (K + U)_f} \quad \text{conserved}$$

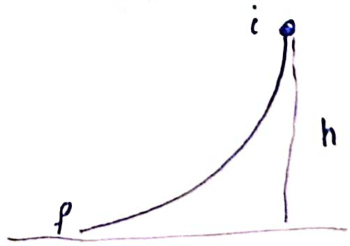
لا تتغير

sample problem 8.3:

mass = m , starting from rest!!

h = 8.5 m

Find v_f ??



Using the Conservation of E_{mech}:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2} m v_i^2 + m g y_i = \frac{1}{2} m v_f^2 + m g y_f$$

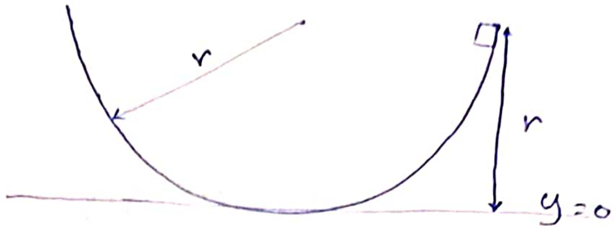
$$\cancel{\frac{1}{2} m (0)^2} + \cancel{m} g (8.5) = \frac{1}{2} \cancel{m} v_f^2 + \cancel{m g (0)}$$

$$v_f = \sqrt{2 g (8.5)} = 13 \text{ m/s}$$

problem 5

$m = 2\,g = 2\times10^{-3}\,kg$

$r = 22\,cm = 0.22\,m$



a) $W_{g\,(top \to bottom)} = -\Delta U$

$= -(U_f - U_i) = U_i - U_f$

$W_g = mg(y_i) - mg(y_f)$

$= mg(r) - mg(0)$

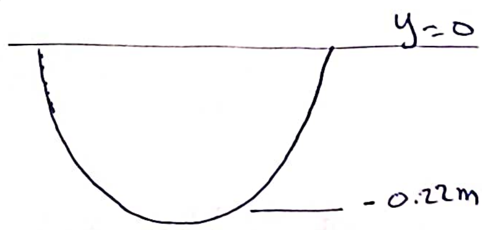
$= 2\times10^{-3} \times 10 \times 0.22 = 0.44\times10^{-2}\,J$

$= 4.4\times10^{-3}\,J = 4.4\,mJ$

b) $\Delta U = U_f - U_i$

$= 0 - 4.4\,mJ = -4.4\,mJ$

d) let the reference point $(y=0)$ at the top.



$U_i = 0$

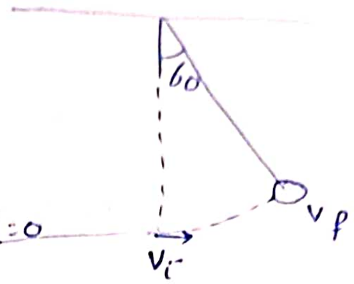
$U_f = mg\,y_f = mg(-0.22)$

$= -4.4\,mg$

e) if the mass is doubled $\Rightarrow$ All the answers will increase

problem 20

$v_i = 8\ m/s$, $m = 2\ kg$, $L = 4.5\ m$



a) Find v_f when the string is at $60°$ to the vertical?

from conservation of E_{mech} :

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2} m v_i^2 + mgy_i = \frac{1}{2} m v_f^2 + mgy_f$$

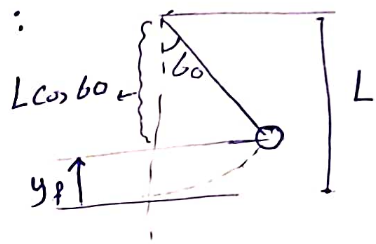
$$\frac{1}{2}(2)(8)^2 + \cancel{mg(0)} = \frac{1}{2}(2) v_f^2 + 2 \times 10 \times 2.25$$

$$64 = v_f^2 + 45$$

$$v_f = 4.3\ m/s$$

, $y_i = 0$,

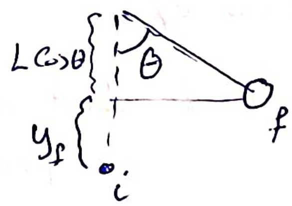
y_f :



$$y_f = L - L\cos 60$$
$$= 4.5 - 2.25$$
$$= 2.25\ m$$

b) what is the greatest angle ?

