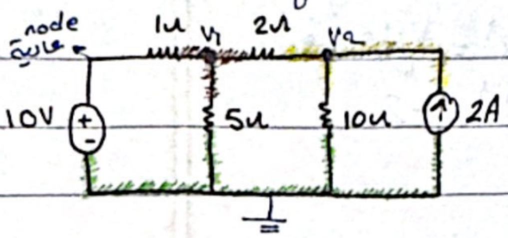
Chapter 4:
Techniques of circuit Analysis

1. Node _ Voltage Method



Step 1: Determine the number of essential nodes, n_e.

$n_e = 3$

Step 2: Select one of these nodes as a reference node

ref ⏚ ground

يفضل اختار النود يلي عليها أكثر تفرعات

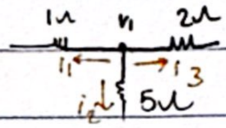
Step 3: Define the node voltages on the circuit

اعطي اسماء للباقي essintial nodes

Step 4: Apply KCL at each node

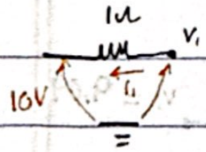
KCL at node V_1:

$i_1 + i_2 + i_3 = 0$ ⁓ (*)



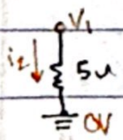
بفرض التيارات طالعين من النقطة عشان أدخلهم موجب

i_1??



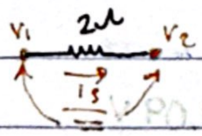
$i_1 = \frac{V_1 - 10}{1Ω}$

i_2 ??



$i_2 = \frac{V_1 - 0}{5}$

i_3??

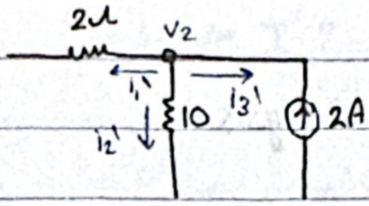


$i_3 = \frac{V_1 - V_2}{2}$

بالتعويض في *

$$\left(\frac{V_1 - 10}{1}\right) + \left(\frac{V_1 - 0}{5}\right) + \left(\frac{V_1 - V_2}{2}\right) = 0 \quad ⁓ (1)$$

KCL at node V_2 :



$i_1' + i_2' + i_3' = 0$

$i_1' = \frac{V_2 - V_1}{2}$ $\qquad$ $i_2' = \frac{V_2 - 0}{10}$ $\qquad$ $i_3' = V$ فش فولتج

$i_3' = -2$

$\left(\frac{V_2 - V_1}{2}\right) + \left(\frac{V_2 - 0}{10}\right) + (-2) = 0$ — ②

$\frac{V_1 - 10}{1} + \frac{V_1 - 0}{5} + \frac{V_1 - V_2}{2} = 0$

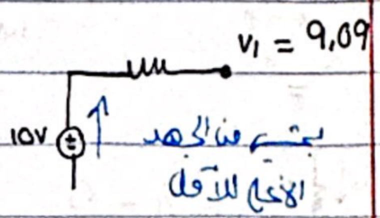
$5(V_2 - V_1) + V_2 = 20$ $\qquad \times ⑩$

$6V_2 - 5V_1 = 20$

$10V_1 - 100 + 2V_1 + 5V_1 - 5V_2 = 0$

$15V_1 + 2V_1 - 5V_2 = 100$

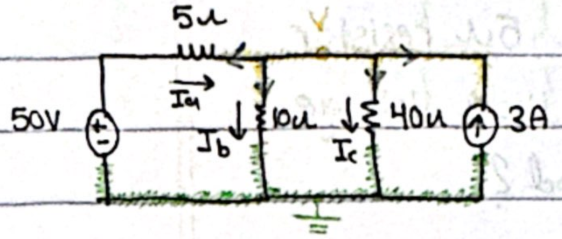
$17V_1 - 5V_2 = 100$



Res $V_1 = 9.09$ V $\qquad$ المصدر 10V يشتغل generator

$V_2 = 10.91$ V

Ex: Find I_a, I_b, I_c using Node - voltage method ?



KCL at V :

$$\frac{V-50}{5} + \frac{V-0}{10} + \frac{V-0}{40} + (-3) = 0 \qquad \times 40$$

$$8V - 400 + 4V + V = 120$$

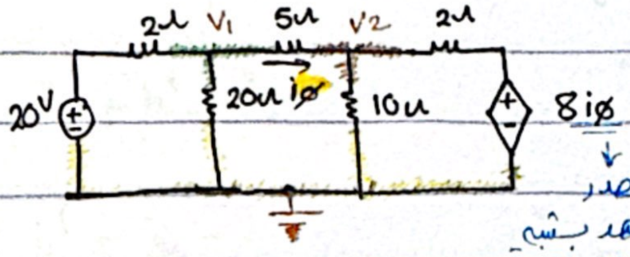
$$13V = 520 \longrightarrow V = 40\,V$$

بنعكس الاتجاه المفروض

$$I_A = \frac{50-V}{5} = \frac{50-40}{5} = 2A$$

$$I_b = \frac{V-0}{10} = 4A \qquad I_c = \frac{V-0}{40} = 1A$$

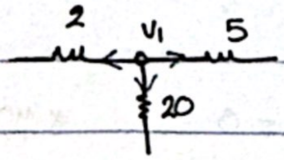
Ex:



Find the power absorbed by the 5Ω resistor using Node voltage method?

