

Digital Systems

Section 2

Chapter (6)

🌀 **Registers** and **counters** are practical examples of clocked **sequential** circuits

🌀 A **register** consists of a group of **flip-flops** each is capable of **storing one bit** of information

🕒 The group of FFs normally share a **common clock**

🕒 **n-bit** register consists of a group of **n** FFs capable of **storing n bits** (Each FF store 1-bit)

🌀 **Registers** may have **combinational logic/gates** that affect its **operation** (Loading/Reading Operations)

🌀 **Flip-flops hold** the data (bits)

🌀 **Gates determine** how the data (bits) is **transferred** into/from the register

🌀 **Registers** are used in computers, calculators, and almost all **electronic equipment** these days. Any device must have registers to store binary data. A **memory unit** is an **array of registers**

🌀 A **counter** is a **register** that goes through a **predetermined sequence** of binary **states**

🕒 The **gates** in the counter are **connected** in such a way as to produce the **prescribed sequence** of states.

🌀 **Counters** are a **special type** of **register**

🌀 **Various** types of **registers** are available

🌀 The simplest register → **4-bit** Register

🕒 Clear (R) is **0** → **clear/reset** the register to **all 0's asynchronously**

★ Note the **bubble** at R → Active **low** clear/reset

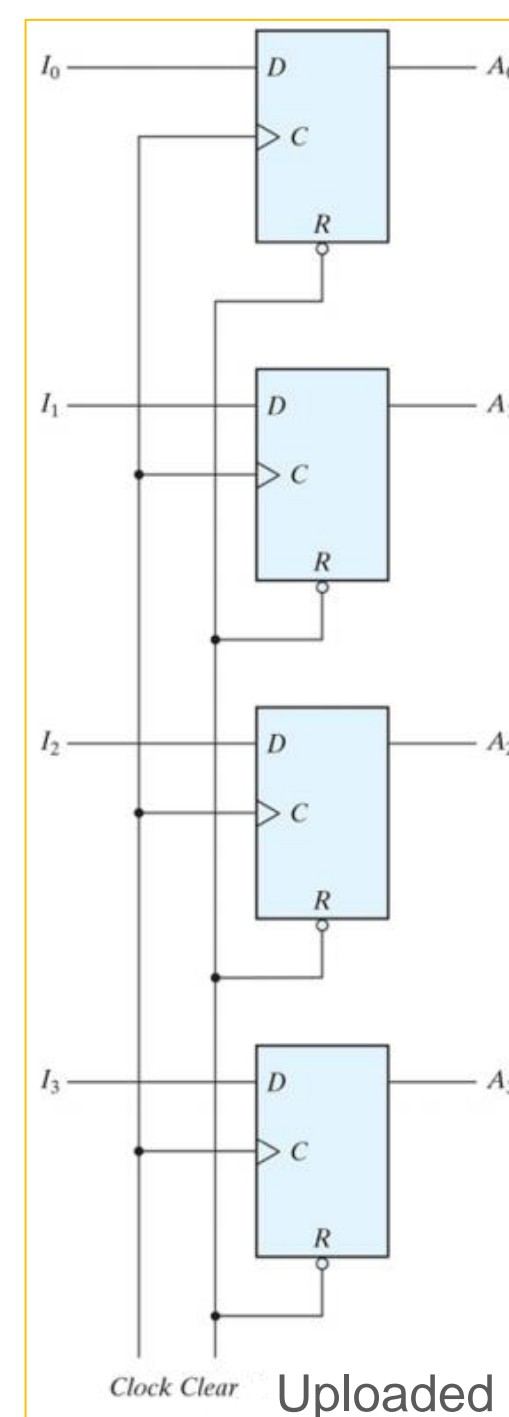
🕒 Clear (R) is **1** → With each **+ve** Edge Clock, the **input** is **stored**

★ The value of ($I_0I_1I_2I_3$) immediately before the clock edge determines the value of ($A_0A_1A_2A_3$) **after** the clock edge

🕒 To **prevent** the **previously stored** value from **changing**:

- 1) The Clock has to be **disabled** **Not Practical**
- 2) Or the **same** value needs to be **passed back** to the inputs

USE: Parallel Load Approach



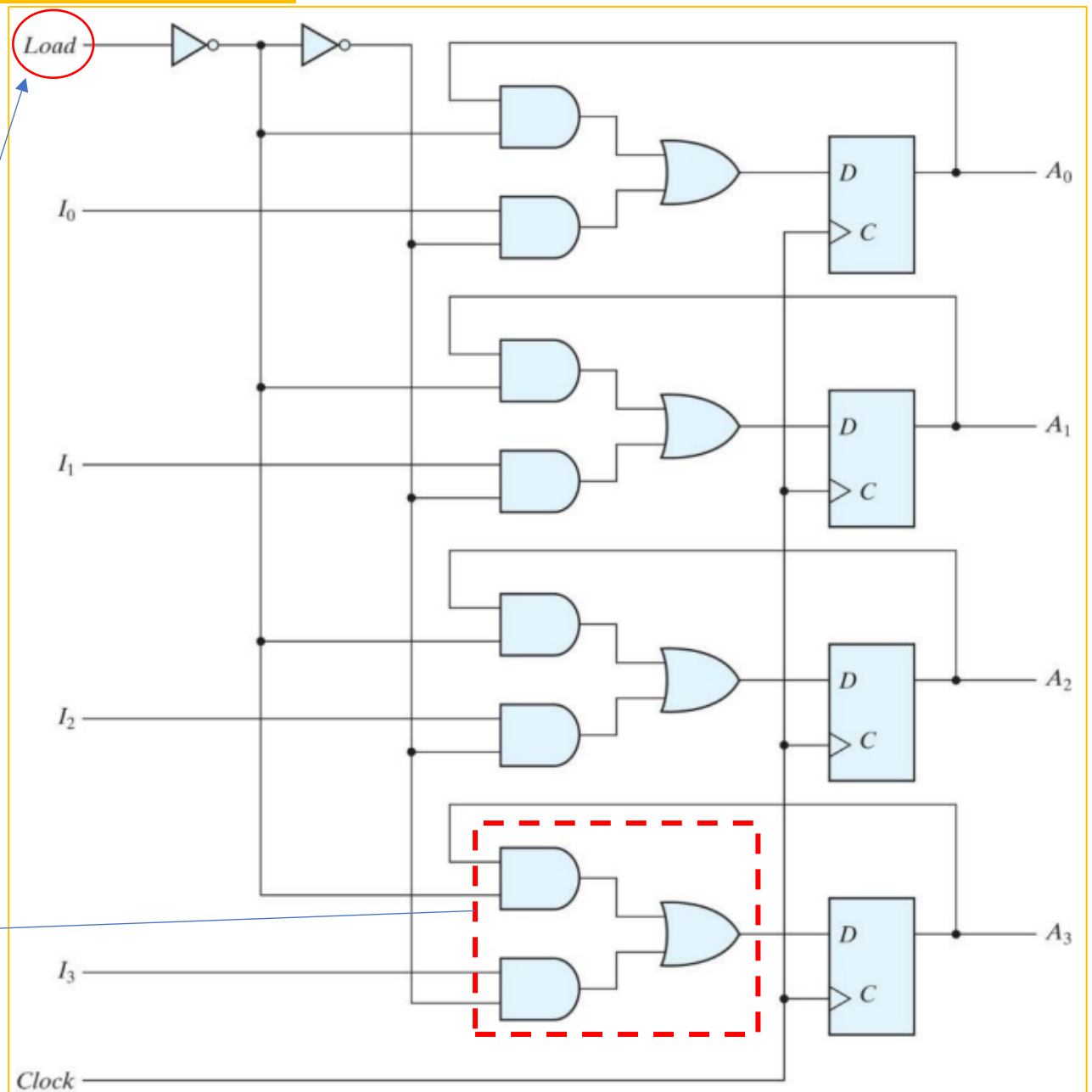
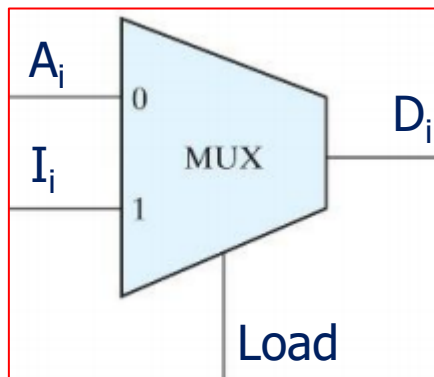
- 🌀 **Registers** with **parallel load** are a fundamental building block in digital systems
- 🌀 Synchronous digital systems have a **master clock generator** that supplies a continuous train of clock pulses
- 🌀 The pulses are applied to **all flip-flops and registers** in the system
- 🌀 The master clock acts like a drum that supplies a constant beat to all parts of the system
- 🌀 A **separate control signal** must be used to decide **which** register **operation** will execute at each clock **pulse**
- 🌀 **loading** or **updating** the register is The **transfer** of new information into a register
- 🌀 The **loading** is in **parallel** if **all** the bits are **loaded simultaneously** at the **same** clock cycle
- 🌀 **Recall:** To keep the **content** of the register **unchanged**:
 - 1) The **inputs** must be held **constant**
 - 2) Or The **clock** must be **disabled** from the circuit

It is advisable to **control** the **operation** of the register **with** the **D inputs**, rather than controlling the clock of the FF's

Use a **combinational** circuit to either **keep previous** value, or **load** a **new** one:

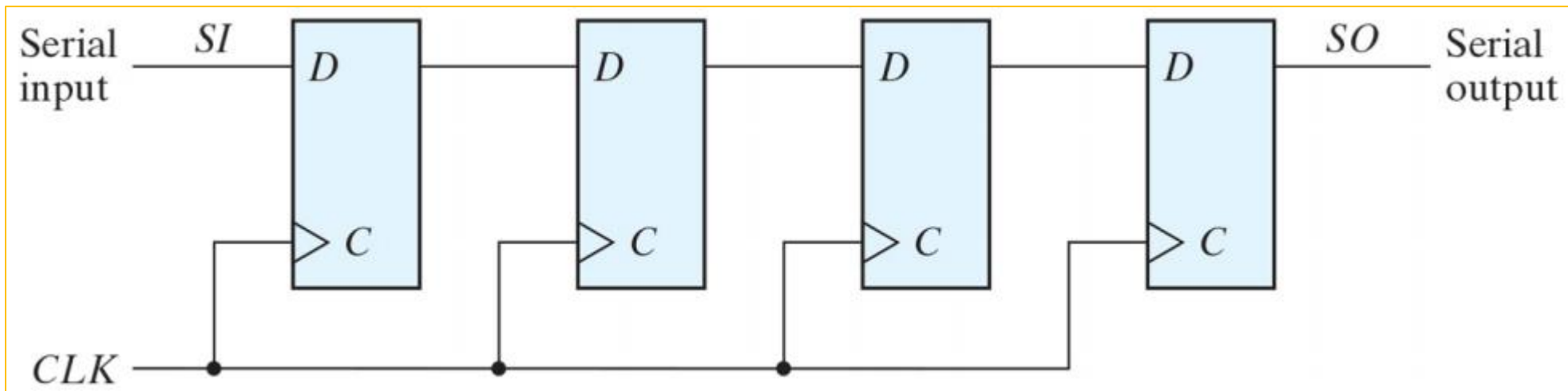
- Load=0 → **keep previous** value
- Load=1 → **accept (load)** new value from **I's**

This can be constructed using a **2x1 mux** with each **FF**



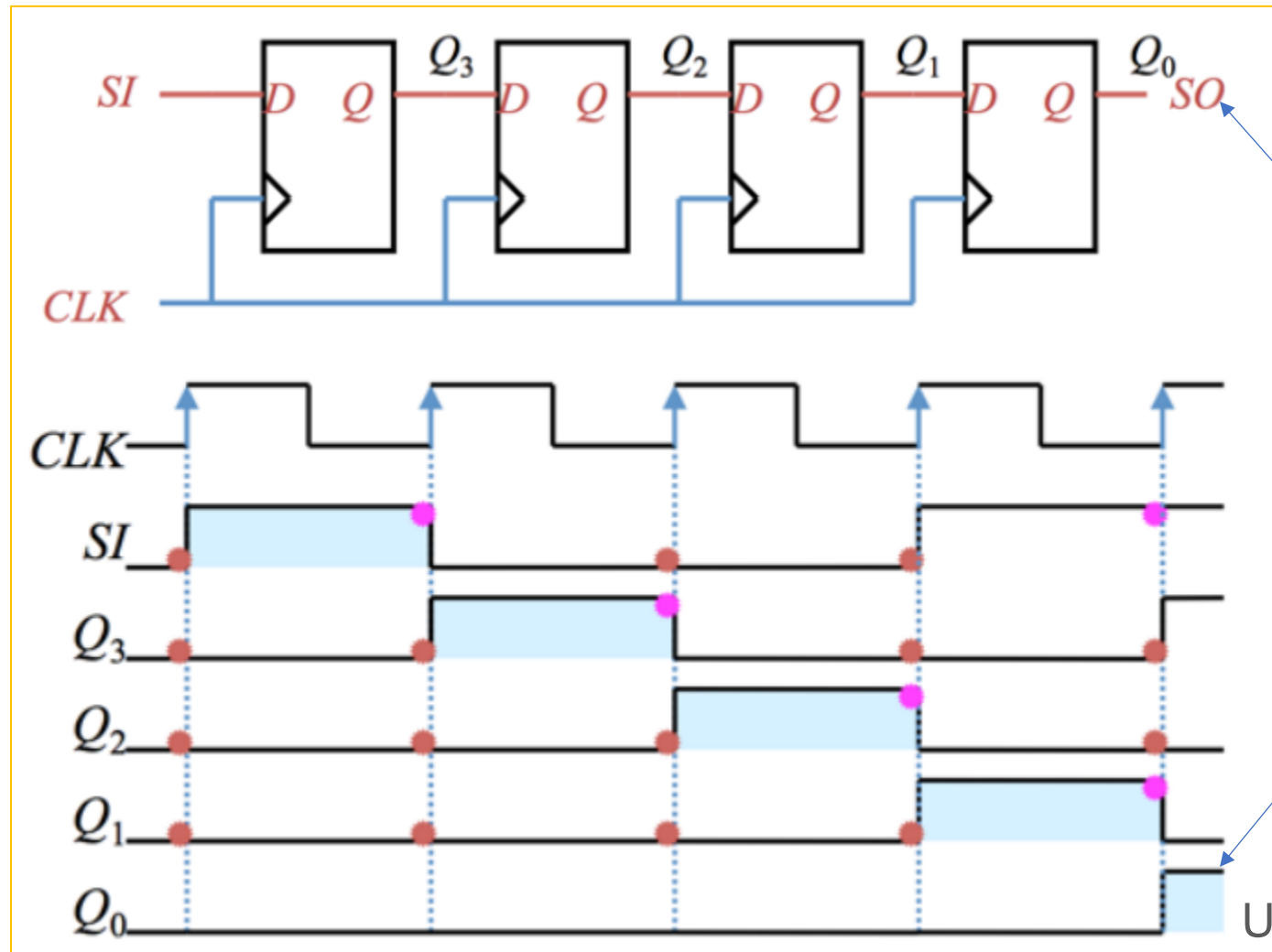
- ⌚ A **shift** register is a register that is **capable of shifting (moving)** the binary information held in each cell to its **neighboring** cell, in a **selected direction**
- ⌚ A **shift** register consists of a **chain** of **flip-flops** in **cascade**, with the **output** of one flip-flop **connected** to the **input** of the next flip-flop
- ⌚ **All** flip-flops receive **common** clock pulses, which **activate** the **shift** of data from one stage to the next

With **each** clock cycle, the data will **move** from one **FF** to its **adjacent** one.
The **SI** will get to the **far-left** FF and the **SO** will get out from the **right-most** FF



Unidirectional (left-to-right) 4-bit Shift register

- 🌀 This shift register is **unidirectional** (Does not support a left shift)
- 🌀 The **serial input** determines what goes into the **leftmost** flip-flop during the shift.
- 🌀 The **serial output** is taken from the output of the **rightmost** flip-flop
- 🌀 A **clear/reset** signal is required in practical designs (to reset the whole register to zero)

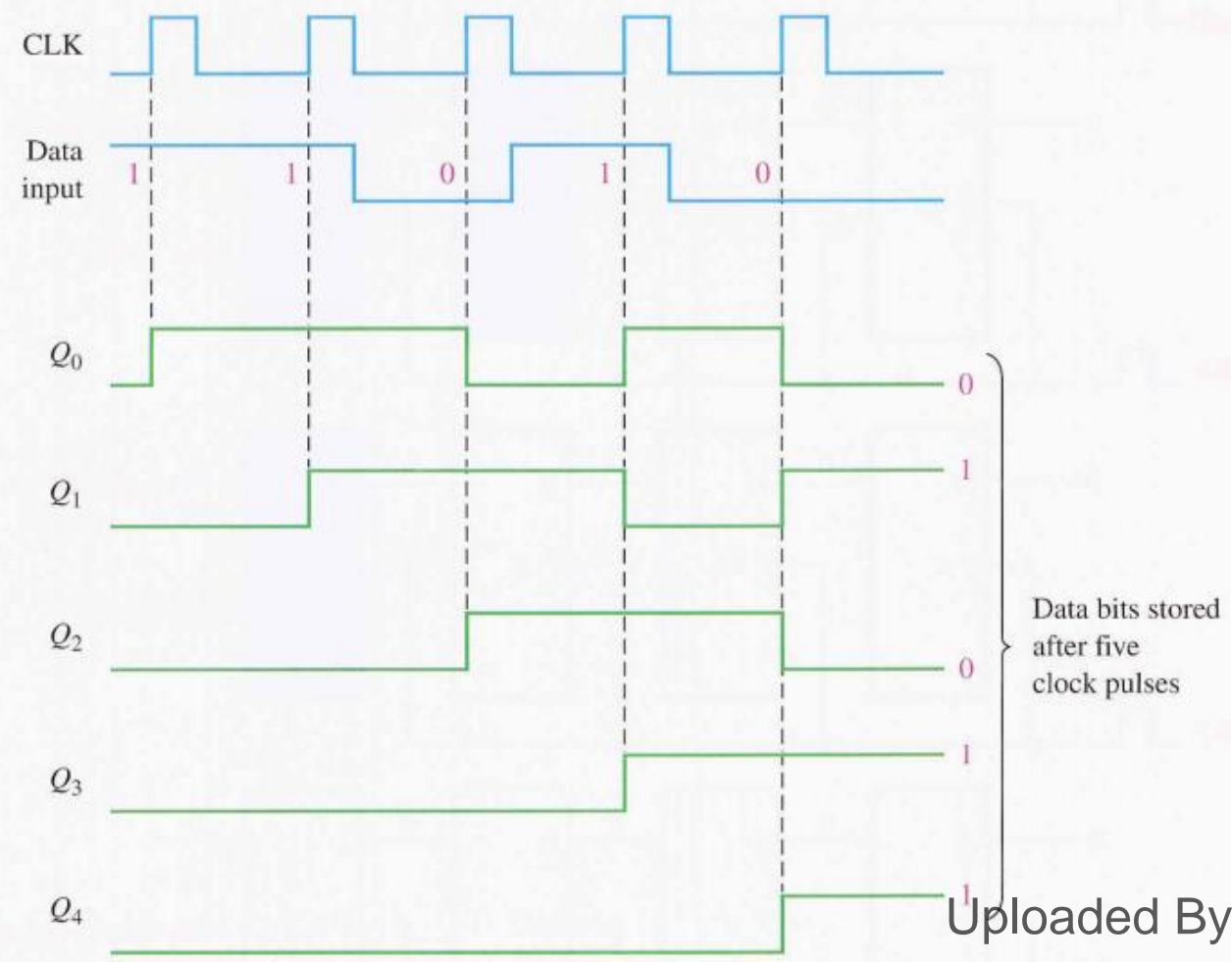
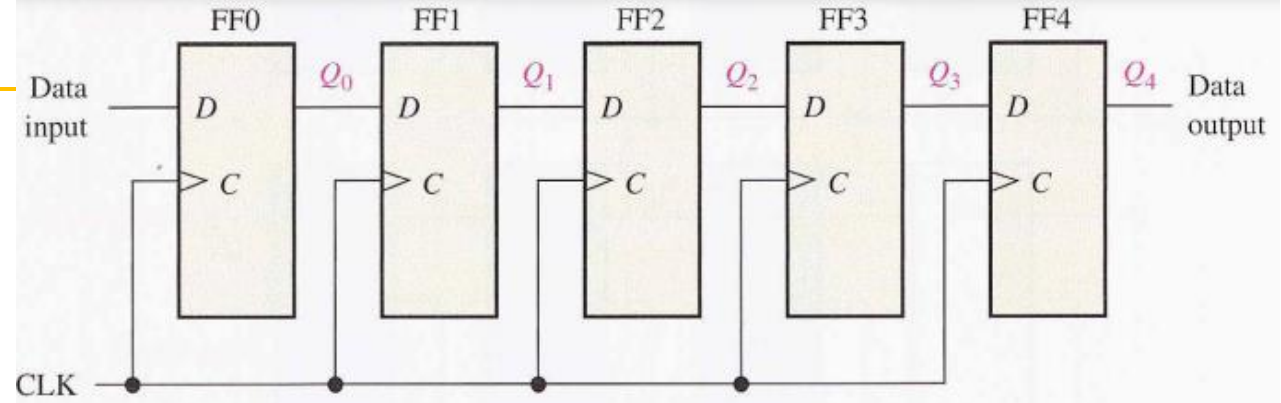


