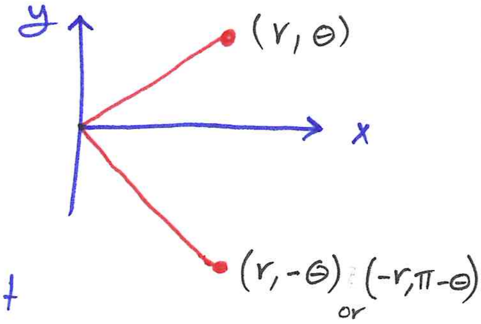
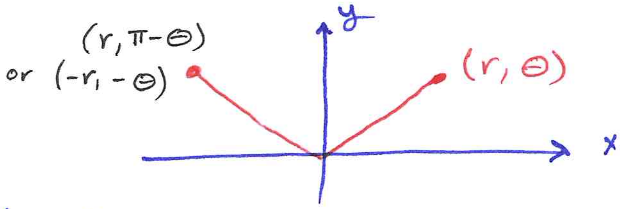
11.4 Graphing in Polar Coordinates

Symmetry Tests for Polar Graphs:

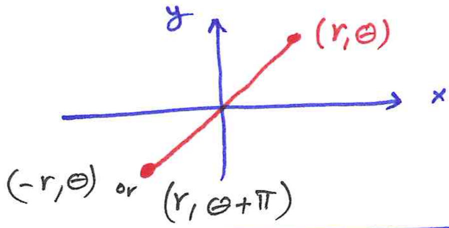
1. Symmetry about x-axis: If the point (r, θ) lies on the graph, then $(r, -\theta)$ or $(-r, \pi-\theta)$ lies on the graph.



2. Symmetry about y-axis: If the point (r, θ) lies on the graph, then the point $(r, \pi-\theta)$ or $(-r, -\theta)$ lies on the graph



3. Symmetry about the origin: If the point (r, θ) lies on the graph, then the point $(-r, \theta)$ or $(r, \theta+\pi)$ lies on the graph.



Slope Let $r = f(\theta)$. Recall the parametric equations:

$$x = r\cos\theta = f(\theta)\cos\theta \quad , \quad y = r\sin\theta = f(\theta)\sin\theta \qquad r' = f'(\theta)$$

slope of the curve $r = f(\theta)$ at (r, θ) is

$$\left.\frac{dy}{dx}\right|_{(r,\theta)} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$$

Proof $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f'\sin\theta + f\cos\theta}{f'\cos\theta - f\sin\theta}$

Note that when the curve $r = f(\theta)$ passes through the origin at $\theta_0 \Rightarrow \left.\frac{dy}{dx}\right|_{(0,\theta_0)} = \tan\theta_0$

Uploaded By: Jibreel Bornat
STUDENTS-HUB.com

Exp Find the slope of $r = \cos 2\theta$ at $\theta = 0, \frac{\pi}{2}$

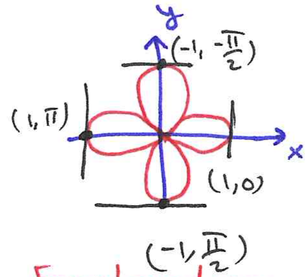
$r' = -2\sin 2\theta$

- when $\theta = 0 \Rightarrow r = 1 \Rightarrow (r, \theta) = (1, 0)$

slope is $\left.\frac{dy}{dx}\right|_{(1,0)} = \left.\frac{r' \sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}\right|_{(1,0)} = \frac{-2\sin(0)\sin(0) + (1)\cos(0)}{-2\sin(0)\cos(0) - (1)\sin(0)} = \frac{1}{0}$ undefined

- when $\theta = \frac{\pi}{2} \Rightarrow r = -1 \Rightarrow (r, \theta) = (-1, \frac{\pi}{2})$

the slope is $\left.\frac{dy}{dx}\right|_{(-1,\frac{\pi}{2})} = \frac{-2\sin(\pi)\sin(\frac{\pi}{2}) + (-1)\cos(\frac{\pi}{2})}{-2\sin(\pi)\cos(\frac{\pi}{2}) - (-1)\sin(\frac{\pi}{2})} = 0$



