Exp Find Maclaurin Series for MS

MS = TS at $x=0$

① $f(x) = \cos x$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \cdots$$

$f(x) = \cos x \Rightarrow f(0) = 1$ $\quad f' = -\sin x \Rightarrow f'(0) = 0$

$f'' = -\cos x \Rightarrow f''(0) = -1$ $\quad f''' = \sin x \Rightarrow f'''(0) = 0$

$f^{(4)} = \cos x \Rightarrow f^{(4)}(0) = 1$ $\quad f^{(5)} = -\sin x \Rightarrow f^{(5)}(0) = 0$

$\sin x$ $\quad 0$

$-\cos x$ $\quad -1$ $\quad 0$

$-\sin x$

$\cos x$ $\quad 1$ $\quad 0$

$-\ \cdots$

$$= 1 + 0 - \frac{1}{2!}x^2 + 0 + \frac{x^4}{4!} + 0 - \frac{x^6}{6!} + \cdots$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$f(x) = \cos x$

Approximation

Poly. of degree 0

$P_0(x) = 1$

$P_2(x) = 1 - \frac{x^2}{2!}$ $\quad$ " " " 2

$P_4(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$ $\quad$ " " " 4

$$\cdots \ \frac{x^4}{} - \frac{x^6}{} + \cdots = \sum^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0} \frac{(-1)\ x}{(2n)!}$$

converges

MS

$$P_n(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + \frac{(-1)^n x^{2n}}{(2n)!}$$

Poly of degree n

$$\lim_{n\to\infty} P_n(x) = \underline{\underline{\cos x}}$$

about 0

② $f(x) = \sin x$

MS = TS at 0

$f = \sin x \Rightarrow f^{(0)}(0) = 0$

$f' = \cos x \Rightarrow f'(0) = 1$

$f'' = -\sin x \Rightarrow f''(0) = 0$

$f''' = -\cos x \Rightarrow f''(0) = -1$

$f^{(4)} = \sin x \Rightarrow f^{(4)}(0) = 0$

$f^{(5)} = \cos x \Rightarrow f^{(5)}(0) = 1$

$\vdots$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \cdots$$

$$= 0 + x + 0 - \frac{x^3}{3!} \quad 0 + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\sin x = \boxed{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{(2n+1)}}{(2n+1)!}}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots \sum_{n=0} \frac{\ }{(2n+1)!}$$

conv

MS

$$P_1(x) = x \quad \checkmark$$

$$P_3(x) = x - \frac{x^3}{3!} \quad \checkmark\checkmark$$

$$P_5(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} \quad \checkmark\checkmark\checkmark$$

$\vdots$

at $x=0$

MS = TS at $x=0$

③ $f(x) = e^x$

$$f = e^x \Rightarrow f(0) = 1$$

$$f' = e^x \Rightarrow f'(0) = 1$$

$$f'' = e^x \Rightarrow f''(0) = 1$$

$\vdots$

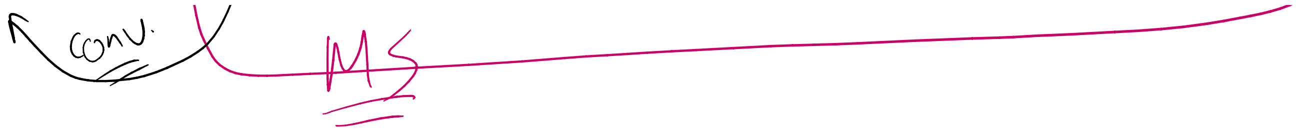
$$e^x = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$e^x = \boxed{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}}$$

conv.

