7.4 Discussion

Wednesday, December 8, 2021 1:49 PM

7.4 | 7.5

25. First-order chemical reactions In some chemical reactions, the rate at which the amount of a substance changes with time is proportional to the amount present. For the change of δ-glucono lactone into gluconic acid, for example,

$$\frac{dy}{dt} = -0.6y$$

$y_0 = y(0)$

when t is measured in hours. If there are 100 grams of δ-glucono lactone present when $t = 0$, how many grams will be left after the first hour?

$y(t)$: amount of lacton available at time t

$$y(t) = y_0 e^{kt}$$

$$= y_0 e^{-0.6t}$$

$$\boxed{y(t) = 100\, e^{-0.6t}}$$

$$y(1) = 100\, e^{-0.6(1)} = 100\, e^{-0.6} \approx 54.88 \text{ gm}$$

Modiling using exp e^x

Growth $k>0$

Pop. ↑

Disease ↑

Decay $k<0$

Radioactive ↓

Pop. ↓

Dis ↓

$$\boxed{y(t) = y_0 e^{kt}}$$

36. Polonium-210 The half-life of polonium is 139 days, but your sample will not be useful to you after 95% of the radioactive nuclei present on the day the sample arrives has disintegrated. For about how many days after the sample arrives will you be able to use the polonium?

$\boxed{\tau = 139}$

⇓

$\boxed{\tau = \frac{\ln 2}{k}}$

$$k = \frac{\ln 2}{\tau} = \frac{\ln 2}{139} \approx 0.005$$

Decay ⇒ $k<0$

$$Q(t) = Q_0 e^{-kt}$$

available

كمية البداية

$$= Q_0 e^{-0.005t}$$

Find time t^* such that $Q(t^*) = \frac{5}{100} Q_0$

use 5% → Decay 95%

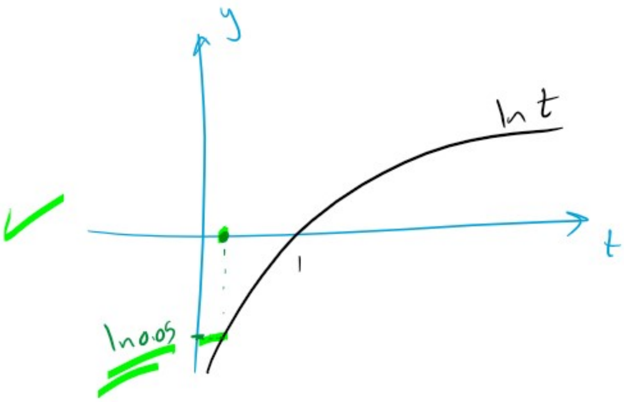
use 5 % → Decay 95%

$$\cancel{Q_0}\, e^{-0.005\, t^*} = \frac{5}{100}\cancel{Q_0}$$

$$\rightarrow \quad e^{-0.005\, t^*} = \frac{5}{100}$$

$$\rightarrow \quad \ln e^{-0.005\, t^*} = \ln 0.05$$

$$\rightarrow \quad -0.005\, t^* \underbrace{(\ln e)}_{1} = \ln 0.05$$



$$t^* = \frac{\overset{\text{سالب}}{\ln 0.05}}{\underset{\text{سالب}}{-0.005}}$$

$$\approx \underline{600} \text{ days}$$