Example
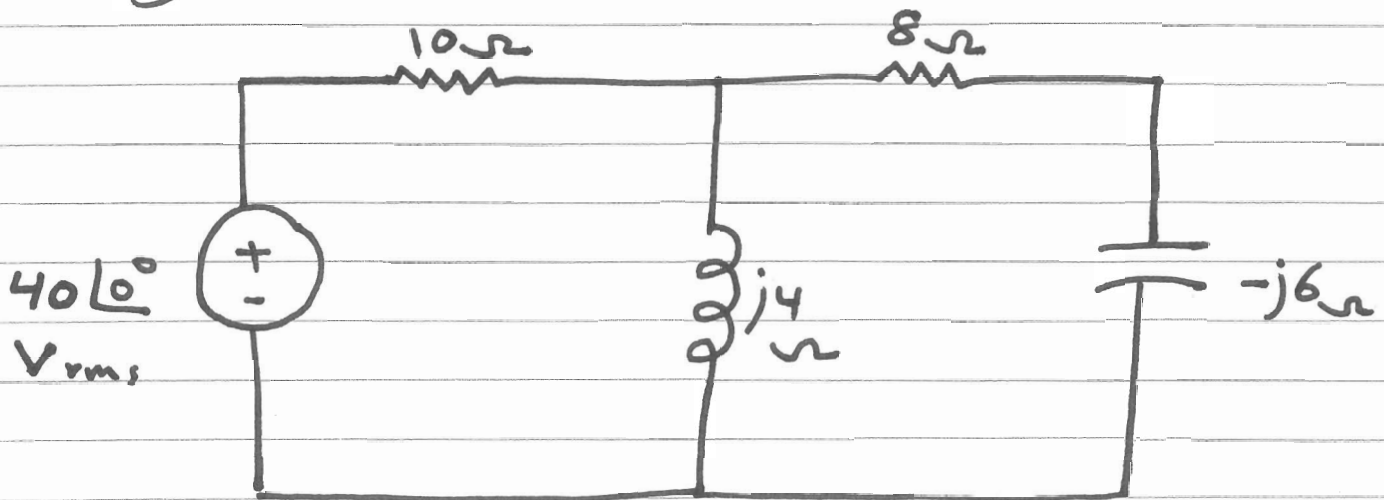
Calculate the power factor seen by the source and the average power supplied by the source.

$$Z = 10 + j4 \parallel (8 - j6)$$

$$Z = 12.69 \; \underline{|20.62°} \quad \Omega$$

$$\vec{I}_s = \frac{40 \underline{|0°}}{Z} = 3.152 \; \underline{|-20.62°} \; A_{rms}$$

$$\Theta_r = 0° \quad , \quad \Phi_i = -20.62°$$

$$PF = \cos(\Theta_r - \Phi_i)$$

$$PF = \cos(20.62°)$$

$$PF = 0.936 \; \text{Lagging}$$

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The average power supply by the source is equal to the average power absorbed by the Circuit

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$$P_{av} = V_{rms} \, I_{rms} \, \cos(\theta_v - \phi_i)$$

$V_{rms} = 40 \, V \, rms$

$I_{rms} = 3.152 \, A \, rms$

$\theta_v = 0°$
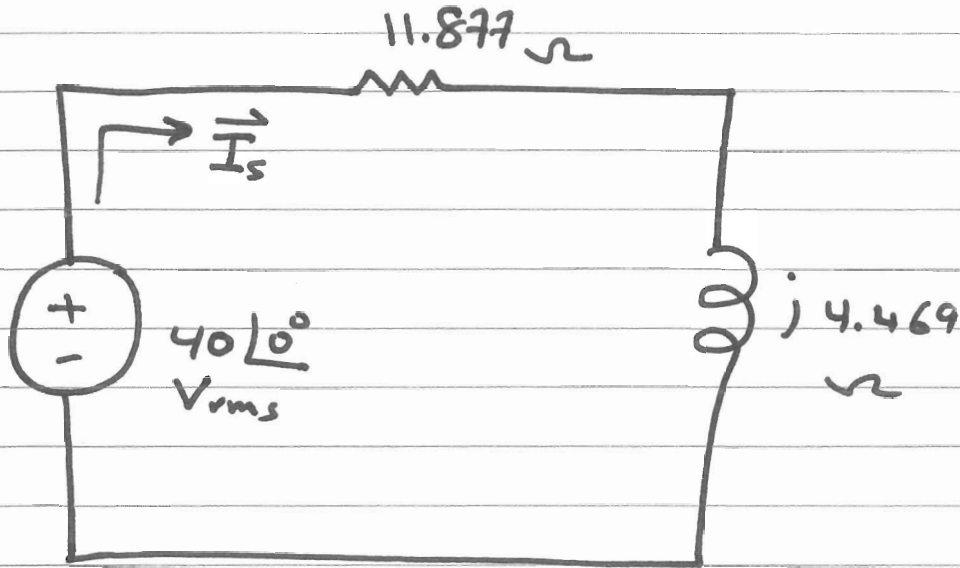
$\phi_i = -20.62°$

$$\therefore P_{av} = (40)(3.152) \cos(0 - (-20.62°))$$

$$\therefore P_{av} = 118 \, Watt$$

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$$Z = 12.69 \,\underline{/20.62°} \ \Omega$$