SO

Shift Registers



- 5-bit shift-right register
- Initially the register is cleared (all 0s)



Two main **Operation mode** for any digital system:

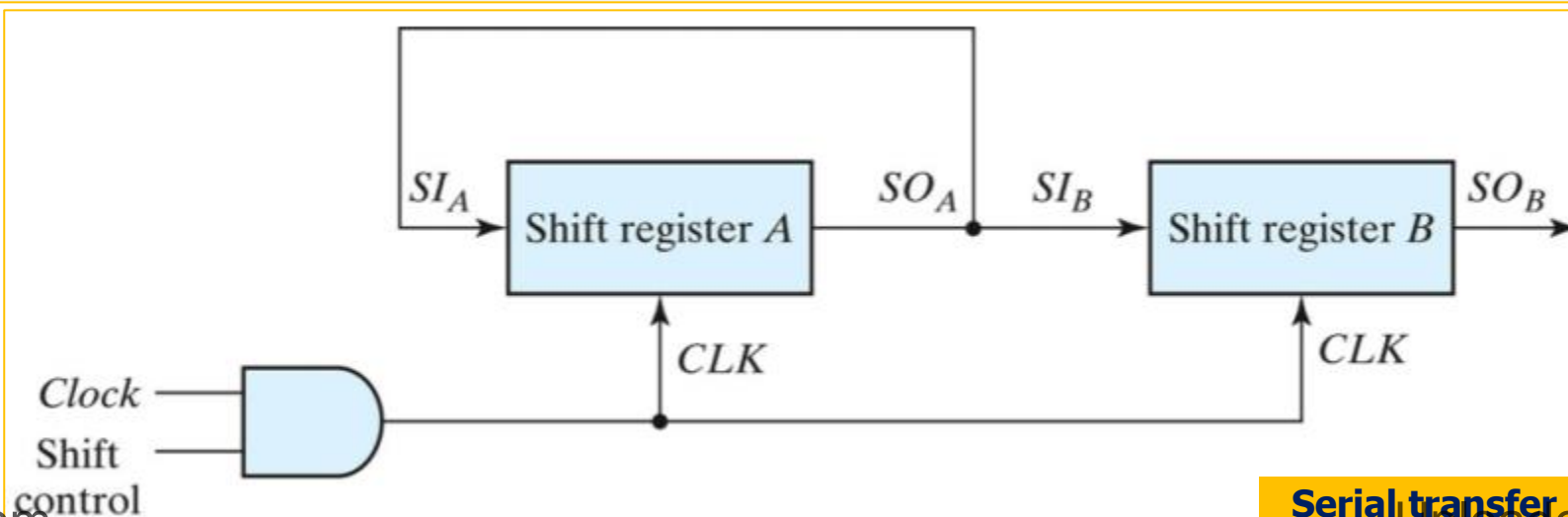
1) Serial mode: data is transferred and processed **one bit at a time**

- Take **more time** to process a collection of data (**Slower**)
- The hardware needed is **for a single bit** (**Simpler**)
- Examples: USB cables, RS232

2) Parallel mode: data is transferred and processed **in parallel** (All at once)

- Take only **1** clock cycle to process a collection of data (**Faster**)
- The hardware needed is **for a n-bits** (**More Complex**)
- Examples: Internal Memory Bus

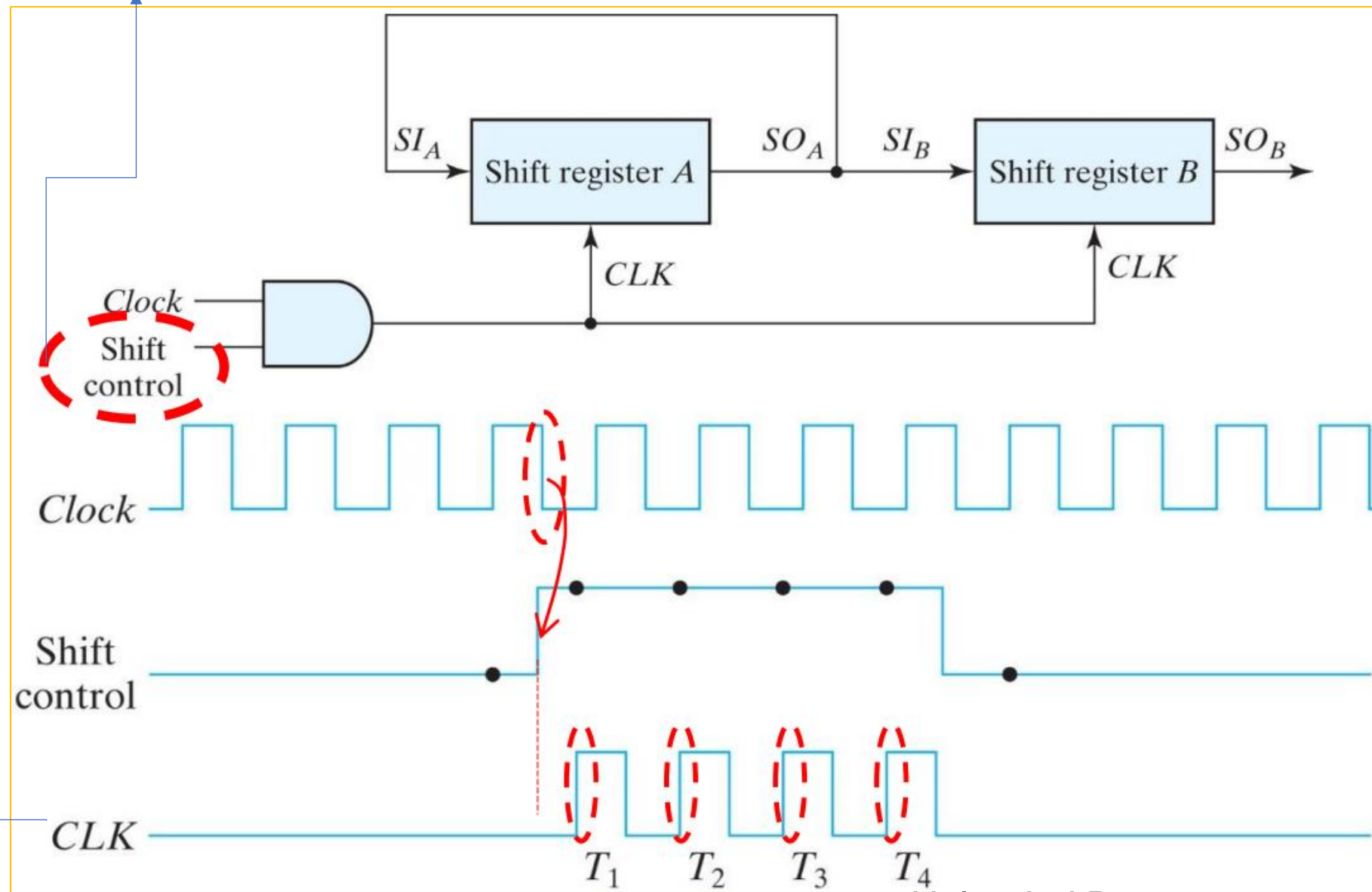
Each Register has **n-bits**. The contents of A are **copied** into B, so that the contents of A remain **unchanged** (i.e., the contents of A are **restored** to their **original** value)



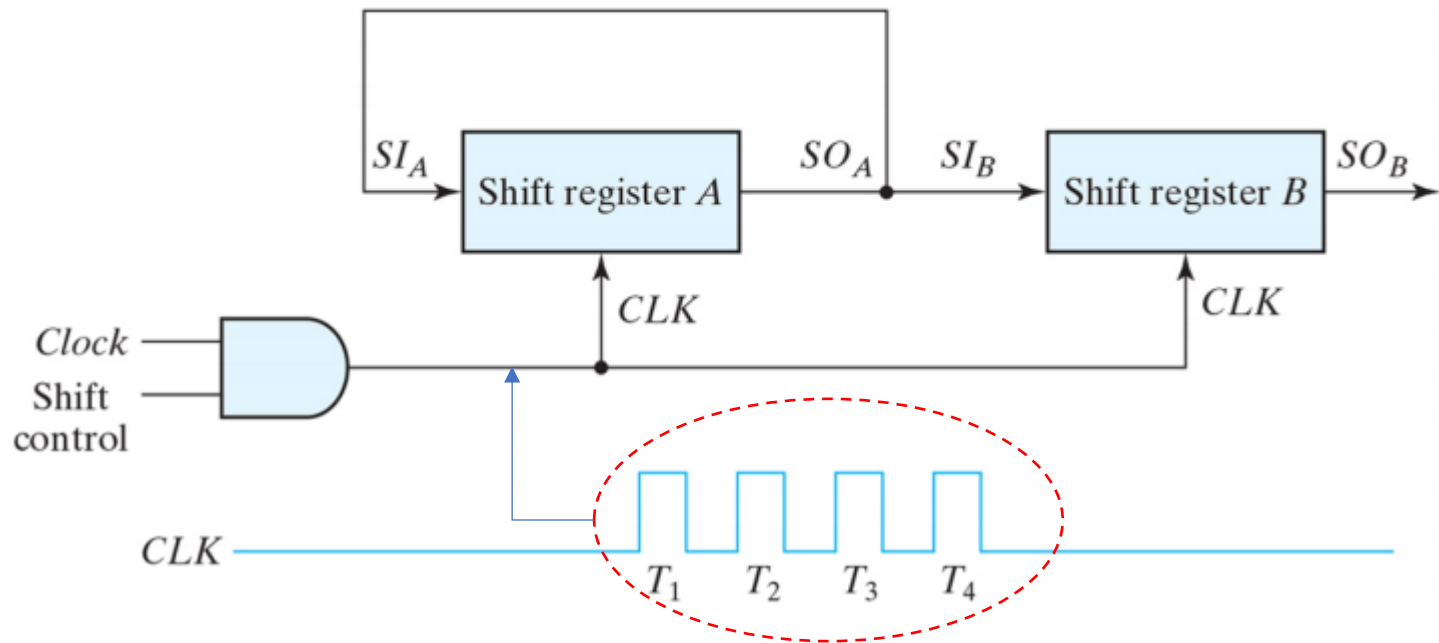
Serial transfer

Control the shift so that it occurs only with **certain** pulses, but not with others.

The 4 pulses perform identical operations, **shifting** the bits of A into B, **one at a time**



- The **4** pulses perform identical operations, **shifting** the bits of A into B, **one at a time**
- After the **fourth** shift, the **shift control** goes to **0**, and registers A and B both have the value **1011**.

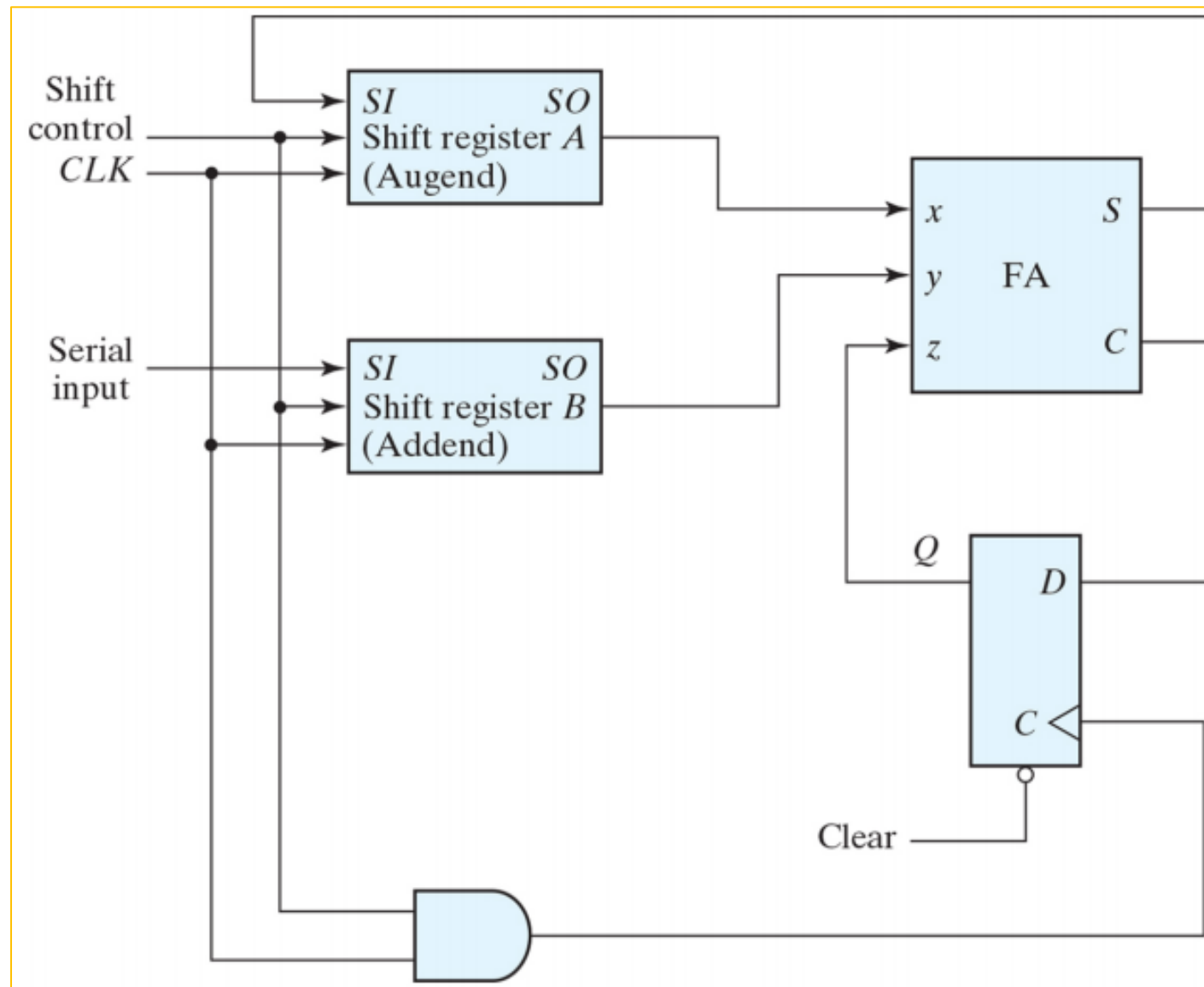


Timing Pulse	Shift Register A	Shift Register B
Initial value	1 0 1 1	0 0 1 0
After T_1	1 1 0 1	1 0 0 1
After T_2	1 1 1 0	1 1 0 0
After T_3	0 1 1 1	0 1 1 0
After T_4	1 0 1 1	1 0 1 1

Serial Adder: sequential circuit that performs serial addition

- The inputs comes from two shift registers (A,B)
- A single full adder is used to add one pair of bits at a time along with the carry
- The result is stored back into one of the shift registers (A)
- The carry out is stored in a D-FF and is then used as the carry input for the next pair of significant bits

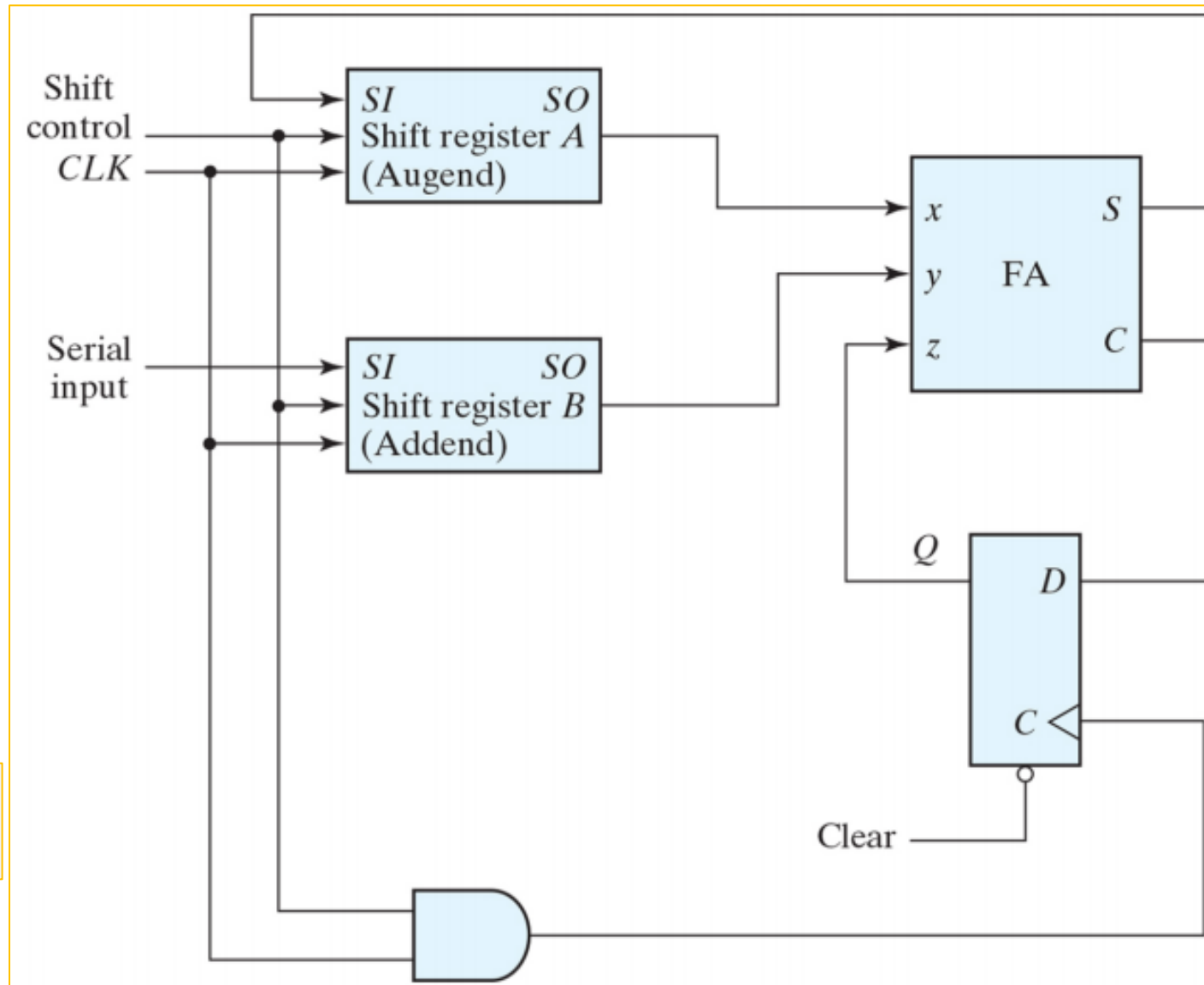
Only 1-FA is used compared to n FAs in the n -bit binary adder



Shift control signal is used to control when the registers will be shifted → control the next 2-bits addition

- Registers A & B hold the **two** binary numbers to be added
- D-FF is reset to **0** (C_{in} is 0)
- The **SO** of both registers are **connected** to the **inputs** of the FA
- The **output** of the FF is connected to the **carry-in** input of the FA
- S** is connected to **SI** of register A
- C** is connected to the D of the FF