Four-leaved rose

Exp sketch the grave of the following curves, Identify the symmetry

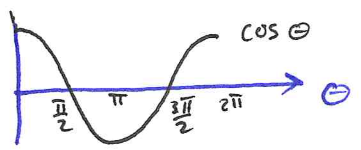
1) $r = 1 - \cos\theta$

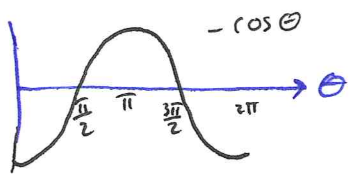
- (r, θ) on the graph $\Rightarrow r = 1 - \cos\theta$
  $\Rightarrow r = 1 - \cos(-\theta)$
  $\Rightarrow (r, -\theta)$ on the graph

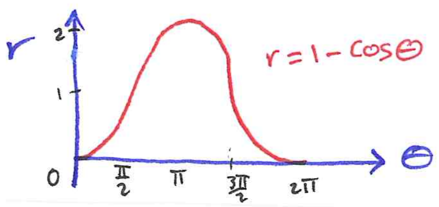
$\Rightarrow$ the curve is symmetric about x-axis.

- $\left.\begin{array}{l} 1 - \cos(-\theta) = 1 - \cos\theta \neq -r \\ 1 - \cos(\pi - \theta) = 1 + \cos\theta \neq r \end{array}\right] \Rightarrow$ the curve is not symmetric about y-axis

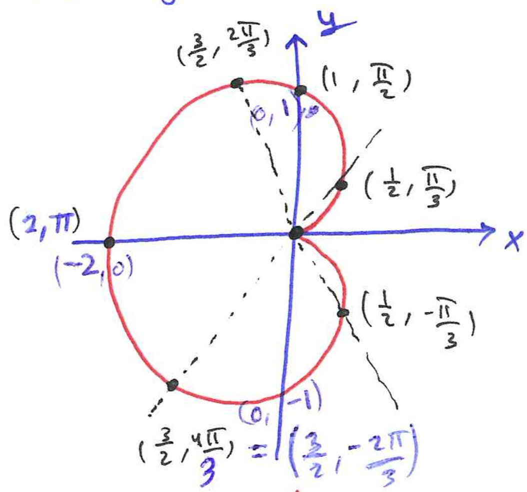
- $\left.\begin{array}{l} 1 - \cos\theta \neq -r \\ 1 - \cos(\theta + \pi) = 1 + \cos\theta \neq r \end{array}\right] \Rightarrow$ the curve is not symmetric about the origin.







θ	$r = 1 - \cos\theta$
0	0
$\frac{\pi}{3}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	$\frac{3}{2}$
π	2



Cardioid

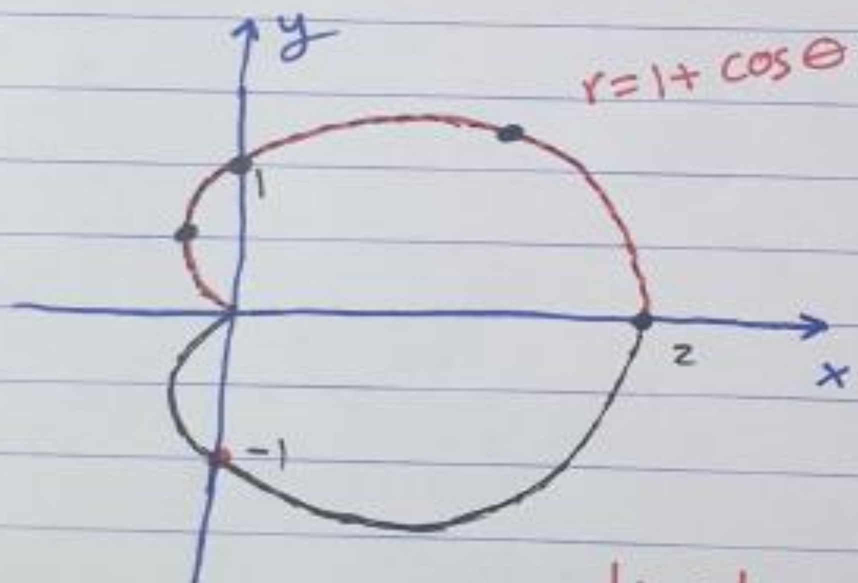
Uploaded By: Jibreel Bornat
STUDENTS-HUB.com

$\boxed{1}$ $r = 1 + \cos\theta$ is

- symmetric about x-axis since
  $1 + \cos(-\theta) = 1 + \cos\theta = r$ ✓

- not symmetric about y-axis since
  $1 + \cos(-\theta) = 1 + \cos\theta \neq -r$ and
  $1 + \cos(\pi - \theta) = 1 + \cos\pi\cos(-\theta) - \sin\pi\sin(-\theta)$
  $= 1 - \cos\theta \neq r$

- Therfore, it is not symmetric about origin.