مصدر جهد بشبه ال 20 ولكن قيمة مش محددة

KCL at V_1



$I_1 + I_2 + I_3 = 0$

$\frac{V_1 - 20}{2} + \frac{V_1 - 0}{20} + \frac{V_1 - V_2}{5} = 0$ ~ ①

KCL at V_2

$\frac{V_2 - V_1}{5} + \frac{V_2 - 0}{10} + \frac{V_2 - 8i_ø}{2} = 0$ ~ ②

بطلع كمان معادلة عشان طلع عندي مجهول $i_ø$

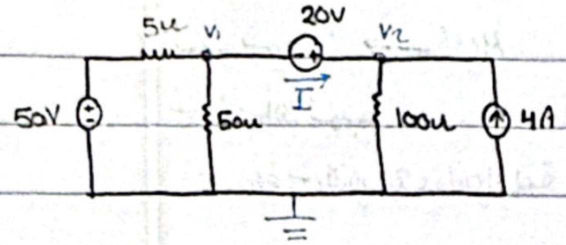
$i_ø = \frac{V_1 - V_2}{5}$ dep source وجود ال لازم يزيد معادلة

res after solving 1 & 2

$V_1 = 16\,V$ $V_2 = 10\,V$ $i_ø = 1.2\,A$

$P = I^2 R = i_ø^2 (5) = (1.2)^2\, 5 = 7.2$ Watt

Ex :



$I_1 + I_2 + I_3 = 0$

$\frac{V_1 - 50}{5} + \frac{V_1 - 0}{50} + I' = 0$ — ①

+ وجود السورس يعطيني تيار I

at Node 2:

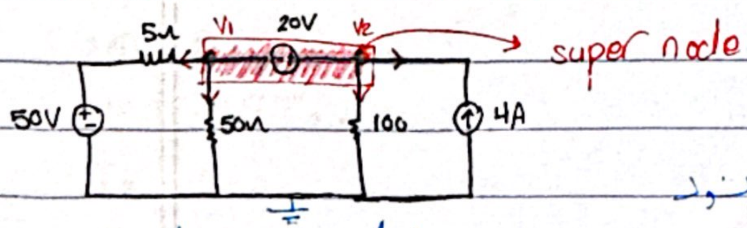
$-I' + \frac{V_2 - 0}{100} + (-4) = 0$ — ②   بجمع المعادلتين

$\frac{V_1}{5} - 10 + \frac{V_1}{50} + \frac{V_2}{100} = 4$ — ③ = 1+2

$V_2 - V_1 = 20\,V$ — ④   بالشتق بـ V_2 لأنه الجهد عند V_2

after solving $V_1 = 60\,V$ $V_2 = 80\,V$

when a voltage source is between two essential nodes we can combine these nodes to form super-node.



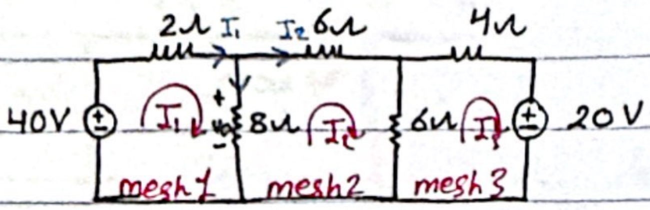
بعدها بكتب معادلة بدخل جوا السوبر نود

$V_2 - V_1 = 20$

KCL at super-node:

$\frac{V_1 - 50}{5} + \frac{V_1 - 0}{50} + \frac{V_2 - 0}{100} + 4 = 0$ هي عبارة عن معادلة 3

② Mesh _ current method



Mesh → لوب بس
فش جواها لوب
احنا افترضنا التيار مع عقارب الساعة

KVL in mesh 1:

$-40 + 2I_1 + 8(I_1 - I_2) = 0$ ~ ①

التيار يلي معي - التيار يلي عكسي

KVL in mesh 2:

$8(I_2 - I_1) + 6I_2 + 6(I_2 - I_3) = 0$ ~ ②

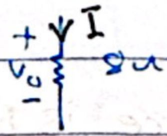
KVL in mesh 3:

$6(I_3 - I_2) + 4I_3 + 20 = 0$ ~ ③

by solving 1, 2, 3

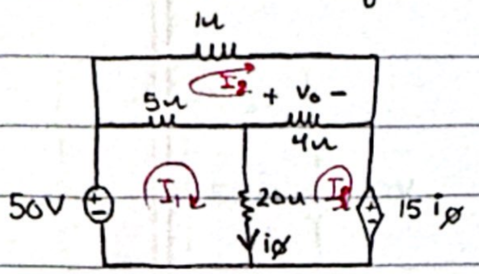
$I_1 = 5.6A$ $I_2 = 2A$ $I_3 = -0.8A$

Find V_o using mesh-current method



$V_o = 8(I_1 - I_2)$
$= 28.8\ V$

Ex: Find V_o using Mesh-current method:



mesh 1:

$-50 + 5(I_1 - I_2) + 20(I_1 - I_3) = 0$

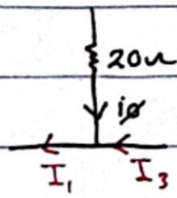
mesh 2:

$5(I_2 - I_1) + 1I_2 + 4(I_2 - I_3) = 0$

mesh 3:

$20(I_3 - I_1) + 4(I_3 - I_2) + 15i_\phi = 0$

$i_\phi = I_1 - I_3$



$25I_1 - 5I_2 - 20I_3 = 50$ — ①

$-5I_1 + 10I_2 - 4I_3 = 0$ — ②

$-5I_1 - 4I_2 + 9I_3 = 0$ — ③

$V_o = 4(I_3 - I_2)$

$= 8V$

2① + ②

$45I_1 - 44I_3 = 100$ — ④

$I_3 = 28$

$I_1 = 29.6$

$I_2 = 26$

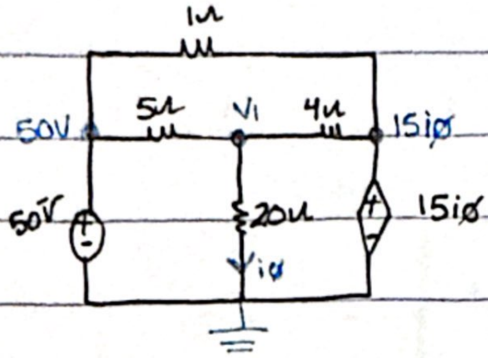
10③ + 4②

$-70I_1 + 74I_3 = 0$

$70I_1 = 74I_3$

$I_1 = 1.057I_3$

بالتعويض في ④

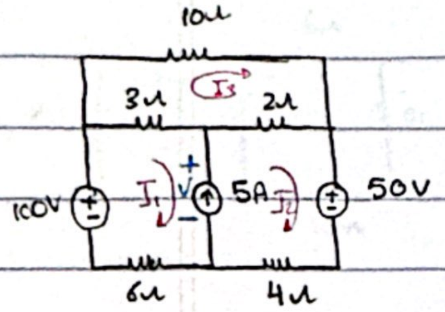


Node voltage method

$$\frac{V_1 - 50}{5} + \frac{V_1 - 0}{20} + \frac{V_1 - 15i_\phi}{4} = 0$$

$$i_\phi = \frac{V_1 - 0}{20} = \frac{V_1}{20}$$

Ex:



$-100 + 3(I_1 - I_3)$ $\boxed{-5}$ $+ 6I_1$

خطأ هيك بجمع voltage مع تيار ← الحل نفرض voltage عليه

$-100 + 3(I_1 - I_3) + V + 6I_1 = 0$

$-V + 2(I_2 - I_3) + 50 + 4I_2 = 0$

$\Rightarrow -100 + 3(I_1 - I_3) + 6I_1 + 2(I_2 - I_3) + 50 + 4I_2 = 0$ —(1)

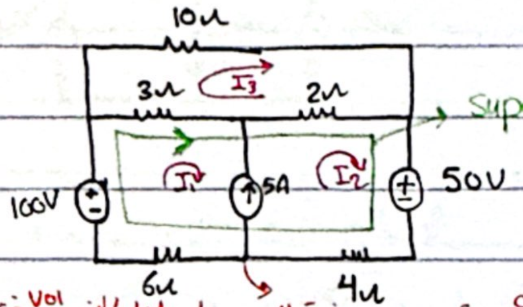
$10I_3 + 2(I_3 - I_2) + 3(I_3 - I_1) = 0$ —(2)

$I_2 - I_1 = 5$ —(3)

by solving

$I_1 = 1.75\ A$ $\quad I_2 = 6.75\ A$ $\quad I_3 = 1.25A$

The concept of super mesh:



المشكلة انها عبارة عن current source مش معروف فرق الجهد عليه إذا كانه Vol دفي مشكلة

KVL on super mesh:

$$-100 + 3(I_1 - I_3) + 2(I_2 - I_3) + 50 + 4I_2 + 6I_1 = 0 \quad (1)$$

هي عبارة عن مجموع المعادلتين اللي قبل

$$I_2 - I_1 = 5 \quad (2)$$

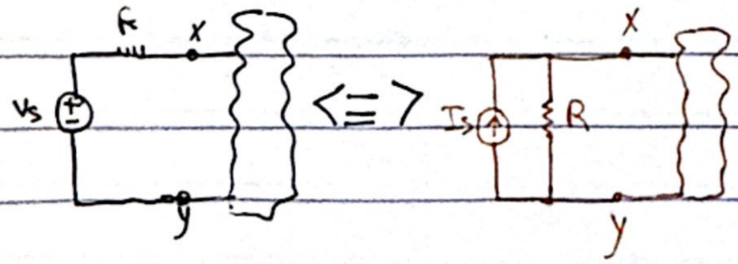
super mesh فوق ال Is

KVL on mesh 3:

$$10I_3 + 2(I_3 - I_2) + 3(I_3 - I_1) = 0 \quad (3)$$

③ Source Transformation:

It allows a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor.



راس السهم مع الاشارة الموجب

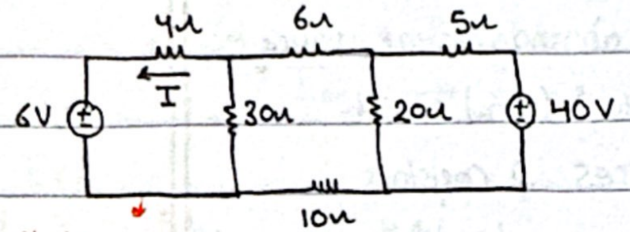
$$V_s = RI_R + V_{xy}$$

$$\frac{V_s}{R} = I_R + \frac{V_{xy}}{R}$$

$$I_s = I_R + \frac{V_{xy}}{R}$$

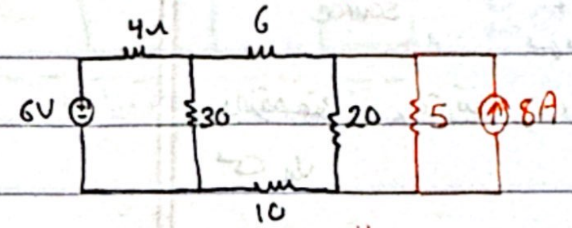
$$I_s = \frac{V_s}{R}$$

Ex :



Find the current I in the circuit using source transformation method.