$$Z = 11.877 + j \, 4.469 \ \Omega$$

$$\therefore \ P_{av} = I_{vms}^2 \, R$$

$$P_{av} = (3.152)^2 \, (11.877)$$

$$P_{av} = 118 \ W$$

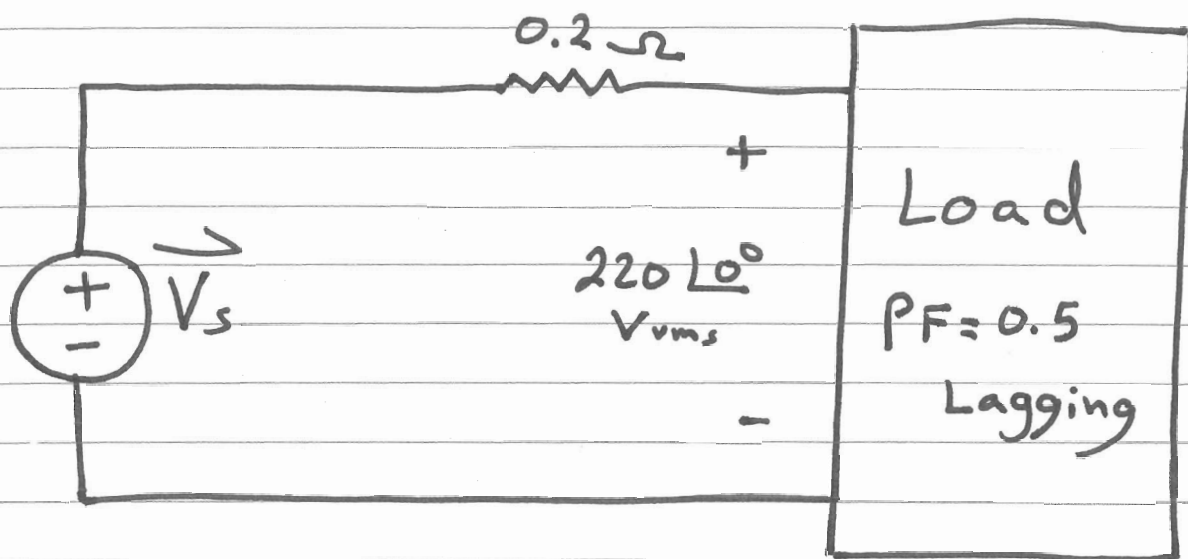also $P_{av} = P_{av}_{10\Omega} + P_{av}_{8\Omega} + P_{av}_{-j6} + P_{av}_{j4}$

$$P_{av} = P_{av}_{10\Omega} + P_{av}_{8\Omega}$$

Example

An industrial Load Consumer 11KW at 0.5 PF Lagging from a 220V rms Line. The transmission Line resistance from the power Company to the plant is 0.2 Ω.

1) Determine the average power that must be Supplied by the power Company

2) Repeat ① if the power factor is Changed to unity.
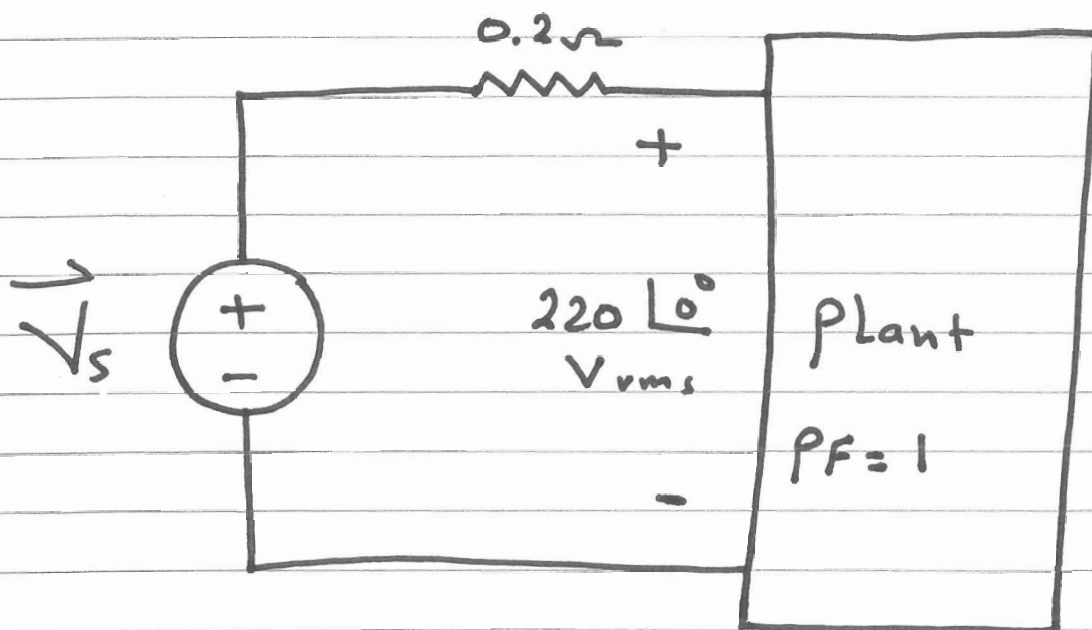
$$P_{av} = V_{rms} \cdot I_{rms} \cdot PF$$
$$\quad L$$

$$\therefore I_{rms} = \frac{P_{av}}{V_{rms} \cdot PF}$$

$$I_{rms} = \frac{11\,KW}{(220)(0.5)} = 100\,A_{rms}$$

$$P_{Loss} = (I_{rms}^{2}) \cdot (0.2) = 2\,KW$$

$$\therefore P_{av} = P_{av} + P_{av}$$
$$\quad sup \qquad L \qquad Loss$$

$$P_{av} = 13\,KW$$
$$\quad sup$$

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$$P_{av_{load}} = V_{rms} \cdot I_{rms} \cdot PF$$

