Laboratory Instructions:

→ grads:

Homeworks	10 %
Quizes	10 %
Reports	40 %
Final exam	40 %

→ Reports:

* Cover page: Name, number, date partener name, experiment name
* Abstract: a brief summery explaining
  - The aim of the experiment
  - The method used
  - The main results
* Theory
* procedure
* Data
* Calculations
* Conclusion.

Measurments & Uncertainties

* Sources of errors :
→ choice of instruments
→ Environment
→ The way the experiment is done
→ Experimenter

* types of errors

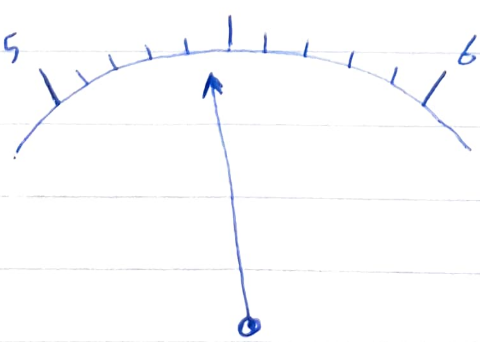
→ Random Errors

→ systematic Errors

4.1 : Random Errors .

4.1.1 Uncertainty in a measurement

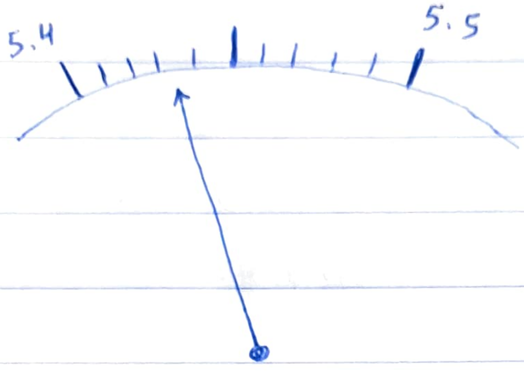
ex1:



5.44 or 5.45
or 5.43

There is uncertainty about the reading of the vol by about ∓ 0.1 Volt. ⇒ $V = 5.4 \mp 0.1$ Volt

ex 2:



5.432 or 5.433

or 5.434

I'm uncertain abut the last figure

There is uncertainty by about ∓ 0.01

⇒ $V = 5.43 \pm 0.01$ volt

$\boxed{\text{The estimated uncertainty depends on how finally the scale is divided}}$

* If only one measurment of a quantity is made??
  The only way to find the uncertainty is to estimate it.

* now, if several measurments of the same quantity are made ; $x_1, x_2, \ldots x_N$

⇒ The best estimate of the true value is the average value, defined as : $\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$

$$= \frac{x_1 + x_2 + x_3 + \cdots \; x_N}{N}$$

- The best estimate of the uncertainty is the sample standard deviation :

$$\delta_s = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2}$$

it means that the probability is about 2/3 (66.7 %) that any one measurement does not differ from another by more than δ_s.

4.1.2 uncertainty in the mean ≡ "standard deviation of the mean"

$$\delta_m = \frac{\delta_s}{\sqrt{N}}$$

- it shows the degree of confidence that the average value is close to the true value.

- clearly the more measurements made, the closer the mean to the true value.

eX: suppose Five students measured the voltage across a resistance (R)

student No	1	2	3	4	5
Voltage (Volts)	5.45	5.44	5.43	5.46	5.43

st. #	Volt	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
1	5.45	5.45 - 5.442	$(0.008)^2$
2	5.44	5.44 - 5.442	$(-0.002)^2$
3	5.43	⋮	⋮
4	5.46		
5	5.43		
$N = 5$	$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$		$\sum_{i=1}^{5} (X_i - 5.442)^2 = \boxed{}$

$\bar{X} = \frac{1}{5}\sum_{i=1}^{5} X_i$

$\bar{X} = 5.442\,V$

$\delta_s = \sqrt{\frac{1}{(5-1)}\sum_{i=1}^{5} (X_i - 5442)^2}$

$\delta_s = 0.013$ volt

$$\delta_m = \frac{\delta_s}{\sqrt{N}} = \frac{0.013}{\sqrt{5}} = 0.0058 \text{ volt}$$

* Back to Appendix C [P 87-88] for using calculator.