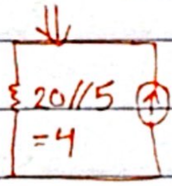
ما بلعب بهاد الجزء
لأنه بدي التيار المطلوب



$\frac{40}{5} = 8A$ قيمة التيار

الاتجاه من العنصر الموجب

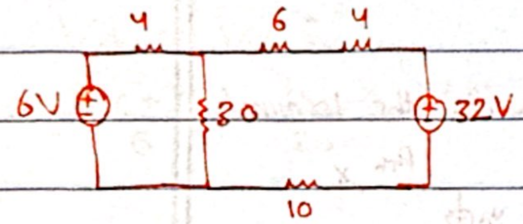
توازي مع نفس المقاومة

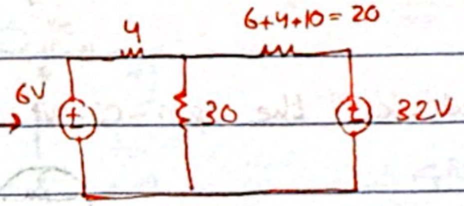


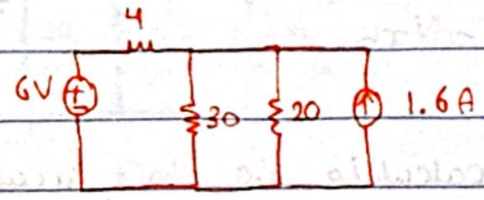
بحولها لـ Voltage S

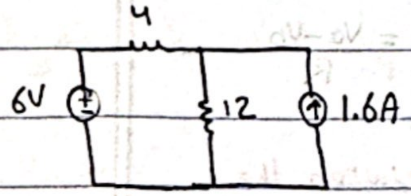
$8 \times 4 = 32$

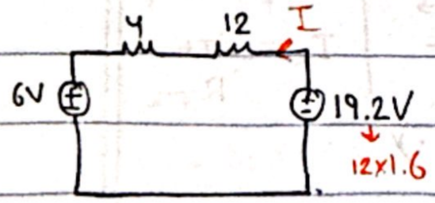
<=>







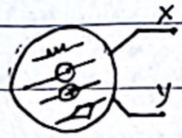




$I = \frac{19.2 - 6}{16} = 0.825A$

① Thevenin Method

Thevenin Equivalent circuit : It is an independent voltage source (V_{Th}) in series with a resistor (R_{Th}) which replace an interconnection of sources & resistors



أي دائرة بقدر أعبر عنها بـ voltage source توالي مع مقاومة

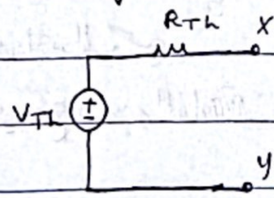
ما كان في indep source ← $0 = V_{Th}$ short

بس بشوف R_{Th}

دائرة مفتوحة ، فش تيار يعني بشوف R_{Th}

بس V_{Th}

⇕

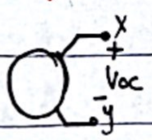


Thevenin Equivalent circuit

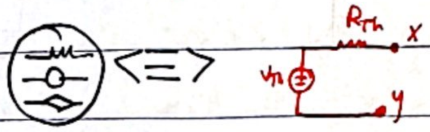
Methods to find The Thevenin equivalent circuit.

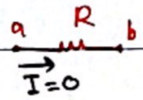
method 1

Step 1 : calculate the open-circuit voltage across the terminals x y



Voc : open-circuit voltage

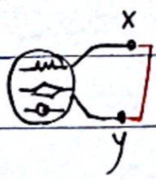




$V_{oc} = V_{Th}$

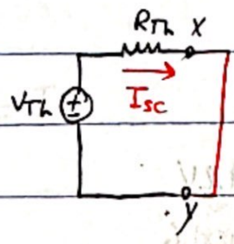
$$I = 0 = \frac{V_a - V_b}{R}$$

Step 2 : calculate the short circuit current between the terminals x & y.



I_{sc} : short circuit current

(بستفيد منه لايجاد R_{Th})

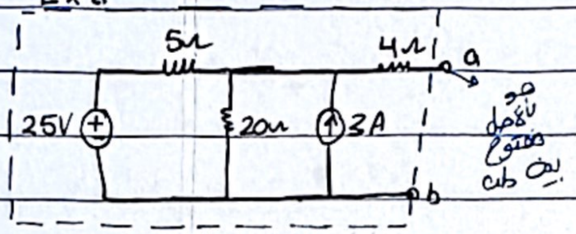


$$I_{sc} = \frac{V_{Th}}{R_{Th}}$$

Step 3: Calculate R_{Th}

$$R_{Th} = \frac{(V_{Th} = V_{oc}) \rightarrow \text{step 1}}{I_{sc} \rightarrow \text{step 2}}$$

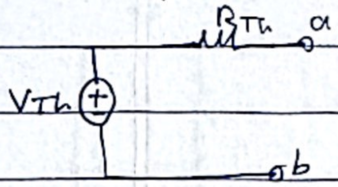
Ex:

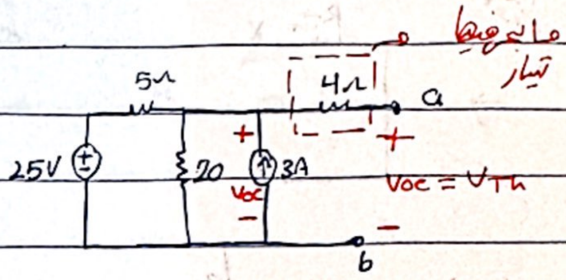


Find the Thevenin equivalent circuit across ab.

Network

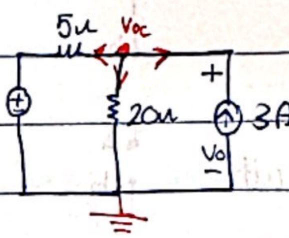
⇕





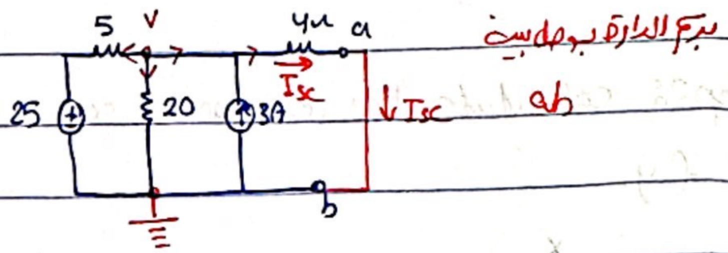
KCL at Voc:

$$\frac{V_{oc}-25}{5} + \frac{V_{oc}-0}{20} + (-3) = 0$$



$V_{oc} = V_{Th} = 32\,V$

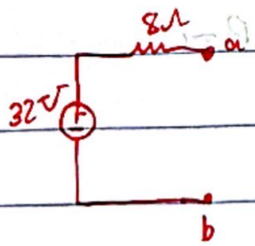
$$\frac{V-25}{5} + \frac{V-0}{20} + -3 + \frac{V-0}{4} = 0$$