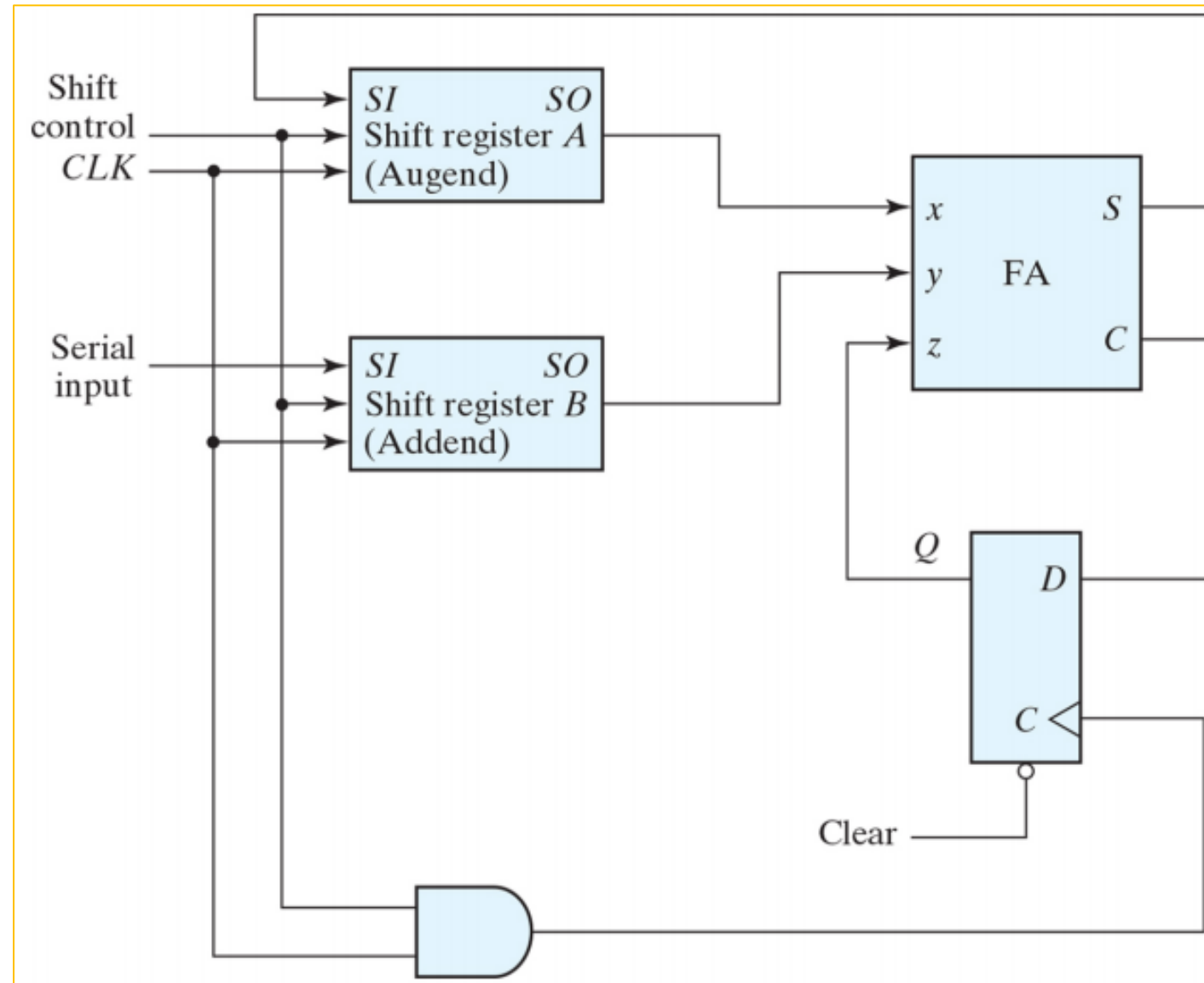
After applying **n** clock pulses the sum **result** is stored in register (**A**)



Example: 4-bit serial adder

CLK	A	B	S	C	Q
	1001	0111	0	1	0
1	0100	0011	0	1	1
2	0010	0001	0	1	1
	0001	0000	0	1	1
	0000	0000			1

$$1001 + 0111 = 10000!$$

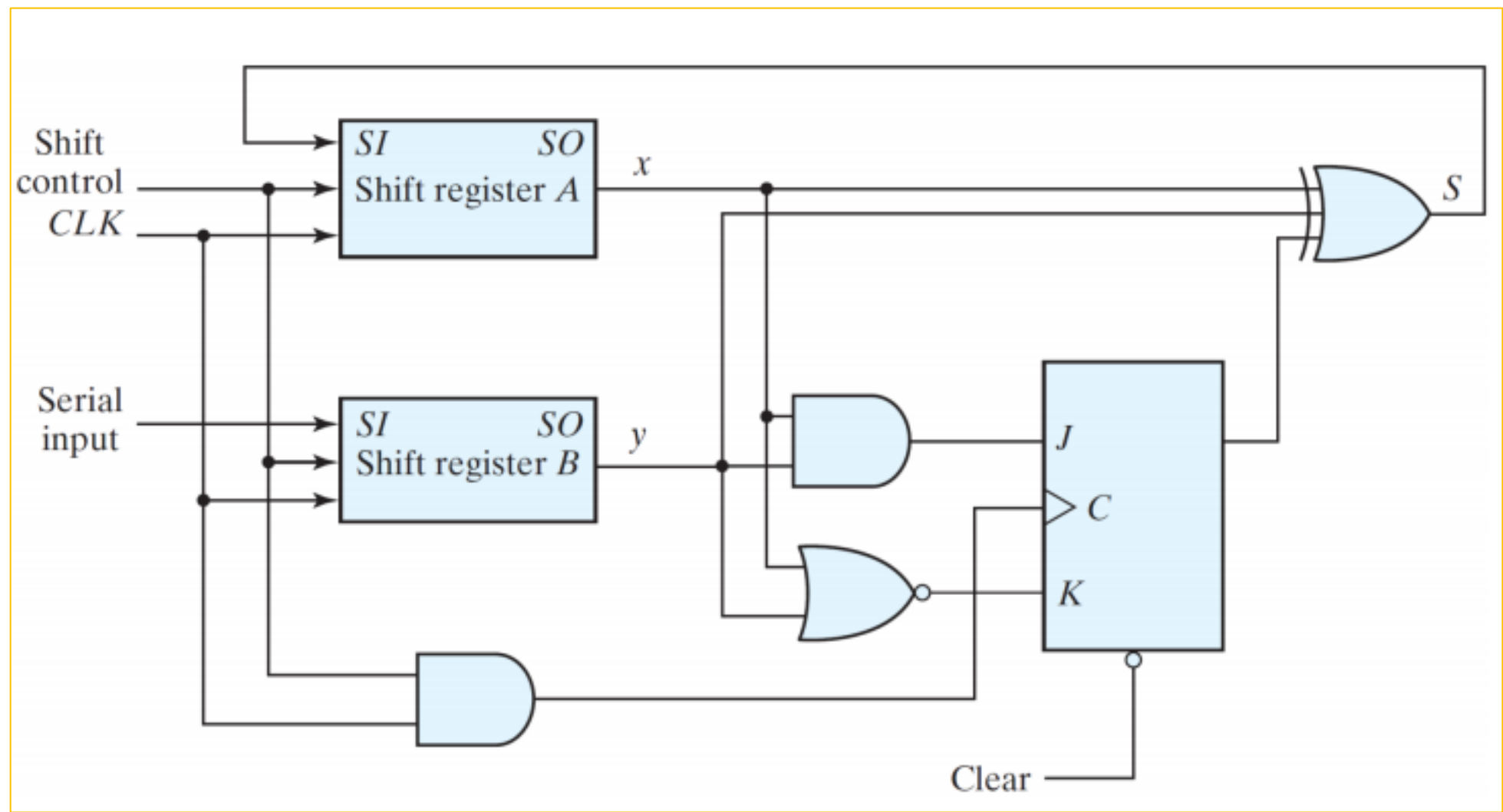


- Follow a sequential circuit **design approach** and use **JK-FF** to store the **carry**
- Let the **LSB** bits from Reg.A and Reg.B \rightarrow **x and y**
- x and y** are **available** for every CLK since the Shift-Reg will provide a **new x and y** for each CLK
- The circuit will have: Two **inputs**, x, y, **Output** S for the sum , Carry that is stored in **Q**

Present State		Inputs		Next State		Output	Flip-Flop Inputs	
C_{in}	Q	x	y	C_{out}	Q	S	J_Q	K_Q
0	0	0	0	0	0	0	0	X
0	0	0	1	0	0	1	0	X
0	0	1	0	0	0	1	0	X
0	0	1	1	1	1	0	1	X
1	1	0	0	0	0	1	X	1
1	1	0	1	1	1	0	X	0
1	1	1	0	1	1	0	X	0
1	1	1	1	1	1	1	X	0

$$J_Q = xy, K_Q = x'y' = (x+y)', S = x \oplus y \oplus Q$$

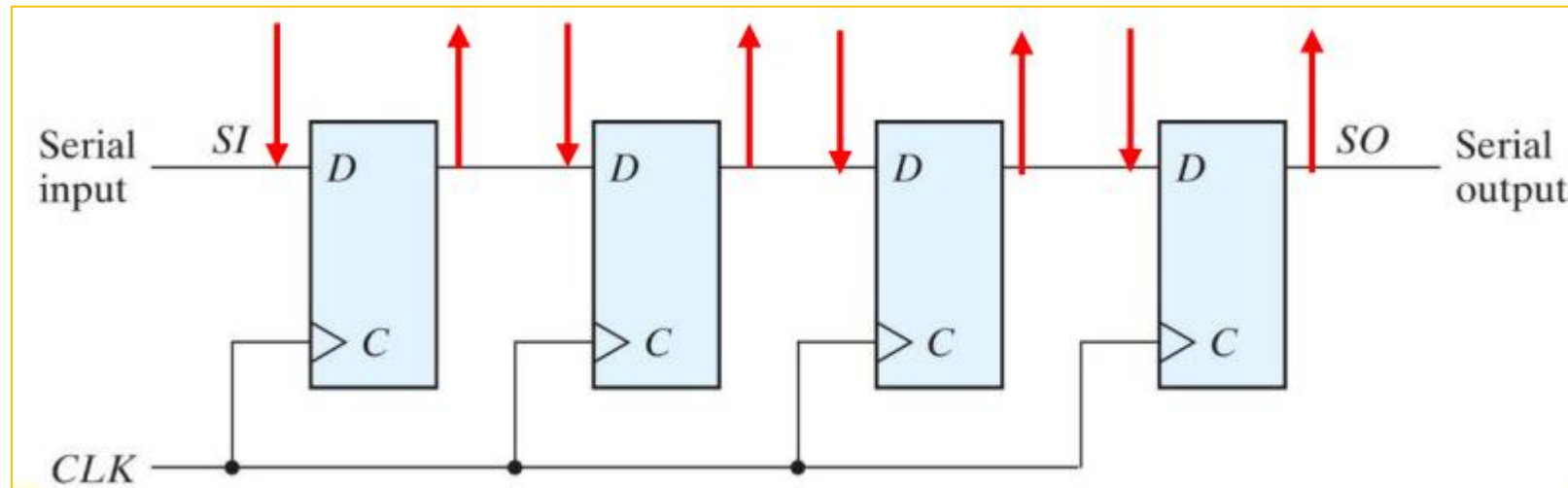
$$J_Q = xy, K_Q = x'y' = (x+y)', S = x \oplus y \oplus Q$$



- ⌚ A register that can shift in **one** direction (left **or** right) is called a **unidirectional** shift register
- ⌚ A register that can shift in **both** directions is called a **bidirectional** shift register
- ⌚ A register that can shift in **both** directions and has a **parallel load** capability is called a **Universal Shift Register**

⌚ In Universal Shift Register:

- ⌚ The data **entered serially** by shifting can be **taken out in parallel** from the outputs of the flip-flops
- ⌚ The data **entered in parallel** can be **taken out in serial** fashion by shifting the data stored in the register

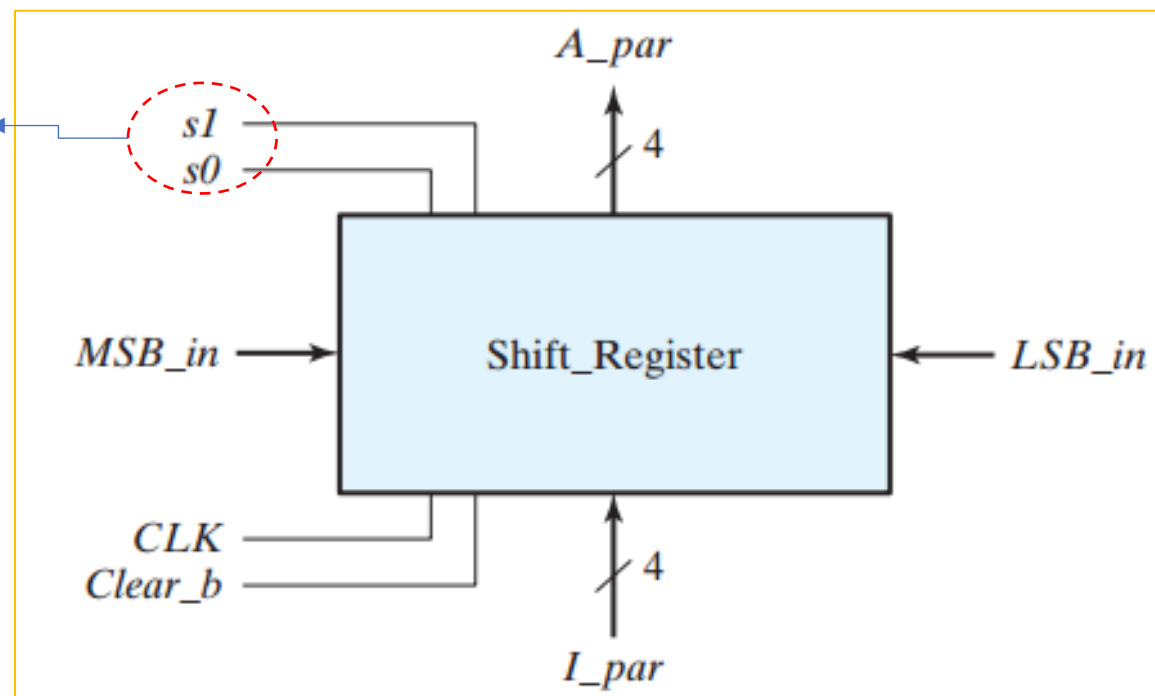


🌀 **The Universal Shift Register** has the following capabilities:

- 🕒 A **clear** control to clear the register to 0
- 🕒 A **clock** input to synchronize the operations
- 🕒 A **shift-right control** to enable the shift-right operation
- 🕒 A **shift-left control** to enable the shift-left operation
- 🕒 A **parallel-load** control to enable a parallel transfer
- 🕒 A **control** that leaves the data in the register **unchanged** in response to the clock
- 🕒 The **serial input and output** lines
- 🕒 **n parallel input lines** associated with the parallel transfer
- 🕒 **n parallel output lines** associated with the parallel transfer

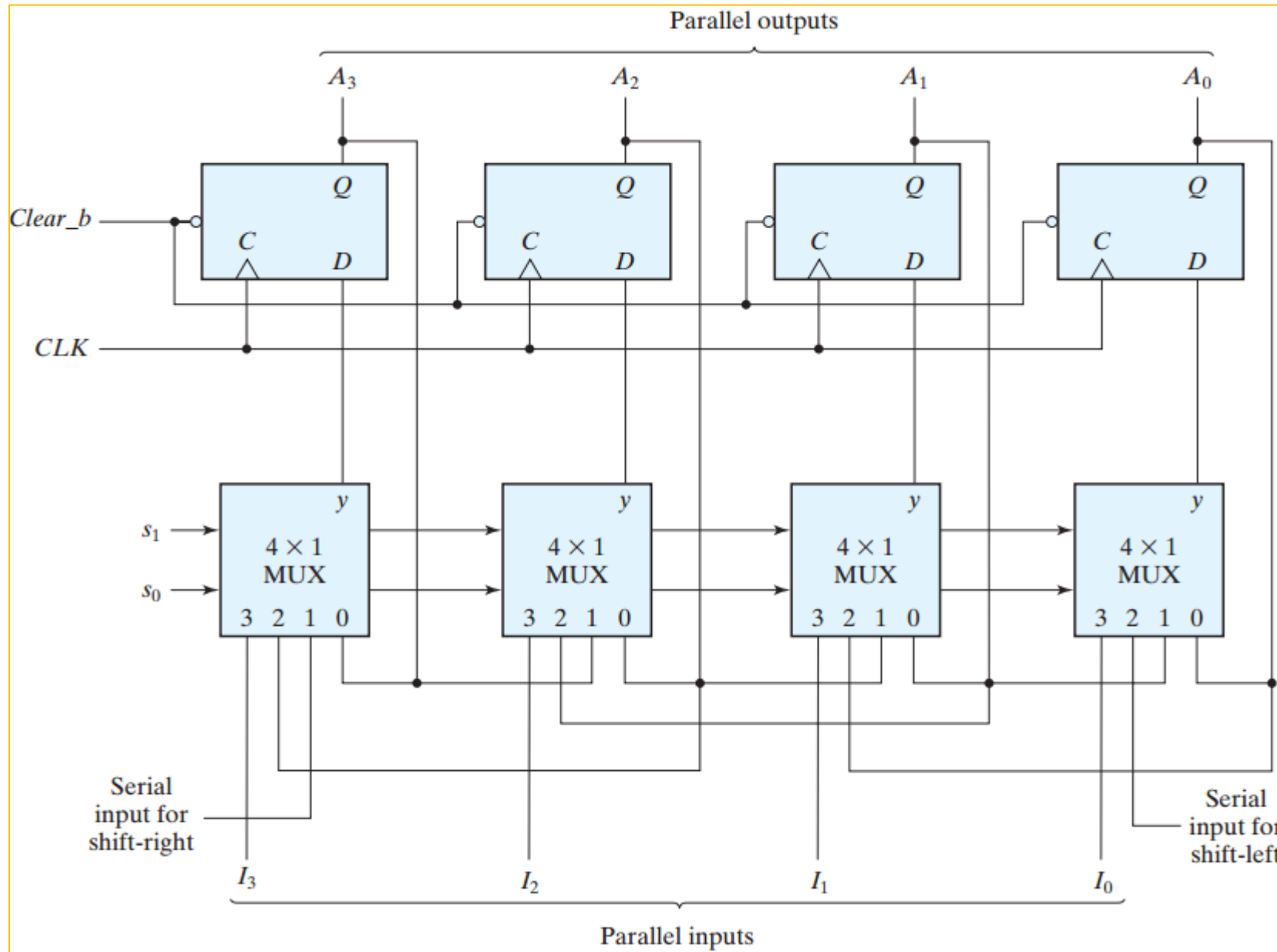
4 Control Signals

2 bits → 4 Combinations
→ (4 control signals)

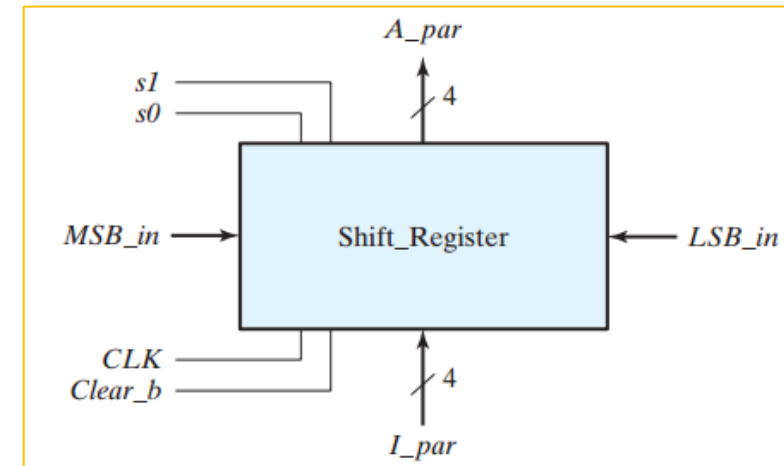


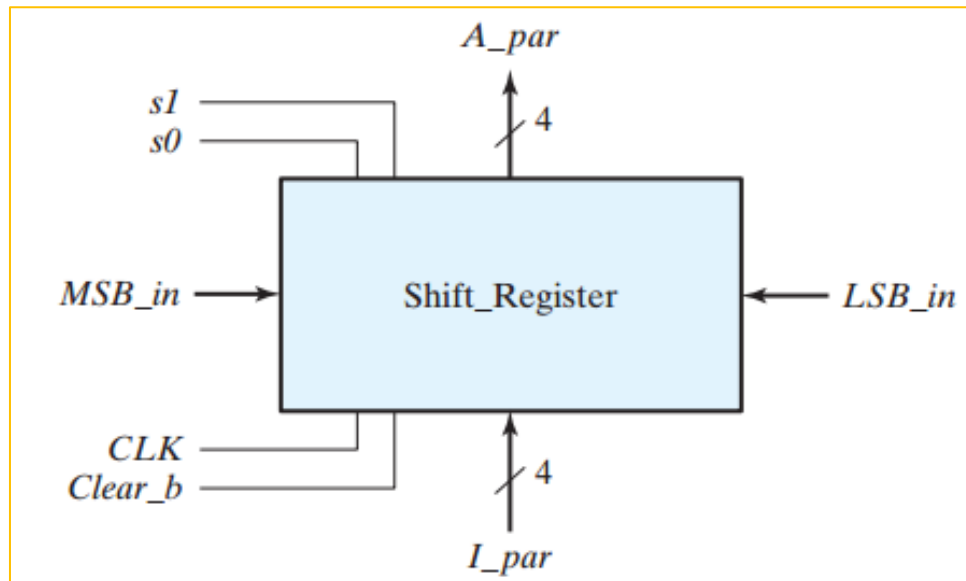


Very Important



Mode Control		
s_1	s_0	Register Operation
0	0	No change
0	1	Shift right
1	0	Shift left
1	1	Parallel load





Block Diagram

Mode Control		
s_1	s_0	Register Operation
0	0	No change
0	1	Shift right
1	0	Shift left
1	1	Parallel load

Function Table

- Universal Shift registers are often used to interface digital systems situated **remotely** from each other
- It will be **expensive** to use **n lines** to **transmit n bits** in **parallel** for a long distance.
- It is more economical to use a **single** line and **transmit** the information **serially**
- The **transmitter** accepts the **n-bit data in parallel** into a **universal shift register** and then **transmits** the data **serially**.
- The **receiver** accepts the data **serially** into a **universal shift register** and then it can be **taken from the outputs** of the register **in parallel**
- The **transmitter** performs a **parallel-to-serial conversion** of data
- The **receiver** does a **serial-to-parallel** conversion



- ☉ In real life, A counter is a device which stores (and sometimes displays) the number of times a particular event or process has occurred.
- ☉ A **Counter** is a **register** that goes through a **prescribed sequence of states** upon the application of input pulses
 - 1) Input pulses may be **clock** pulses (CLK)
 - 2) Input may originate from **external** sources (Example: other signals like A, B, etc.)
- ☉ There are two types of counters:
 - 1) **Synchronous** Counters
 - ★ Input sequence occurs at a **fixed** interval of time
 - ★ **All flip-flops are triggered by the same clock pulses.**
 - 2) **Asynchronous** Counters (**Ripple** Counters)
 - ★ Input sequence occurs at **random** (input occurrence time is not pre-defined)
 - ★ **Each flip flop is triggered by the transition of other flip-flops.**
- ☉ A counter that follows the **binary** number **sequence** is called a **binary counter**
 - ★ (**n**-bit binary counter: count from **0** to **$2^n - 1$**)

☞ In **Ripple** Counters, a flip-flop output **transition serves as a trigger** for other flip-flops. In other words, the **CLK** input of some or all flip-flops **are triggered by the transition** that occurs in other flip-flop outputs.

