2. from the equation $x(t)$:

$$\vec{V}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{dx}{dt} \quad \text{(first derivative)}$$

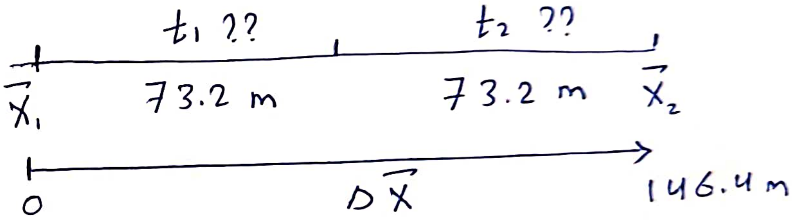
المشتقة الأولى

problem (2.2) :

Find your average velocity in the following :-

a) walk for 73.2 m at a speed of 1.22 m/s , then run 73.2 m at a speed of 2.85 m/s (along a straight track) في خط مستقيم

$$\vec{V}_{avg} = \frac{\Delta \vec{x}}{\Delta t}$$



Δt : time interval (الزمن المستغرق في الانتقال من نقطة البداية الى نقطة النهاية)

$= t_1 + t_2$

we want to find t_1 , t_2 ?

$$S_{1\,avg} = \frac{\text{Distance}}{t_1} = \frac{73.2\text{ m}}{t_1} = 1.22 \text{ m/s}$$

$$\boxed{t_1 = 60 \text{ s}}$$

$$S_{2\,avg} = \frac{73.2}{t_2} = 2.85 \text{ m/s}$$

$$\boxed{t_2 = 20.2 \text{ s}}$$

$\Rightarrow \Delta t = 60 + 20.2$

$$\boxed{\Delta t = 80.2 \text{ s}}$$

ch 2 : Lc 2

2.4 : Constant Acceleration :

a particle moves from x_0, with V_0 at t_0. at later time t, it has V, X.

initial	final
$t=0$	t
X_0	X
V_0	V

if a is const

$$a = a_{avg} = \frac{\Delta V}{\Delta t}$$

$$= \frac{V - V_0}{t - 0}$$

$$at = V - V_0$$

$$\boxed{V = at + V_0} \quad — (1)$$

we have two values of velocity (V, V_0)

$$V_{avg} = \frac{V + V_0}{2}$$

$$V_{avg} = \frac{\Delta X}{\Delta t} = \frac{X - X_0}{t - 0} \Rightarrow X - X_0 = V_{avg}\, t$$

$$X - X_0 = \frac{V + V_0}{2}\, t$$

but $V = at + V_0$

$$X - X_0 = \frac{at + 2V_0}{2}\, t$$

$$\boxed{X - X_0 = V_0 t + \frac{1}{2} a t^2} \quad — (2)$$

from ① $t = \frac{V - V_0}{a}$

$$X - X_0 = V_0\left(\frac{V - V_0}{a}\right) + \frac{1}{2} a \left(\frac{V - V_0}{a}\right)^2$$

$$= \frac{V_0 V - V_0^2}{a} + \frac{1}{2a}\left(V^2 - 2VV_0 + V_0^2\right)$$

$$a(X - X_0) = V_0 V - V_0^2 + \frac{1}{2}\left(V^2 + V_0^2\right) - VV_0$$

$$\boxed{V^2 = V_0^2 + 2a(X - X_0) \quad - \quad ③}$$

Table 2-1 Page 22

* $X \xrightarrow{\frac{dx}{dt}} V \xrightarrow{\frac{dV}{dt}} a$

$X \xleftarrow{\int V dt} V \xleftarrow{\int a dt} a$

ex: (Variable acceleration)

Let $a = 5t$

at $t = 2\,s$, $V_1 = 17\ m/s$

$t = 4\,s$ Find V_2 ?

$$a = \frac{dV}{dt} \Rightarrow \int dV = \int a\,dt$$

$$V = \int 5t\,dt$$

$$V = \frac{5t^2}{2} + C$$

To find C :

$$V(2\,s) = V_1 = \boxed{17\ m/s = \frac{5}{2}(2)^2 + C}$$

$$C = 7$$

$$\Rightarrow V = \frac{5}{2}t^2 + 7$$

$$V_2 = V(4\,s) = \frac{5}{2}(4)^2 + 7$$

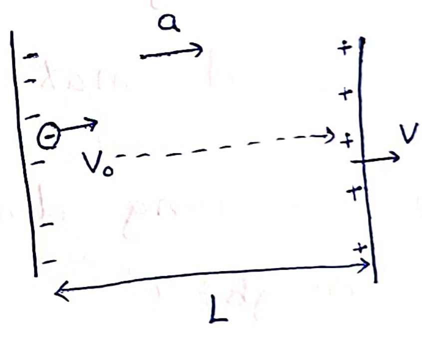
$$= 47\ m/s$$

ex: Const. acceleration:

$V_o = 1.5 \times 10^5\ m/s$

$L = 1\ cm = 10^{-2}\ m$

$V = 5.7 \times 10^6\ m/s$



Find a ??

