سارة بزار

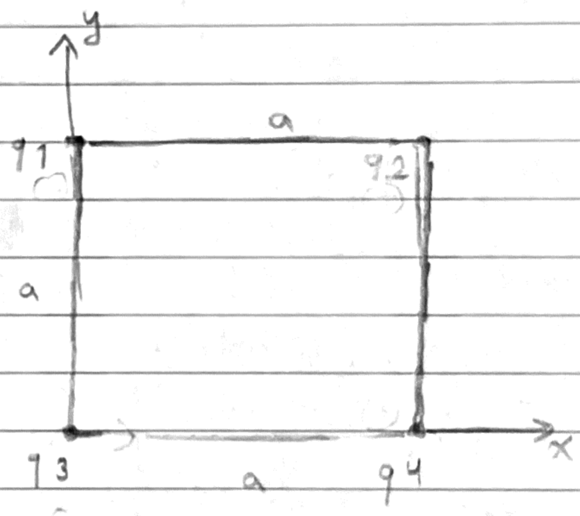
Principles of Physics (10th addition)

Phy 132

CH21 : coulomb's law

Problems : 3, 6, 13, 31, 35, 37

P_3 : In Fig 21.11, the particles have charges $q_1 = -q_2 = 300\ nC$ and $q_3 = -q_4 = 200\ nC$ and distance $a = 5.0\ cm$. What are the (a) magnitude and (b) angle (relative to +x direction) of the net force on particle 3?



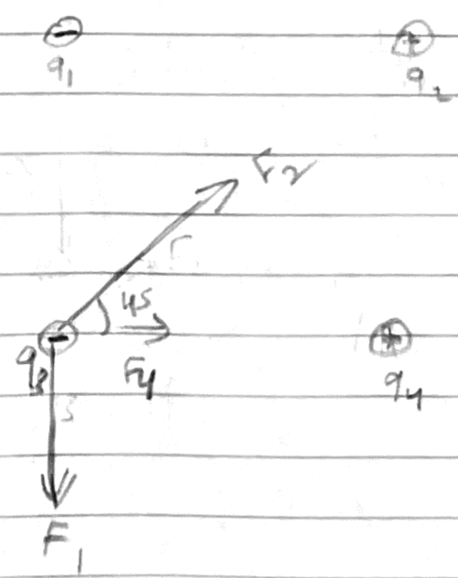
Sol : $a = 5\ cm = 5\times10^{-2}\ m$, $q_1 = -q_2 = 300\ nC = 300\times10^{-9}\ C$, $q_3 = -q_4 = 200\ nC = 200\times10^{-9}\ C$

$$\vec{F}_{13} = \frac{k\, q_1 q_3}{a^2}$$

$$= \frac{9\times10^{9} \times 300\times10^{-9} \times 200\times10^{-9}}{(5\times10^{-2})^2}$$

$$= \frac{5.4\times10^{-4}}{25\times10^{-4}}$$

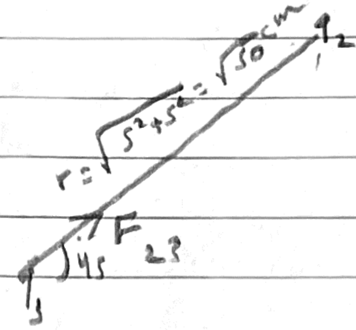
$$\vec{F}_{13} = 0.216\ N\ (-y)$$



ساعدنا

$$\vec{F}_{23} = \frac{k q_2 q_3}{r^2} = \frac{9\times10^9 \times 300\times10^{-9} \times 200\times10^{-9}}{(\sqrt{50}\times10^{-2})^2}$$

