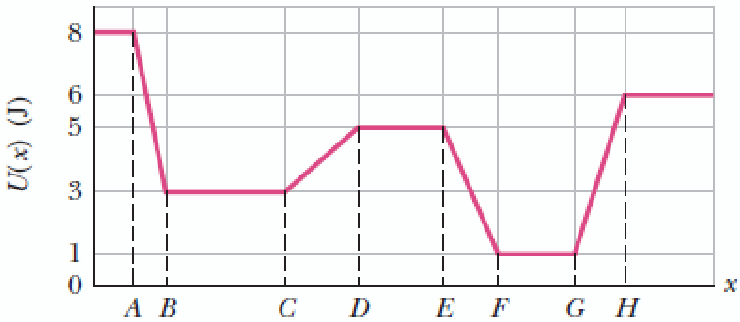
Chapter 8: Potential Energy and Conservation of Energy

Q-2 The below figure gives the potential energy function of a particle.
(a) Rank regions AB, BC, CD and DE according to the magnitude of the force on the particle, greatest first. What value must the mechanical energy E_{mec} of the particle not exceed if the particle is to be (b) trapped in the potential well at the right, (c) trapped in the potential well at the Left, and (d) able to move between the two potential wells but not to the right of point H? For the situation of (d), in which of regions BC, DE, and FG will the particle have (e) the greatest Kinetic energy and (f) the least speed?



(a) $F(x) = -\dfrac{dU(x)}{dx}$

$AB > CD > BC = DE = \text{Zero}$

(b) Particle is trapped in the potential well at the right and cannot go out; the particle has zero Kinetic energy $K=0$
($E_{mec} = K + U$; for $K=0$; $E_{mec} = U$)
$\Rightarrow$ E_{mec} does not exceed : **5** J

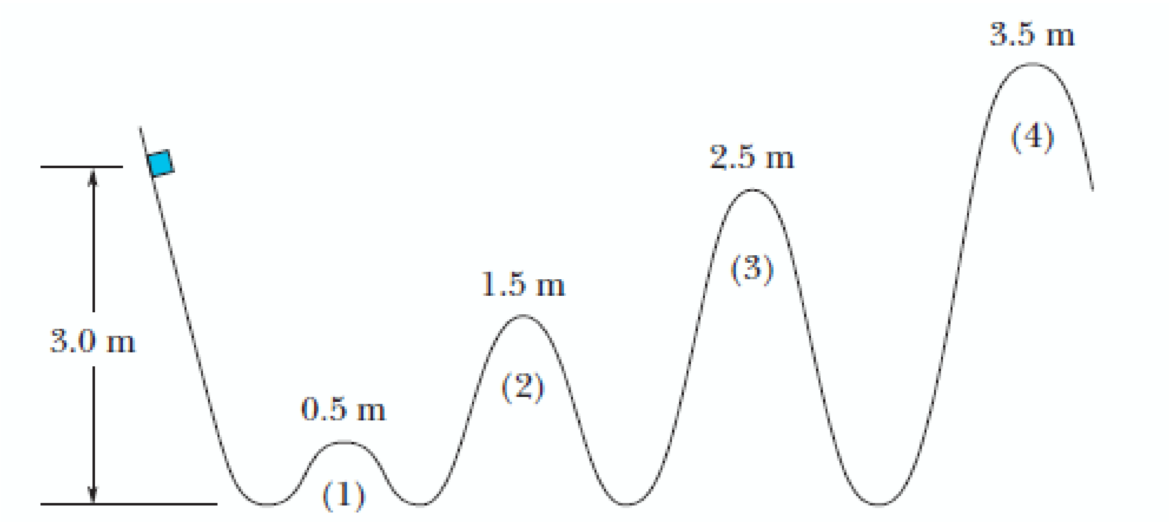
c) The particle trapped in the left potential well if E_{mec} does not exceed **5** J

d) Move between the two potential wells but not to the right of point H if mechanical energy is between **5** J and **6** J, so it should not exceed **6** J.

e) $E_{mec} = K + U$; the particle will have the greatest Kinetic energy where potential energy is the least, which in region FG

f) Least speed $\rightarrow$ Least Kinetic energy $\rightarrow$ Largest potential energy
region DE

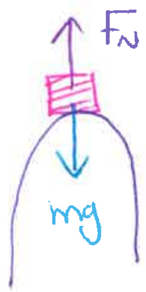
Q-4 A small, initially stationary block is released on a frictionless ramp at a height of 3.0 m. Hill heights along the ramp are as shown in the figure. The hills have identical circular tops, and the block does not fly off any hill. (a) Which hill is the first the block cannot cross? (b) What does the block do after failing to cross that hill? Of the hills that the block can cross, on which hilltop is (c) the centripetal acceleration of the block greatest and (d) the normal force on the block least?