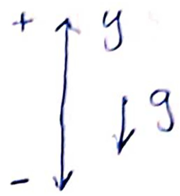
$V^2 = V_o^2 + 2a\,\Delta X$, ΔX: Displacement.

$$(5.7 \times 10^6)^2 = (1.5 \times 10^5)^2 + 2a(1 \times 10^{-2})$$

$$a = 1.51 \times 10^{15}\ m/s^2$$

Free Fall Accleration :-

The best example of Const. accleration is the Free falling.



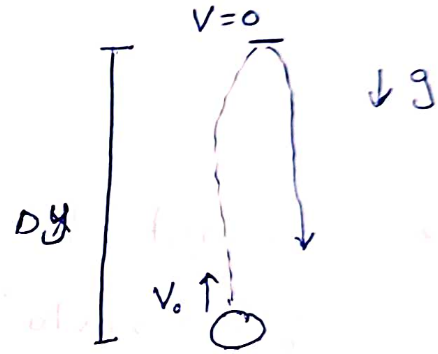
$a = -g = -9.8 \ m/s$

$V = V_0 - gt \quad - ①$

$V^2 = V_0^2 - 2g\Delta y \quad - ②$

$\Delta X = V_0 t - \frac{1}{2} g t^2 \quad - ③$

Sample problem 2.05 :-



$V_0 = 12 \ m/s$

$a = -g = -9.8 \ m/s^2$

V at maximum height = 0

a) How long does the ball take to reach the max height ?

$V_0 = 12 \ m/s \quad , \quad V = 0$

$V = V_0 - gt \Rightarrow 0 = 12 - 9.8t$

$t = 1.2 \ Sec$

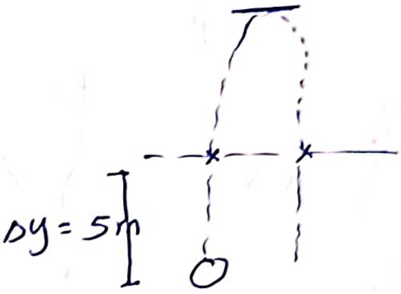
b) max height ?

$\Delta y = V_0 t - \frac{1}{2} g t^2$

$\Delta y = 12 \times 1.2 - \frac{1}{2}(9.8)(1.2)^2$

$= 7.3 \ m$

c) How long does the ball take to reach a point 5 m?



$$\Delta y = V_0 t - \frac{1}{2} g t^2$$

$$5 = 12t - \frac{1}{2}(9.8)t^2$$

$$4.9t^2 - 12t + 5 = 0$$

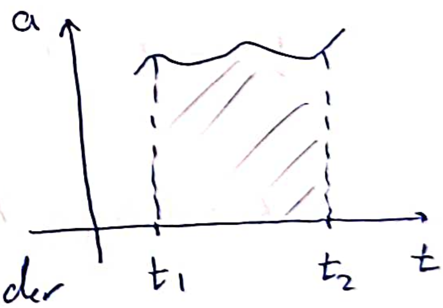
$$t = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-12) \mp \sqrt{(-12)^2 - 4(4.9)(5)}}{2(4.9)}$$

$t = 0.53$ sec (up) & $t = 1.9$ sec (down)

2.6 Graphical Integration in Motion Analysis :-

$$a = \frac{dV}{dt} \Rightarrow dV = a\,dt$$

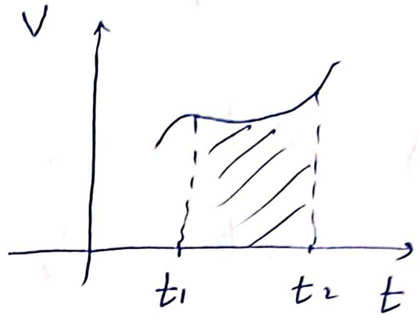


$$\int_{V_1}^{V_2} dV = \int_{t_1}^{t_2} a\,dt$$ [The area under the acceleration curve]

$$V_2 - V_1 = \int_{t_1}^{t_2} a\,dt$$

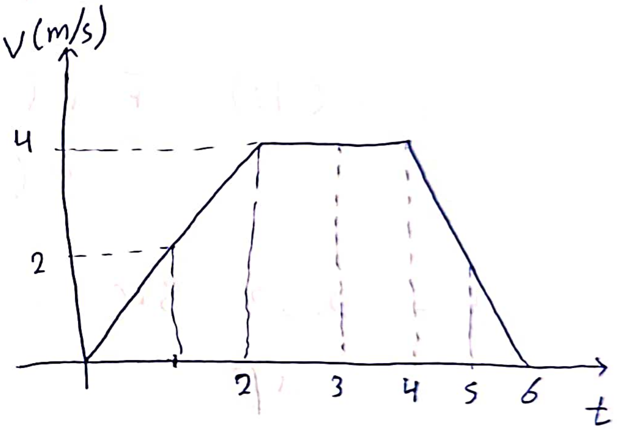
$$V = \frac{dx}{dt}$$

$$\int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v\, dt$$



$$x_2 - x_1 = \int_{t_1}^{t_2} v\, dt$$ = The area under the Velocity curve

ex:



a) Find x at $t = 4$ sec?