$$= 0.108\ N$$



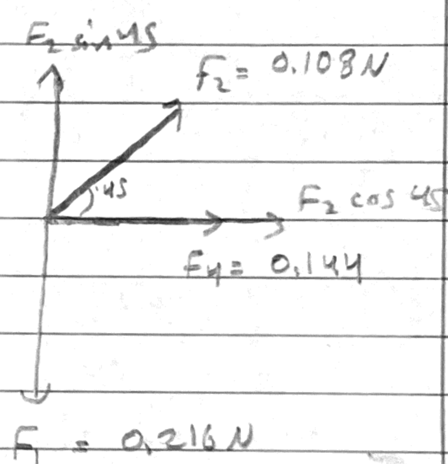
$$\vec{F}_{43} = \frac{k q_4 q_3}{a^2} = \frac{9\times10^9 \times 200\times10^{-9} \times 200\times10^{-9}}{(5\times10^{-2})^2}$$

$$= 0.144\ N\ (+x)$$

$$\Sigma F_x = F_4 + F_2 \cos 45$$

$$= 0.144 + 0.108 \times \frac{1}{\sqrt{2}}$$

$$= 0.22036\,N\ (+x)$$



$$\Sigma F_y = F_2 \sin 45 - F_1$$

$$= -0.1396$$

$$= 0.1396\,N\ (-y)$$

$$|F_{Net}| = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(0.2203)^2 + (0.1396)^2}$$

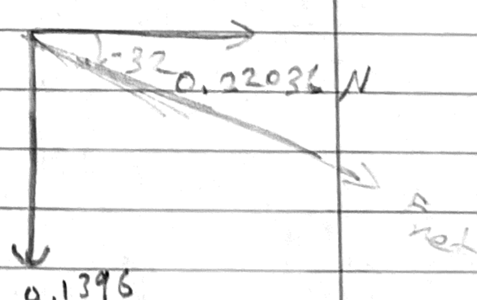
$$= 0.2608\ N$$

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} \Rightarrow \theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{0.1396}{0.22036}\right) = \tan^{-1}(0.633)$$

$= -32$ clock wise

or $360 - 32 = 328$ counterclockwise



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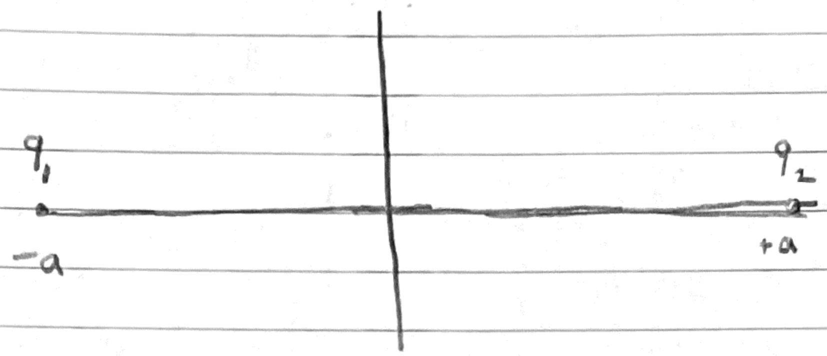
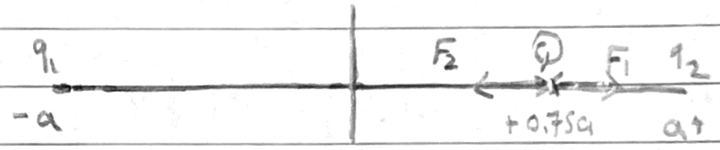
P_6: Three particles are fixed on an x axis. Particle 1 of charge q_1 is at $x = -a$, and particle 2 of charge q_2 is at $x = +a$. If their net electrostatic force on particle 3 of charge $+Q$ is to be zero, what must be the ratio q_1/q_2 when particle 3 is at (a) $x = +0.750a$ and (b) $x = +1.50a$?