4.1.3 Importance of knowing the uncertainty:

Consider the measurments of the length of a rod at two different temperatures:

length (cm)	98.025	98.034
Temperature (C°)	10.0	20.0

Does the length of the rod depend on the tem. ?

if the uncertainty is ∓ 0.01 cm
$\Rightarrow$ we can't answer the question

since the difference between the two measurments is smaller than their uncertainties.

i.e: if we take the length 98.025 cm then the true value is 98.025 ∓ 0.01 cm

$\Downarrow$

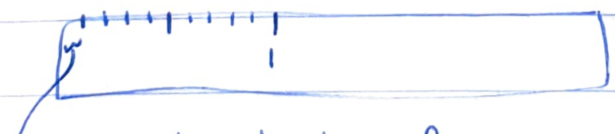
[98.025 - 0.01 , 98.025 + 0.01]

The true value lay between [98.015 , 98.035]

4.2 systematic Errors:

Random errors	systematic errors
* are always present in an experiment	it depends on the experimenter & environment.
* They are equally likely to be positive or negative & cause several measurements to spread out around the true value	shift the measured values to be bigger or smaller of the true value

* examples:
  1. uncalibrated tools
  2. worn off ruler used to measure the length of a rod.



Does not start from zero.

  3. the way the quantity is measured.

H.W $\boxed{\text{solve Ex1 , Ex2 p [20,22]}}$

4.2.1 Precision and Accuracy.
الدقة ⇓ الصحة

* Small Random Errors means High precision.

ex: two students made two measurments of the voltage
$V_1 = 5.443 \pm 0.002$ Volt
$V_2 = 5.44 \pm 0.03$ Volt

st. 1 has less random error compared with st. 2
V_1 more precise than V_2.

* Negligible systematic Errors means High Accuracy

ie: if the measured value is close to the true value ⇒ the measurment is said to be accurate

⇒ $\boxed{\text{precise but not accurate}}$

ie. $D = 2.13 \mp 0.01$ cm [the true value is 2.15 cm]

Due to systematic errors.

$\boxed{\text{accurate but not precise}}$

i.e. $g = 9.9 \pm 0.2\ m/s^2$, The true value is $9.82\ m/sec^2$

Due to the big uncertainty, but the measurement is accepted since $g \in [9.9-0.2,\ 9.9+0.2]$

$g \in [9.7,\ 9.11]$

4.2.2 A Comparison between measured & accepted values !!

* <u>Discrepancy test</u> : $|\text{measured value} - \text{accepted value}| \leq 2\ \text{uncertainty}$

ex: Two students A & B measured the speed of sound, $V_A = 338 \pm 2\ m/s$

$V_B = 325 \pm 5\ m/s$

The true value $V = 331\ m/s$

which measurement is accepted ??

By using Discrepancy test.

$D_A = |338 - 331| \overset{??}{\leq} 2(\overset{\Delta V_A}{2})$

$7 \not\leq 4 \Rightarrow$ not accepted.

$D_B = |331 - 325| \overset{??}{\leq} 2(5)$

$6 \leq 10$ its accepted ✓

* Or in other words, if the result $= \bar{X} \pm \Delta_m$

; $\Delta_m = DX =$ uncertainty

The result is accepted if

$\bar{X} - 2\Delta_m \leq$ True value $\leq \bar{X} + 2\Delta_m$

$-\cancel{\bar{X}}$ $\quad -\bar{X}$ $\quad -\cancel{\bar{X}}$

$-2\Delta_m \leq$ True value $- \bar{X} \leq 2\Delta_m$

$\Rightarrow |$ True value $- \bar{X}| \leq 2\Delta_m$

4.3 Significant figures:

if I have $\Delta L = \mp 0.1$ cm , $L = 2.435$ cm

meaningless

Because 4 is uncertain by ∓ 0.1

$\Rightarrow$ 2, 4 are significant figures

$\Rightarrow L = 2.4 \mp 0.1$ cm

if I use micrometer $\Delta L = \mp 0.01\,mm$
$= \mp 0.001\,cm$

$\Rightarrow$ 2, 4, 3, 5 are significant figures.