$$\therefore I_{rms} = \frac{P_{av}}{V_{rms} \cdot PF} = 50 \, A_{rms}$$

$$P_{Loss} = I_{rms}^2 \cdot R = (50)^2 \cdot (0.2) = 0.5 \, Kw$$

$$\therefore P_{av_{sup}} = 0.5 \, Kw + 11 \, Kw$$

$$P_{av_{sup}} = 11.5 \, Kw$$

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Example

Find the power factor of the two loads
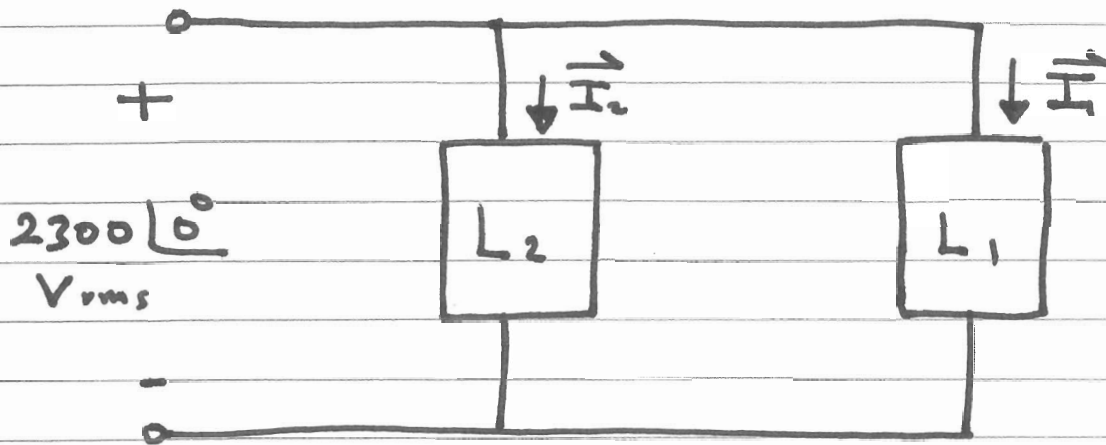
Load 1 : 10 KW , 0.9 Lagging PF

Load 2 : 5 KW , 0.95 Leading PF

$$\vec{I_1} = \frac{10,000}{(2300)(0.9)} \; \underline{/ -\cos^{-1} 0.9}$$

$$\vec{I_1} = 4.83 \; \underline{/ -25.84°} \quad A_{vms}$$

$$\vec{I_2} = \frac{5000}{(2300)(0.95)} \; \underline{/ +\cos^{-1} 0.95}$$

$$\vec{I_2} = 2.288 \; \underline{/ 18.195°} \quad A_{vms}$$

$$\vec{I_s} = \vec{I_1} + \vec{I_2} = 6.78 \; \underline{/ -12°} \quad A_{vms}$$

$$PF = \cos(\theta_v - \theta_i) = \cos(0 - (-12°))$$

$$PF = 0.978 \; Lagging$$

Complex Power
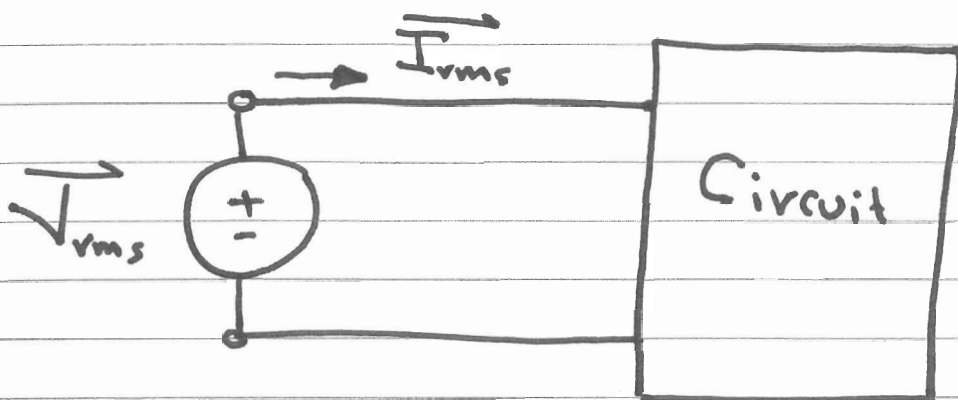
$$\vec{V}_{rms} = V_{rms} \angle \theta_v$$

$$\vec{I}_{rms} = I_{rms} \angle \phi_i$$

$$\vec{S} \equiv \text{Complex Power}$$

$$\vec{S} = \vec{V}_{rms} \cdot \vec{I}_{rms}^{*}$$

$$\vec{S} = V_{rms} I_{rms} \angle \theta_v - \phi_i$$

$$\vec{S} = V_{rms} I_{rms} \cos(\theta_v - \phi_i)$$
$$\qquad\qquad + j V_{rms} I_{rms} \sin(\theta_v - \phi_i)$$