☞ **Binary ripple counter** consists of a series of **complementing flip-flops**.

- ★ Either **Count-up** or **Count-down**
- ★ Could be implemented using **D, T,** or **JK** FFs.
- ★ **Negative** Edge or **Positive** Edge FFs could be used

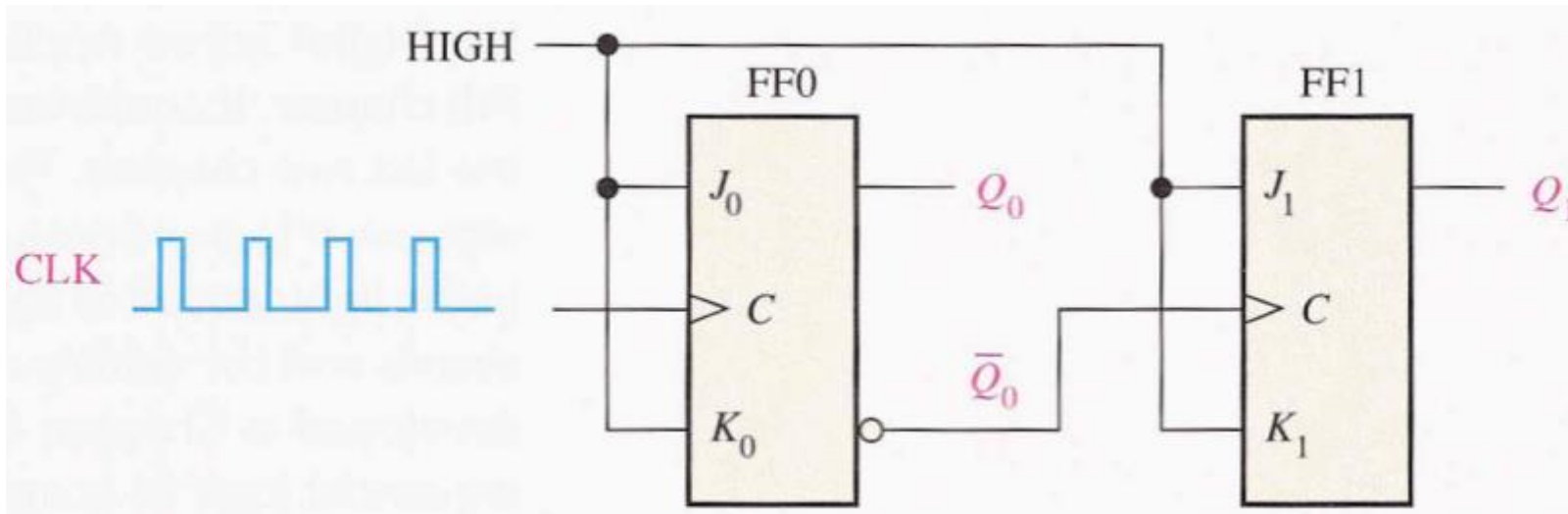
Binary Count-up

A_3	A_2	A_1	A_0
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0

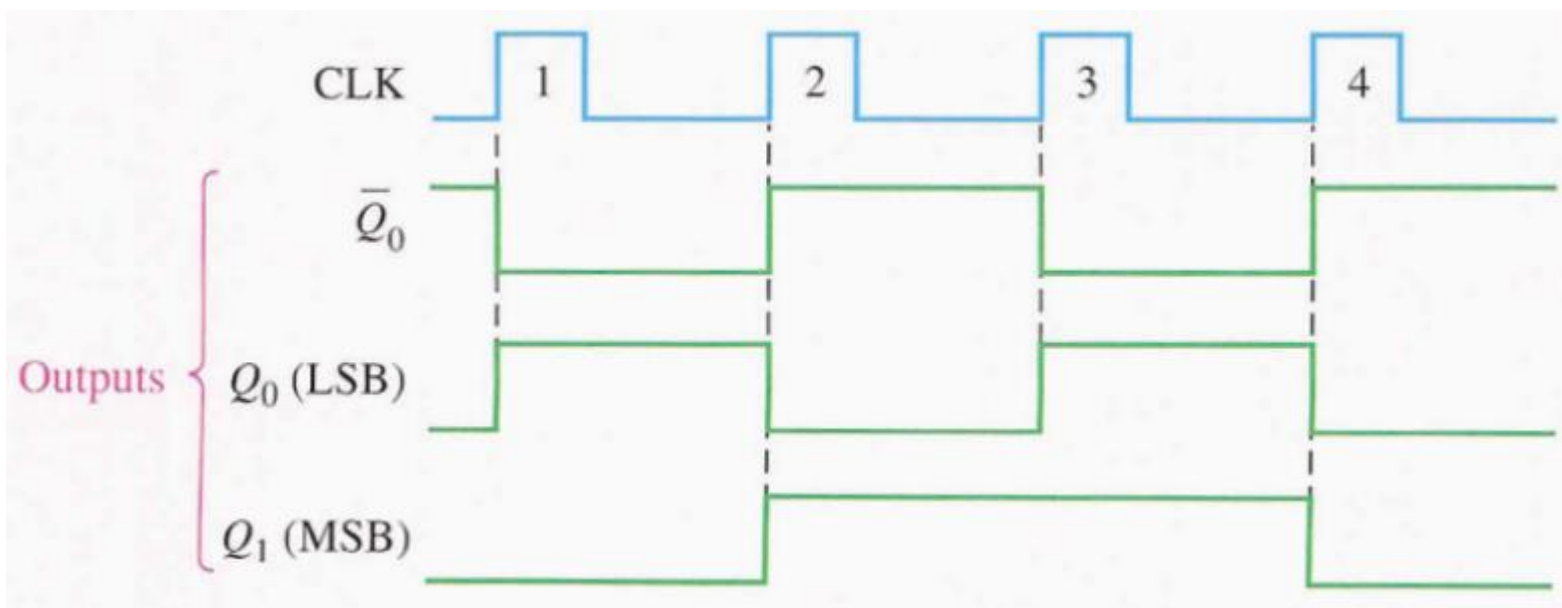
1. The **first** FF is complemented **every** Clock Edge
2. The **second** FF is complemented when the **first** FF is **changed from 1 → 0**.
(The **first** FF **acts** as the **Clock** for the **second** FF).

☞ **Same** Applied for **higher** FFs.

2-bit Ripple Counter (JK-FF)



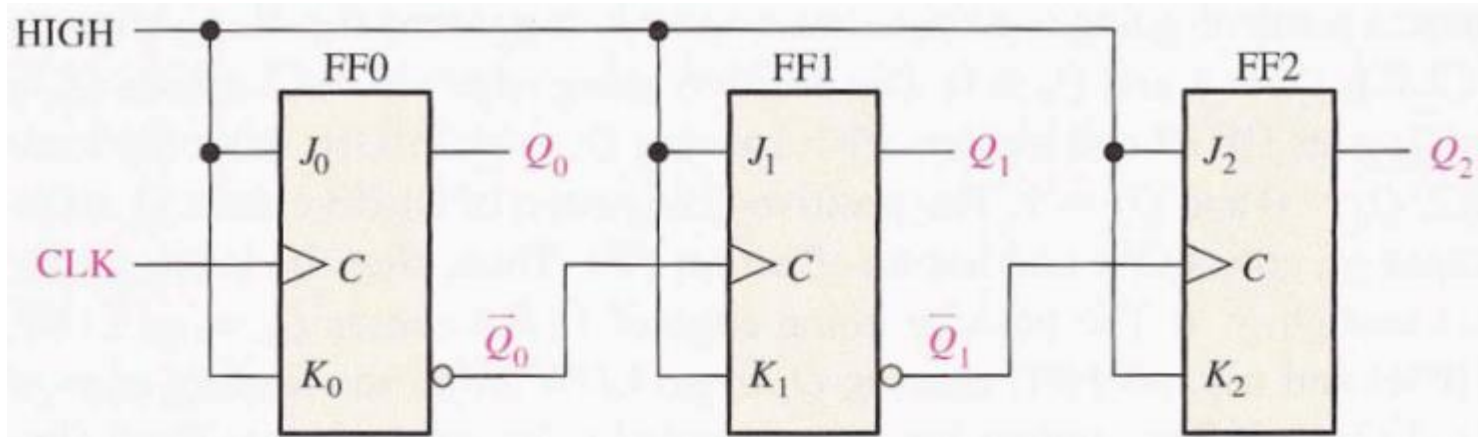
CLOCK PULSE	Q_1	Q_0
Initially	0	0
1	0	1
2	1	0
3	1	1
4 (recycles)	0	0



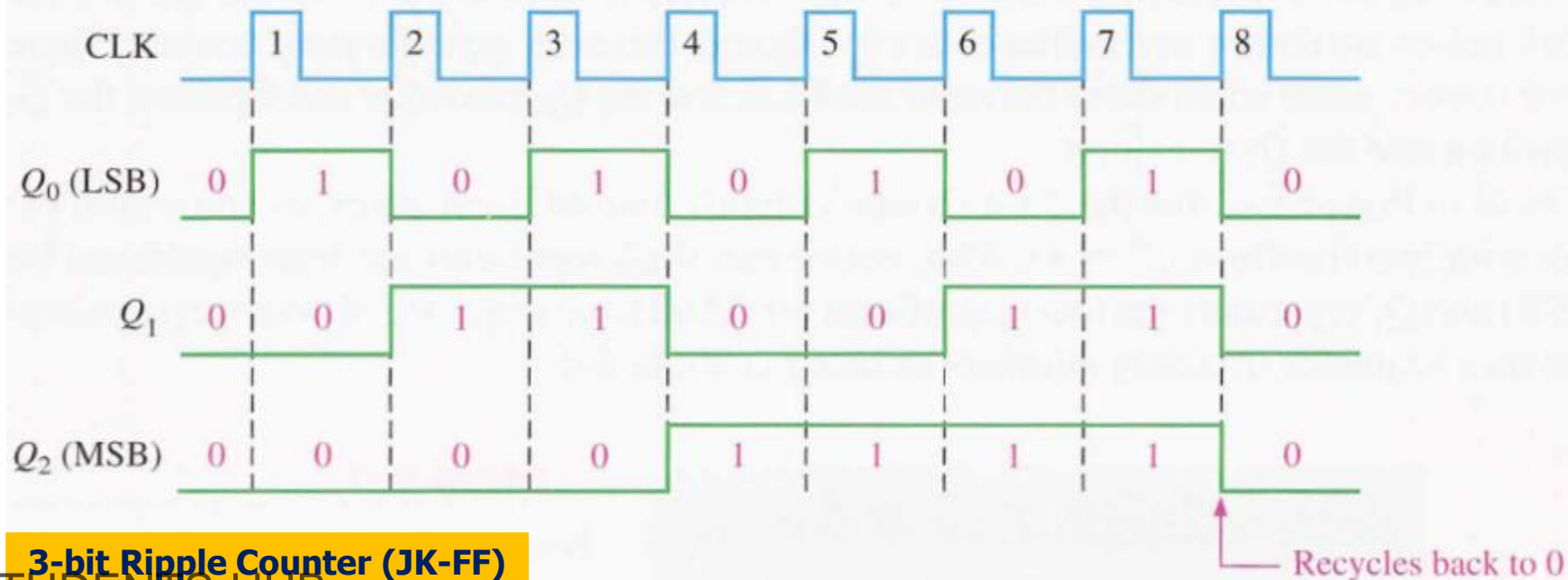
Notice:

When **+Ve** Edge FF is used → Connect **Q'** to the clock of the **next** FF

When **-Ve** Edge FF is used → Connect **Q** to the clock of the **next** FF



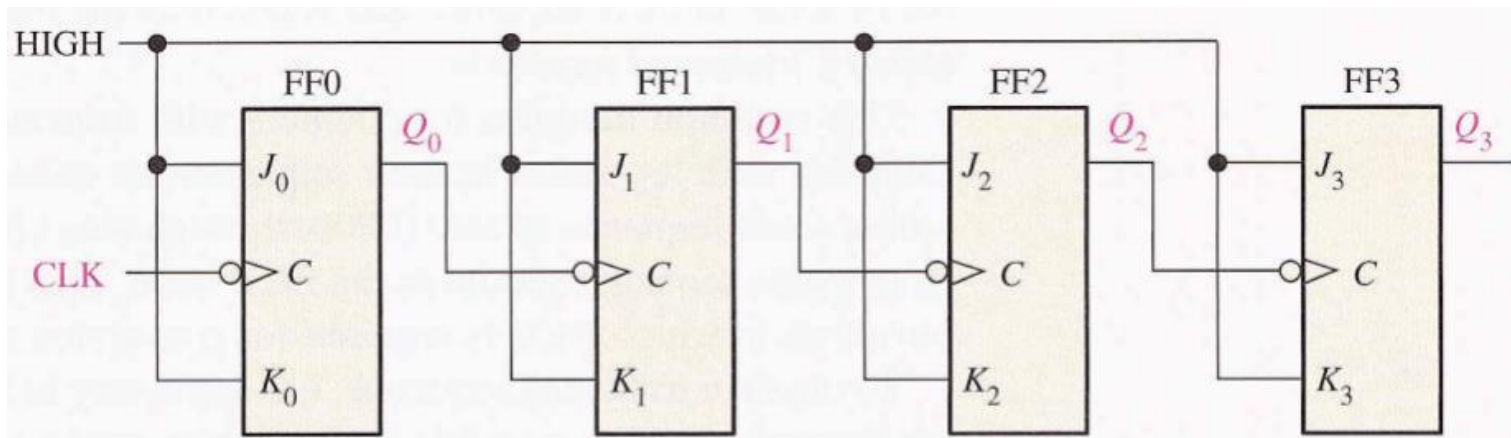
(a)



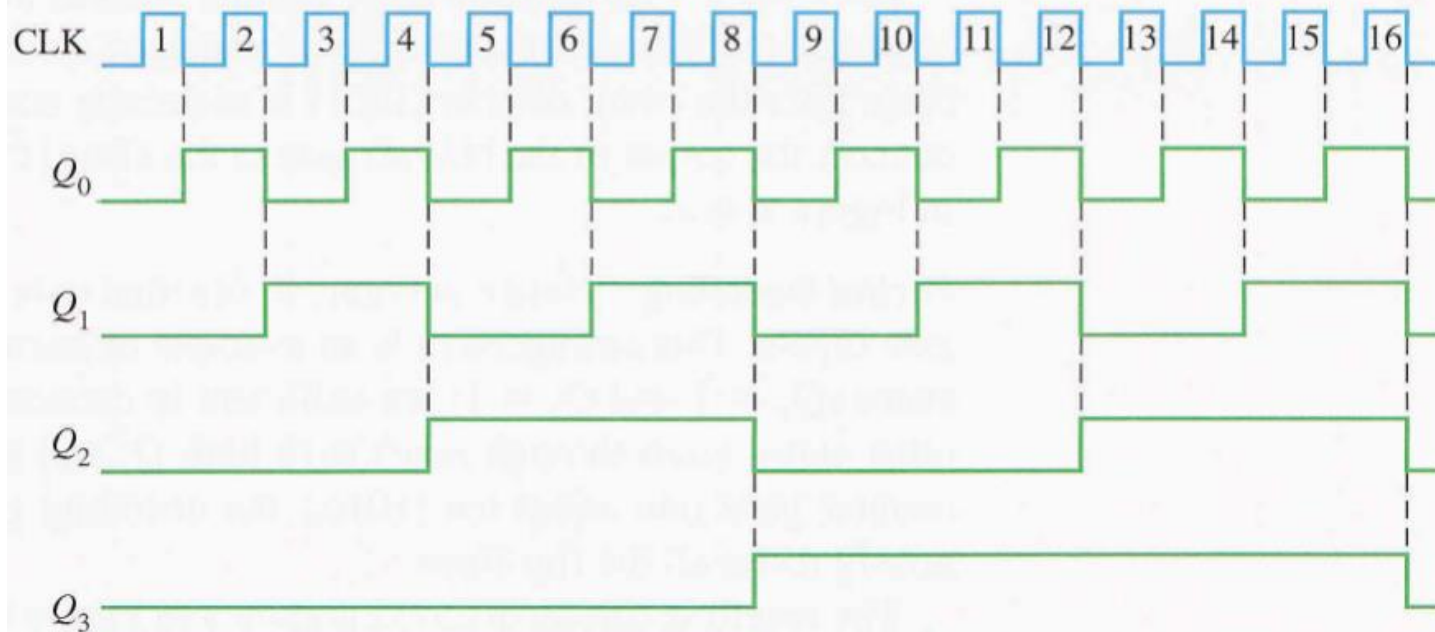
- Transition of Q_0 from **1 to 0** triggers FF1 and its output is complemented (Q_1 is complemented)
- Similarly, as Q_1 changes from **1 to 0**, FF2 is triggered (Q_2 is complemented)
- This is exactly what happens in binary counting

Notice:

When **+Ve** Edge FF is used →
Connect Q' to the clock of the **next** FF



(a)

**Notice:**

When **-Ve** Edge FF is used → Connect **Q** to the clock of the **next** FF

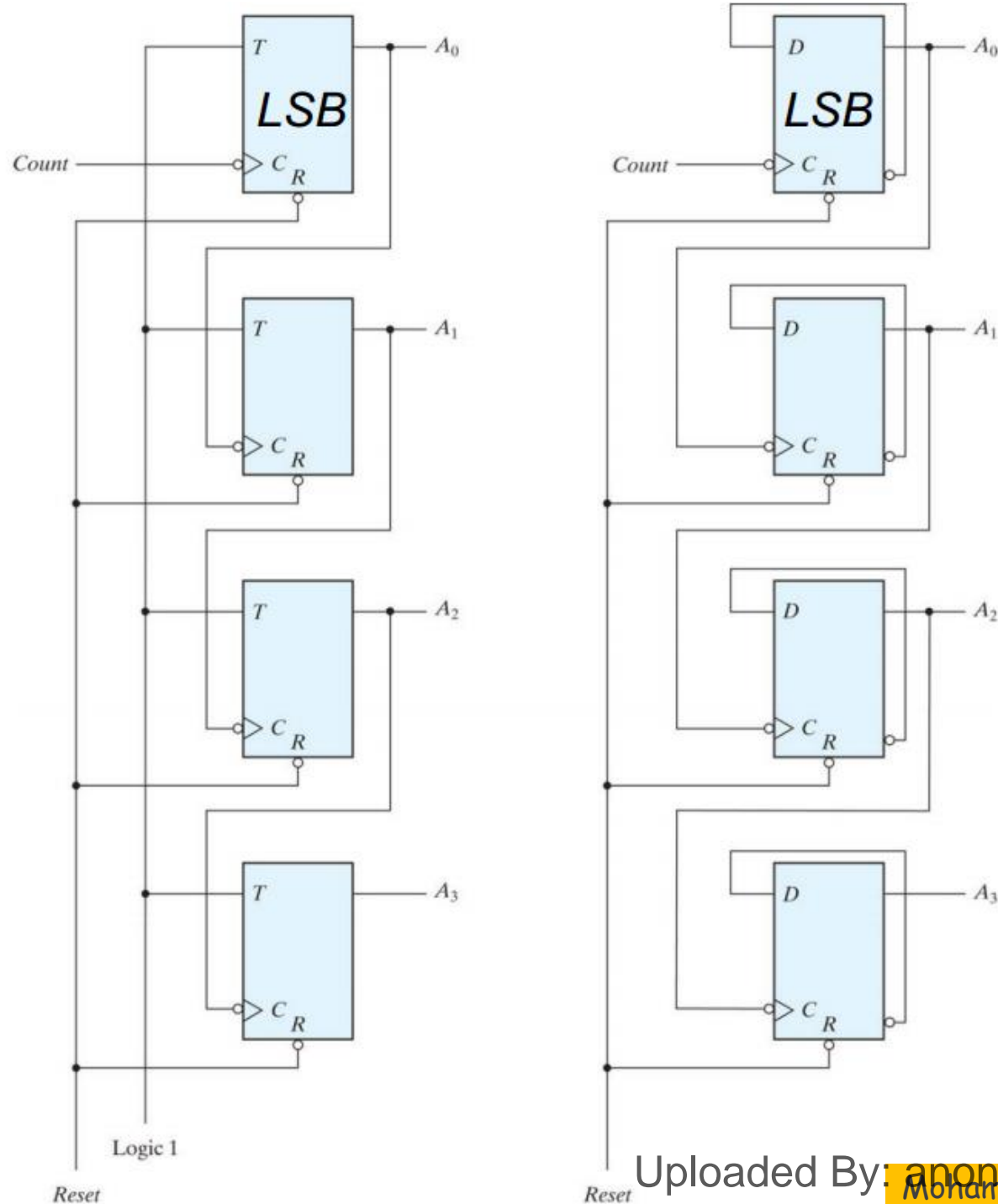
4-bit Ripple Counter (T-FF & D-FF)