θ	r
0	2
$\frac{\pi}{3}$	1.5
$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	$\frac{1}{2}$
π	0

cardioid

STUDENTS-HUB.com Uploaded By:

$\boxed{11.4}$ Lecture Problems

$\boxed{4}$ $r = 1 + \sin\theta$ is

- not symmetric about x-axis since

$$1 + \sin(-\theta) = 1 - \sin\theta \neq r \quad \text{and}$$

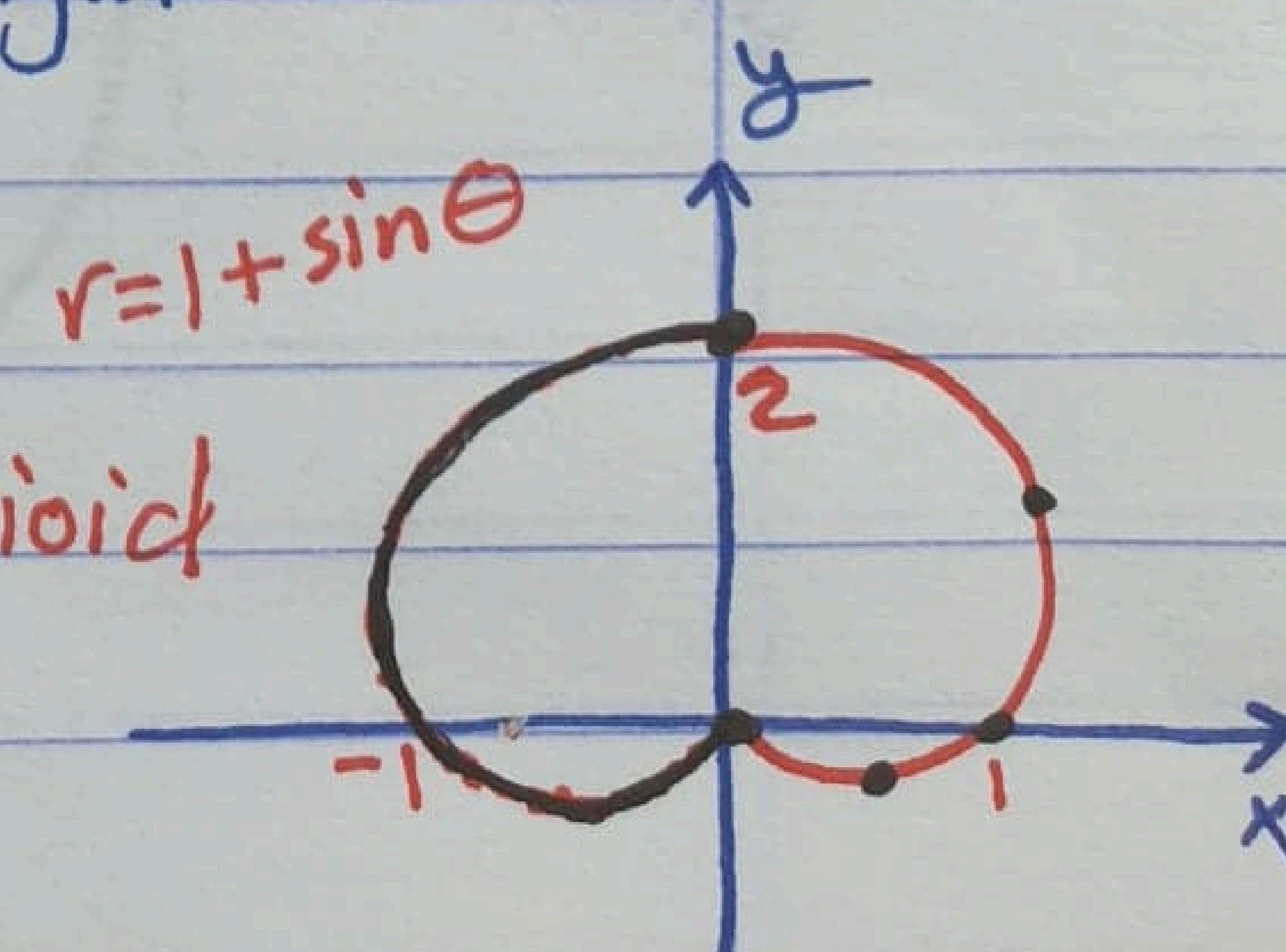
$$1 + \sin(\pi - \theta) = 1 + \sin\pi\cos(-\theta) + \sin(-\theta)\cos\pi$$
$$= 1 + 0 - \sin\theta\,(-1)$$
$$= 1 + \sin\theta \neq -r$$