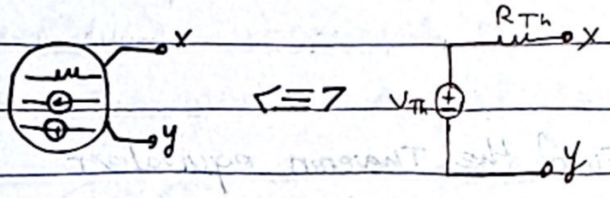
$V = 16\,V$

$$I_{sc} = \frac{V}{4} = \frac{16}{4} = 4A$$

$$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{32}{4} = 8\,\Omega$$

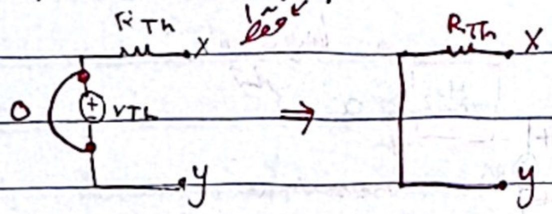


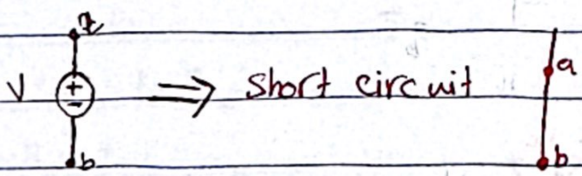
Method 2: It is useful if the network contains only indep sources & resistors.

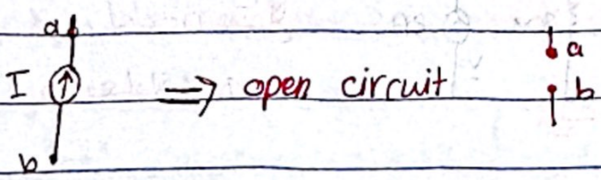


Step1: نفس يلي قبل

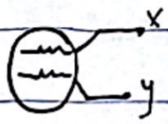
Step2: Kill all indep (فقط) sources in the circuit → $0 = V_{Th}$ بتصفي
بتصير الدائرة فقط مقاومات







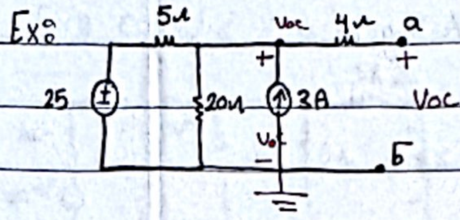
Step3: calculate the resistance seen at terminals x & y



$R_{eq} = R_{Th}$

بنستخدم الميثود حسب ظروف الدائرة كلهم رح يعطوني نفس النتيجة ولكن وحدة رح تكون أسهل

Ex:



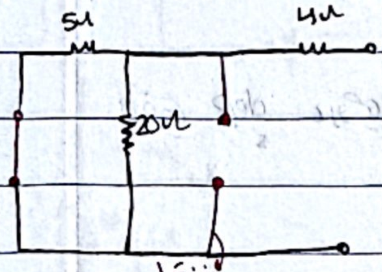
الدائرة بالأطراف مفتوحة

Find the Thevenin Equivelent circuit across the terminals a & b.

$$\frac{V_{oc}-25}{5} + \frac{V_{oc}}{20} + (-3) + 0 = 0$$

$$V_{Th} = V_{oc} = 32\ V$$

2. Kill indep



كأني قطعت السلك

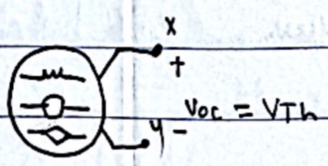
إذا فش ولا dep source ← بروح على Method 2

إذا في dep source ← إما Method 1 or 3

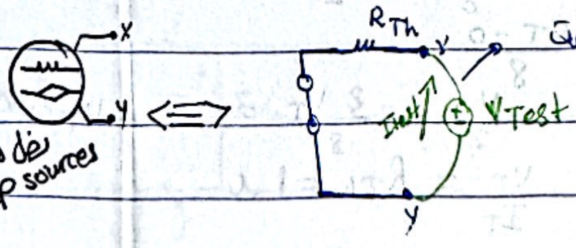
Method 3: It is useful if the circuit contains dep - sources

Step 1: calculate the open circuit voltage

نفس يلي قبل



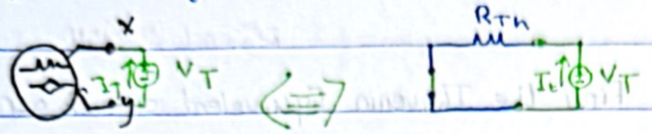
Step 2: Kill all indep sources in the circuit



حطي مصدر خارجي وأعطيه أي قيمة

بضل مقاومات dep sources

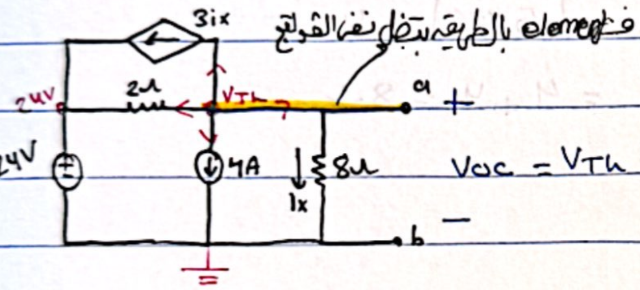
Step 3: Apply a test source on the terminals x y.