$L = 2.435 \pm 0.001$ cm [4 significant fig]
[I'm uncertain about "5"].

ex: 4.7 ± 0.3 (2 sig. fig.)
473 ± 2 (3 sig. fig.)
472.84 ± 0.03 (5 sig. fig)
0.47 ± 0.03 (2 sig. fig)
$472 \pm 3 = (4.72 \pm 0.03) \times 10^2$ (3 sig. fig)

* wrong way to write sig. fig.

$\rightarrow$ 4.7 ± 0.03 X $\Rightarrow$ 4.70 ± 0.03
[3 sig. fig]

$\rightarrow$ 472.8 ± 0.04 X $\Rightarrow$ 472.80 ∓ 0.04
[5 sig. fig]

$\rightarrow$ 0.47 ± 0.3 X $\Rightarrow$ 0.5 ± 0.3
[1 sig. fig]

$\rightarrow$ 473 ± 30 X $\rightarrow$ $(47.3 \pm 3) \times 10$
meaningless
$(47 \pm 3) \times 10$ (2 sig. fig)
$= 470 \pm 30$

4.3.1 Significant figures in calculated values:

4.3.1.1 Rounding:

eX: X = 4.374 ∓ 0.1

we must round the last significant fig "3" !!

if the first non significant fig "7" > 5, we round "3" ⇒ X = 4.4 ∓ 0.1

if the first nonsignificant fig < 5, we fix the last sig. fig

eX: L = 4.351 ∓ 0.03

the last sig. fig → < 5

⇒ L = 4.35 ∓ 0.03

eX: 3.7 ∓ 1 ⇒ 4 ∓ 1

last sig. fig → 7 > 5

51.36 ∓ 0.2 ⇒ 51.4 ∓ 0.2

last sig. fig → 6 > 5

33.1 ∓ 10 ⇒ (3.31 ∓ 1) x 10

last sig. fig, 3 < 5

⇒ (3 ∓ 1) x 10 = 30 ∓ 10

* if the first non-significant fig. = 5 ??

① if the last sig. fig is odd ⇒ round up

ex: $4.\underset{\sim}{3}5 \mp 0.1 \Rightarrow 4.4 \mp 0.1$

odd

② if the last sig. fig is even ⇒ fix it

ex: $4.\underline{6}5 \mp 0.1 \Rightarrow 4.\underline{6} \mp 0.1$

even fix it

4.3.1.2 Addition & subtraction

* The number with the fewest decimal places limits the number of decimal places in the result.

ex: R = 10.3 cm + 108.76 cm + 0.0349 cm

= 119.0949 cm

but the fewest decimal is 0.3 so we must keep one decimal place

R = 119.1 cm

4.3.1.3 Multiplication & division

* The number of significat fig. in the result will be given by the fewest number of sig. fig.

ex: $R = \frac{1.8 \times 2.346}{7.86} = 0.537$

but fewest number of sig. fig [1.8] = 2

⇒ the result is rounded to only two sig. fig.

$R \simeq 0.54$

ex: $\sin(12^\circ) = 0.21$ (θ has 2 sig. fig ⇒ the result must has 2 sig. fig).

the same for Cos

ex $\sqrt{\frac{2.3 \times 4.57}{1.2}} = 3.0$ (must has 2 sig. fig)

4.3.2 Significant figures in the uncertainty

* The uncertainty in a result should be rounded to one significant fig.

ex: $\Delta X = \mp 0.0237 \Rightarrow \Delta X = \mp 0.02$

$\Delta L = \mp 0.0412 \Rightarrow \Delta L = \mp 0.04$

* An exception to this rule if the leading digit in the uncertainty is "1", then we keep two sig. fig.