- symmetric about y-axis since

$$1 + \sin(\pi - \theta) = 1 + \sin\pi\cos(-\theta) + \sin(-\theta)\cos\pi$$
$$= 1 + 0 - \sin\theta\,(-1)$$
$$= 1 + \sin\theta$$
$$= r \checkmark$$

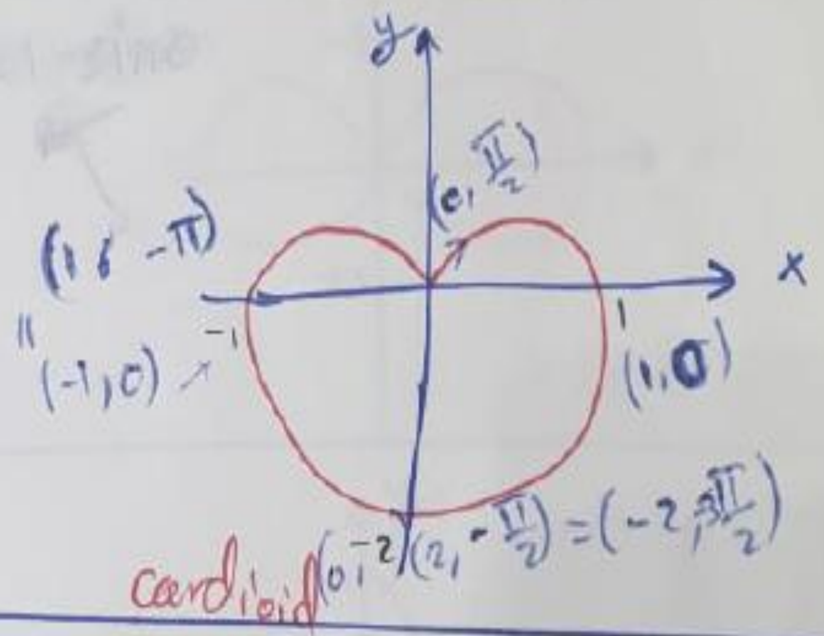
- Therefore, no symmetry about origin

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$
r	0	$-\frac{1}{2}$	1	$\frac{3}{2}$	2



