Maclurin $f(x)$

Exp show that Taylor series for $\cos x$ at $x=0$ converges to $\cos x$ for every value of x.

TF $\quad f(x) = P_{2n}(x) + R_{2n}(x)$

$$\cos x = \boxed{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!}} + R_{2n}(x)$$

$$0 \le \left| R_{2n}(x) \right| = \left| \frac{f^{(2n+1)}(c)}{(2n+1)!} x^{2n+1} \right|$$

$f = \cos x$

$\left| f^{(2n+1)}(c) \right| \le \underline{1}$

$$\le \left| \frac{x^{2n+1}}{(2n+1)!} \right|$$

$$\lim_{n\to\infty} 0 \le \lim_{n\to\infty} \left| R_{2n}(x) \right| \le \lim_{n\to\infty} \frac{|x|^{2n+1}}{(2n+1)!} = 0$$

10.1

$$\lim_{n\to\infty} R_{2n}(x) = 0$$

Hence, $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

Exp For what values of x can we replace $\sin x$ by $x - \frac{x^3}{3!}$ with an error of magnitude no more than 3×10^{-4}?

TF $\quad f(x) = P_n(x) + R_n(x)$

TF

$f(x) = P_n(x) + R_n(x)$

$\boxed{\sin x \approx x - \frac{x^3}{3!}} + \frac{x^5}{5!}$

Alternating Series Estimation Theorem

Error $< \left|\frac{x^5}{5!}\right|$

Error $< 3 \times 10^{-4}$

$\left|\frac{x^5}{5!}\right| < 3 \times 10^{-4}$ $\Rightarrow \left(|x^5|\right)^{\frac{1}{5}} < \left(3 \times 5! \times 10^{-4}\right)^{\frac{1}{5}}$

$|x| < 0.514$

$-0.514 < x < 0.514$ $\Rightarrow$ $x = 0.1 \Rightarrow$ Estimate $\sin(0.1)$

$\sin(0.1) \approx 0.1 - \frac{(0.1)^3}{3!}$

Maclurin $\Rightarrow c = 0$

Exp Estimate the error if $P_3(x) = x - \frac{x^3}{6}$ is used to estimate the value of $\sin x$ at $x = 0.1$

TF

$f(x) = P_3(x) + R_3(x)$

$\sin x = x - \frac{x^3}{6} + \frac{f^{(4)}(c)}{4!} x^4$ $\rightarrow (x-0)^4$

$f = \sin$

$f' =$

$f'' =$

$|f^{(4)}(c)| \le \overset{M}{1}$

$\text{Error} = |R_3(x)| = \left|\frac{f^{(4)}(c)\, x^4}{4!}\right| \le \frac{|x^4|}{4!} = \frac{x^4}{4!}$

$\text{Error} \le \frac{x^4}{4!} = \frac{(0.1)^4}{4!} < 4.2 \times 10^{-6}$

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