

Name.....

Number.....

Section

(Q1) [100 points] Fill the blanks with true (T) or false (F).

- [] (1) If $A^2 = I$, then $\det(A) = \pm 1$
- [] (2) If A and B are $n \times n$ nonsingular matrices, then $\det(A - B) = \det(A) - \det(B)$.
- [] (3) If A is an 2×2 matrix, then $|\alpha A| = \alpha^4 |A|$.
- [] (4) If $\det(A) = 1$, then $A^{-1} = \text{adj} A$.
- [] (5) If A and B are $n \times n$ matrices such that AB is singular, then at least A or B is singular.
- [] (6) If $A = \begin{bmatrix} 2 & 5 & 7 \\ 1 & 3 & 4 \\ 2 & 1 & 6 \end{bmatrix}$, then the $(2, 3)$ entry of A^{-1} is $-\frac{1}{3}$.
- [] (7) If A and B are 2×2 matrices such that $\det(BA) = 0$, then $\det(A) = 0$ and $\det(B) = 0$.
- [] (8) If A and B are $n \times n$ matrices, then $\det((AB)^T) = \det(A)\det(B)$.
- [] (9) Row equivalent matrices have the same determinants.
- [] (10) If A is singular, then $\text{adj} A$ is also singular.
- [] (11) Cramer's rule can be used to solve any square linear system.
- [] (12) If A and B are $n \times n$ matrices and A is singular, then AB is singular.
- [] (13) $\det(AB) = \det(A)\det(B)$ only when A and B are nonsingular.
- [] (14) If A is an $n \times n$ matrix, then $|A^n| = |A|^n$.
- [] (15) Every diagonal matrix is nonsingular.
- [] (16) Every Elementary matrix is nonsingular.
- [] (17) $\det(-I) = -\det(I)$.
- [] (18) If A is a 5×5 skew-symmetric matrix, then the system $Ax = 0$ has a nontrivial solution.
- [] (19) $|AB| = |BA|$ for any $n \times n$ matrices A and B .
- [] (20) If A, B, S are $n \times n$ matrices such that S is nonsingular and $A = SBS^{-1}$, then $|A| = |B|$.
- [] (21) If A is a 7×7 nonsingular matrix, then the RREF of A has 7 nonzero rows.
- [] (22) If $|A| = 1$, then $A = I$.
- [] (23) If $A = LU$ is the LU factorization of A and U is nonsingular, then A is nonsingular.
- [] (24) If A is a singular matrix and U is the REF of A , then $|U| = 0$.
- [] (25) If A is a square and nonsingular matrix with $|\text{adj} A| = |A|$, then A is 2×2 .
- [] (26) If A and B are square nonzero matrices with $AB = 0$, then both A and B are singular.
- [] (27) If $\det(A) = 0$, the A is a zero matrix.
- [] (28) If the diagonal entries of a square matrix are all zero, then it is singular.

- [] (29) If A is a 3×3 with $a_1 = a_3$, then $\det(A) = 0$.
- [] (30) If the system $A^3x = 0$ has a nontrivial solution, then A is singular.
- [] (31) If E is a 4×4 elementary matrix, then the linear system $Ex = b$ is consistent for any $b \in \mathbb{R}^4$.
- [] (32) If A is a square matrix and one of the rows is a linear combination of the others, then $|A| = 0$.
- [] (33) If A is an $n \times n$ matrix with $n > 1$, then $|\text{adj}A| = |A|^{n-1}$.
- [] (34) If A is an $n \times n$ matrix, then $\det(A^T A) \geq 0$.
- [] (35) There is a matrix A such that $A^{-1} = \begin{bmatrix} 4 & 2 \\ 6 & 3 \end{bmatrix}$.
- [] (36) If A^T is singular, then A^2 is also singular.
- [] (37) There exists a nonsingular matrix with two identical columns.
- [] (38) A matrix having a zero row cannot be row equivalent to I .
- [] (39) If E and F are 2×2 elementary matrices of type I and III respectively, then $\det(-2E^T F^{-1}) = 4$.
- [] (40) If A is a nonsingular diagonal matrix, then A^{-1} is also diagonal.
- [] (41) $\det(AB^T) = \det(A^T B)$ for any $n \times n$ matrices A and B .
- [] (42) If $\det(A - B) = 0$, then $A = B$.
- [] (43) If $\det(A - B) = 0$, then the matrix equation $Ax = Bx$ has a nonzero solution.
- [] (44) A triangular matrix is nonsingular if and only if its diagonal elements are all nonzero.
- [] (45) If A is a nonzero matrix with $A^k = 0$ for some positive integer k , then A is singular.
- [] (46) If x and y are two distinct vectors in \mathbb{R}^n such that $Ax = Ay$, then $\det(A) = 0$.
- [] (47) If A and B are 3×3 matrices with $|A| = 2$ and $|B| = -6$, then $|-3AB^{-1}| = 9$.
- [] (48) If A is a nonsingular matrix, then $\text{adj}A^{-1} = (\text{adj}A)^{-1}$.
- [] (49) If A is a symmetric matrix, then $\text{adj}A$ is also symmetric.
- [] (50) If E and F are 3×3 elementary matrices of type I and A is 3×3 , then $|-AEF| = |A|$.