at $\theta_{max} \Rightarrow v_f = 0$



$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2} m v_i^2 + 0 = 0 + mgy_f$$

$$\frac{1}{2}(2)(8)^2 = 2 \times 10\ y_f$$

$$y_f = 3.2\ m \Rightarrow L\cos\theta = L - y_f$$

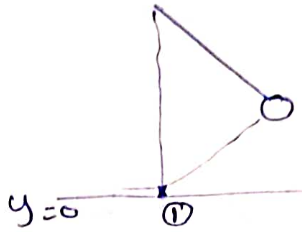
$$= 4.5 - 3.2$$

$$4.5 \cos\theta = 1.3$$

$$\cos\theta = 0.29$$

$$\theta = 73°$$

c) Find E_{mech}?

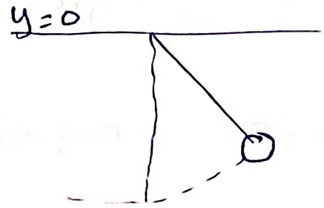


$E_{mech} = U_1 + K_1$

$= \cancel{mg y_{(1)}}^{0} = \frac{1}{2} m (v_1)^2$

$= \frac{1}{2} (2) (8)^2 = 64 \text{ J}$

Solve This problem taking y=0 at the top.

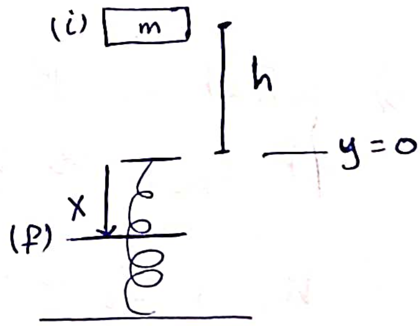


problem 24

$m = 2 \text{ kg}$, $h = 50 \text{ cm}$

$k = 1960 \text{ N/m}$

Find the max. distance the spring is compressed?



$E_i = E_f$

$\cancel{K_i}^{0} + U_i = \cancel{K_f}^{0} + U_f$

$mg(h) = (U_f)_g + (U_f)_s$

$mgh = mg(-x) + \frac{1}{2} k x^2$

$\Rightarrow \frac{1}{2} k x^2 - mgx - mgh = 0$

$\cancel{\frac{1}{2} k x^2 - m}$

$$\frac{1}{2}(1960)x^2 - 2(10)x - 2(10)(0.5) = 0$$

$$\boxed{980x^2 - 20x - 10 = 0}$$

$$98x^2 - 2x - 1 = 0$$

$$x = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a} = 0.11\ m$$

$W_{cons} = -\Delta U$,

$W = F \cdot \Delta x$

$\Rightarrow \Delta U = -F \cdot \Delta x$

$dU = -F\,dx$

$\Rightarrow \boxed{F = -\frac{dU}{dx}}$ in one dimensional motion

Ex: $U(x) = -4x\,e^{-x/4}$ J

→ Find $F(x)$? $F(x) = -\frac{dU}{dx}$

$= -4x\left(\frac{-1}{4}\right)e^{-x/4} + e^{-x/4}(-4)$

$= x\,e^{-x/4} - 4\,e^{-x/4}$ N

→ For what value of x does $F(x) = 0$?

$F(x) = x\,e^{-x/4} - 4\,e^{-x/4} = 0$

$(x-4)\,e^{-x/4} = 0$

$\Rightarrow x - 4 = 0 \Rightarrow x = 4$ m

→ at $x = 5$ m the body has a kinetic energy of 2 J,

Find E_{mech} ??