ex: $\Delta y = \mp 0.147 \Rightarrow \Delta y = \mp 0.15$

4.4 Combining Uncertainties:

4.4.1 Addition & subtraction:

if $R = X \pm Y \Rightarrow \Delta R = \Delta X \pm \Delta Y$

ex: $X = 3.4 \mp 0.2$

$Y = 7.5 \mp 0.3$

$R = X - Y = 3.4 - 7.5 = -4.1$

$\Delta R = \Delta X + \Delta y = 0.2 + 0.3 = 0.5$

4.4.2 Constant multipliers

if $R = aX \pm by$; a, b constants

$\Delta R = a\Delta X + b\Delta y$

eX: $\cancel{X} = 57 \pm 2$

$Y = 23 \pm 3$

$R = X - 2Y = 57 - 2 \times 23 = 11$

$\Delta R = \Delta X + 2\Delta y = 2 + 2 \times 3 = 8$

$R = 11 \pm 8$

4.4.3 Multiplication & division

if $A = XY$ we find ΔA by direct differentiation with respect to X & Y

$$\Delta A = \frac{\partial A}{\partial X}\Delta X + \frac{\partial A}{\partial y}\Delta y$$

$$\Delta A = Y\Delta X + X\Delta y$$

dividing by A $\Rightarrow \dfrac{\Delta A}{A} = \dfrac{\cancel{Y}}{X\cancel{y}}\Delta X + \dfrac{\cancel{X}\Delta y}{\cancel{X}y}$

$$\Rightarrow \frac{\Delta A}{A} = \frac{\Delta X}{X} + \frac{\Delta y}{y}$$

ex: $L = 8.27 \mp 0.05$, $W = 5.12 \pm 0.02$

$A = LW = 8.27 \times 5.12 = 42.3424$

$$\frac{\Delta A}{A} = \frac{\Delta L}{L} + \frac{\Delta W}{W}$$

$$\frac{\Delta A}{42.3424} = \frac{0.05}{8.27} + \frac{0.02}{5.12}$$

$\Rightarrow \Delta A = 0.4214 \Rightarrow \Delta A = 0.4$ (1 sig. fig)

$A = 42.3 \pm 0.4$

4.4.4 Raising to a power

$R = x^n y^m z^l$ by differentiation

$$\Delta R = \left|\frac{\partial R}{\partial x}\right| \Delta x + \left|\frac{\partial R}{\partial y}\right| \Delta y + \left|\frac{\partial R}{\partial z}\right| \Delta z$$

$$\Delta R = |n x^{n-1} y^m z^l| \Delta x + |x^n m y^{m-1} z^l| \Delta y + |x^n y^m l z^{l-1}| \Delta z$$

dividing by R

$$\frac{\Delta R}{R} = \left|\frac{n}{x}\right| \Delta x + \left|\frac{m}{y}\right| \Delta y + \left|\frac{1}{z}\right| \Delta y$$

ex: $R = x^2$

$$\Rightarrow \frac{\Delta R}{R} = 2 \frac{\Delta x}{x}$$

ex: $R = z^2 y^3 / x^4$

$$\frac{\Delta R}{R} = 2 \frac{\Delta z}{z} + 3 \frac{\Delta y}{y} + 4 \frac{\Delta x}{x}$$

4.4.5 Other functions:

1.

$R = \sin(x) \Rightarrow \Delta R = |\cos(x)| \Delta x$

$R = \cos(x) \Rightarrow \Delta R = |\sin(x)| \Delta x$

$R = \tan(x) \Rightarrow \Delta R = |\sec^2(x)| \Delta x$

$R = \cot(x) \Rightarrow \Delta R = |\csc^2(x)| \Delta x$

$R = \sec(x) \Rightarrow \Delta R = |\sec(x)\tan(x)| \Delta x$

* x must be in Radians

if x in degrees $\Rightarrow$ multiply by $\frac{\pi}{180}$

ex: $\theta = 80^\circ \pm 1^\circ$

$R = \sin\theta = \sin(80^\circ) = 0.984810$

$\Delta R = \cos\theta \left(\Delta\theta \frac{\pi}{180} \right) = \cos(80^\circ) \left(\frac{\pi}{180} \times 1 \right)$

$= 0.0030307$

$\Delta R = 0.003$

$R = 0.985 \pm 0.003$

2. Natural logarithm

$R = \ln(x) \Rightarrow \Delta R = \frac{\Delta X}{X}$

3. exponential function

if $R = e^{X} \Rightarrow \Delta R = e^{X} \Delta X$

H.W $\boxed{\begin{array}{ll} \text{Solve: Ex 3} & \text{P 32} \\ \text{Ex 4} & \text{P 36} \end{array}}$