Step 4: calculate R_{Th}

$$R_{Th} = \frac{V_T}{I_T}$$

Ex: Find the Thevenin equivalent circuit with respect to terminal ab.



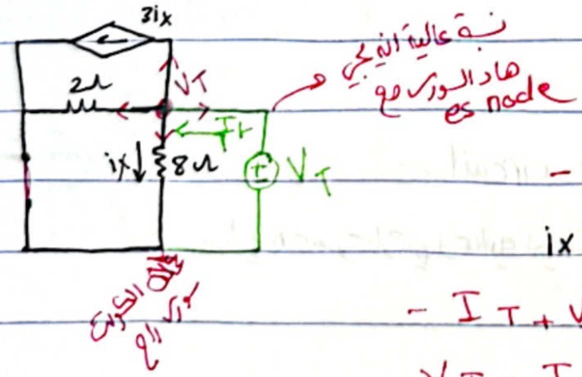
KCL at V_{Th}:

$$3i_x + 4 + \frac{V_{Th}}{8} + \frac{V_{Th} - 24}{2} = 0$$

$$i_x = \frac{V_{Th}}{8}$$

$$\frac{3}{8} V_{Th} + 4 + \frac{V_{Th}}{8} + \frac{V_{Th}}{2} - 12 = 0$$

$$V_{Th} = 8\,V$$



KCL at V_T:

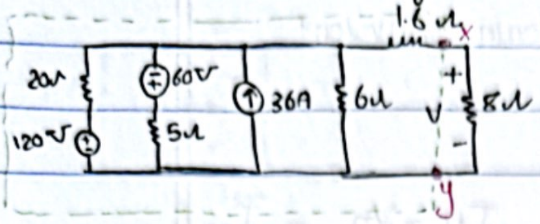
$$-I_T + \frac{V_T}{8} + \frac{V_T}{2} + 3i_x = 0$$

$$i_x = \frac{V_T - 0}{8}$$

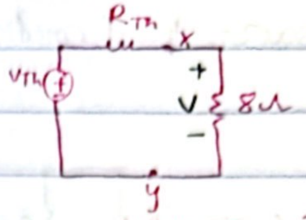
$$-I_T + \frac{V_T}{8} + \frac{V_T}{2} + \frac{3}{8} V_T = -I_T + V_T = 0$$

$$V_T = I_T \qquad \frac{V_T}{I_T} = R_{Th} = 1\,\Omega$$

Ex: calculat V using Thevenin Method



⇐⇒

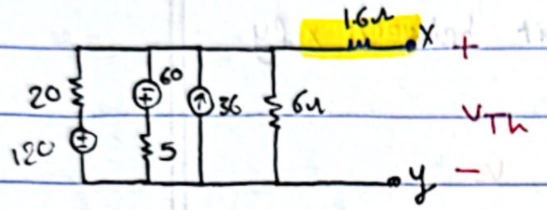


Thevenin Equivalent circuit

Voltage div (مقسم جهد)

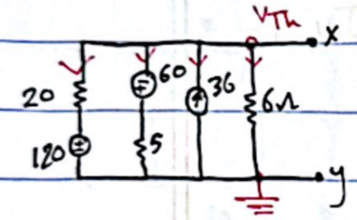
$$V = \frac{8}{8+R_{Th}} \cdot V_{Th}$$

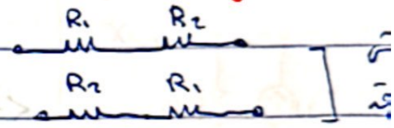
⇓



note: when x & y are open no current flows through the external load because there's no closed path for the current to circulate, As a result there's no current flowing through 1.6 Ω

⇓



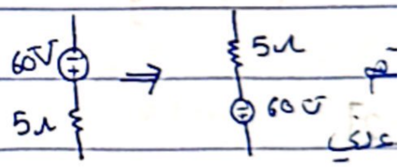


انتبه للإشارة

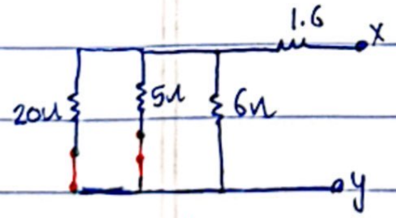
$$\frac{V_{Th}}{6} - 36 + \frac{(V_{Th} - 60)}{5} + \frac{(V_{Th} - 120)}{20}$$

$V_{Th} = 72$

solve for V_{Th}



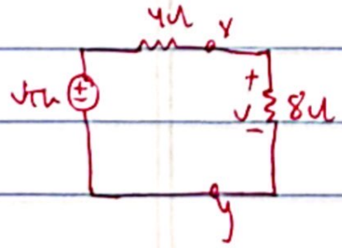
to find R_{Th} → Method 1 or 2 no need to use method 3 there is no indep



$R_{Th} = (20 \parallel 5) = 4$

$(4 \parallel 6) = 2.4$

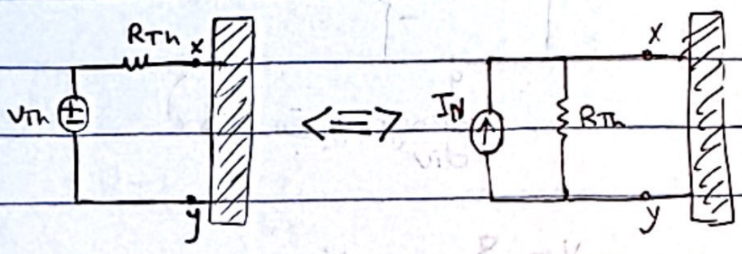
$(2.4 + 1.6) = 4\Omega$



$$V = \frac{8}{8+4} \cdot V_{Th} = 48\,V$$

Norton Equivalent circuit

It is the source transformation of thevenin equivalent circuit



$I_N = \frac{V_{Th}}{R_{Th}} = I_{sc}$ ↓ short circuit current

Methods to find the Norton equivalent circuit between x & y

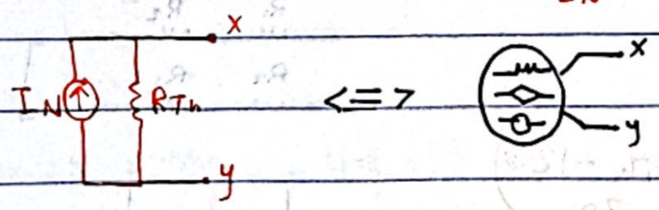
1]