$E_{mech} = K.E + P.E$, $U(5) = -4(5)\,e^{-5/4}$

$= 2 - 5.7$ $= -5.7$ J

$= -3.7$ J

→ Find the velocity when the body at $x = 4\,m$?

$$E_{mech(f)} = E_{mech(i)}$$

$$K(5m) + U(5m) = K(4m) + U(4m)$$

$$-3.7 = K_f + (-5.92)$$

$$U(4) = -4(4)e^{-1} = -5.92\ J$$

$$K_f = 2.2\ J$$

$$\frac{1}{2} m V_f^2 = 2.2 \quad , \text{if } m = 2\,kg$$

$$V_f = 1.48\ m/s$$

* Finding U from F ??

$$W = \int F \cdot dx \quad , \quad W = -\Delta U$$

$$\Rightarrow \Delta U = -\int_{x_i}^{x_f} F \cdot dx \quad ; \text{ or } U = -\int F \cdot dx$$

EX: (8^{P}-104)

$$F_{cons} = -3x - 5x^2 \quad , \text{ at } x = 0 \ , \ U = 0$$

$$m = 20\,kg$$

1) Find U at $x = 2\,m$?

$$U = -\int F \cdot dx = -\int -3x - 5x^2\, dx$$

$$U = \frac{3x^2}{2} + \frac{5x^3}{3} + C$$

$$U(0) = 0 + 0 + C = 0$$

$$U = \frac{3x^2}{2} + \frac{5x^3}{3}$$

$$U(2) = \frac{3}{2}(2)^2 + \frac{5}{3}(2)^3 = 19.6 \text{ J}$$

2) at $x = 5$ m , $v = -4$ m/s , Find v at $x = 0$

$$(E_i)_{x=5} = (E_f)_{x=0}$$

$$(K_i + U_i)_{x=5} = (K_f + U_f)_{x=0}$$

$$\frac{1}{2} m v_i^2 + U(5) = \frac{1}{2} m v_f^2 + U(0)$$

$$\frac{1}{2}(20)(-4)^2 + \left[\frac{3}{2}(5)^2 + \frac{5}{3}(5)^3\right] = \frac{1}{2}(20) v_f^2$$

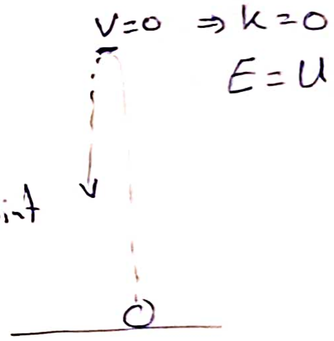
$$v_f = -6.37 \text{ m/s}$$

* potential energy curve :

→ turning points : ① gravity

& Equilibrium points : $F = 0$

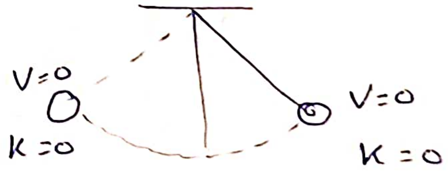
one turning point



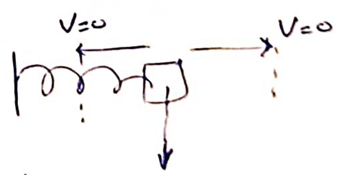
② Pendulum :

2 turning points

E = U



③ spring



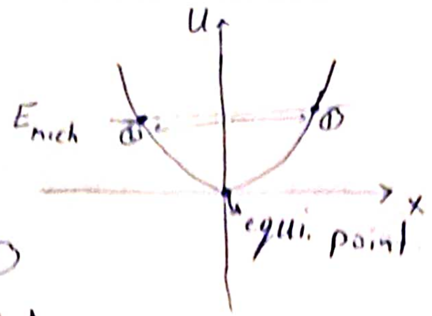
two turning points

equilibrium point

$U_s = \frac{1}{2} k x^2$

at ① & ②

$E_{mch} = U \Rightarrow$ ① & ② turning points



~~In the previous example (8-104)~~

~~Find turning points:~~

Ex: $U(x) = 8x^2 + 2x^4$

at $x = 1$ m, $V = 5$ m/s, $m = 0.2$

Find turning points:

$E = K + U$

$E_{x=1} = \frac{1}{2} m V_{x=1}^2 + U(x=1)$

$= \frac{1}{2}(0.2)(5)^2 + (8(1)^2 + 2(1)^4)$

$= 12.5$ J

at turning points $K = 0 \Rightarrow E = U$

$12.5 = 8x_t^2 + 2x_t^4$

$\Rightarrow x_t^2 = \frac{-8 \mp \sqrt{64 - 4(2)(-12.5)}}{2(2)}$

$x_t = \mp 1.1$ m

Find The equilibrium points:

$\cancel{F} = 0$, $F = -\frac{dU}{dx} = -16x - 8x^3$

$\Rightarrow -16x - 8x^3 = 0$

$x(-16 - 8x^2) = 0$

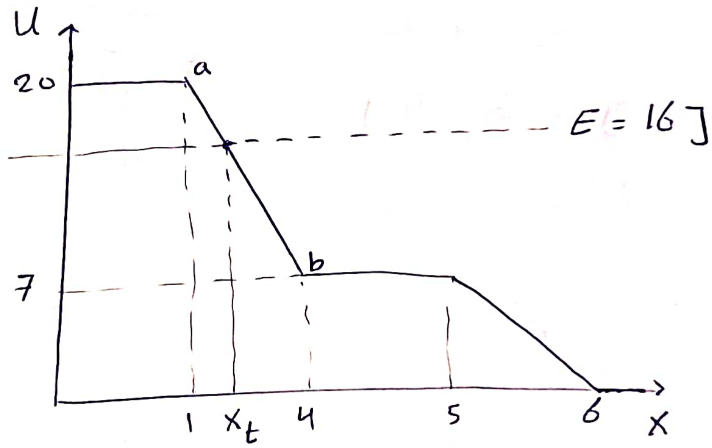
$\Rightarrow \boxed{x = 0}$ or $-16 - 8x^2 = 0$ ✗

equilibrium point.

Sample problem 8.04:

$m = 2\ kg$

at $x = 6.5\ m$, $\vec{v} = -4\hat{i}\ m/s$



a) Find v at $x = 4.5\ m$?

* at $x = 6.5\ m$:

, $v = -4\ m/s \Rightarrow K = \frac{1}{2}mv^2$

$= \frac{1}{2}(2)(4)^2 = 16\ J$

$\Rightarrow U = 0$

$\Rightarrow E = K + U = 16\ J$

* at $x = 4.5\ m$:

$U = 7\ J$, but $E = 16$

$E = U + K$

$16 = 7 + K \Rightarrow K = 9\ J$

$K = \frac{1}{2}mv^2$

$9 = \frac{1}{2}(2)v^2 \Rightarrow v = \mp 3\ m/s$

b) Find turning point ? (x_t)

$K=0 \Rightarrow E=U$

$\boxed{U=16}$

(1, 20), (4, 7) (x_t, 16), (4, 7)

$$\text{slope (a-b)} = \frac{20-7}{1-4} = \frac{16-7}{x_t-4}$$

$x_t = 1.9\ m$

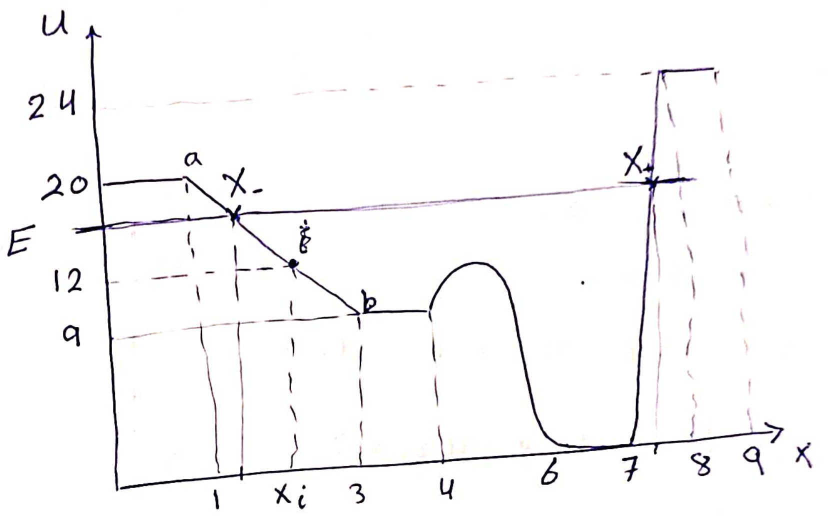
problem 38 :