Sol:



a)



$\Sigma F = 0$ (net electrostatic force equal zero)

$F_1 = F_2$

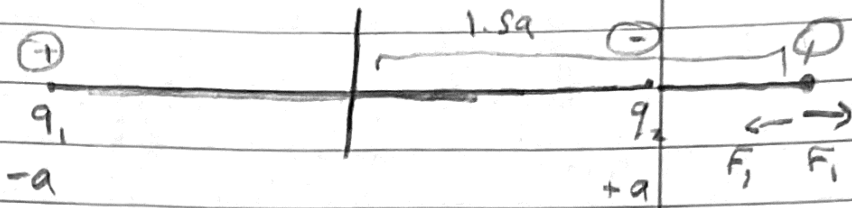
$$\frac{\cancel{k} q_1 \cancel{Q}}{(a+0.75a)^2} = \frac{\cancel{k} q_2 \cancel{Q}}{(a-0.75a)^2}$$

$$\frac{q_1}{(1.75a)^2} = \frac{q_2}{(0.25a)^2}$$

$$\frac{q_1}{q_2} = \frac{(1.75a)^2}{(0.25a)^2} = 49$$

سارة بكار

(b) بس q_1, q_2 مختلفين
بالاشارة كمية q_1 و q_2 مختلفتان
بالاتجاه



$$F_1 = F_2$$

$$\frac{\cancel{k}\, q_1 \cancel{Q}}{(a+a+0.5a)^2} = \frac{\cancel{k}\, q_2 \cancel{Q}}{(0.5a)^2}$$

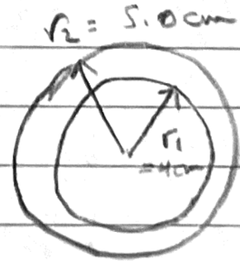
$$\frac{q_1}{(2.5a)^2} = \frac{q_2}{(0.5a)^2}$$

$$\frac{q_1}{q_2} = \frac{(2.5)^2}{(0.5)^2}\,\frac{a^2}{a^2}$$

$$\frac{q_1}{q_2} = 25$$

since q_1 (+) q_2 (−) then $\frac{q_1}{q_2} = -25$

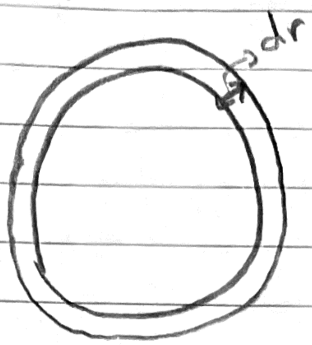
P13: A nonconducting spherical shell, with an inner radius of 4.0 cm and outer radius of 5.0 cm, has charge spread nonuniformly through its volume between its inner and outer surfaces. The volume charge density ρ is the charge per unit volume with the unit coulomb per cubic meter. For this shell $\rho = b/r$, where r is the distance in meters from the center of the shell and $b = 3.0\ \mu C/m^2$. What is the net charge in the shell?



