10.10 Part 1

Monday, May 10, 2021 1:08 PM

$(1+x)^2 = (1+x)(1+x) = 1 + 2x + x^2$

$(1+x)^3 = (1+x)^2(1+x) = (1+2x+x^2)(1+x) = \cdots$

$(1+x)^{20} = \cdots$

$(1+x)^{\frac{1}{2}} = \cdots$

$$(1+x)^m = 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k$$

$\binom{m}{1} = m$

$\binom{m}{2} = \frac{m(m-1)}{2}$

$$\binom{m}{k} = \frac{m(m-1)(m-2)(m-3)\cdots(m-k+1)}{k!}, \quad k \geq 3$$

Exp Find the first four terms of the binomial series for

① $\sqrt{1+x} = (1+x)^{\frac{1}{2}}$ $m = \frac{1}{2}$

$$= 1 + \sum_{k=1}^{\infty} \binom{\frac{1}{2}}{k} x^k$$

$$= 1 + \binom{\frac{1}{2}}{1} x^1 + \binom{\frac{1}{2}}{2} x^2 + \binom{\frac{1}{2}}{3} x^3 + \cdots$$

$m-k+1$

$\frac{1}{2} - 3 + 1$

?

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$$= 1 + \frac{1}{2}x + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2}x^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}x^3 + \cdots$$

$m-k+1$

$\frac{1}{2}-3+1$

$$= 1 + \frac{x}{2} - \frac{1}{8}x^2 + \frac{x^3}{16} + \cdots$$

(2) $\sqrt[3]{1+x} = (1+x)^{\frac{1}{3}}$ $\qquad m = \frac{1}{3}$

$$= 1 + \sum_{k=1}^{\infty} \binom{\frac{1}{3}}{k} x^k$$

$$= 1 + \binom{\frac{1}{3}}{1} x^1 + \binom{\frac{1}{3}}{2} x^2 + \binom{\frac{1}{3}}{3} x^3 + \cdots$$

$$= 1 + \frac{1}{3}x + \frac{\frac{1}{3}(\frac{1}{3}-1)}{2}x^2 + \frac{\frac{1}{3}(\frac{1}{3}-1)(\frac{1}{3}-2)}{3!}x^3 + \cdots$$

$\frac{(\frac{1}{3})(-\frac{2}{3})}{2}$

$$= \boxed{1 + \frac{x}{3} - \frac{x^2}{9} + \frac{5x^3}{81}} + \cdots$$

Exp Find Poly. that approximates $F(x) = \int_0^x \sin t^2 \, dt$ on $[0,1]$. with error less than 10^{-3}

Maclurine of $\sin t = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \cdots$

Maclurine of $\sin t = t - \frac{}{3!} + \frac{}{5!} - \frac{}{7!} + \cdots$

$$F(x) = \int_0^x \sin t^2 \, dt = \int_0^x \left[t^2 - \frac{t^6}{3!} + \frac{t^{10}}{5!} - \frac{t^{14}}{7!} + \cdots \right] dt$$

$$= \left[\frac{t^3}{3} - \frac{t^7}{(7)(3!)} + \frac{t^{11}}{(11)(5!)} - \frac{t^{15}}{(15)(7!)} + \cdots \right]_0^x$$

$$= \left(\frac{x^3}{3} - \frac{x^7}{(7)(3!)} + \frac{x^{11}}{(11)(5!)} - \frac{x^{15}}{(15)(7!)} + \cdots \right) - (0)$$

$x \in [0,1]$

$F(x) \approx \frac{x^3}{3}$ with error $< \left| \frac{x^7}{(7)(3!)} \right| < \frac{1}{(7)(3!)} \approx 0.0238 > 0.001$

$F(x) \approx \frac{x^3}{3} - \frac{x^7}{(7)(3!)}$ with error $< \left| \frac{x^{11}}{(11)(5!)} \right| < \frac{1}{(11)(5!)} \approx \boxed{0.00076} < 0.001$

Hence, $P_7(x) = \frac{x^3}{3} - \frac{x^7}{(7)(3!)} \approx F(x) = \int_0^x \sin t^2 \, dt$ on $[0,1]$

$P_{11}(x) = \frac{x^3}{3} - \frac{x^7}{(7)(3!)} + \frac{x^{11}}{(11)(5!)}$ with error $< \left| \frac{x^{15}}{(15)(7!)} \right| < \frac{1}{(15)(7!)} \approx 0.0000013 < 0.001$

Error