(a) Hill (4), The block cannot cross it

(b) Reverse direction, crossing hills 2, 3 and 1, and returing to height 3.0 m (starting point) then continuing forward and back forward.

(c)



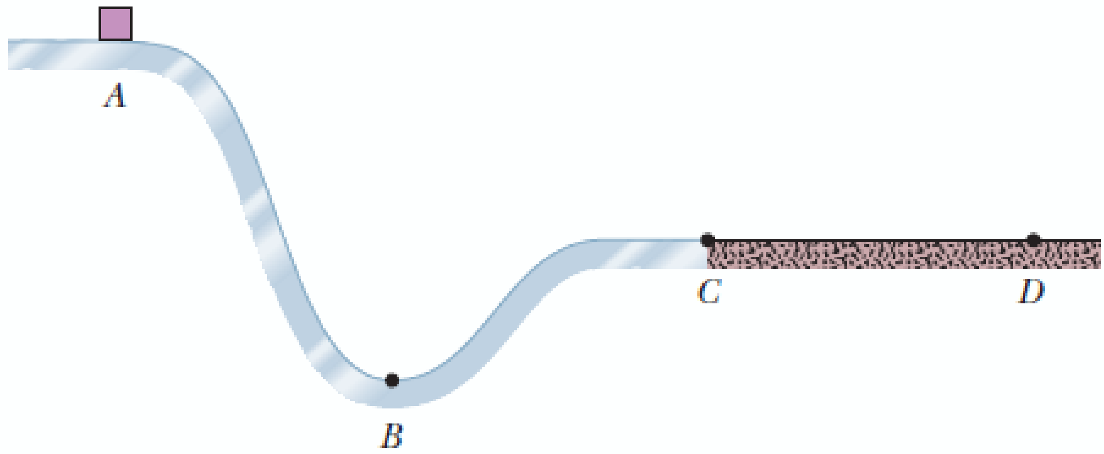
$$F_c = mg - F_N = m\frac{v^2}{R} = m a_c$$

· Larger acceleration ⟷ Larger speed ⟷ Smaller F_N

$$F_N = mg - m\frac{v^2}{R}$$

On hill 1 the centripetal acceleration is the Largest and the normal force is the least.

Q-5 A block slides from A to C along a frictionless ramp, and then it passes through horizontal region CD, where a frictional force acts on it. Is the block's kinetic energy increasing, decreasing, or constant in (a) region AB, (b) region BC, and (c) region CD? (d) Is the block's mechanical energy increasing, decreasing, or constant in those regions?



* A → C, No Friction ⇒ $\Delta E_{mec} = $ zero
C → D, Friction ⇒ The mechanical energy is not conserved

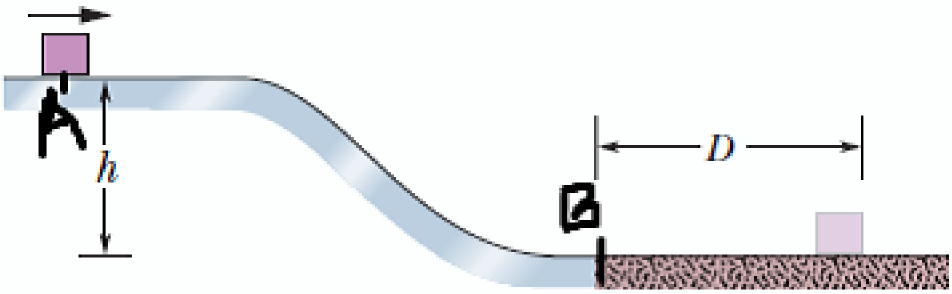
(a) $K_{A\to B}$ increasing

(b) $K_{B\to C}$ decreasing

(c) $K_{C\to D}$ decreasing

(d) Block's mechanical energy is constant in AB and BC regions and decreasing in region CD

Q-8 A block slides along a track that descends through distance h. The track is frictionless except for the lower section. There the block slides to a stop in a certain distance D because of the friction. (a) If we decrease h, will the block now slide to a stop in a distance that is greater than, less than, or equal to D? (b) If, instead, we increase the mass of the block, will the stopping distance now be greater than, less than, or equal to D?



(a) If we decrease h, the block will slide to a stop in a distance less than D.