Important Points:

- 1) To activate the **complement** Mode in **JK/T** FFs → connect Logic **1** (High) to the **JK/T** inputs.
- 2) To activate the **complement** Mode in **D** FFs → connect the **inverted FF output** (Q') back to the **D** input.
- 3) **Reset** Signal usually used to clear/reset the counter to the **initial** count/state at **any** time.
- 4) The **CLK** signal connected to the **first** FF is usually named **Count**

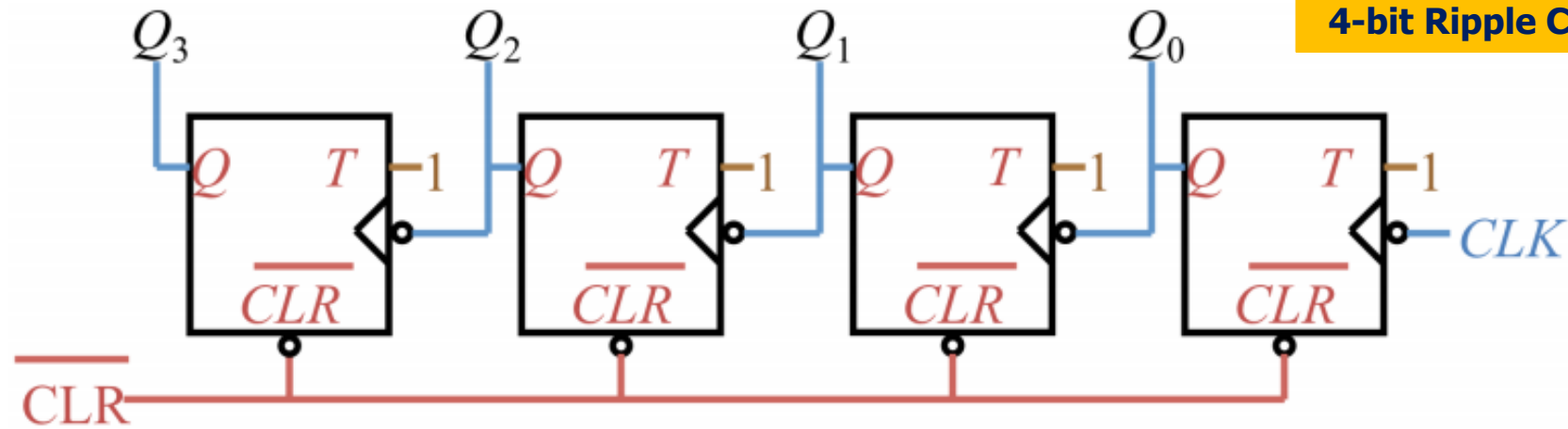
Notice:

When **-Ve** Edge FF is used → Connect **Q** to the clock of the **next** FF

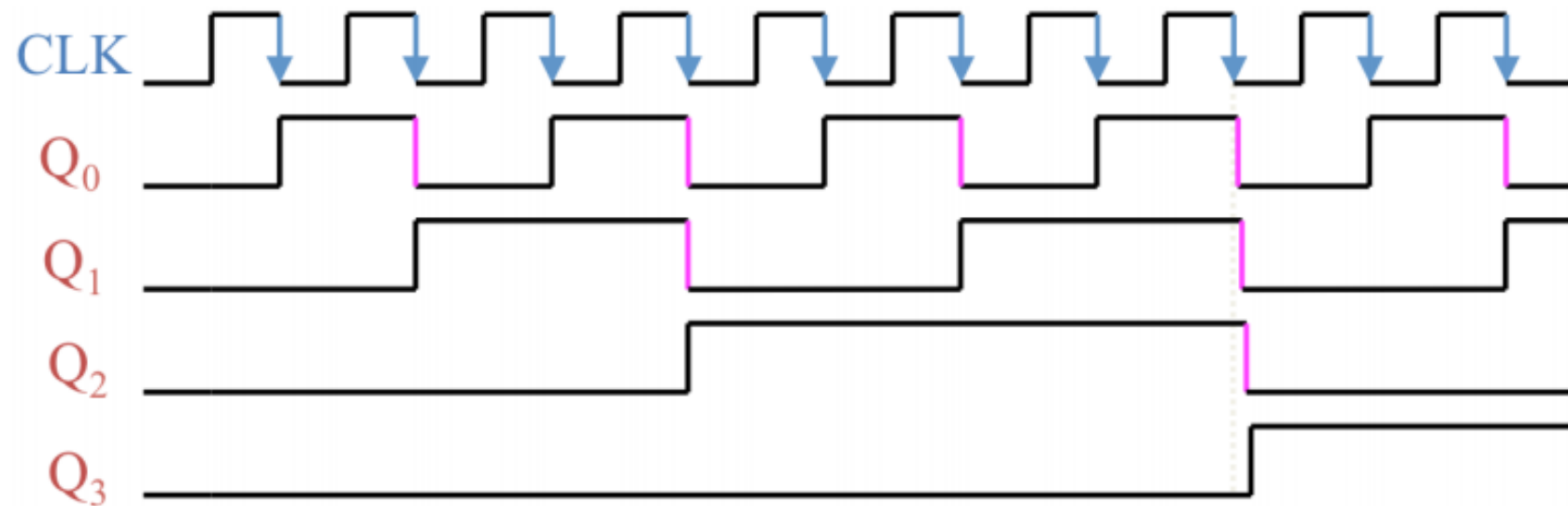




4-bit Ripple Counter (T-FF)

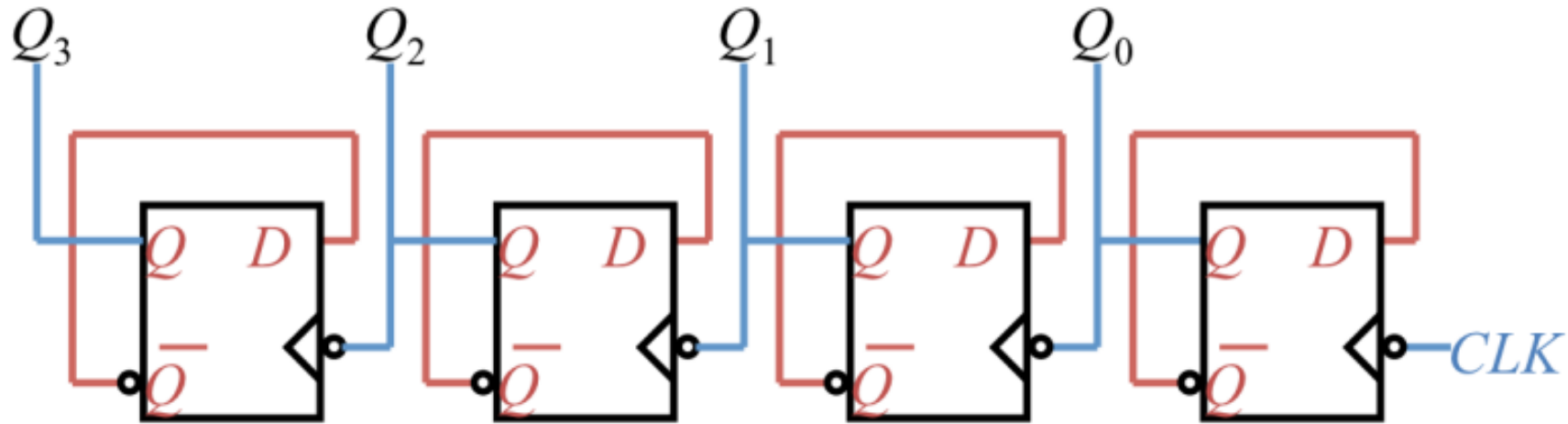


Timing Diagram

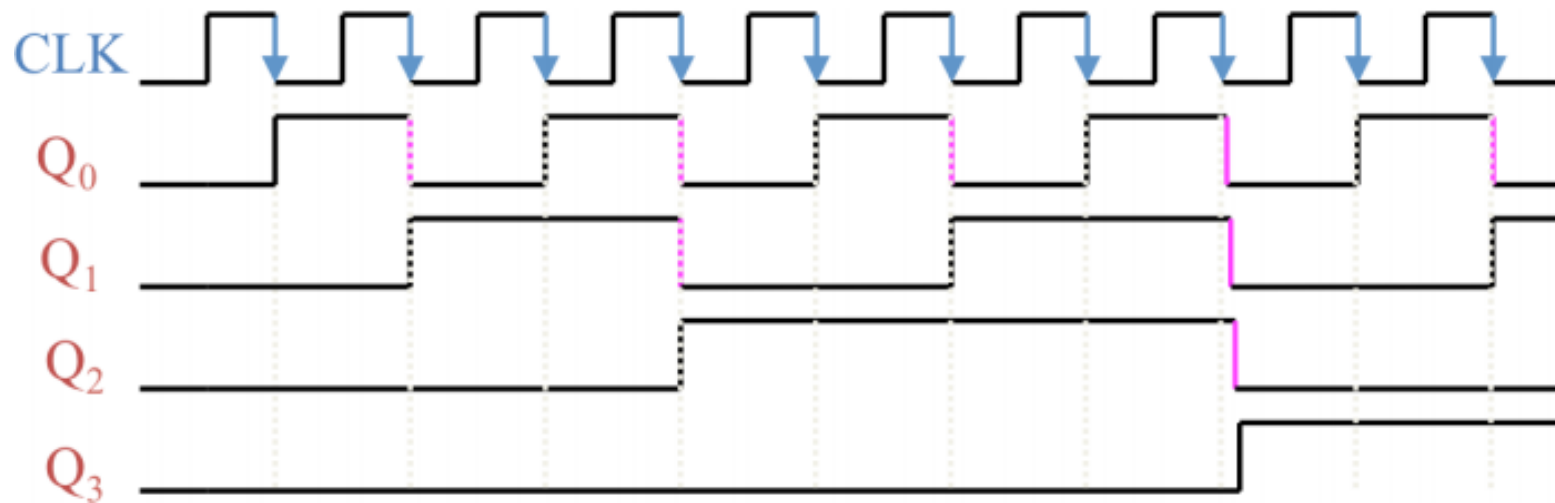




4-bit Ripple Counter (D-FF)



Timing Diagram



- ⌚ A binary **count-down** counter is a binary counter with a **reverse** count
- ⌚ **Initial** State is **All 1s** (4-bit counter starts with **1111**)
- ⌚ Any bit in the sequence is **complemented** if its **previous** least significant bit goes from **0 to 1**

Same design as Count-up **Except:**

When **+Ve** Edge FF is used →
Connect **Q** to the clock of the **next** FF

When **-Ve** Edge FF is used → Connect
Q' to the clock of the **next** FF

Common Practice:

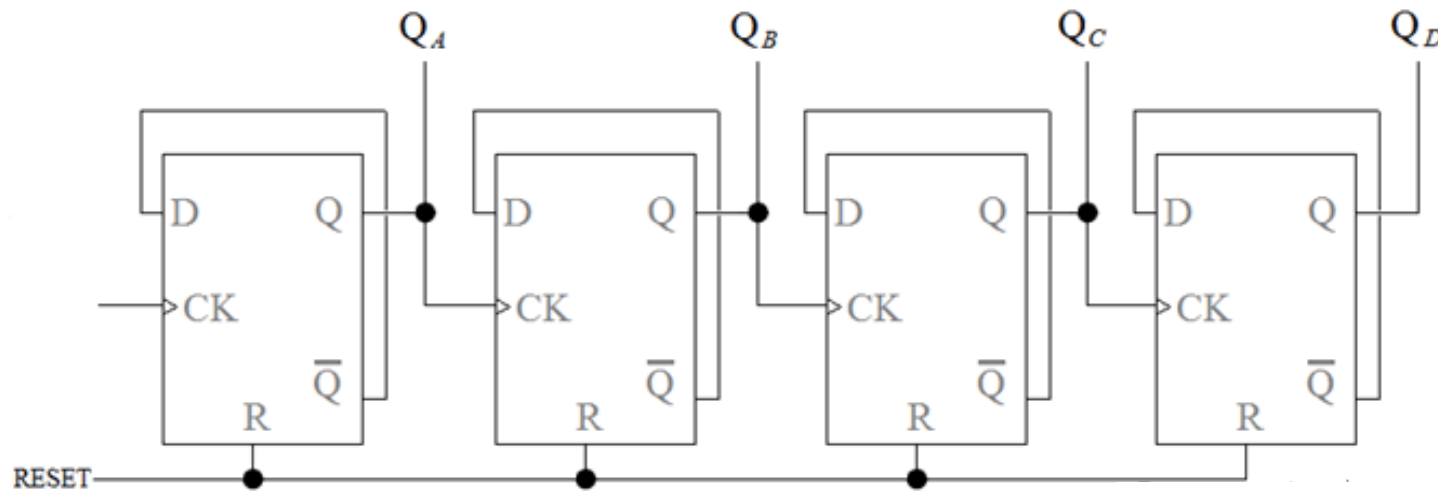
Count-Up: Use **Negative** Edge

Count-Down: Use **Positive** Edge

Binary Count Down

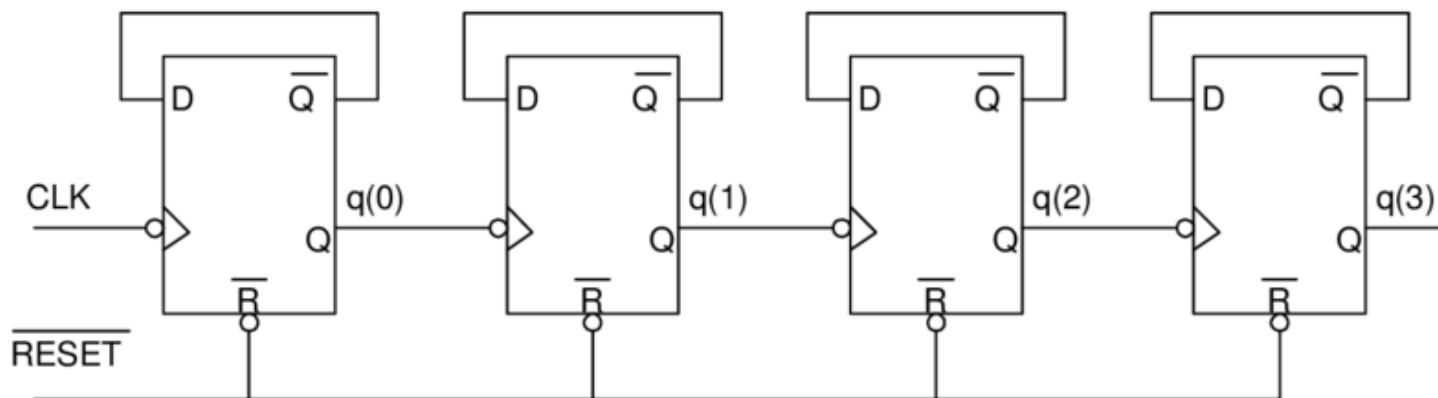
A ₃	A ₂	A ₁	A ₀
1	1	1	1
1	1	1	0
1	1	0	1
1	1	0	0
1	0	1	1
1	0	1	0
1	0	0	1
1	0	0	0
0	1	1	1
0	1	1	0

Binary Count Down +Ve Edge

**Common Practice:**

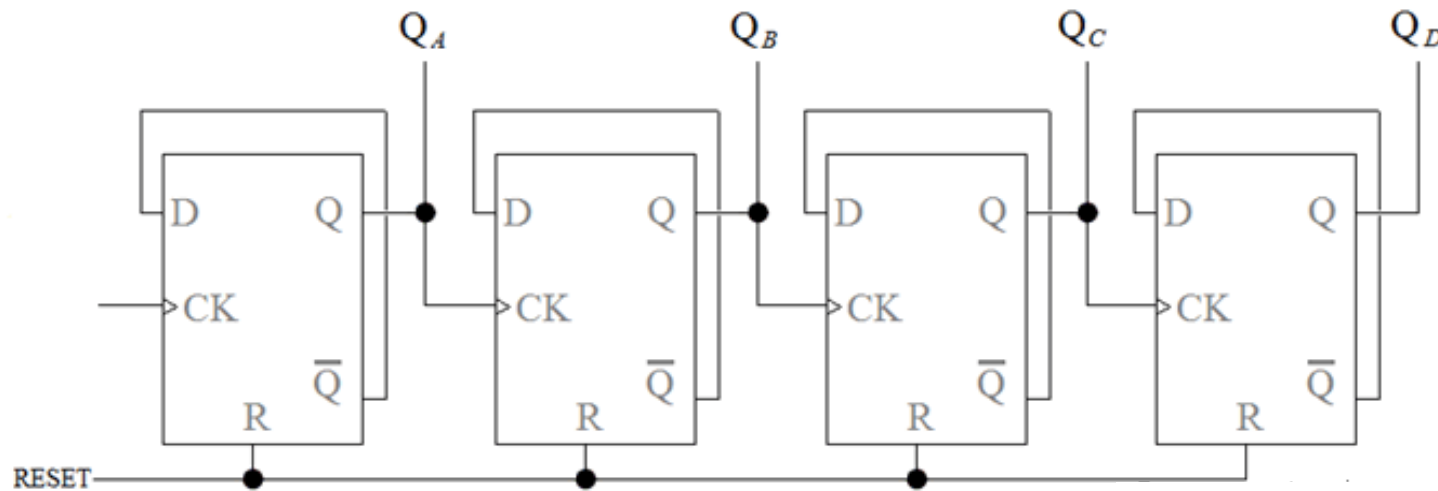
Use **+Ve** Edge FF → Connect **Q** to the clock of the **next** FF

Binary Count UP (-Ve Edge)

**Common Practice:**

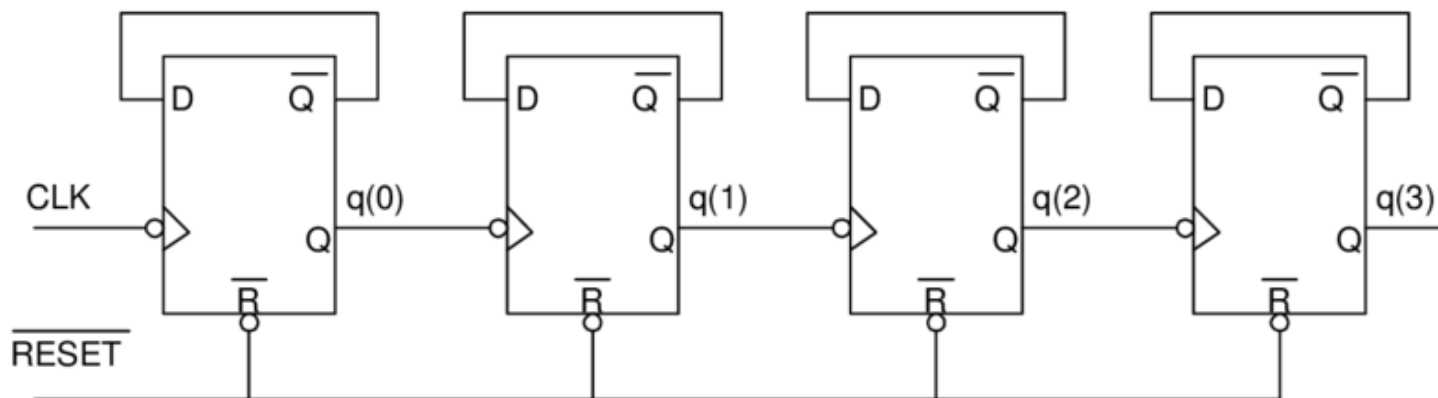
Use **-Ve** Edge FF → Connect **Q** to the clock of the **next** FF