$m = 0.2\ kg$

at initial state

$U = 12\ J$, $K = 4\ J$

a) Find v at $x = 3.5\ m$?



$E_i = E_{3.5\ m}$

$U_i + K_i = U(3.5) + K(3.5)$

$12 + 4 = 9 + K$

$K = 7\ J \Rightarrow \frac{1}{2}mv^2 = 7$

$v = 8.3\ m/s$

b) Find v at $x = 6.5\ m$?

c) Find the turning points ?

$E = U$

x_- : slope of ab : [(1, 20), (3, 9)] , [(1, 20), (x_-, 16)]

$$\frac{20-9}{1-3} = \frac{20-16}{1-x_-} \Rightarrow x_- = 1.8\ m$$

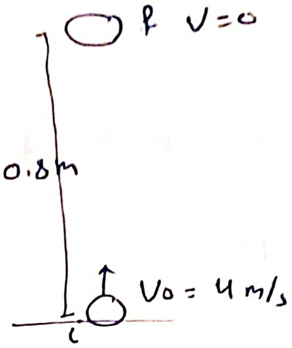
* work done by a non conservative force:

$$\boxed{W_{non\,cons} = \Delta E} = E_f - E_i$$

friction, drag force,

Ex: (8-97)

$m = 0.5\,kg$



* Find $W_{drag\,force}$?

$E_i \neq E_f$

$$W_{drag\,force} = E_f - E_i$$
$$= (K+U)_f - (K+U)_i$$
$$= (0 + mgy) - (\tfrac{1}{2} m V_0^2 + 0)$$
$$= -0.08\,J$$

* Find W_g? → conser

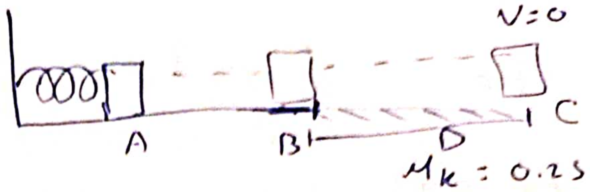
$$W_g = -\Delta U$$
$$= U_i - U_f$$
$$= 0 - mgy$$
$$= -3.9\,J$$

$$power = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$

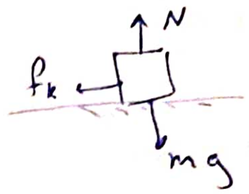
problem 43

$k = 640\ N/m$, $D = 7.8\ m$

$m = 3.5\ kg$



a) Find W done by the friction?



$$W_{friction} = \vec{f_k} \cdot \vec{D}$$

$$= (\mu_k N)\, D \cos 180$$

$$= -\mu_k m g D$$

$$= -67\ J \qquad (E_{thermal} = 67\ J)$$

b) the max. kinetic energy?

$$K_{max} = K_B$$

$W_{net} = \Delta K$, f_K is the only force (from B to C)

$$-67 = \cancel{K_C}^{0} - K_B$$

$$-67 = -K_B \Rightarrow K_{max} = 67\ J$$

c) Find the compression distance of the spring?

$E_A = E_B$ (conservative force)

$$(\cancel{K}^{0} + U)_A = (K + \cancel{U}^{0})_B$$

$$\frac{1}{2} k x^2 = \frac{1}{2} m V^2$$

$$\frac{1}{2}(640)\, x^2 = 67 \Rightarrow x = 0.46\ m$$