sol: $b = 3 \times 10^{-6}\ c/m^2$

$\rho = \frac{Q}{V}$

ساره بار



$$q = \rho V$$
$$dq = \rho\, dV$$

but $V =$ Area x hight or thickness

$$V = A \cdot L$$

but in spherecal shell

$$L = dr$$
$$A = 4\pi r^2$$ مساحة سطح الكرة

$$dq = \rho\, A\, dr$$
$$dq = \rho\, 4\pi r^2 dr$$

$$\int dq = \int_{r_1}^{r_2} \rho\, 4\pi r^2 dr \quad , \quad \rho = b/r$$

$$q = \int_{r_1}^{r_2} \frac{b}{r}\, 4\pi r^2 dr$$

$$q = 4\pi b \int_{r_1}^{r_2} r\, dr$$

$$q = 4\pi b \left.\frac{r^2}{2}\right]_{r_1}^{r_2}$$

$$q = 2\pi b \left. r^2 \right]_{r_1}^{r_2}$$

$$q = 2\pi b\,(r_2^2 - r_1^2)$$

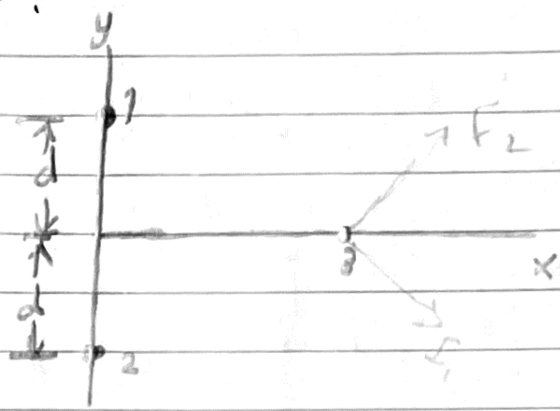
$$q = 2\pi \times 3\times10^{-6} \left((5\times10^{-2})^2 - (4\times10^{-2})^2\right)$$

$$q = 2\pi \times 3\times10^{-6} \times 9\times10^{-4}$$

$$q = 1.7\times10^{-8}\ C$$

ساره ذكار

P_{31}: In Fig 21-21, particles 1 and 2 of charge $q_1 = q_2 = +4e$ are on Y axis at distance $d = 17.0$ cm from the origin. Particle 3 of charge $q_3 = +8e$ is moved gradually along the x axis from $x=0$ to $x = +5.0$ m. At what values of x will the magnitude of electrostatic force on the third particles be (a) minimum (b) maximum? What are the (c) minimum and (d) maximum magnitudes?



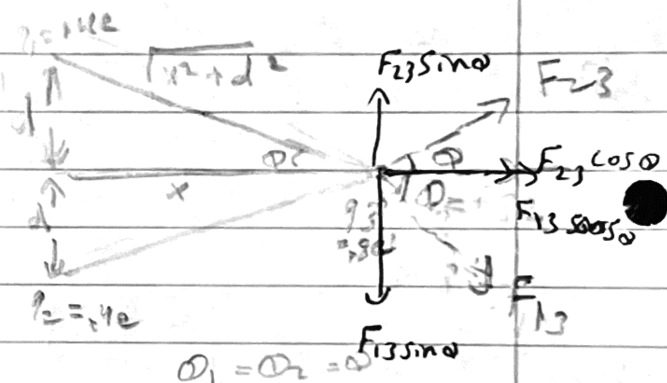
Given

$q_1 = q_2 = +4e \Rightarrow d = 17$ cm

$q_3 = +8e \Rightarrow x \quad 0 \rightarrow 5$ m

Sol:

* The magnitude of the electrostatic force between any two particles



$\theta_1 = \theta_2 = \theta$

$|F_{13}| = |F_{23}|$

$$F = k\frac{|q_1||q_2|}{r^2} \qquad (1)$$

θ: angle between F and x-axis

$$\cos\theta = \frac{x}{\sqrt{x^2+d^2}} \qquad (2)$$

* due to symmetry, there is no y component to the net force on the third particle

$\Sigma F_x = F_{13}\cos\theta + F_{23}\cos\theta = 2F_{31}\cos\theta$, $\Sigma F_y = F_{23}\sin\theta - F_{13}\sin\theta = 0$

$$F_{net} = F_{1x} + F_{2x} = 2F\cos\theta = \frac{2k(4e)(8e)}{(\sqrt{x^2+d^2})^2}\frac{x}{\sqrt{x^2+d^2}} = \frac{64ke^2x}{(x^2+d^2)^{3/2}}$$

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ساره بخار

لا ينياه

$$F_{net} = \frac{64ke^2 x}{(x^2+d^2)^{3/2}} \quad — ③$$

$F_{net} = 0$ at $x = 0$ according to eq ③

a So the minimum value for x is zero

b taking the derivative of eq 3 and equating it to zero to find the max value

$$\Rightarrow x = \frac{d}{\sqrt{2}} = \frac{17 \text{ cm}}{\sqrt{2}} = 12 \text{ cm}$$

(c) $F_{net} = \frac{64ke^2 x}{(x^2+d^2)^{3/2}} = \frac{64ke^2 (0)}{(0+d^2)^{3/2}} = 0$