Binary Count Down +Ve Edge

**Common Practice:**

Use **+Ve** Edge FF → Connect **Q** to the clock of the **next** FF

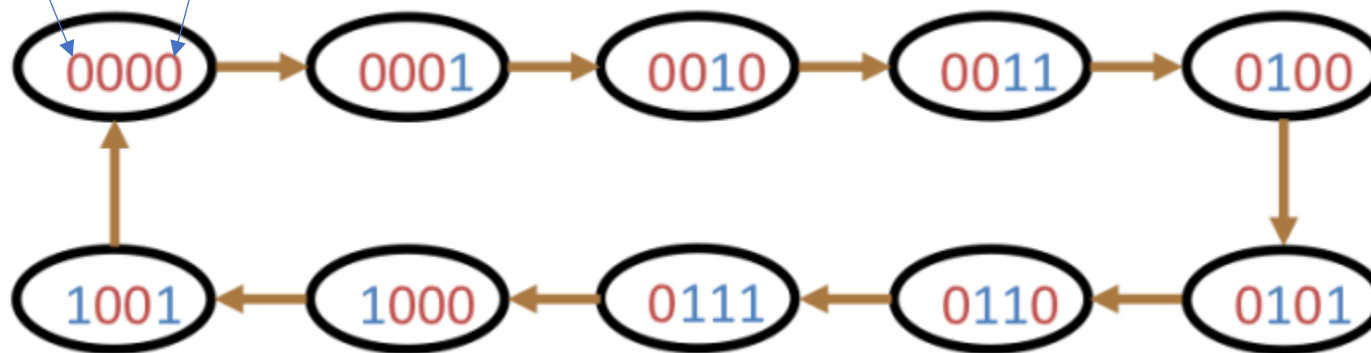
Binary Count UP (-Ve Edge)

**Common Practice:**

Use **-Ve** Edge FF → Connect **Q** to the clock of the **next** FF

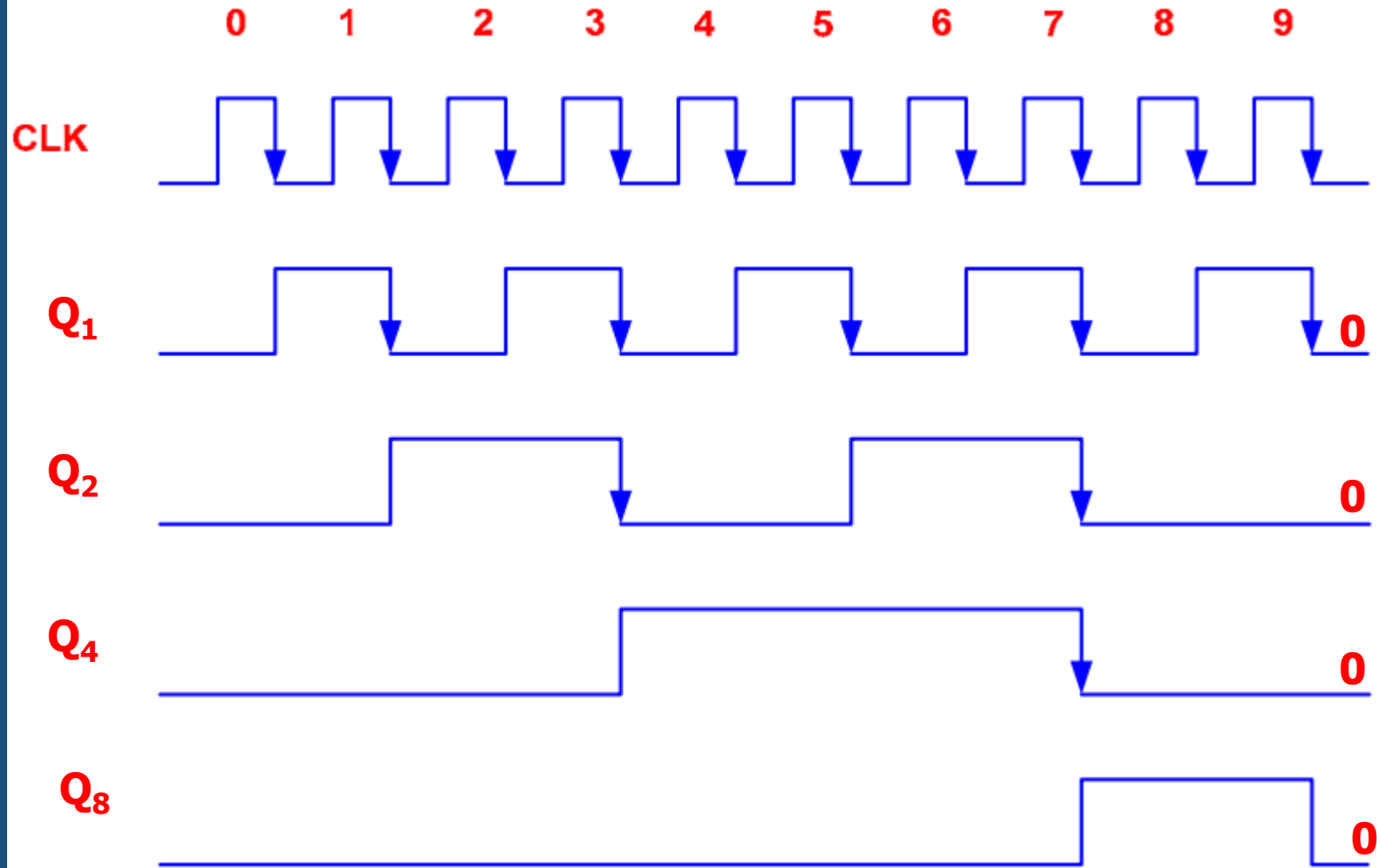
- 🌀 A **decimal (BCD)** counter follows a sequence of **10** states and **returns to 0** after the count of **9**
 - ★ (Count 0→9 then restart to 0)
- 🌀 **10** States → **4** FFs are needed ($2^3 = 8, < 10 \rightarrow 4$ is needed)
- 🌀 **BCD** counter sequence: **0000** → **1001** then restart to **0000**
- 🌀 **Common** Names: Decimal / BCD/ Decade

The **4** FFs are named according to their decimal **weight** →
 $Q_8 Q_4 Q_2 Q_1$



BCD Counter

Notice: Q_8 and Q_2 FFs **reset** after the **10th** pulse
In Binary Counter: 1001 → 1010 (Both $Q_3, Q_1 = 1$)

**Notice:**

- 1) **Q₁** complements after **each** clock pulse.
- 2) **Q₂** complements every time **Q₁** goes from **1 to 0**, as long as **Q₈ = 0**.
- 3) When **Q₈ = 1**, **Q₂** remains at **0**.
- 4) **Q₄** complements every time **Q₂** goes from **1 to 0**.
- 5) **Q₈** remains at **0** as long as **Q₂** or **Q₄** is **0**.
- 6) When both **Q₂** and **Q₄** become **1**, **Q₈** complements when **Q₁** goes from **1 to 0**.
- 7) **Q₈** is cleared on the next transition of **Q₁**.

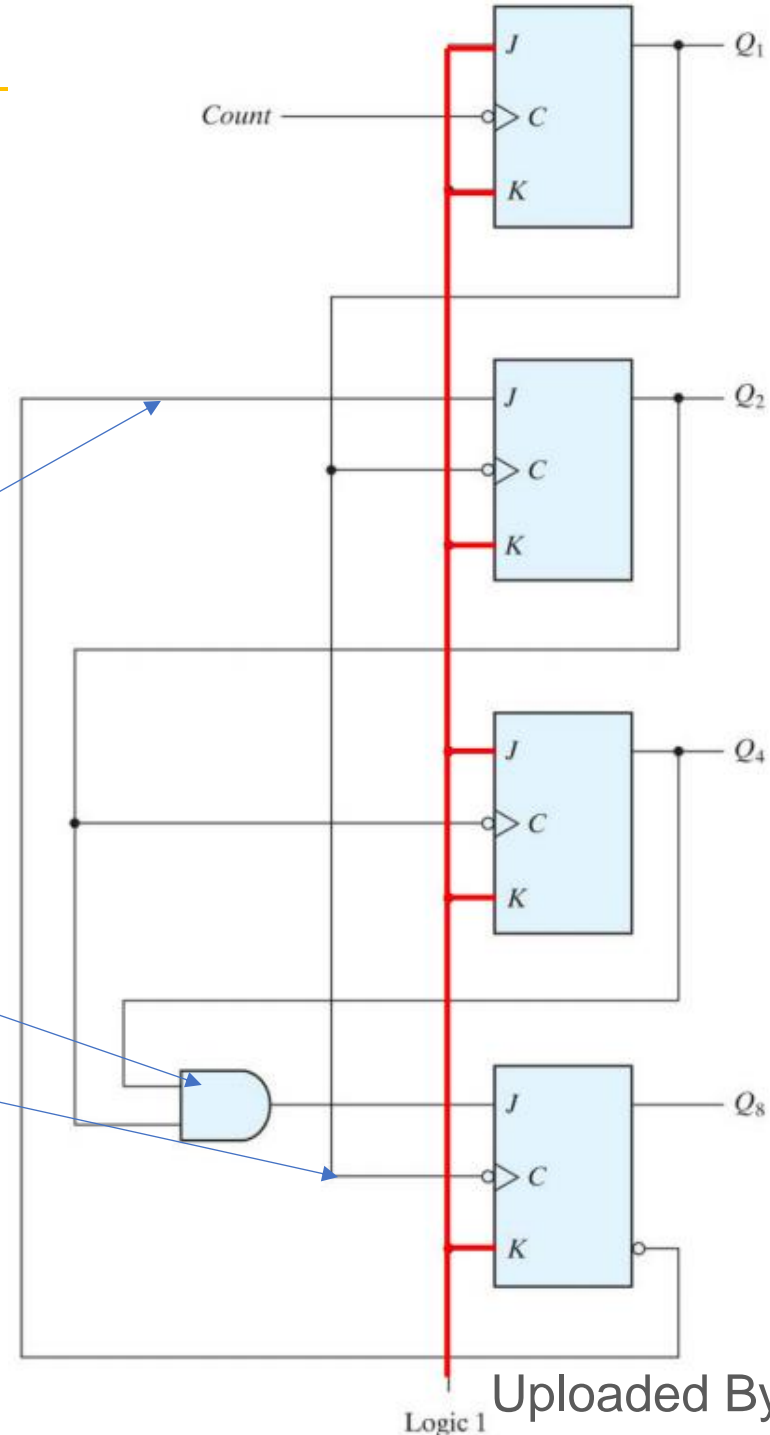
BCD Counter - Timing Diagram

- Point 3: → Connect **Q₈'** to **J** input of **Q₂**
 Point 6: → Connect **AND(Q₂, Q₄)** to **J** input of **Q₈**
 Point 7: → Connect **Q₁** to the Clock of **Q₈**

Use **JK** FF, **-ve** Edge (As in Binary Ripple Counter)

Apply the following Modifications:

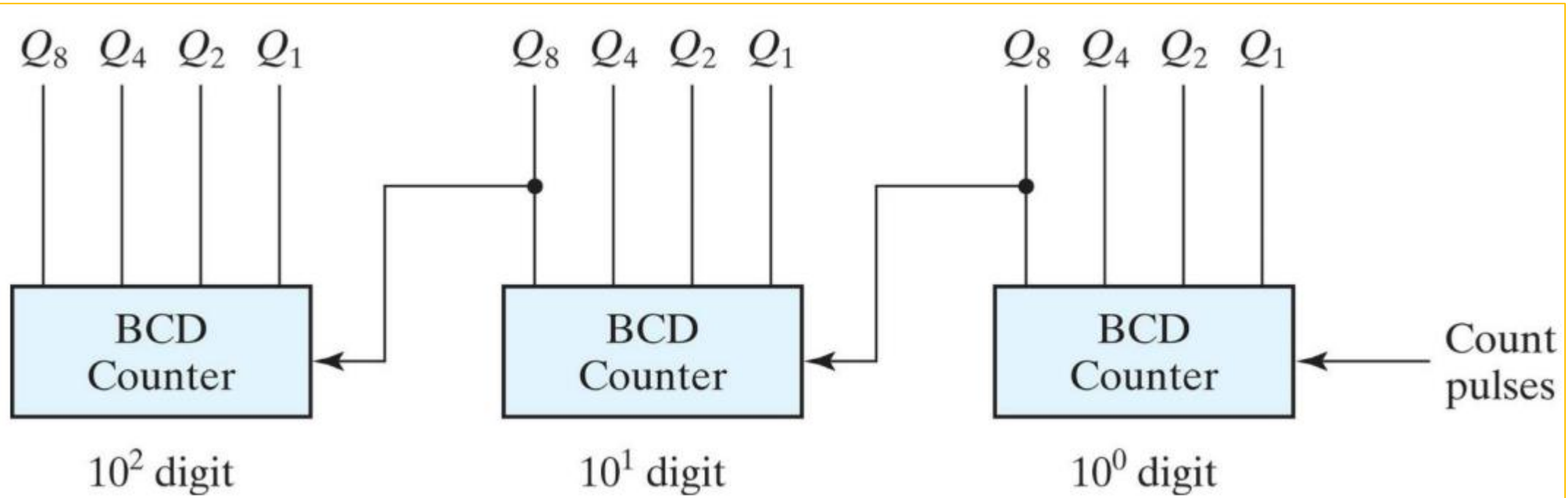
- 1) Connect Q_8' to **J** input of Q_2
- 2) Connect **AND**(Q_2, Q_4) to **J** input of Q_8
- 3) Connect Q_1 to the Clock of Q_8



🌀 We can **cascade** BCD counters (connect in **series**) to obtain **multiple** decimal digits counter

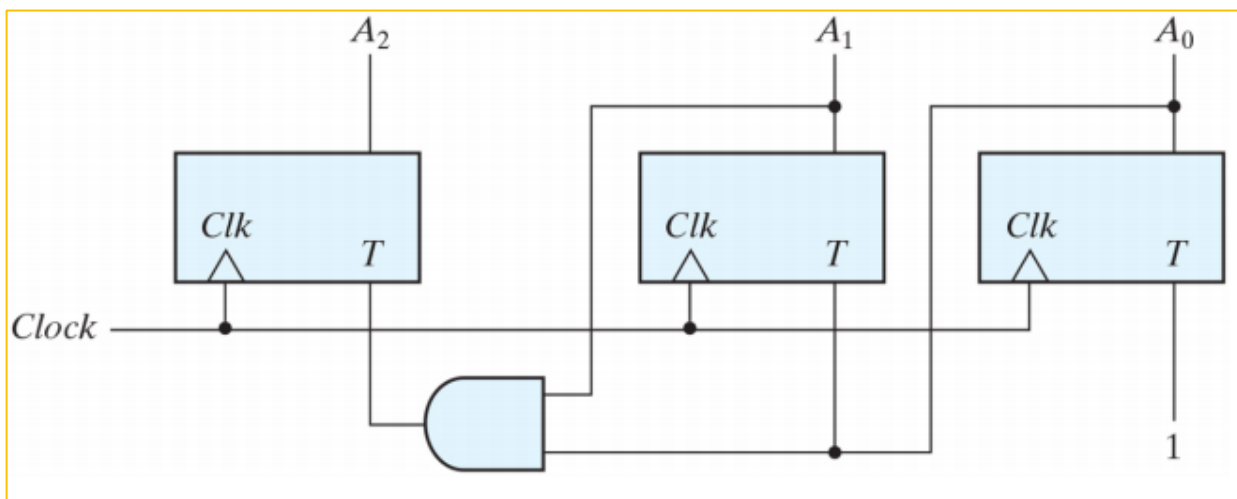
🌀 To count from **0** to **999**

- 📌 **Three** BCD counters is needed (**One** for **Each** Digit)
- 📌 Constructed by **connecting** the BCD counters in **cascade**



Remember: All **ripple** counters are an **asynchronous** sequential circuits.

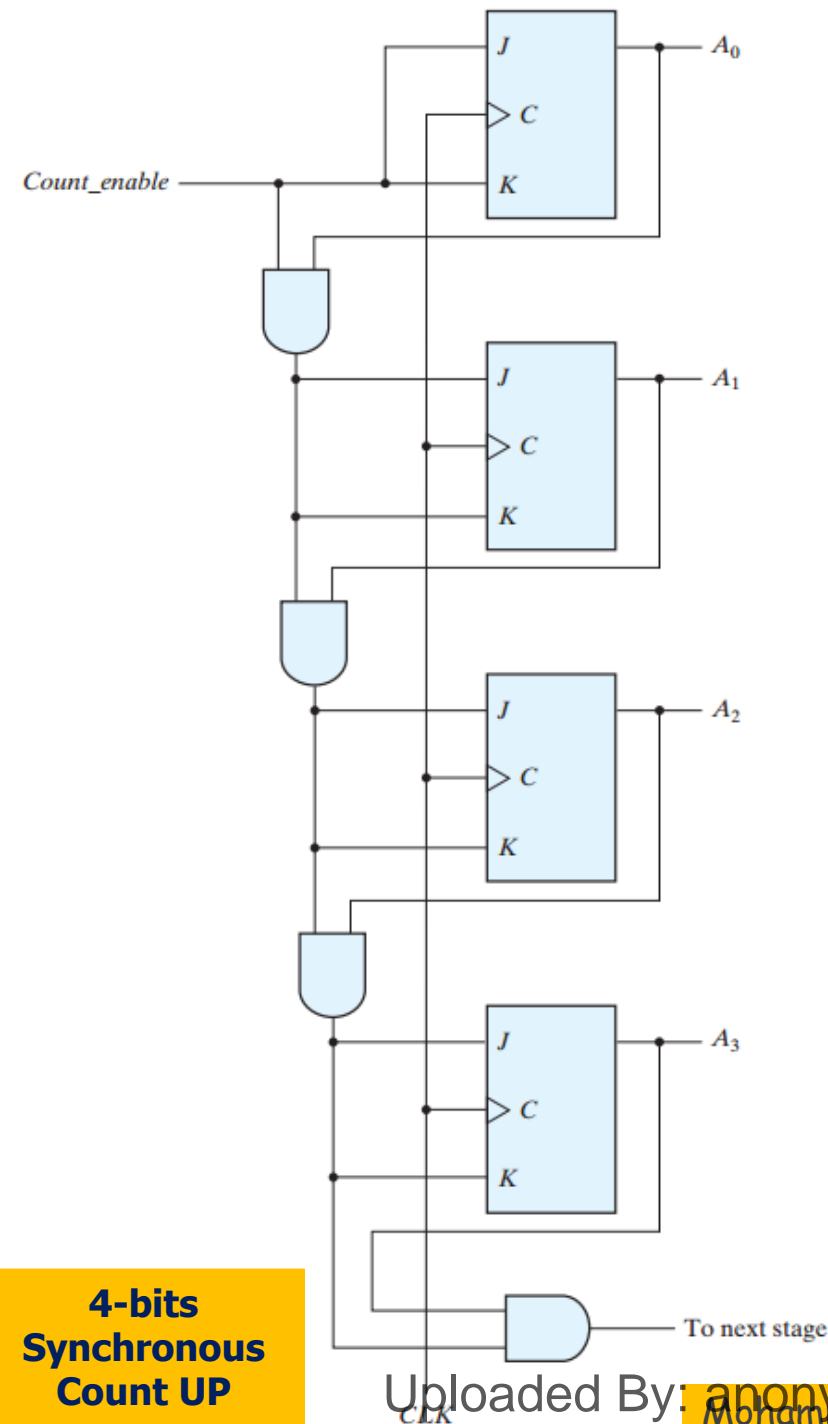
- ⌚ A **common** clock **triggers all** flip-flops simultaneously
- ⌚ Typically, their design procedure follows that for Sequential circuits
- ⌚ The design of a synchronous binary counter is so **simple** that there is **no need** to go through a sequential logic design process
- ⌚ In a synchronous binary counter, the flip-flop in the **least** significant position (**LSB**) is **complemented** with **every** pulse
- ⌚ A flip-flop in any **other** position is **complemented** when **all the bits** in the **lower** significant **positions** are **equal to 1**