$$\vec{S} = P_{av} + j Q$$

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$$\vec{S} = P_{av} + j Q$$

$P_{av} \equiv$ Average power in Watt

$Q \equiv$ Reactive power in VAR

$\therefore P_{av} = \text{Real} \left[\vec{S} \right]$

$Q = \text{Imajinary} \left[\vec{S} \right]$

$Q = V_{rms} I_{rms} \sin (\Theta v - \phi_i)$

1) For pure resistance

$\Theta v - \phi_i = 0$

$\therefore Q_R = 0$

2) For pure inductance

$\Theta v - \phi_i = + 90°$

$\therefore Q_L = V_{rms} I_{rms}$

$V_{rms} = WL I_{rms}$

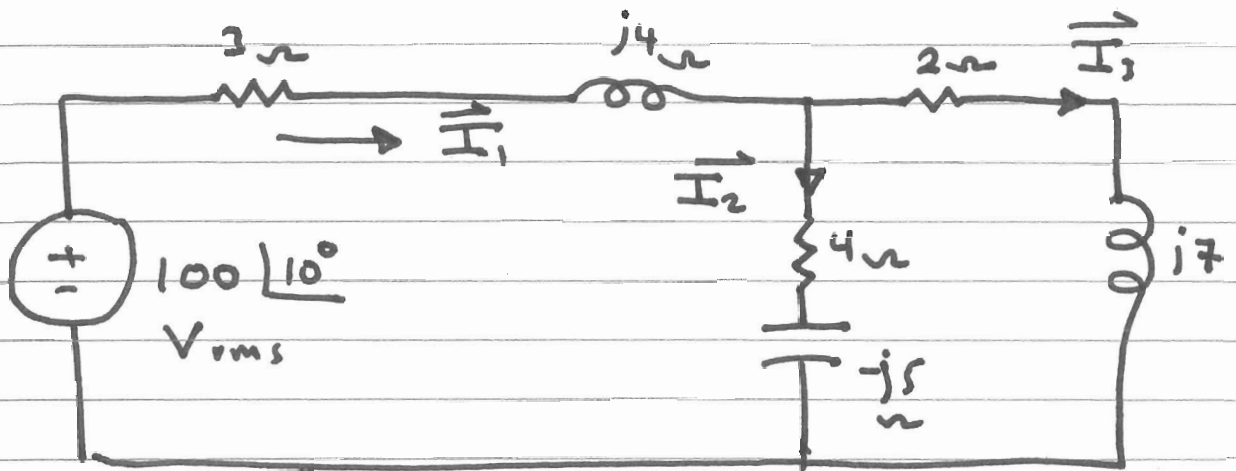$\therefore Q_L = WL I_{rms}^2 = \dfrac{V_{rms}^2}{WL}$

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3) For pure Capacitance

$$\theta_v - \theta_i = -90°$$

$$Q_c = - V_{rms} I_{rms}$$

$$I_{rms} = WC V_{rms}$$

$$\therefore Q_c = - \frac{I_{rms}^2}{WC} = - WC V_{rms}^2$$

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What are the VARS Consumed by the Circuit

$$Q = V_{rms} \, I_{rms} \, \sin(\theta_v - \phi_i)$$

$$\vec{I_1} = \frac{\vec{V_s}}{Z}$$

$$Z = (2+j7) \| (4-j5) + 3 + j4$$

$$Z = 10.35 + j4.55 = 11.3 \angle 27.7°$$

$$\therefore \vec{I_1} = \frac{100 \angle 10°}{11.3 \angle 27.7°} = 8.84 \angle -13.7° \; A_{rms}$$

$$\therefore Q = (100)(8.84) \sin\left(10 - (-13.7°)\right)$$

$$\therefore Q = 355 \; VARS$$

$$I_2 = 10.2 \; A_{rms}$$

$$I_3 = 8.95 \; A_{rms}$$

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$$P_{av} = V_{rms} \, I_{rms} \, \cos(\theta_v - \phi_i)$$

$$Q = V_{rms} \, I_{rms} \, \sin(\theta_v - \phi_i)$$

$$\frac{Q}{P_{av}} = \tan(\theta_v - \phi_i)$$

$$\therefore Q = P_{av} \tan(\theta_v - \phi_i)$$

$$Q = P_{av} \tan\left[\cos^{-1}(PF)\right]$$

$$\vec{S} = P_{av} + j\,Q$$

$$\vec{S} = \sqrt{P_{av}^2 + Q^2} \; \left\lfloor \tan^{-1}\frac{Q}{P_{av}} \right.$$

$$\vec{S} = V_{rms} \, I_{rms} \; \left\lfloor \theta_v - \phi_i \right.$$

$$\therefore P_a = |S| = \sqrt{P_{av}^2 + Q^2}$$

$$\theta_v - \phi_i = \tan^{-1}\frac{Q}{P_{av}}$$

$$\theta_v - \theta_i = \tan^{-1} \frac{Q}{P_{av}}$$

To increase P.F, we need to decrease Q

$\therefore$ For inductive circuit, we

add a Capacitor in parallel

to increase the power factor
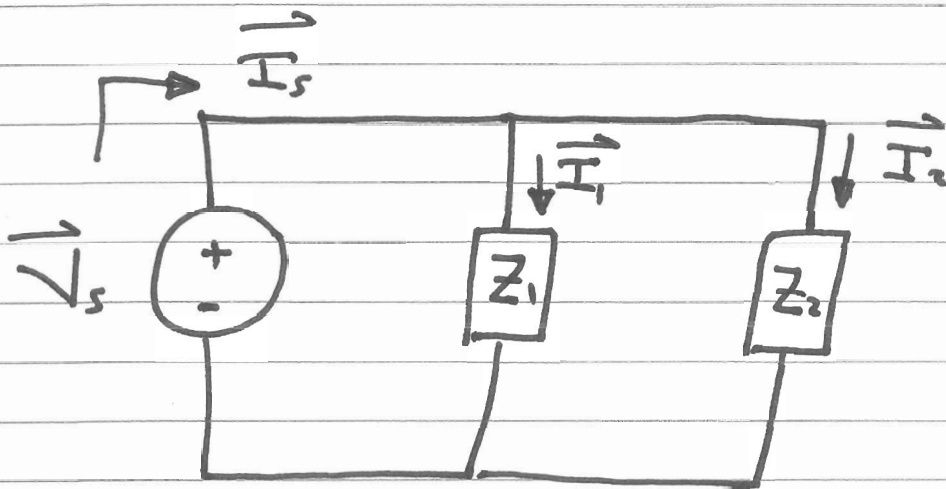
$$P_{av_T} = P_{av_1} + P_{av_2} + \text{---} P_{av_n}$$