Step1: calculate the open circuit voltage, V_{oc}

$V_{Th} = V_{oc}$

Step2: calculate the short circuit current, I_{sc} $\quad I_N = I_{sc}$

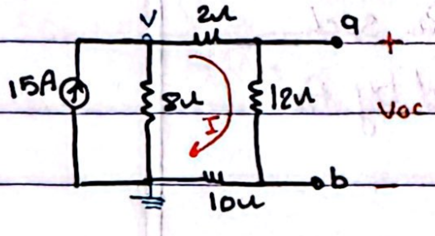
Step3: calculate the $R_{Th} = \frac{V_{Th}}{I_N}$



2]

using the methods of thevenin equivalent circuit with source transformation

Ex: Find the Norton equivalent circuit with respect to the terminals ab



current divider $I = \frac{8}{8+(2+12+10)} \cdot 15 = 3.75\ A$

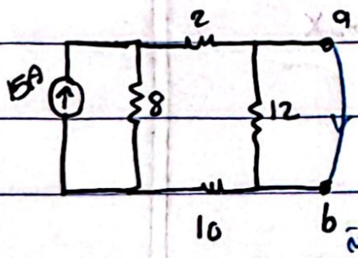
$V_{Th} = V_{oc} = 12I = 45\ V$

or by Node

$-15 + \frac{V}{8} + \frac{V}{24} = 0 \qquad \frac{4V}{24} = 15$

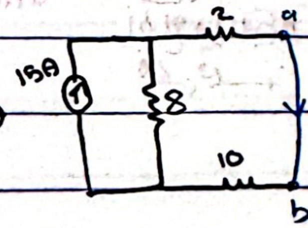
الجهد الكلي , $V = 90$

$V_{oc} = \frac{(12)}{(12)+2+10} \cdot 90 = 45\ V$



$I_N = I_{sc}$

الفولتج عند a
نفس عند b
الفولتج على المقاومة
$0 = 12\Omega$
$I = \frac{V}{R} = 0$

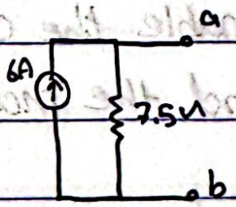


بمجرد short مقطوعة

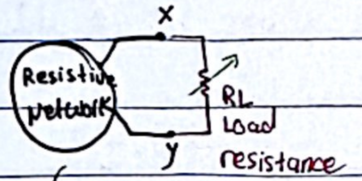
يلغي المقاومة

current $I_N = \frac{8}{8+12} \cdot 15 = 6A$

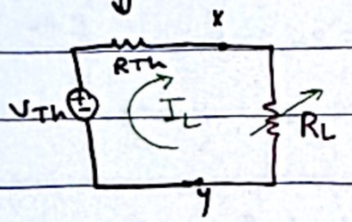
$R_{Th} = \frac{V_{Th}}{I_N} = \frac{45}{6} = 7.5\Omega$



Maximum power Transfer



What is the value of R_L such that the power absorbed by R_L is maximum?



I_L: load current

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

The power absorbed by R_L:

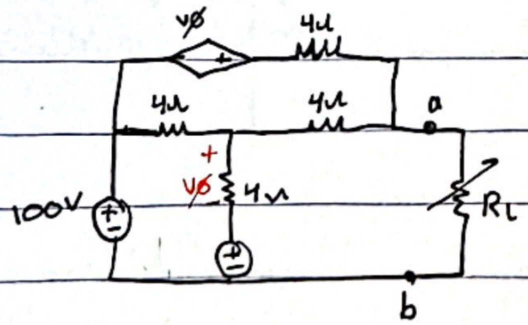
$$P_L = I^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 \cdot R_L$$

$$P(x) = \left(\frac{V_{Th}}{x + R_{Th}}\right)^2 \cdot x \xrightarrow{\text{بشتق}} \frac{dP_L}{dR_L} = 0 \Rightarrow R_L = R_{Th} = X$$

عشان أطلع وين تكون أكبر قيمة

$$P_{L\,max} = \frac{V_{Th}^2}{4R_{Th}}$$

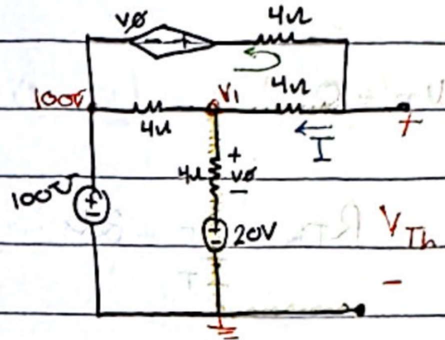
Ex: Find the value of R_L to enable the circuit to deliver maximum power and find the maximum power delivered to R_L.



assisment problems

$R_L = R_{Th}$

$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$



$\frac{V_1 - 20}{4}$

جهد النقطة $V_1 = V_\phi + 20$

$\frac{V_1 - 20}{4} + \frac{V_1 - (V_\phi + 100)}{8} + \frac{V_1 - 100}{4}$

عوض عن $V_1 = V_\phi + 20$

$\frac{V_\phi + 20 - 20}{4} + \frac{V_\phi + 20 - (V_\phi + 100)}{8} + \frac{V_\phi + 20 - 100}{4}$

