Exercises:

12. $n = 10$, $SST = 6724.125$, $SSR = 6216.375$, $S_{b1} = 0.0813$, $S_{b2} = 0.0567$.

$\hat{y} = 29.1270 + 0.5906\, x_1 + 0.4980\, x_2$. $SSE = SST - SSR = 507.75$

a. Compute MSR and MSE :

$$MSR = \frac{SSR}{p} = \frac{6216.375}{2} = 3108.19.$$

$$MSE = \frac{SSE}{n-p-1} = \frac{507.75}{10-2-1} = 72.54$$

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b. Compute F and perform the appropriate F-test. use $\alpha = 0.05$.

H_0 : $\beta_1 = \beta_2 = 0$

H_1 : Not all β_j are Zero

$$F = \frac{MSR}{MSE} = \frac{3108.19}{72.54} = 42.85 .$$

$F_{0.05}$ with $df_1 = 2$ and $df_2 = 3 \rightarrow F_{0.05} = 4.74$.

$F > F_\alpha$ so we reject H_0 ($\alpha = 0.05$) $\Rightarrow$ the Model is significance.

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c. perform a t-test for the significance of β_1 . Use $\alpha = 0.05$.

H_0' : $\beta_1 = 0 \rightarrow t' = \frac{b_1}{S_{b1}} = \frac{0.5906}{0.0813} = 7.26$.

$t_{\frac{\alpha}{2}} = t_{0.025}$ with $df = 7 \rightarrow F_{0.025} = 2.365$

$|t| > t_{\frac{\alpha}{2}}$ so we reject H_0' ($\alpha = 0.05$)

$\rightarrow \beta_1 \neq 0$ ($\alpha = 0.05$).

$\rightarrow X_1$ is a significance variance ($\alpha = 0.05$).

d. perform a t-test for the significance of β_2. use $\alpha = 0.05$.

$H_1^2 : \beta_2 = 0$

$\rightsquigarrow t^{(2)} = \frac{b_2}{S_{b_2}} = \frac{0.4980}{0.0567} = 8.78$.

$\rightsquigarrow t_{\frac{\alpha}{2}}$ with $df = 7 \rightsquigarrow t_{\frac{\alpha}{2}} = 2.365$.

$\rightsquigarrow t > t_{\frac{\alpha}{2}}$ so we Reject H_0 $(\alpha = 0.05)$.

$\rightsquigarrow \beta_2 \neq 0$ $(\alpha = 0.05)$

$\rightsquigarrow X_2$ is significance variable .

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13. $n = 10$, $\hat{y} = -18.4 + 2.01 X_1 + 4.74 X_2$

$SST = 15\,182.9$, $SSR = 14052.2$, $S_{b_1} = 0.2471$, $S_{b_2} = 0.9484$.

a. test for a significance relationship among X_1, X_2 and y. use $\alpha = 0.05$, F-test.

- $MSR = \frac{SSR}{p} = \frac{14052.2}{2} = 7026.1$
- $MSE = \frac{SSE}{n-p-1} = \frac{1130.7}{7} = 161.53$.
- $F = \frac{MSR}{MSE} = \frac{7026.1}{161.53} = 43.5$
- F_α with $df_1 = 2$ and $df_2 = 7 \rightsquigarrow F_{0.05} = 4.74$
- $F > F_\alpha$ so we reject H_0 $(\beta_1 = \beta_2 = 0)$ $(\alpha = 0.05)$.

$\rightsquigarrow$ The Model is significance $(\alpha = 0.05)$.

B.b. Is β_1 significance ? $\alpha = 0.05$.

- $t' = \frac{b_1}{S_{b_1}} = \frac{2.01}{0.2471} = 8.13$.

- $t_{\frac{\alpha}{2}}$ with $df = 7 \rightsquigarrow t_{0.025} = 2.365$.

- $t^{(1)} > t_{\frac{\alpha}{2}}$ so we Reject $H_0^{(1)}$ $(\beta_1 = 0)$ $(\alpha = 0.05)$

  $\rightsquigarrow \beta_1 \neq 0$ $(\alpha = 0.05)$

  $\rightsquigarrow \beta_1 X_1$ is significance variable.

C. Is β_2 significance ? $\alpha = 0.05$.

- $t^{(2)} = \frac{b_2}{S_{b_2}} = \frac{4.74}{0.9484} = 5$

- $t_{\frac{\alpha}{2}}$ with $df = 7 \rightsquigarrow t_{0.025} = 2.365$.

- $t^{(2)} > t_{\frac{\alpha}{2}}$ so we Reject H_0 $(\beta_2 = 0)$ $(\alpha = 0.05)$.

  $\rightsquigarrow \beta_2 X_2$ is significance variable.

14.

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15. $\hat{y} = 25 + 10X_1 + 8X_2$. $n = 10$

$SST = 16000$, $SSR = 12000$

a. compute SSE, MSE, MSR.

- $SSE = SST - SSR = 2000$

- $MSR = \frac{SSR}{p} = \frac{12000}{2} = 6000$.

- $MSE = \frac{SSE}{n-p-1} = \frac{16000}{7} = 2285.71$.

b. use F-test and $\alpha = 0.05$ to determine whether there is a relationship among the variable.

- $F = \frac{MSR}{MSE} = \frac{6000}{2285.71} = 2.63$

- $F_{0.05}$ with $df_1 = 2$ and $df_2 = 7 \rightsquigarrow F_{0.05} = 4.74$.

- $F < F_\alpha$ so don't Reject H_0 ($B_1 = B_2 = 0$) ($\alpha = 0.05$).

  $\rightsquigarrow$ the Model is Not significance ($\alpha = 0.05$).