MS




**MS** $e^x$ , $\sin x$ , $\cos x$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

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**Exp** Find Talyor Series of $\boxed{f(x) = 2^x}$ at $x = 1$

$a = 1$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(1)}{n!}(x-1)^n = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \cdots$$

$f = 2^x \Rightarrow f(1) = 2$

$f' = 2^x(\ln 2) \Rightarrow f'(1) = 2\ln 2$

$f'' = 2^x(\ln 2)^2 \Rightarrow f''(1) = 2(\ln 2)^2$

$f''' = 2^x(\ln 2)^3$

$\vdots$

$f^{(n)} = 2^x(\ln 2)^n$

 

$f^{(n)} = 2^x (\ln 2)$

$$= 2 + (2\ln 2)(x-1) + \frac{2(\ln 2)^2}{2!}(x-1)^2 + \frac{2(\ln 2)^3}{3!}(x-1)^3 + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{2(\ln 2)^n}{n!}(x-1)^n$$

Exp Find MS for cosh x → f(x)    $x = 0$

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{1}{2}\left[e^x + e^{-x}\right]$$

$$= \frac{1}{2}\left[1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \left(1 + (-x) + \frac{(-x)^2}{2!} + \frac{(-x)^3}{3!} + \frac{(-x)^4}{4!} + \cdots\right)\right]$$

$$= \frac{1}{2}\left[1 + \cancel{x} + \frac{x^2}{2!} + \cancel{\frac{x^3}{3!}} + \frac{x^4}{4!} + \cdots + 1 \cancel{- x} + \frac{x^2}{2!} \cancel{- \frac{x^3}{3!}} + \frac{x^4}{4!} + \cdots\right]$$

$$= \cancel{\frac{1}{2}}\left[2 + 2\frac{x^2}{2!} + 2\frac{x^4}{4!} + 2\frac{x^6}{6!} + \cdots\right]$$

$$= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots$$

Exp Find the first 4 nonzero terms in the MS of

Exp Find the first 4 nonzero terms in the MS of

① $\frac{1}{3}(2x + x\cos x)$

$= \frac{2}{3}x + \frac{x}{3}\cos x$

$= \frac{2}{3}x + \frac{x}{3}\left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\right]$

$= \boxed{\frac{2}{3}x + \frac{x}{3}} - \frac{x^3}{3(2!)} + \frac{x^5}{3(4!)} - \frac{x^7}{3(6!)} + \cdots$

$= x - \frac{x^3}{3(2!)} + \frac{x^5}{3(4!)} - \frac{x^7}{3(6!)} + \cdots$

The $1^{st}$ 4 non zero terms: $\left\{x, -\frac{x^3}{3(2!)}, \frac{x^5}{3(4!)}, -\frac{x^7}{3(6!)}\right\}$

② $e^x \sin x$

$= \left[1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \cdots\right]\left[x - \frac{x^3}{3!} + \frac{x^5}{5!} \frac{x^7}{7!} \cdots\right]$

$= x + x^2 + \frac{x^3}{2!} + \cancel{\frac{x^4}{3!}} + \frac{x^5}{4!} + \cdots - \frac{x^3}{3!} - \cancel{\frac{x^4}{3!}} - \frac{x^5}{3!(2!)} - \frac{x^6}{(3!)(3!)} + \cdots + \frac{x^5}{5!} + \frac{x^6}{5!} + \cdots$

$$= x + x^2 + \left(\frac{x^3}{2!} - \frac{x^3}{3!}\right) + \left(\frac{x^5}{4!} - \frac{x^5}{3!\,(2!)} + \frac{x^5}{5!}\right) + \cdots$$

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③ $\cos 2x$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n}}{(2n)!}$$

$$= 1 - \frac{\overbrace{(2x)^2}^{4x^2}}{2!} + \frac{\overbrace{(2x)^4}^{16}}{4!} - \frac{(2x)^6}{6!} + \cdots$$

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$f(x) = \cos 2x \Rightarrow f(0) = 1$

$f' = -2\sin 2x$

$f'' = -4\cos 2x \Rightarrow f''(0) = -4$

$f''' = 8\sin 2x$

$f^{(4)} = 16\cos 2x \Rightarrow f^{(4)}(0) = 16$

$f^{(5)} = -32\sin 2x$