$= \frac{V_\phi}{4} + (-10) + \frac{V_\phi}{4} + (-20)$

$= \frac{V_\phi}{2} = 30$

$V_\phi = 60\ V$

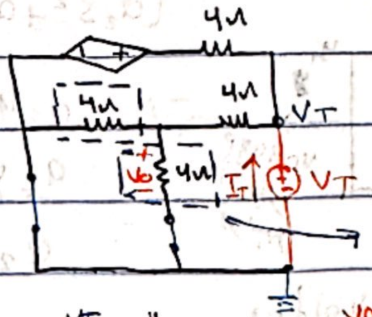
$V_{Th} = 4I + V_\phi + 20$

عوض عن I بالأخضر: $I = \frac{(V_\phi + 100) - (V_\phi + 20)}{8} = 10A$

$V_{Th} = 40 + 60 + 20 = 120\ V$

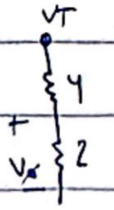
R_{Th} "Method 3"

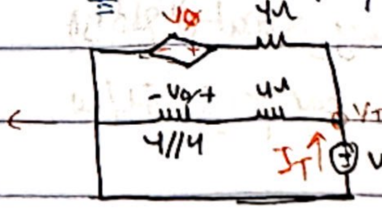
بنحط V_T على أطراف الدارة.



هاي توازي مع هدول إذا الهم نفس Vø

$\frac{V_T - 0}{6} + (-I_T) + \frac{V_T - V_\phi}{4} = 0$

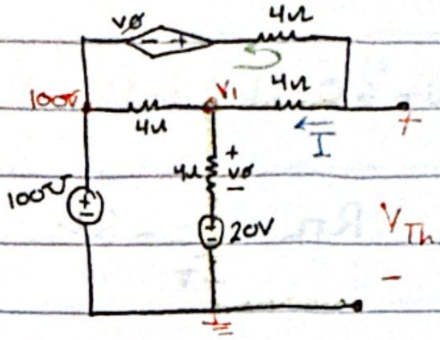




$V_\phi = \frac{2}{2+4} V_T = \frac{1}{3} V_T$

$R_L = R_{Th}$

$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$



$\frac{V_1 - 20}{4}$

نعوض بالقطع $V_1 = V_\phi + 20$

$\frac{V_1 - 20}{4} + \frac{V_1 - (V_\phi + 100)}{8} + \frac{V_1 - 100}{4}$

عوض بدل $V_1 = V_\phi + 20$

$\frac{V_\phi + 20 - 20}{4} + \frac{V_\phi + 20 - (V_\phi + 100)}{8} + \frac{V_\phi + 20 - 100}{4}$

$= \frac{V_\phi}{4} + (-10) + \frac{V_\phi}{4} + (-20)$

$= \frac{V_\phi}{2} = 30$

$V_\phi = 60\ V$

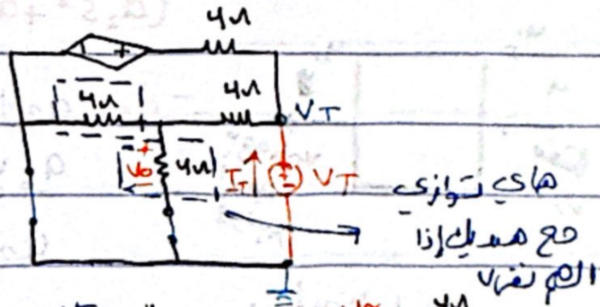
$V_{Th} = 4I + V_\phi + 20$

نحسب بالأخضر $I = \frac{(V_\phi + 100) - (V_\phi + 20)}{8} = 10\ A$

$V_{Th} = 40 + 60 + 20 = 120\ V$

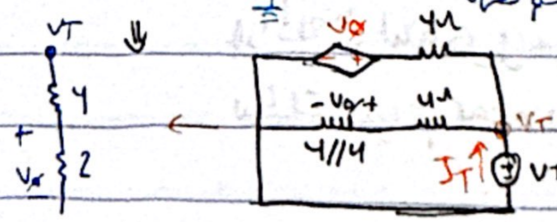
R_{Th} "Method 3"

بنعطي V_T على أطراف السورس.



$\frac{V_T - 0}{6} + (-I_T) + \frac{V_T - V_\phi}{4} = 0$

$V_\phi = \frac{2}{2+4} V_T = \frac{1}{3} V_T$



$$-I_T + \frac{V_I}{6} + \frac{1}{6} V_T = 0$$

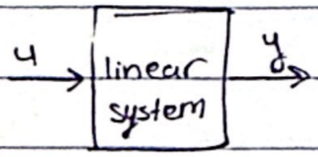
$$I_T = \frac{V_T}{3} \qquad R_{Th} = \frac{V_I}{I_T} = 3\Omega$$

$R_L = 3\Omega$

$$p_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{120^2}{4(3)} = 1.2\,KW$$

⑤ Superposition:

For any linear system is driven by more than one indep source, the total response is the sum of the individual responses.



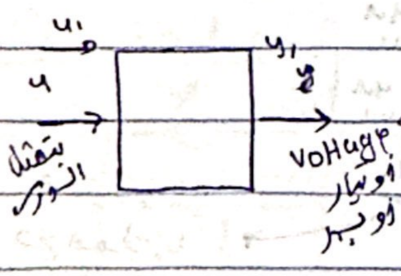
linear differential equation

$$a_2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = u(t)$$

$$\frac{y(s)}{u(s)} = H(s)$$

$$(a_2 s^2 + a_1 s + a_0)\, y(s) = u(s)$$

$$y = \left(\frac{1}{a_2 s^2 + a_1 s + a_0}\right) u(s)$$



$$a_2 y'' + a_1 y' + a_0 = u$$

$$a_2 y_1'' + a_1 y_1' + a_0 = u_1$$

$$a_2 y_2'' + a_1 y_2' + a_0 = u_2$$

إذا طلع ال output $y_1 + y_2$ بنطلق عليها linear