$\boxed{4}$ $r = 1 - \sin\theta$

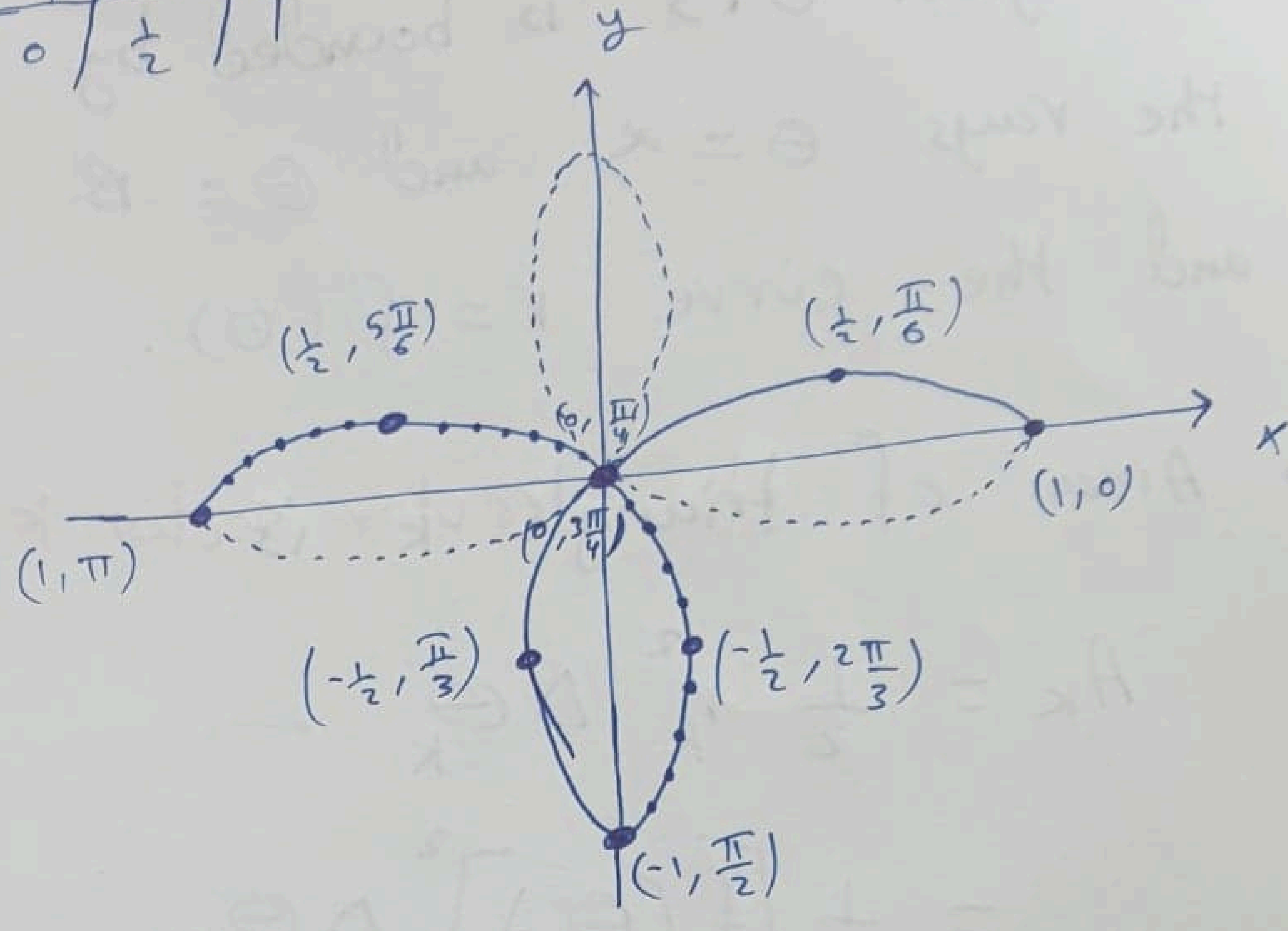
symmetric about y-axis



* $r = \cos 2\theta$

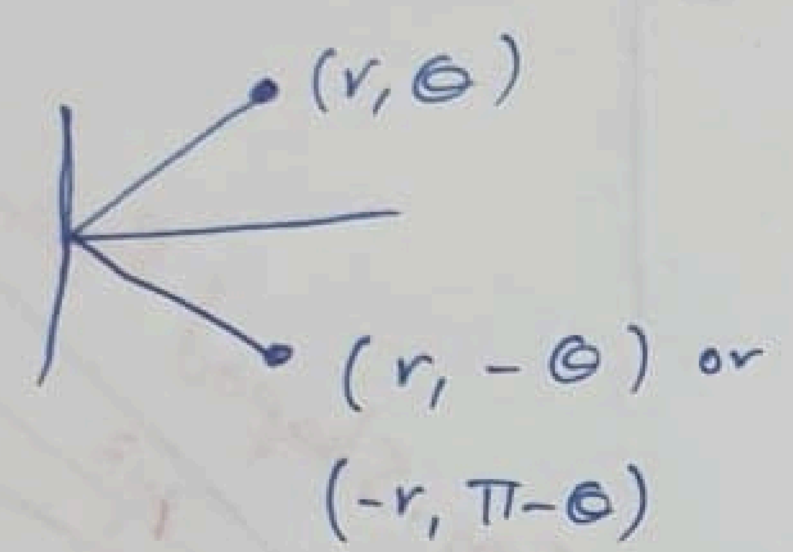
θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
r	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1