(d) $F_{net} = \frac{64ke^2 x}{(x^2+d^2)^{3/2}}$

$$= \frac{64 \times 9\times10^9 \times (1.6\times10^{-19})^2 \times (0.12)}{((0.12)^2 + (0.17)^2)^{3/2}}$$

$$= \frac{1.769 \times 10^{-27}}{9.010 \times 10^{-3}}$$

$$= 1.96 \times 10^{-25} \text{ N}$$

$$\simeq 2.0 \times 10^{-25} \text{ N}$$

derivative for part (b)

$$F = \frac{64e^2kx}{(d^2+x^2)^{3/2}}$$

$$F = 64e^2k \; x \; (d^2+x^2)^{3/2}$$

$$\frac{dF}{dx} = 64e^2k\left[(d^2+x^2)^{-3/2} + x\left(\frac{-3}{2}\right)(d^2+x^2)^{-5/2} \; 2x\right]$$

$$= 64e^2k\left[\frac{1}{(d^2+x^2)^{3/2}} - \frac{3x^2}{(d^2+x^2)^{5/2}}\right]$$

$$= 64e^2k\left[\frac{d^2+x^2-3x^2}{(d^2+x^2)^{5/2}}\right]$$

$$\frac{dF}{dx} = 64e^2k\left[\frac{d^2-2x^2}{(d^2+x^2)^{5/2}}\right]$$

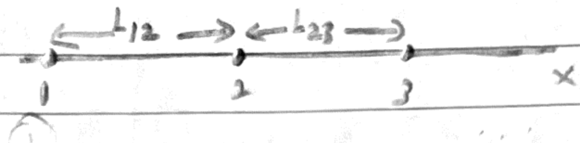
$$0 = 64e^2k\left[\frac{d^2-2x^2}{(d^2+x^2)^{5/2}}\right]$$

$$d^2-2x^2=0 \Rightarrow d^2 = 2x^2$$

$$x = \sqrt{\frac{d}{2}}$$

سادوبكار

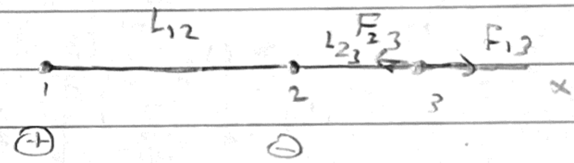
P35: In Fig 21-24, three charged particles lie on an x axis. Particles 1 and 2 are fixed in place. Particle 3 is free to move, but the net electrostatic force on it from particles 1 and 2 happens to be zero. If $2.0 L_{23} = L_{12}$. What is the ratio q_1/q_2?



بما أن محصلة القوى = صفر لنقطة
تقع خارج الشحنتين
فإن الشحنتين مختلفتان بالإشارة
وتكون المحصلة أقرب للأصغر
أي $q_1 > q_2$ إذن نتوقع
$\left|\frac{q_1}{q_2}\right| > 1$
وإشارتها سالبة

فرضًا

Sol:



$F_{net} = F_{13} - F_{23}$

$\ominus$ q_2 ، $\oplus$ q_1 فرضًا

but $F_{net} = 0$

$\Rightarrow F_{13} = F_{23}$

$$\frac{\cancel{k}\, q_1 \cancel{q_3}}{(L_{12}+L_{23})^2} = \frac{\cancel{k}\, q_2 \cancel{q_3}}{(L_{23})^2}$$

$$\frac{q_1}{(L_{12}+L_{23})^2} = \frac{q_2}{L_{23}^3} \qquad \text{but } L_{12} = 2L_{23}$$