$$Q_T = Q_1 + Q_2 + \text{---} Q_n$$

$$\vec{S}_T = P_{av_T} + j Q_T$$

$$\vec{S}_T = \vec{S}_1 + \vec{S}_2 + \text{-----} \vec{S}_n$$

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Conservation of Ac Power

The Complex, real, and reactive powers of the Sources equal the respective Sum of the Complex, real, and reactive powers of the individual Loads.
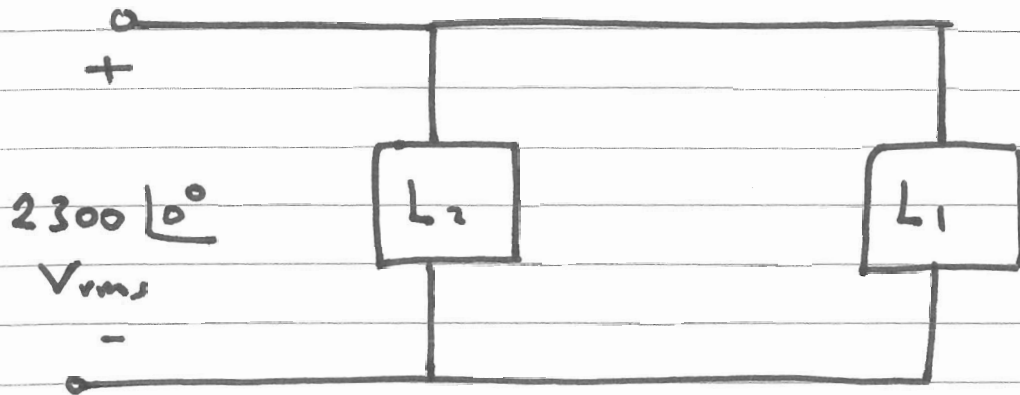

$$\vec{S}_{Source} = \vec{V}_s \cdot \vec{I}_s^*$$

$$\vec{S}_{Source} = \vec{V}_s \cdot \left(\vec{I}_1^* + \vec{I}_2^* \right)$$

$$\vec{S}_{Source} = \vec{V}_s \cdot \vec{I}_1^* + \vec{V}_s \cdot \vec{I}_2^*$$

$$\vec{S}_{Source} = \vec{S}_1 + \vec{S}_2$$

The Same results Can be obtained for a series Connection.

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Find the power factor of the two Loads

Load 1 :   10 KW , 0.9 Lagging P.F

Load 2 :   5 KW , 0.95 Leading P.F

$$\vec{S_1} = P_{av_1} + j\,Q_1$$

$$Q_1 = P_{av_1} \tan\left[\cos^{-1}(P.F_1)\right] = 4843 \text{ VARS}$$

$$\therefore \vec{S_1} = 10000 + j\,4843$$

$$\vec{S_2} = P_{av_2} + j\,Q_2$$

$$Q_2 = -P_{av_2} \tan\left[\cos^{-1}(P.F_2)\right]$$

$$Q_2 = -1643 \text{ VARS}$$

$$\therefore \vec{S_2} = 5000 - j\,1643$$

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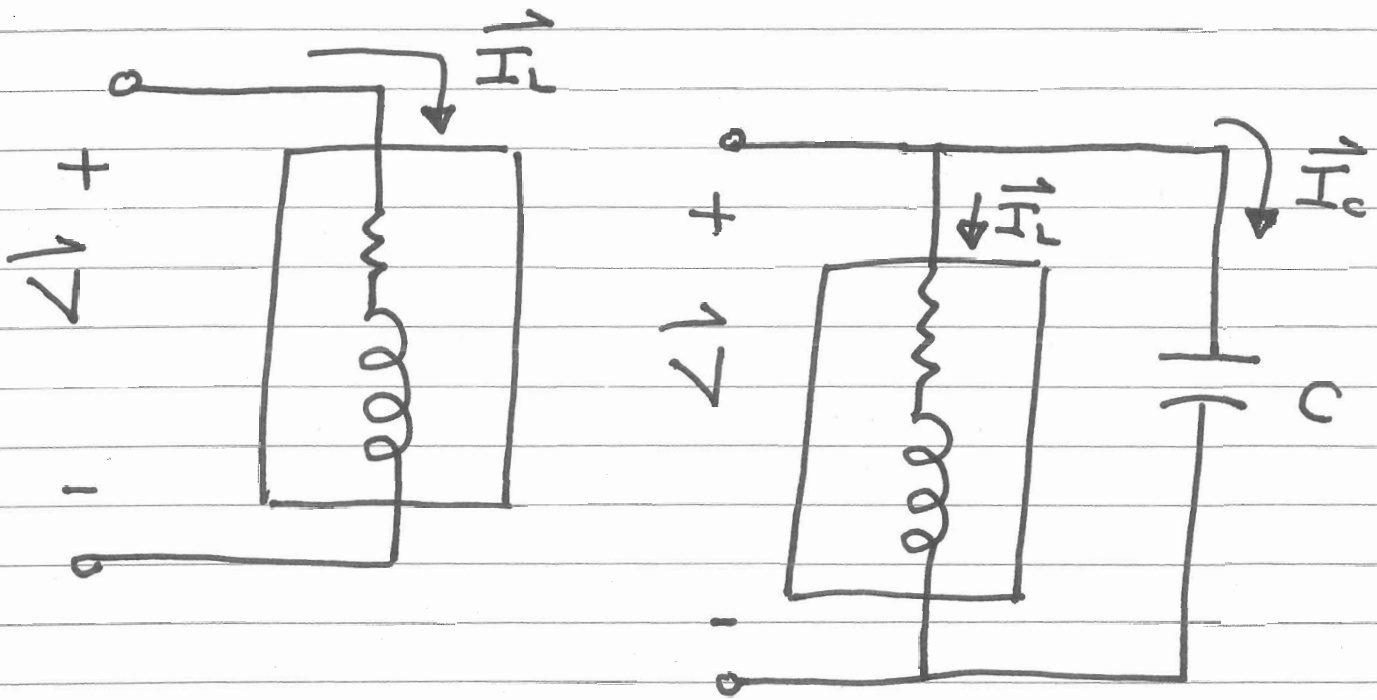$$\vec{S}_T = \vec{S}_1 + \vec{S}_2$$