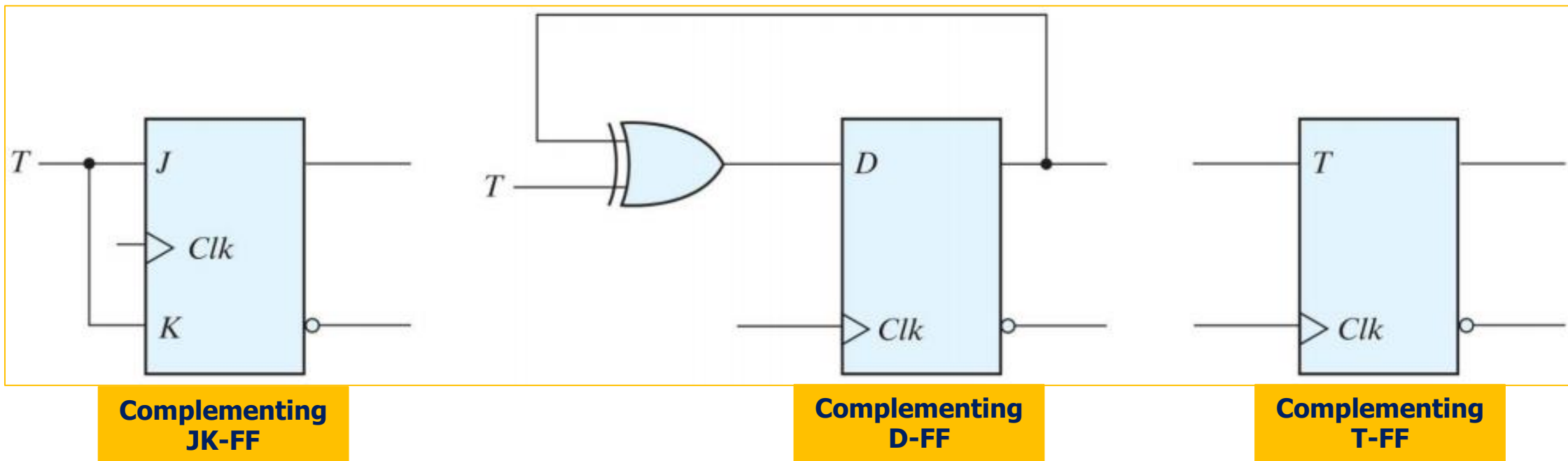
Binary Count UP			
A_3	A_2	A_1	A_0
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0

- ⌚ A_0 complements every time the count pulses go from 1 to 0
- ⌚ A_1 complements only when A_0 is 1 and goes to 0
- ⌚ A_2 complements only when A_1 and A_0 are 1 and going to 0
- ⌚ A_3 complements only when A_2 , A_1 and A_0 are all 1 and going to 0

- ☉ Synchronous binary counters have a **regular pattern** and can be constructed with **complementing flip-flops** and **gates**
- ☉ The counter is enabled by **Count_enable**
 - ★ $\text{Count_enable} = 0 \rightarrow \text{No Change}$
- ☉ The counter can be **extended** to **any** number of stages, with **each** stage having an **additional FF** and an **AND** gate
- ☉ The synchronous counter can be designed with either the positive or the negative clock edge



- As stated earlier, Synchronous counters need to be designed with **complementing** Flip-Flops
- The **complementing** FF can be of either the **JK** type, the **T** type, or the **D** type with **XOR** gates



Notice:

T-FF: is a **complementing** FF by its **nature**

JK-FF: can be converted to a **complementing** FF (T-FF), by **connecting both J,K to same input**

D-FF: can be converted to a **complementing** FF (T-FF), by **XOR both the Present State and the input**

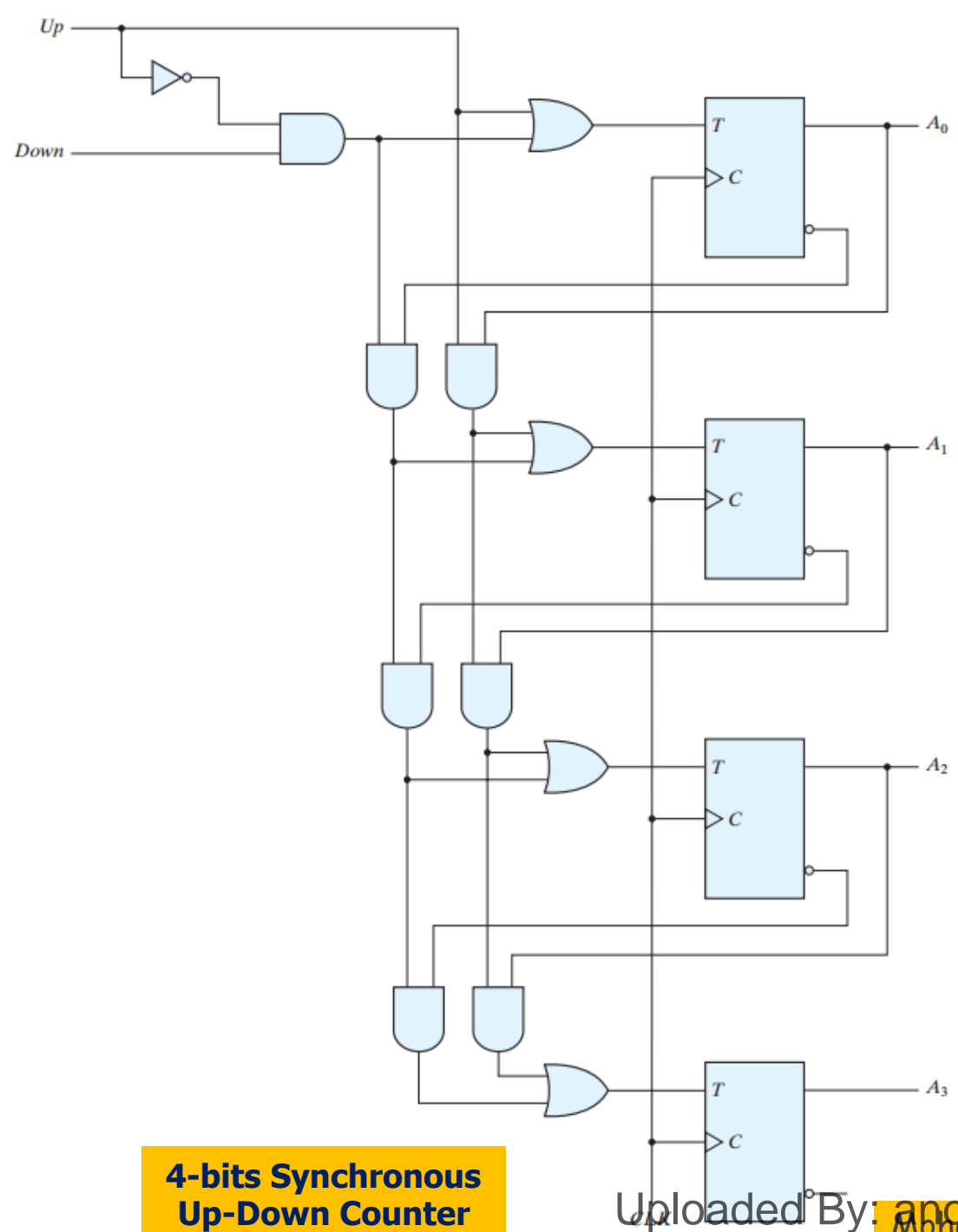


- ⌚ A synchronous **count-down** binary counter goes through the binary states in **reverse** order, from 1111 down to 0000 and back to 1111
- ⌚ The bit in the **least** significant position(LSB) is **complemented** with **each** pulse
- ⌚ A bit in **any** other position is **complemented** if **all lower** significant bits are **equal to 0**
- ⌚ A countdown binary counter can be constructed as up counter, **except** that the **inputs** to the **AND** gates must come from the **complemented outputs** of the **FF (Q')**

Binary Count Down			
A ₃	A ₂	A ₁	A ₀
1	1	1	1
1	1	1	0
1	1	0	1
1	1	0	0
1	0	1	1
1	0	1	0
1	0	0	1
1	0	0	0
0	1	1	1
0	1	1	0

- ☞ The two (Up & Down) counters can be combined in one circuit to form a counter capable of counting either up or down
- ☞ It has an Up and Down **control** Inputs
 1. Up = 0 & Down = 0 → **Don't Count**
 2. Up = 0 & Down = 1 → Count **Down**
 3. Up = 1 & Down = X → Count **Up**
- ☞ This set of conditions ensures that only **one operation** is **performed** at any given time
- ☞ The **Up** input has **priority**

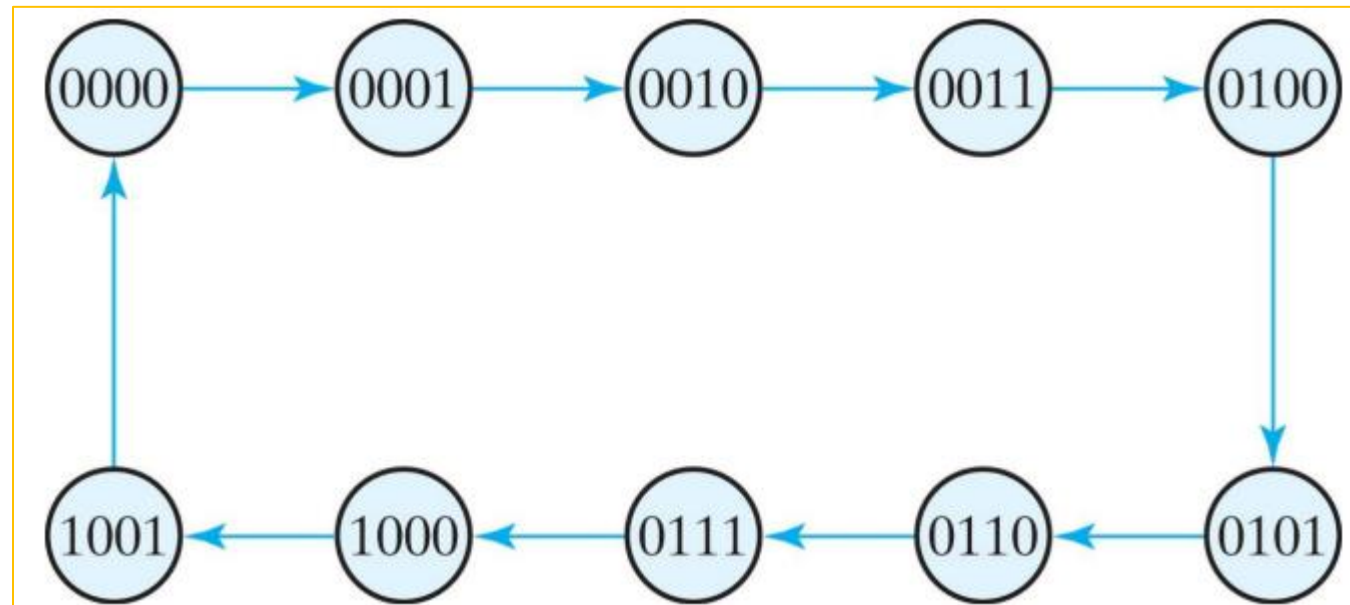
Up	Down	Operation
0	0	Same
0	1	Down
1	x	Up

4-bits Synchronous
Up-Down Counter



- ☞ A **BCD** counter counts in binary-coded decimal from 0000 to 1001 and back to 0000
- ☞ Because of the return to 0 after a count of 9, a BCD counter **does not have a regular pattern**, unlike a straight binary count
- ☞ To derive the circuit of a BCD synchronous counter, it is necessary to **go through a sequential circuit design** procedure
- ☞ An **output** is defined, to **enable** the **next** decade counter (stage)

**BCD Counter –
State Diagram**



🌀 The **output** y , equal to **1** when the **present** state is **1001**, to **enable** the **count** of the **next-higher** significant decade while the same pulse switches the present decade from 1001 to 0000

Present State				Next State				Output	Flip-Flop Inputs			
Q_8	Q_4	Q_2	Q_1	Q_8	Q_4	Q_2	Q_1	y	TQ_8	TQ_4	TQ_2	TQ_1
0	0	0	0	0	0	0	1	0	0	0	0	1
0	0	0	1	0	0	1	0	0	0	0	1	1
0	0	1	0	0	0	1	1	0	0	0	0	1
0	0	1	1	0	1	0	0	0	0	1	1	1
0	1	0	0	0	1	0	1	0	0	0	0	1
0	1	0	1	0	1	1	0	0	0	0	1	1
0	1	1	0	0	1	1	1	0	0	0	0	1
0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	0	0	1	0	0	0	0	1
1	0	0	1	0	0	0	0	1	1	0	0	1

🌀 The **unused** states (**6 States**) for minterms **10 to 15** are taken as **don't-care** terms

🌀 The **simplified T-FF** and **Output** functions (using K-Maps) are:

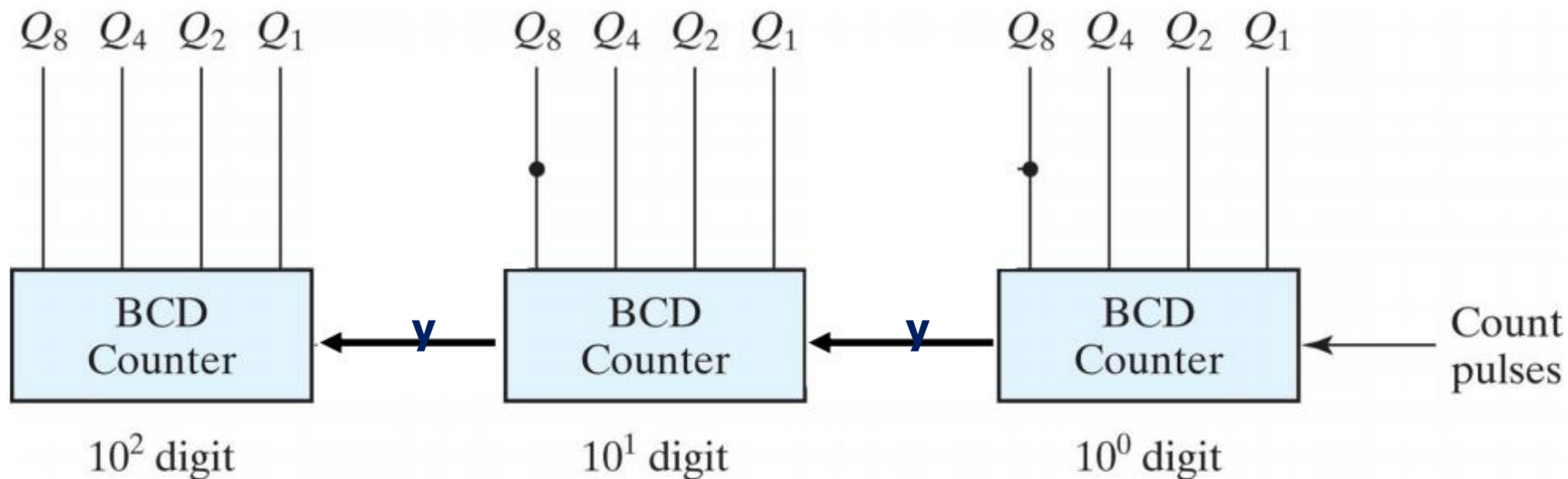
$$T_{Q_1} = 1$$

$$T_{Q_2} = Q'_8 Q_1$$

$$T_{Q_4} = Q_2 Q_1$$

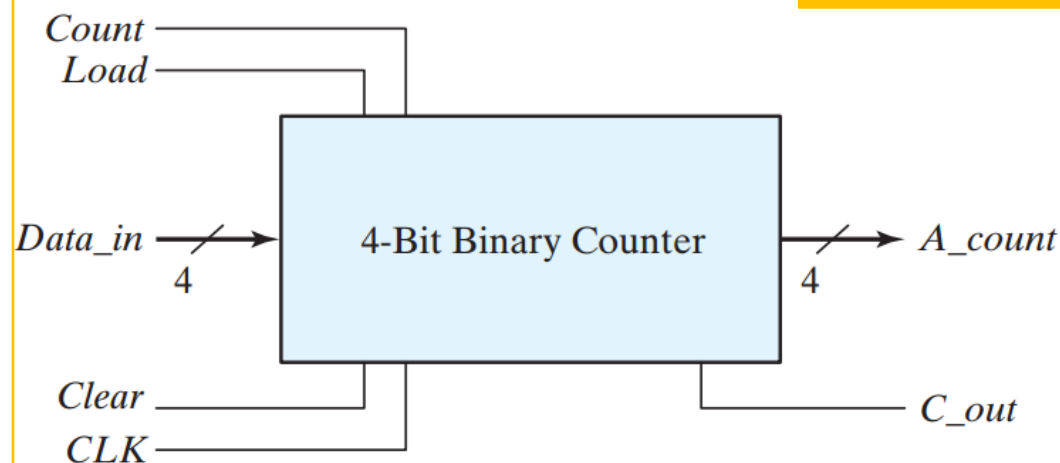
$$T_{Q_8} = Q_8 Q_1 + Q_4 Q_2 Q_1$$

$$y = Q_8 Q_1$$



- Parallel load is used to **start** counting from a **defined** state (does not always have to be ZERO)
- Clear input is used to **clear** (reset to 0) the counter **asynchronously**
- Carry bit is set when the count **reaches all 1's (saturate)**, to give indication to a **following** stage to **start** the count

4-bit binary counter with parallel load



Block Diagram

Clear	CLK	Load	Count	Function
0	X	X	X	Clear to 0
1	↑	1	X	Load inputs
1	↑	0	1	Count next binary state
1	↑	0	0	No change

Function Table

3 Control Inputs (With the following Priority: **High** → **Low**: Clear → Load → Count)

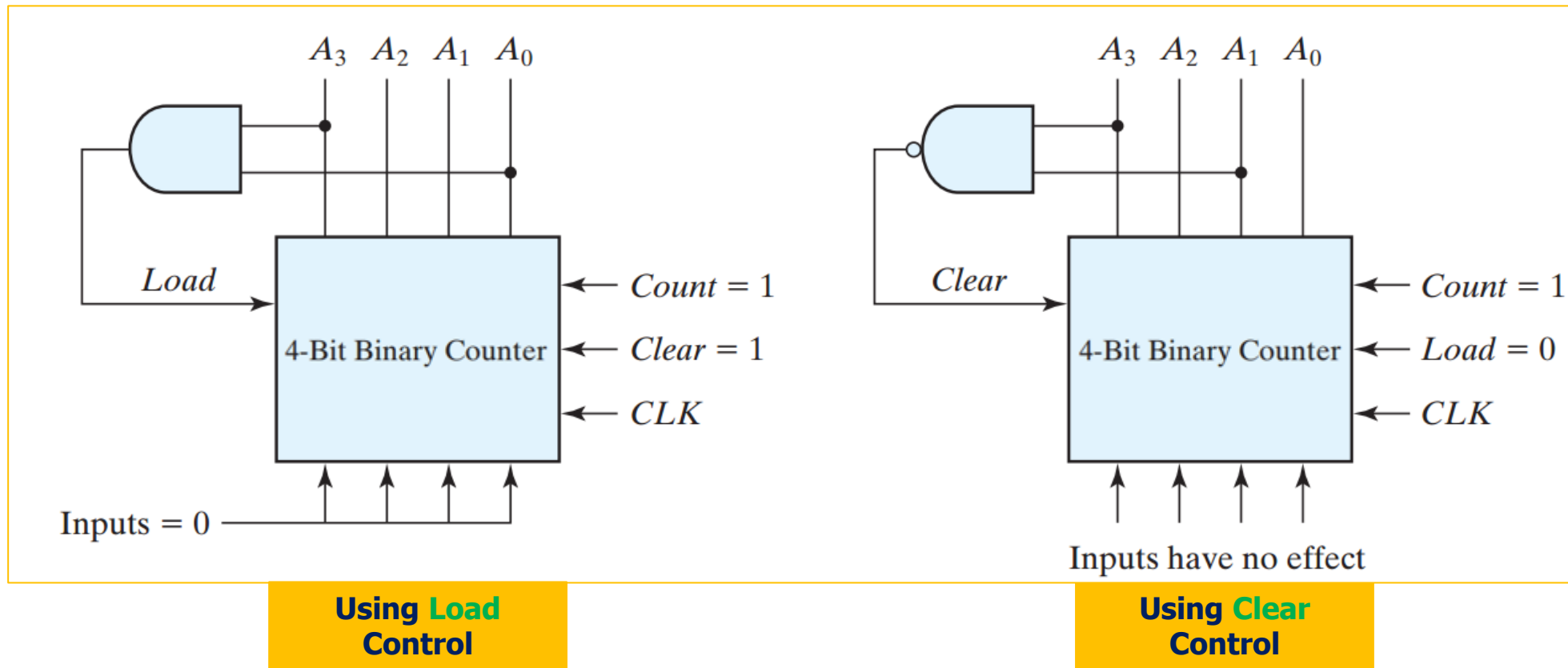
1 Clock input

n-bits data/load input, **n-bits count value**/output (here n=4)