is enough



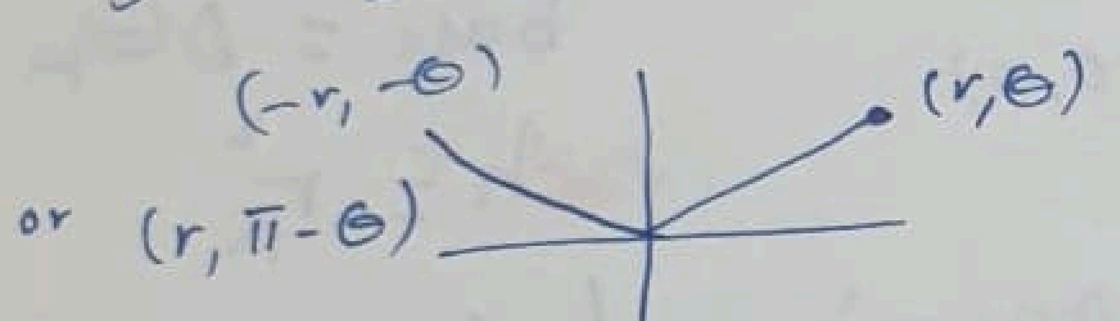
- symmetry about x-axis since

$$r = \cos 2\theta = \cos 2(-\theta)$$



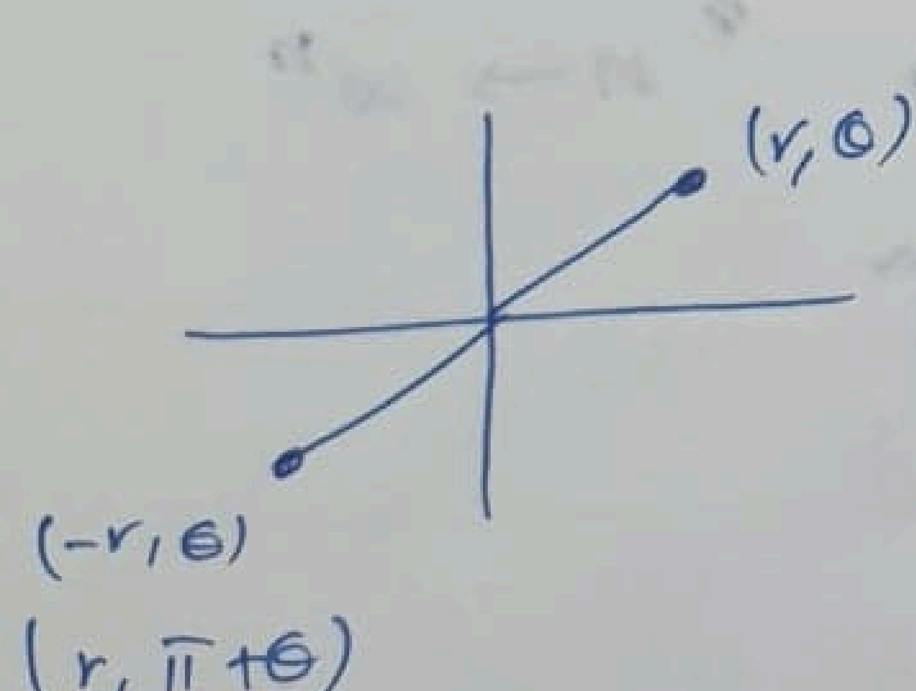
- symmetry about y-axis since $r = \cos 2\theta = \cos 2(\pi - \theta)$

$$= \cos(2\pi - 2\theta)$$
$$= \cos 2\pi \cos(-2\theta) - \sin 2\pi \sin(-2\theta)$$
$$= (1) \cos 2\theta - 0$$
$$= \cos 2\theta$$
$$= r$$



- Hence, symmetry about origin too. That is $r = \cos 2\theta = \cos 2(\pi + \theta)$

$$= \cos(2\pi + 2\theta)$$
$$= \cos 2\pi \cos 2\theta - \sin 2\pi \sin 2\theta$$
$$= (1) \cos 2\theta - 0$$
$$= \cos 2\theta$$
$$= r$$