$$\vec{S}_T = 15000 + j\,3200$$

$$\vec{S}_T = 15337.5 \;\underline{/12.02^\circ} \qquad VA$$

$$P.F = Cos(12.02)$$

$$P.F = 0.978 \; Lagging$$

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Power Factor Correction

Power factor Correction is the process of increasing the power factor without altering the Voltage or Current to the original Load.

Power factor Correction is necessary for Economic Reason.

$$PF = \cos(\theta_v - \phi_i)$$

For R

$$P.F = 1$$
$$Q_R = 0$$

∴ To improve the power factor we must decrease the Reactive power

∴ For inductive Circuit, we add a Capacitor in parallel to the load.

$$Q_c = Q_{Final} - Q_{init}$$

$$C = -\frac{Q_c}{\omega V_{rms}^2}$$

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Example

A certain industrial plant Consumer
1 MW at 0.7 Lagging power factor
and a 2300 V rms.
What is the minimum Capacitor required
to improve the power factor to
0.9 Lagging. $\omega = 377 \ v/s$

$Q_{ini} = P_{av} \tan \left[\cos^{-1} PF_1 \right]$

$Q_{ini} = 1M \tan \left[\cos^{-1} 0.7 \right]$

$Q_{ini} = 1.02 \ MVARS$

$Q_{Fin} = P_{av} \tan \left[\cos^{-1} PF_2 \right]$

$Q_{Fin} = P_{av} \tan \left[\cos^{-1} 0.9 \right]$

$Q_{Fin} = 0.484 \ MVARS$
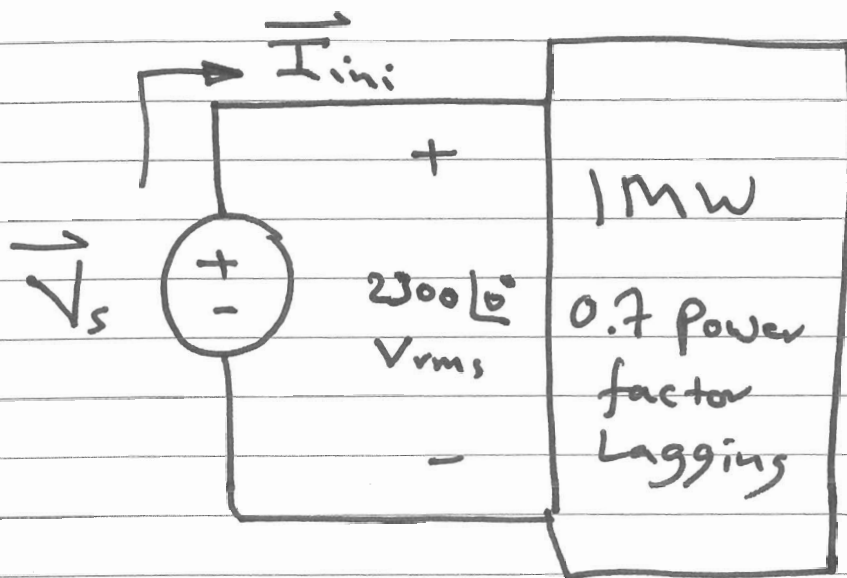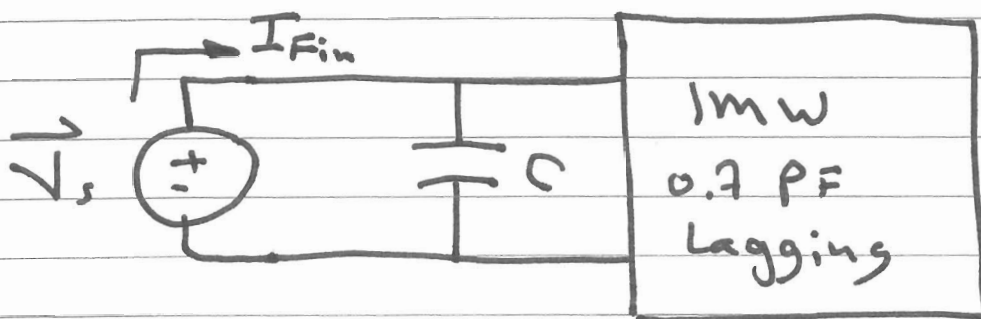
$Q_c = Q_{Fin} - Q_{ini}$