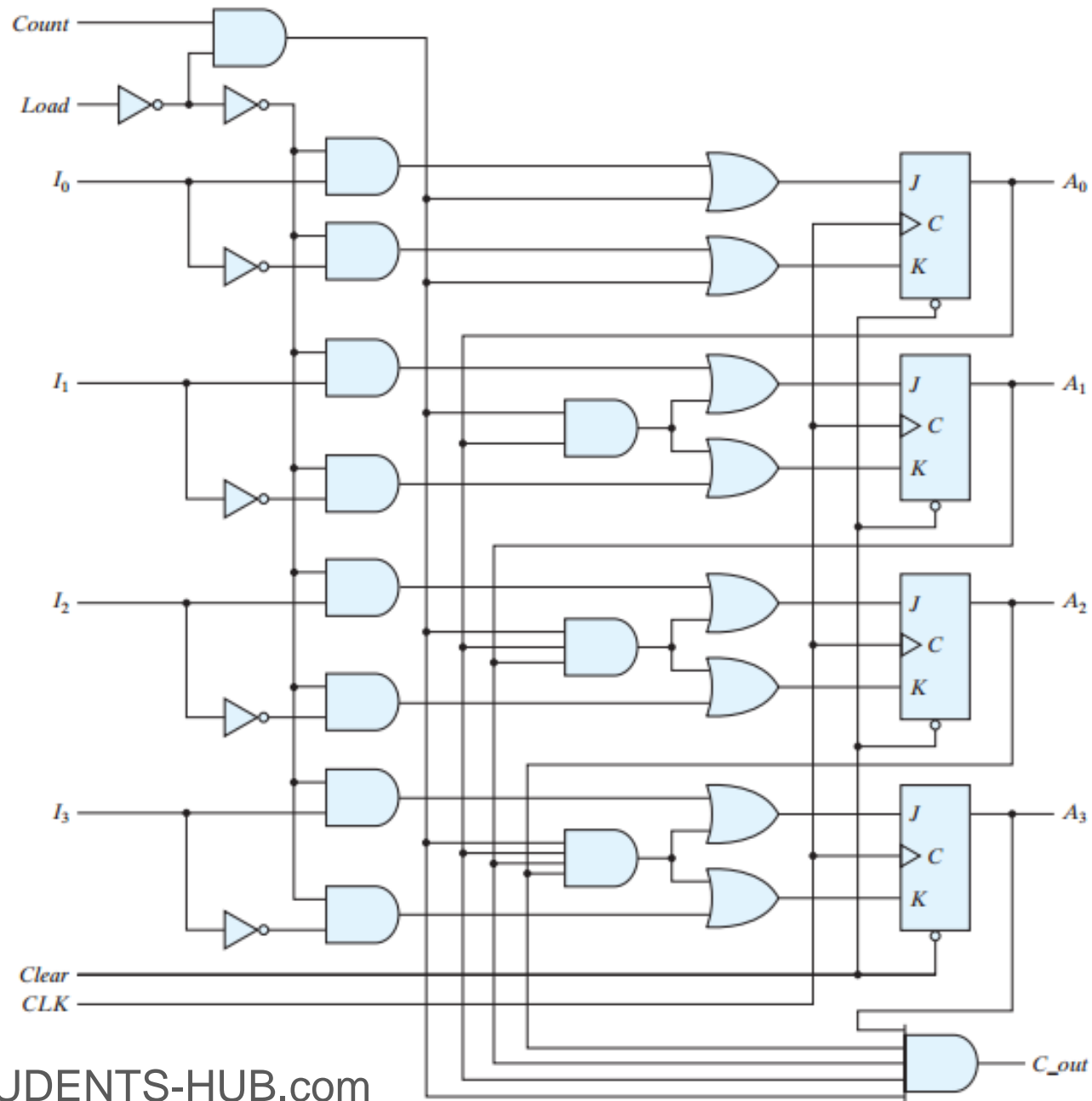
1-bit carry output

🌀 A counter with a parallel load can be used to **generate any** desired **count sequence**

Two ways to achieve a BCD counter using a counter with parallel load



Notice: **Clear** has the **highest** priority



Circuit Diagram
4-bit binary counter with parallel load



- ☉ Counters can be designed to generate **any** desired **sequence** of states
- ☉ The sequence may follow the **binary** count or may be any other **arbitrary** sequence
- ☉ Counters are used to generate **timing signals** to control the sequence of operations in a digital system
- ☉ Counters can also be constructed **by** means of **shift registers**
- ☉ We will briefly discuss:
 - ☐ Counter with **Unused** States
 - ☐ **Ring** Counter
 - ☐ **Johnson** Counter

Example: Design a counter that repeats (0,1,2,4,5,6) using **JK** flip-flops

2 Unused States (**011,111**). These shall be considered as **Don't Care** in the state table

Present State			Next State			Flip-Flop Inputs					
A	B	C	A	B	C	J_A	K_A	J_B	K_B	J_C	K_C
0	0	0	0	0	1	0	X	0	X	1	X
0	0	1	0	1	0	0	X	1	X	X	1
0	1	0	1	0	0	1	X	X	1	0	X
1	0	0	1	0	1	X	0	0	X	1	X
1	0	1	1	1	0	X	0	1	X	X	1
1	1	0	0	0	0	X	1	X	1	0	X

Counter State Table

$$J_A = B \quad K_A = B$$

$$J_B = C \quad K_B = 1$$

$$J_C = B' \quad K_C = 1$$

Simplified JK equations
(using k-map)

K-map for J_A (as an example)

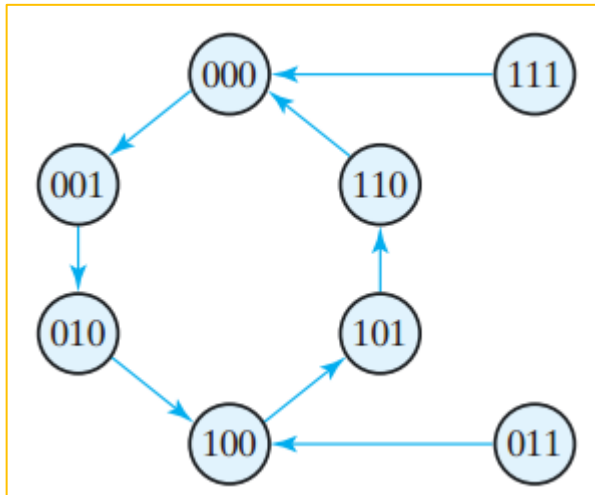
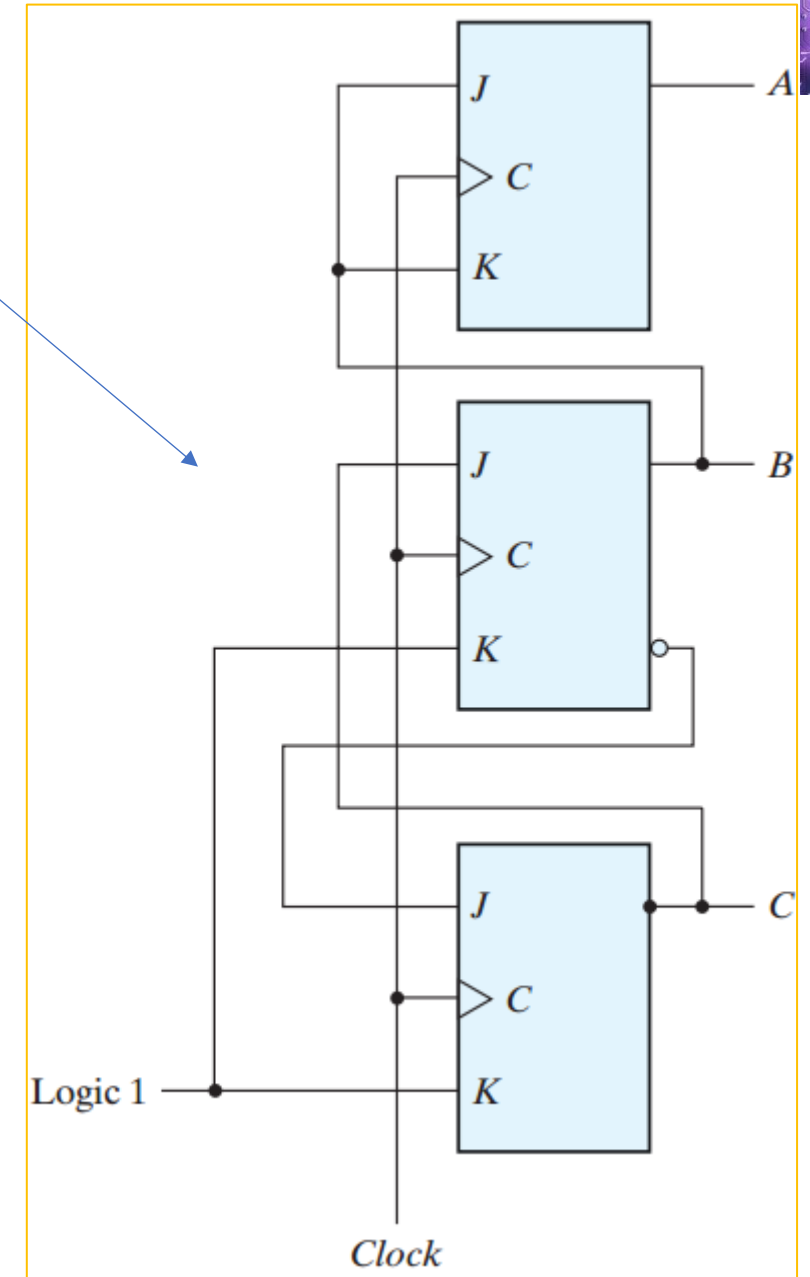
		BC			
		00	01	11	10
A	0			x	1
	1	x	x	x	x

X: Original from JK Excitation Table
x: From Don't Care of States: 3,7

Example Continue:

Since there are **two** unused states, we **analyze** the **circuit diagram** to determine their **effect**:

- If the circuit happens to be in state **011** because of an error signal, the circuit goes to state **100** after the application of a clock pulse
 - $(B=1 \rightarrow J_A=K_A=1, J_C=0) \rightarrow A$ will be complemented ($1 \rightarrow 0$), $C=0$
 - $(C=1 \rightarrow J_B=1) \rightarrow B$ will be complemented ($0 \rightarrow 1$)
- If the circuit happens to be in state **111** because of an error signal, the circuit goes to state **000** after the application of a clock pulse
 - Same as above

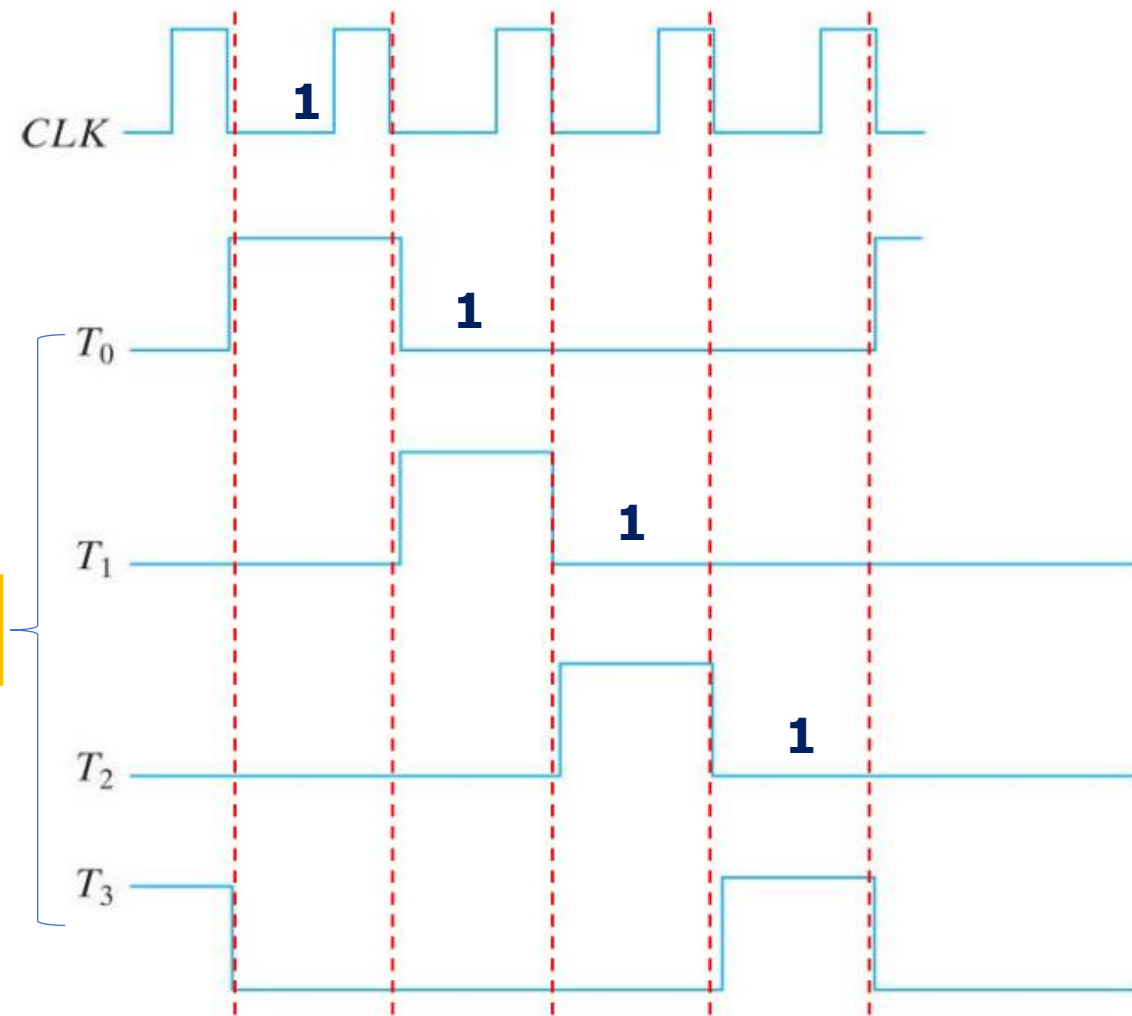
Self-Correcting Counter**Counter Circuit Diagram**



- As shown in the previous example, if the circuit ever goes to one of the **unused** states because of an error, the **next count pulse transfers it to one of the valid states** and the circuit **continues** to count **correctly**. This kind of counters is called **Self-Correcting Counter**
- In a **self-correcting counter**, if the counter happens to be in one of the **unused** states, it **eventually** reaches the **normal count sequence** after **one or more** clock **pulses**
- An alternative design could use **additional** logic to **direct/enforce every unused** state to a **specific next** state

- ☉ In digital systems, we sometimes need a control signal that will **trigger** every $2^n - 1$ cycles
- ☉ Such circuit is called a **ring counter**
- ☉ A **ring counter** is a circular shift register with **only one** flip-flop being **set** at **any** particular **time**; **all** others are **cleared**
- ☉ The **single** bit (which = 1) is **shifted** from one flip-flop to the next to produce the sequence of timing signals

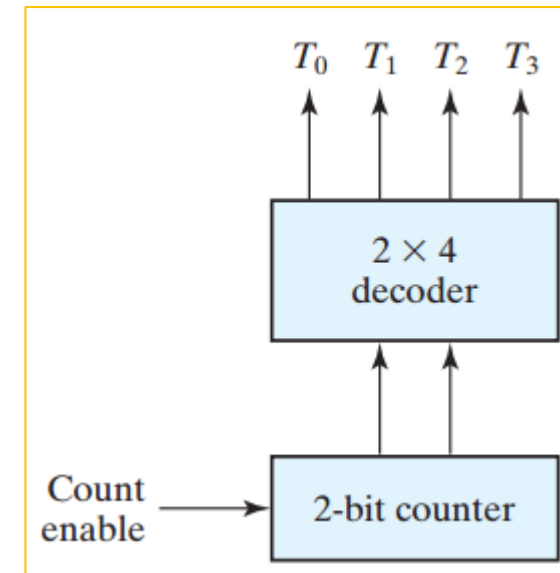
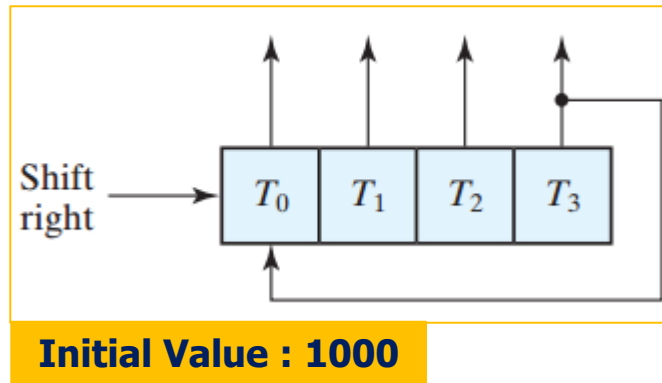
Four different timing signals generated using the same CLK



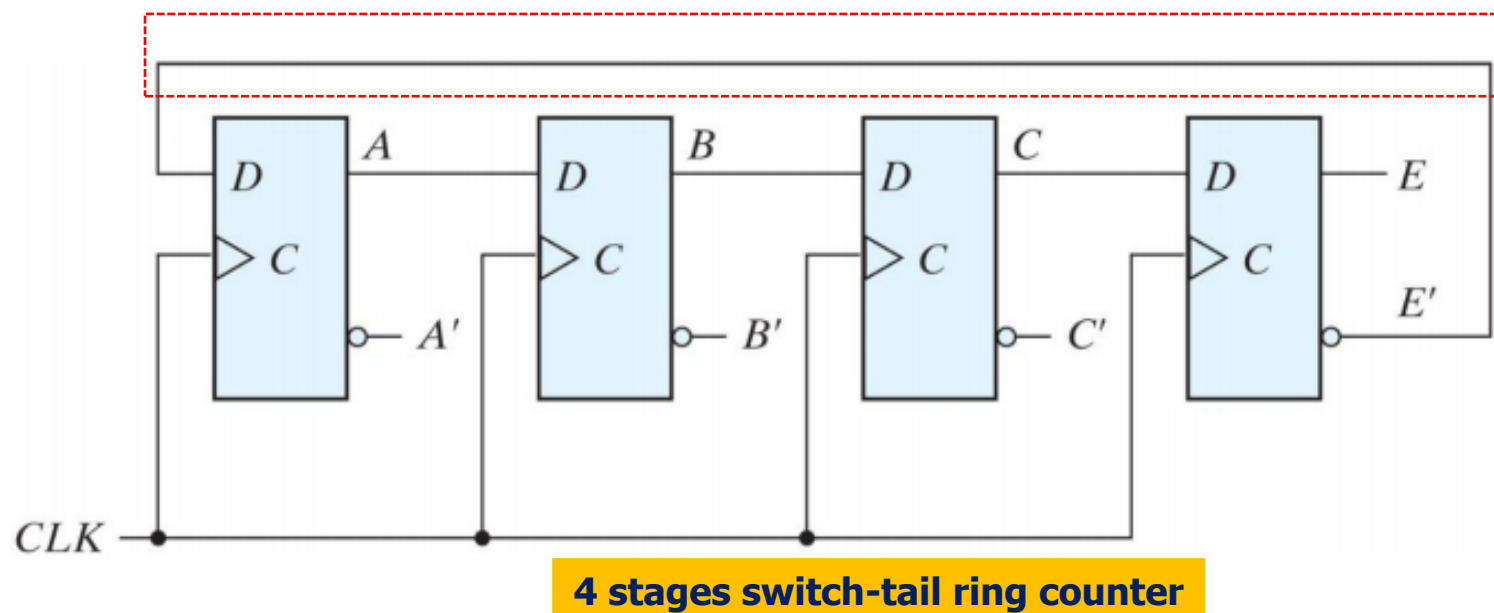
$n=4$ Each signal ($T_0 \rightarrow T_3$) is **triggered** (set to 1) after every **3** cycles ($2^2 - 1 = 3$)

☯ To generate 2^n timing signals, we can use:

1. A **shift register** with 2^n **flip-flops**
2. An **n-bit** binary **counter** together with an **n-to- 2^n decoder**



- ⌚ **k-bit ring counter circulates a single bit among the flip-flops to provide k distinguishable states**
- ⌚ The **number** of states can be **doubled** if the shift register is **connected** as a **switch-tail ring counter**
- ⌚ A **switch-tail ring counter** is a **circular** shift register with the **complemented output** of the **last** flip-flop **connected** to the **input** of the **first** flip-flop



- ⌘ A **Johnson** counter is a **k-bit** switch-tail ring counter with **2k** decoding gates to **provide** outputs for **2k timing signals**
- ⌘ The **eight AND** gates listed in the table below, when **connected** to the circuit, will **complete** the construction of the Johnson counter
- ⌘ **Each gate** is **enabled** during **one particular state sequence**
 - ★ The **outputs** of the gates **generate eight timing** signals in succession

Sequence number	Flip-flop outputs				AND gate required for output
	A	B	C	E	
1	0	0	0	0	$A'E'$
2	1	0	0	0	AB'
3	1	1	0	0	BC'
4	1	1	1	0	CE'
5	1	1	1	1	AE
6	0	1	1	1	$A'B$
7	0	0	1	1	$B'C$
8	0	0	0	1	$C'E$

**K=4 → 2K = 8 →
8 States/Timing Signals**

Only one gate will produce 1 at any given state

4-bit Johnson Counter



- ☉ One **disadvantage** of Johnson counter is that if it finds itself in an unused state, it will keep in moving from one invalid state to another and **never find its way to a valid state (NOT Self-Correcting)**
- ☉ This can be corrected by modifying the circuit to avoid this undesirable condition
- ☉ One possible **correction**: $D_C = (A + C)B$ (instead of $D_C = B$)
- ☉ Johnson counters can be constructed for any number of timing sequences.
 1. The number of **flip-flops** needed is **one-half** the number of **timing** signals.
 2. The number of **decoding gates** is **equal** to the number of **timing** signals, and only **two-input** gates are needed.