STUDENTS-HUB.com Uploaded By: Jibreel Bornat

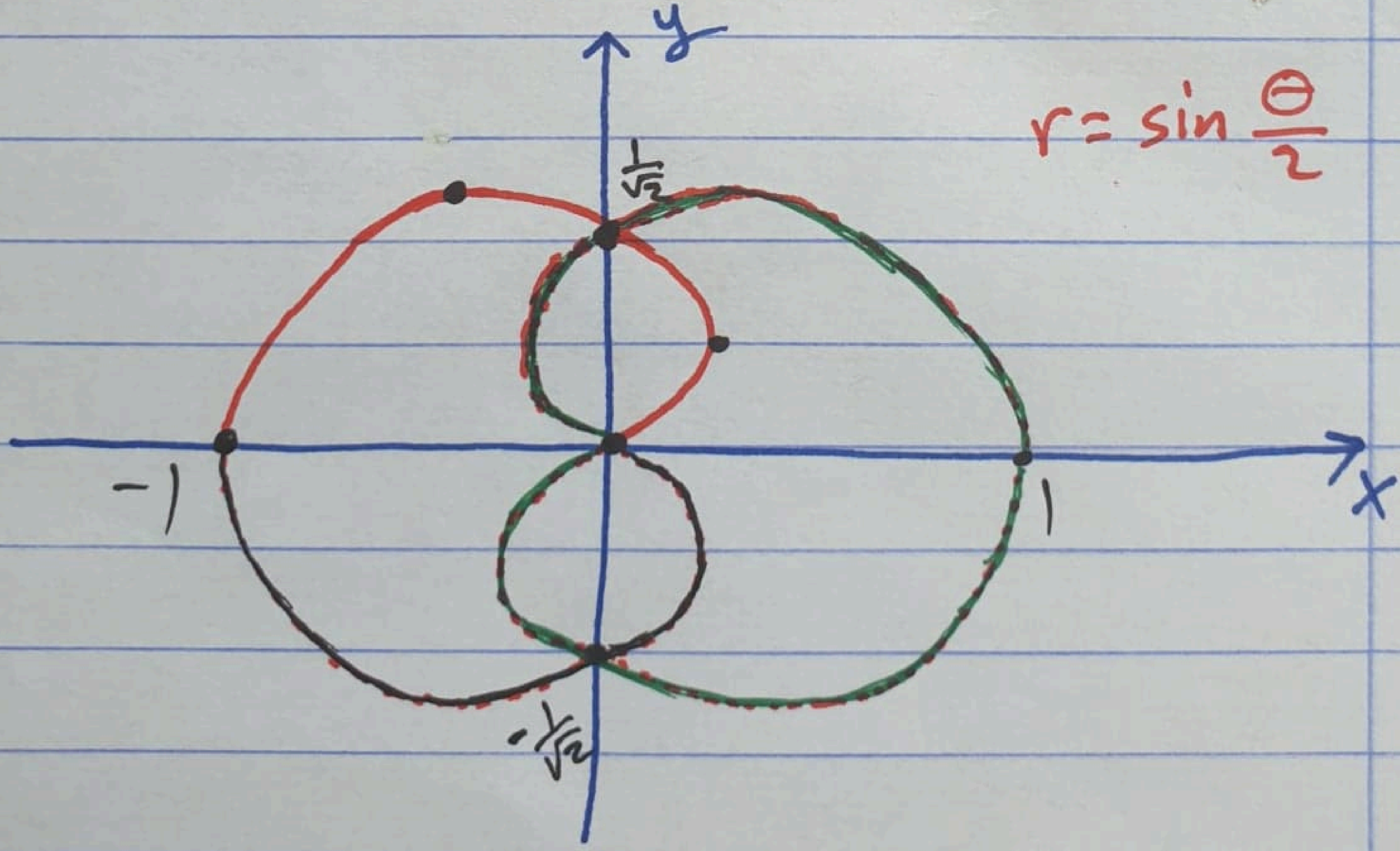
$\boxed{7}$ $r = \sin\left(\frac{\theta}{2}\right)$ is $\boxed{2}$

- symmetric about y-axis since
  $\sin\left(-\frac{\theta}{2}\right) = -\sin\left(\frac{\theta}{2}\right) = -r$ ✓

- symmetric about x-axis since
  $\sin\left(\pi - \frac{\theta}{2}\right) = \sin\pi \cos\left(-\frac{\theta}{2}\right) + \sin\left(-\frac{\theta}{2}\right)\cos(\pi)$
  $= 0 - \sin\left(\frac{\theta}{2}\right)(-1) = \sin\left(\frac{\theta}{2}\right) = r$ ✓

- Hence, it is symmetric about origin ✓

θ	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{4\pi}{3}$	π
r	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1



STUDENTS-HUB.com Uploaded By: Jibreel Bornat