$$\Delta X = \int_{t_0}^{t} v\, dt$$

= Area under the curve (V vs. t) from $t_0 \to t$

$t_0 = 0$, $X_0 = 0$, $V_0 = 0$ $\Rightarrow$ Initial Conditions

$$\Delta X = X_4 - X_0 \int_0^4 v\, dt \equiv \text{Area from } t=0 \to t=4$$

مساحة شبه المنحرف = $\frac{1}{2}$ (مجموع القاعدتين) * الارتفاع

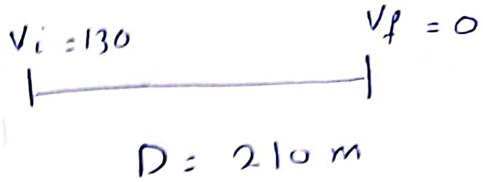
$$= X_4 = \frac{1}{2}(4+2) * 4 = 12 \text{ m}$$

b) at $t = 4$ s ; Find V, a?

$V_4 = 4$ m/s , a = slope = 0

Find for $t = s$, x, V, a?

Problem 21: $V_i = 130$ km/h



$$V_i = \frac{130 \times 1000}{3600} \text{ m/s}$$

$$= 36.1 \text{ m/s}$$

a) Find the deceleration of the car?

$$V_f^2 = V_i^2 + 2a\Delta x$$

$$0 = (36.1)^2 + 2a(210)$$

$$\Rightarrow a = -3.1 \text{ m/s}^2$$

b) How long does it take for the car to stop?

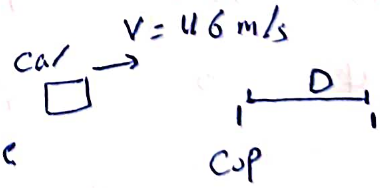
$$V_2 = V_1 + at$$

$$0 = 36.1 - 3.1t \Rightarrow t = 11.6 \text{ sec}$$

39| $V_{car} = 46$ m/s , reaction time 1. sec

$a_{cop} = 4$ m/s^2

How much time does the cop need to over take the car?



in 1 sec the car moved a distance D!! $V = \frac{D}{t} \Rightarrow D = Vt$

$$= 46 \times 1 = 46 \text{ m}$$

The cop reachs the car when $D_{cop} = D_{car}$

D_{car} : $V = \frac{dx}{dt} \Rightarrow \int dx = \int v\, dt$

$D \leftarrow x = vt + c$

at $t=0 \Rightarrow D = v(0) + c$

but we find $D = 46\,m = c$ (The moment we start monitoring (يراقب) the motion } when the cop start moving

$$\boxed{D_{car} = 46t + 46}$$

$$D_{cop} = V_0 t + \frac{1}{2} a t^2$$

$$= 0(t) + \frac{1}{2}(4) t^2$$

$$\boxed{D_{cop} = 2t^2}$$

$$D_{car} = D_{cop}$$

$$46t + 46 = 2t^2 \Rightarrow t^2 - 23t + 23 = 0$$

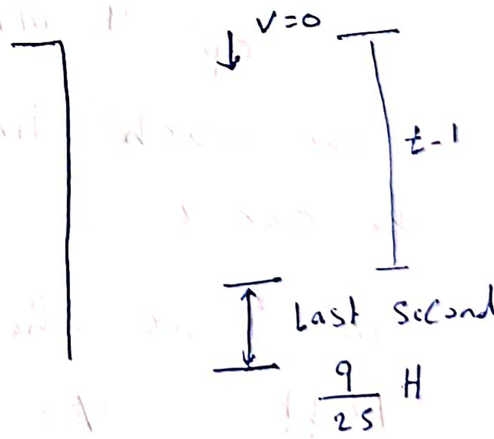
$$t = 24\ s$$

45) Find H ?

$$D = V_0 t + \frac{1}{2} a t^2$$

$$-H = 0 - \frac{1}{2} g t^2$$

$$\boxed{t = \sqrt{\frac{2H}{g}}} \quad \text{— } (1)$$



$$D_{t-1} = V_0 (t-1) - \frac{1}{2} g (t-1)^2$$

$$-\left(H - \frac{9}{25} H\right) = 0 - \frac{1}{2} g (t-1)^2$$

$$(t-1)^2 = \frac{2}{g} \frac{16}{25} H$$

$$\boxed{t-1 = \sqrt{\frac{2}{g} \frac{16}{25} H}} \quad - \text{ (2)}$$

sub (1) in (2)

$$\sqrt{\frac{2H}{g}} - 1 = \frac{4}{5} \sqrt{\frac{2H}{g}}$$

$$\frac{1}{5} \sqrt{\frac{2H}{g}} = 1$$

$$\frac{2H}{g} = 25 \Rightarrow \boxed{H = \frac{25g}{2}}$$