$Q_c = - 0.536 \ MVARS$

$$Q_c = -\frac{V_{rms}^2}{X_c}$$

$$Q_c = -wc\, V_{rms}^2$$

$$\therefore\ C = -\frac{Q_c}{w\, V_{rms}^2} = 269\, MF$$
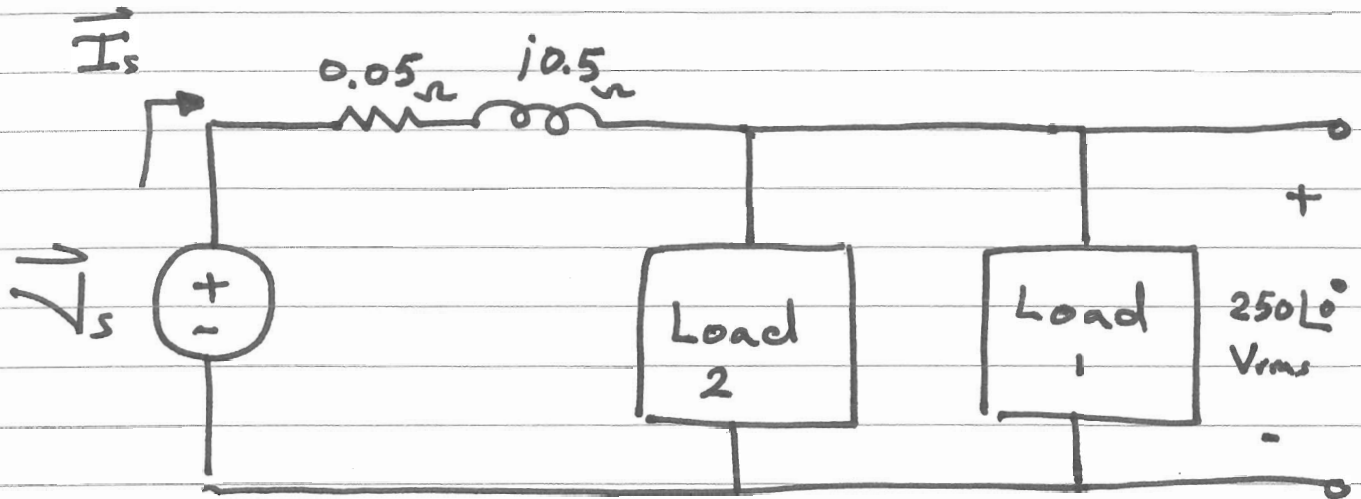

$$I_{ini} = \frac{P_{av}}{(V_{rms})(PF_1)} = 621\ A\,vms$$

$$I_{Fin} = \frac{P_{av}}{(V_{rms})(PF_2)} = 483\ A\,vms$$

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Example

Load 1 : 8 KW ; 0.8 PF Leading

Load 2 : 20 KVA ; 0.6 PF Lagging

1) Determine the power factor of the two Loads in parallel

2) Determine the apparent power required to supply the loads ; the magnitude of the current I_s ; the average power loss in the transmission Line

3) Compute the value of the Capacitor that would Correct the power factor to 1 if

placed in parallel with the two loads

$$\omega = 377 \text{ r/s}$$

4) Repeat step 2

_____

Load 1 : 8 KW ; 0.8 PF_1 leading

Load 2 : 20 KVA ; 0.6 PF_2 Lagging

P_{av_1} = 8000 W

$\therefore$ Q_1 = $-P_{av_1} \tan\left[Cos^{-1}(PF_1)\right]$ = -6000 VARS

$\therefore$ $\vec{S_1}$ = $P_{av_1} + j\, Q_1$

$\vec{S_1}$ = $8000 - j\, 6000$ VA

P_{a_2} = 20000 VA ; PF_2 = 0.6 Lagging

$\therefore$ P_{av_2} = $P_a \cdot PF_2$ = 12000 W

Q_2 = $P_{av_2} \tan\left[Cos^{-1}(PF_2)\right]$ = $+16000$ VARS

$\therefore$ $\vec{S_2}$ = $P_{av_2} + j\, Q_2$

$\vec{S_2}$ = $12000 + j\, 16000$ VA

$\vec{S_T}_L$ = $\vec{S_1} + \vec{S_2}$

$\vec{S_T}_L$ = $20000 + j\, 10000$ VA

$\vec{S_T}_L$ = $22360 \underline{/26.565°}$ VA

$\therefore$ PF = $Cos(26.565)$ = 0.8944 Lagging

$$\vec{S_{LT}} = \vec{V}_{rms} \cdot \vec{I_s}^*$$

$$\therefore \vec{I_s}^* = \frac{\vec{S_{LT}}}{\vec{V}_{rms}} = \frac{22360 \, \lfloor 26.565°}{250}$$

$$\therefore \vec{I_s} = 89.44 \, \lfloor -26.565° \quad A \, rms$$

Since $\vec{S_{LT}} = 22360 \, \lfloor 26.565°$

$$\therefore P_a = |\vec{S_{LT}}| = 22360 \quad \sqrt{A}$$

$$P_{av \atop Loss} = |\vec{I_s}|^2 \cdot (0.05)$$

$$= 400 \, W$$

3) Since $\vec{S_{LT}} = 20000 + j \, 10000$

$$\therefore Q_{ini} = 10000 \, VARS$$

$$Q_{Fin} = 0$$

$$\therefore Q_c = Q_{Fin} - Q_{ini} = -10000 \, VARS$$

$$\therefore C = \frac{-Q_c}{W \, V_{rms}^2} = 424.4 \, \mu F$$

4) Since $Q_{Fin} = 0$

$\therefore \vec{S_F} = P_{av} = 20000 \text{ VA}$

$\therefore P_a = P_{av} = 20000 \text{ VA}$

$\therefore \vec{S_F} = 20000 \underline{|0^\circ} \text{ VA}$

$\vec{S_F} = \vec{V_{rms}} \cdot \vec{I_s^*}$

$\therefore \vec{I_s^*} = \dfrac{20000\underline{|0^\circ}}{250\underline{|0^\circ}} = 80\underline{|0^\circ} \text{ Arms}$