$$\frac{q_1}{(2L_{23}+L_{23})^2} = \frac{q_2}{(L_{23})^2}$$

$$\frac{q_1}{9L_{23}^2} = \frac{q_2}{L_{23}^2}$$

سارة بخار

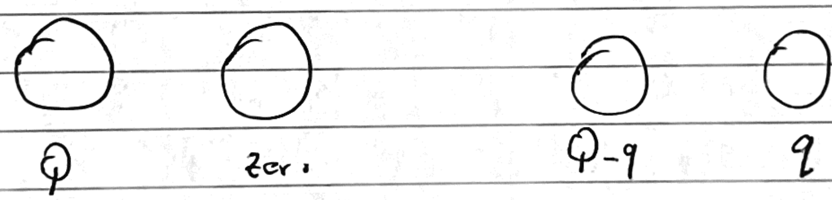
$$\frac{q_1}{q_2} = \frac{9\cancel{L_{23}^2}}{\cancel{L_{23}^2}}$$

$$\Rightarrow \left|\frac{q_1}{q_2}\right| = 9 \qquad \Rightarrow \frac{q_1}{q_2} = -9$$

becuow q_1 (+)
q_2 (−)

P_{37}: Of the change Q initially on a tiny sphere, a portion q is to be transferred to a second, nearby sphere. Both spheres can be treated as particles and are fixed with a certain separation. (a) For what value of q/Q will the electrostatic force between the two spheres be maximized? What are (b) smaller and (c) larger values of q/Q that give a force magnitude that is 75% of that maximum?

sol: Befor After



$$|F| = \frac{k\, q_1 q_2}{r^2} = \frac{k\,(Q-q)\,q}{r^2}$$

$$\Rightarrow |F| = \frac{k\,(Qq - q^2)}{r^2}$$

a) $$\frac{dF}{dQ} = \frac{k}{r^2}\,\frac{d}{dq}\,(Qq - q^2) = \frac{k}{r^2}\,(Q - 2q)$$

سادة بختار

maximized $\frac{dF}{dq} = 0$

$\Rightarrow \frac{k}{r^2}(Q - 2q) = 0$, $\frac{k}{r^2}$ const

$Q - 2q = 0$

$Q = 2q$

$$\boxed{\frac{q}{Q} = \frac{1}{2}}$$

b) F_{max} at $\frac{q}{Q} = \frac{1}{2} \Rightarrow$ at $q = \frac{Q}{2}$

sub $q = Q/2$ in $F = \frac{k(Qq - q^2)}{r^2}$

$$F_{max} = \frac{k\left(\frac{Q \times Q}{2} - \left(\frac{Q}{2}\right)^2\right)}{r^2}$$

$$= \frac{k\left(\frac{Q^2}{2} - \frac{Q^2}{4}\right)}{r^2}$$

$$F_{max} = \frac{kQ^2}{4r^2}$$

$\Rightarrow$ 75% of F_{max} is $\frac{3}{4}$ of F_{max}

$\Rightarrow$ 75% of $F_{max} = \frac{3}{4} * \frac{kQ^2}{4r^2} = \frac{3}{16}\frac{kQ^2}{r^2}$

* to find the smaller value and larger value of q/Q of 75 F_{max}

$|F| =$ 75% F_{max}

$$\frac{\cancel{k}(Qq - q^2)}{\cancel{r^2}} = \frac{3}{16}\frac{\cancel{k}Q^2}{\cancel{r^2}}$$

سادة بحار

$$Qq - q^2 = \frac{3}{16} Q^2$$

$$q^2 - Qq + \frac{3}{16} Q^2 = 0$$

$a = 1 \quad , b = -Q \quad , c = \frac{3}{16} Q^2$ معادلة تربيعية

$$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{+Q \mp \sqrt{Q^2 - 4 \times \frac{3}{16} Q^2}}{2}$$

$$= \frac{Q \mp \sqrt{Q^2 - \frac{12}{16} Q^2}}{2}$$

$$= \frac{Q \mp \sqrt{\frac{1}{4} Q^2}}{2}$$

$$q = \frac{Q \mp \frac{Q}{2}}{2}$$

$$q = \frac{Q + \frac{Q}{2}}{2} = \frac{3}{4} Q$$

$$q = \frac{Q - \frac{Q}{2}}{2} = \frac{Q}{2}$$

so b) smaller value $\Rightarrow q = Q/2$

c) larger value $\Rightarrow q = \frac{3}{4} Q$