

Q29: single policyholders

Married Policyholders

$$n_1 = 400$$

$$n_2 = 900$$

# of making claims = 76

# of making claims = 90

9.  $\alpha = 0.05$

$$H_0: \pi_1 = \pi_2$$

$\pi_1$ : pop. prop. of single policyholders making claims

$$H_1: \pi_1 \neq \pi_2$$

$\pi_2$ : pop. prop. of Married

$$p_1 = \frac{76}{400} = 0.19$$

$$p_2 = \frac{90}{900} = 0.10$$

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{76 + 90}{1300} = 0.13$$

$$Z = \frac{0.19 - 0.10}{\sqrt{0.13(1-0.13)\left(\frac{1}{400} + \frac{1}{900}\right)}} = 4.45$$

$$\Rightarrow Z_{\frac{\alpha}{2}} = 1.96 \rightarrow \mp Z_{\frac{\alpha}{2}} = \pm 1.96$$



df = ∞  
α = 0.05  
two-tail  
t

$|Z| > Z_{\frac{\alpha}{2}}$  Reject  $H_0$  ( $\alpha = 0.05$ )

$\rightarrow (\pi_1 \neq \pi_2)$  ( $\alpha = 0.05$ )

b. 95% CI:

$$0.95 \text{ CI} = (p_1 - p_2) \mp Z_{\frac{\alpha}{2}} \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

$$= 0.09 \mp 1.96 \sqrt{\frac{0.19(1-0.19)}{400} + \frac{0.10(1-0.10)}{900}}$$

$$= 0.09 \mp 0.04 = (0.05, 0.13)$$

## Exercises:

Q23:  $n_1 = 400$   $n_2 = 300$   
 $p_1 = 0.48$   $p_2 = 0.36$

a. What is the point estimator of the difference between two pop. proportions.

$$\text{point estimator} = p_1 - p_2 = 0.48 - 0.36 = 0.12$$

b. construct a 90 percent CI for the difference between the two pop. proportions.

$$0.90 \text{ CI} = (p_1 - p_2) \pm E$$

$$1 - \alpha = 0.9 \rightarrow \alpha = 0.1, \frac{\alpha}{2} = 0.05$$

$$E = Z_{\frac{\alpha}{2}} \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}} = Z_{0.05} \sqrt{\frac{0.48(0.52)}{400} + \frac{0.36(0.64)}{300}}$$
$$= 1.645 \sqrt{0.001392} = 0.037309516$$

$$E = 0.061$$

$$\Rightarrow 0.90 \text{ CI} = 0.12 \pm 0.061$$
$$= (0.059, 0.181)$$

c. construct a 95% CI for the difference between the two pop. proportions:

$$E = Z_{\frac{\alpha}{2}} (0.037309516)$$

$$1 - \alpha = 0.95 \rightarrow \alpha = 0.05$$

$$= Z_{0.025} (0.037309516)$$

$$\frac{\alpha}{2} = 0.025$$

$$= 1.96 (0.037309516) = 0.073$$

$$\Rightarrow 0.95 \text{ CI} = 0.12 \pm 0.073$$
$$= (0.047, 0.193)$$

Q24: Consider the Hypotheses test  $H_0: \pi_1 - \pi_2 \leq 0$  upper tail test

$$H_1: \pi_1 - \pi_2 > 0$$

The following results are for independent samples taken from the two pop.

$$n_1 = 200$$

$$n_2 = 300$$

$$p_1 = 0.22$$

$$p_2 = 0.16$$

Q. What is the p-value?

$$Z = \frac{(p_1 - p_2)}{\sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{200(0.22) + 300(0.16)}{500} = \frac{92}{500} = 0.184$$

$$\Rightarrow Z = \frac{0.22 - 0.16}{\sqrt{0.184(1-0.184)\left(\frac{1}{200} + \frac{1}{300}\right)}} = 1.7$$

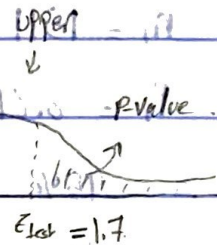
$$p\text{-value} = P(Z \geq Z_{\text{test}})$$

$$= P(Z > 1.7)$$

$$= 1 - P(Z < 1.7)$$

$$= 1 - 0.9554$$

$$p\text{-value} = 0.0446$$



احد بطل اللذة  
على اليسار، على  
اليمين، على

b. With  $\alpha = 0.05$ , what is your hypothesis testing conclusion:

$$p\text{-value} \square \alpha$$

$$\rightarrow 0.0446 \leq 0.05$$

So we Reject  $H_0$  ( $\alpha = 0.05$ )

Q25:  $n_1 = 1511$   $n_2 = 1050$   
 $p_1 = 0.61$   $p_2 = 0.78$

Provide a 95% CI interval estimate for the difference between pop. proportion in the two countries.

$$1 - \alpha = 0.95$$

$$\alpha = 0.05 \quad \alpha/2 = 0.025$$

$$E = Z_{\alpha/2} \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

$$= Z_{0.025} \sqrt{\frac{0.61(0.39)}{1511} + \frac{0.78(0.22)}{1050}}$$

$$= 1.96 (0.017912955)$$

$$E = 0.035$$

$$\Rightarrow 0.95 \text{ CI} = (p_1 - p_2) \pm E$$

$$= -0.17 \pm 0.035$$

$$= (-0.135, -0.205) \leftarrow \alpha$$

$$= (-0.205, -0.135) \leftarrow \begin{matrix} \text{هنا} \\ \text{الترتيب} \end{matrix}$$

Q26:

$$n_1 = 400$$

$$n_2 = 700$$

$$p_1 = 0.69$$

$$p_2 = 0.57$$

Construct 95% CI for the difference between the proportion . . .

$$E = Z_{\frac{\alpha}{2}} \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

$$= Z_{0.025} \sqrt{\frac{0.69(0.31)}{400} + \frac{0.57(0.43)}{700}}$$

t-table  
df = ∞  
α = 0.025

$$= 1.96 (0.029747148)$$

$$E = 0.058$$

$$\Rightarrow 0.95 \text{ CI} = (0.69 - 0.57) \pm 0.058$$

$$= 0.12 \pm 0.058$$

$$= (0.062, 0.178)$$

Q27 :

Q28 :

$$n_1 = 1103$$

$$n_2 = 1065$$

$$p_1 = 0.74$$

$$p_2 = 0.66$$

Two tail test :

Test the hypothesis  $\pi_1 - \pi_2 = 0$  with  $\alpha = 0.05$ .

$$H_0 : \pi_1 - \pi_2 = 0$$

What is p-value? What is your conclusion?

$$H_1 : \pi_1 - \pi_2 \neq 0$$

$$\rightarrow \bar{p} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{1103(0.74) + 1065(0.66)}{2168} = \frac{1519.12}{2168} = \underline{\underline{0.7}}$$

$$\rightarrow Z = \frac{(p_1 - p_2)}{\sqrt{p(\bar{p})(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{0.08}{\sqrt{0.7(0.3)(\frac{1}{1103} + \frac{1}{1065})}} = \frac{0.08}{0.019686872} = \underline{\underline{4.06}}$$

df = 2168

$$\rightarrow \begin{array}{r} \alpha = 0.05 \\ \hline \infty \quad | \quad \underline{\underline{2.576}} \quad \boxed{4.06} \end{array}$$

So p-value  $< 0.005$

Two tailed 2p-value  $< 0.01 < \alpha$

So we reject  $H_0$  ( $\alpha = 0.05$ )