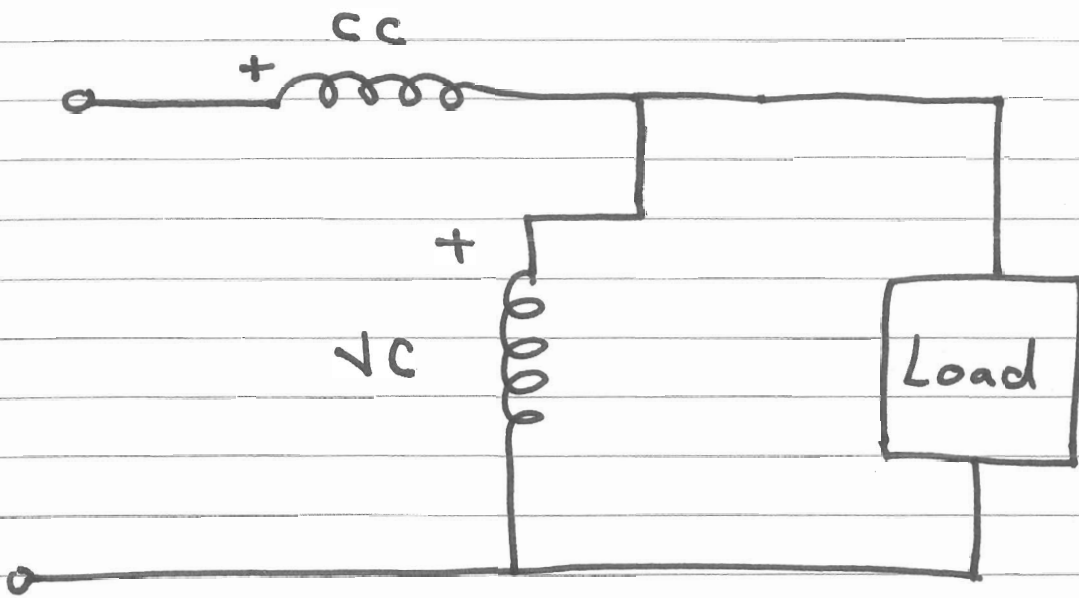$\therefore \vec{I_s} = 80\underline{|0^\circ} \text{ A rms}$

$P_{Loss} = |I_s|^2 \cdot (0.05)$

$P_{Loss} = 320 \text{ W}$

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Power Measurement

Wattmeter is the instrument for measuring the average power
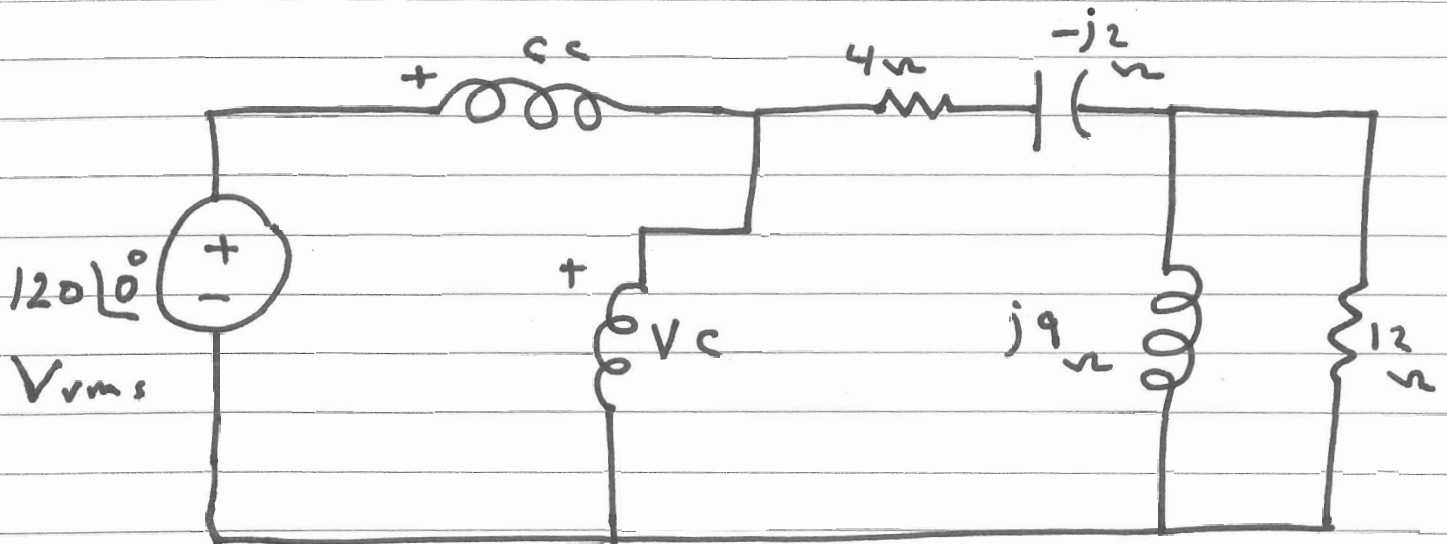
Two Coils are used, the high impedance Voltage Coil and the Low impedance Current Coil.

$$P = V_{rms} I_{rms} \cos(\theta_v - \phi_i)$$

Example

Find the wattmeter reading

$$Z = 4 - j2 + \left(j9 \| 12 \right)$$

$$Z = 9.13 \underline{/24.32°}$$

$$\vec{I} = \frac{120 \underline{/0°}}{9.13 \underline{/24.32°}} = 13.14 \underline{/-24.32°} \; A_{vms}$$

$$P = (120)(13.14) \cos \left(0 + 24.32° \right)$$

$$P = 1436.9 \; w$$

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