During sliding over a frictionless part of the track ⇒ the mechanical energy of the system is conserved $\Delta E_{mec} = \text{Zero}$

$$K_A + U_A \longrightarrow K_B \iff mgh + \frac{1}{2} m v_A^2 = \frac{1}{2} m v_B^2$$

When the block cross the friction part, the block's mechanical energy will be dissipated through the work of the friction $(W = \vec{f}_k \cdot \vec{D} = -f_k D)$

$W = -\Delta E_{th}$

(b) If we increase the mass of the block, the stopping distance will be equal to distance D.

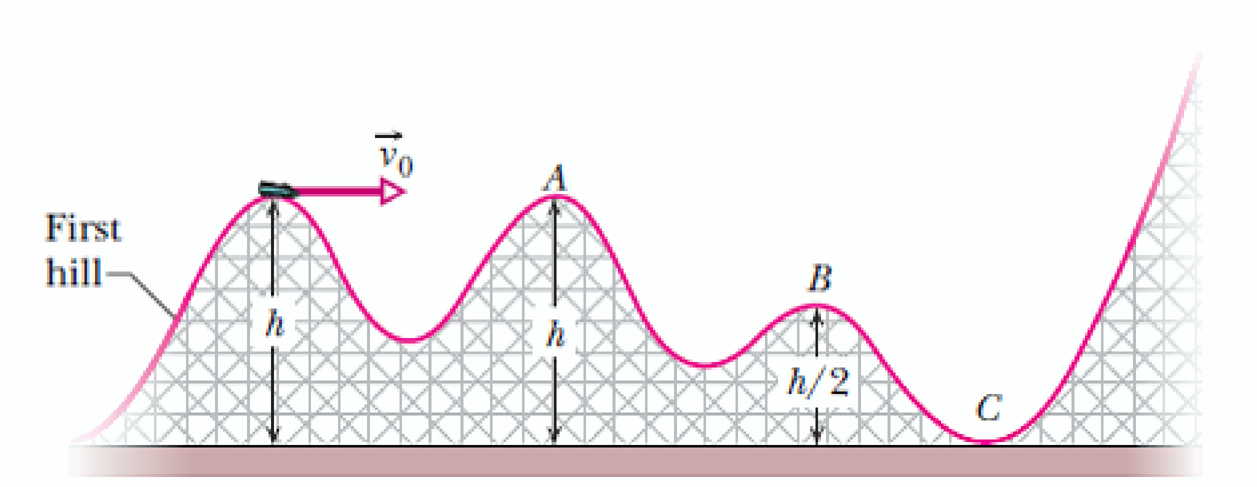
$v_A = v_i$

$$mgh + \frac{1}{2} m v_i^2 = \mu_k mg D$$

$$D = \frac{gh + \frac{v_i^2}{2}}{\mu_k g} = \frac{h + (v_i^2/2g)}{\mu_k}$$

not depends on $\underline{\underline{m}}$ the mass

P-2 A single frictionless roller-coaster car of mass $m = 825$ kg tops the first hill with speed $v_0 = 17.0$ m/s at height $h = 42.0$ m. How much work does the gravitational force do on the car from that point to (a) Point A, (b) point B, and (c) Point C? If the gravitational potential energy of the car-Earth system is taken to be zero at C, what is its value when the car is at (d) B and (e) A? (f) If mass m were doubled, would the change in the gravitational potential energy of the system between point A and B increase, decrease, or remain the same?



(a) $W_g = mgd\cos\phi$ "Work done by gravitational force"

$\phi = 90^\circ$ (mg downward and d is horizontal)

$\boxed{W_g = \text{Zero}}$

b) the vertical displacement $= h/2$ downward $(\phi = 0^\circ)$

$$W_g = \vec{F}_g \cdot \vec{d} = mgd\cos\phi = mg\frac{h}{2}$$

$$W_g = (825\text{ kg})(9.8\text{ m/s}^2)((42.0/2)\text{m})$$

$$\boxed{W_g = 1.7 \times 10^5\text{ J}}$$

(c) The vertical displacement = h downward ($\phi = 0°$)

$$W_g = mgh = (825\text{ kg})(9.8\text{ m/s}^2)(42.0\text{ m})$$

$$\boxed{W_g = 3.4\times10^5\text{ J}}$$

(d) U_c = zero

$U_B = \frac{1}{2}mgh = 1.7\times10^5$ J

(e) $U_A = mgh = 3.4\times10^5$ J

(f) If the mass is doubled, the gravitational potential energy is doubled.

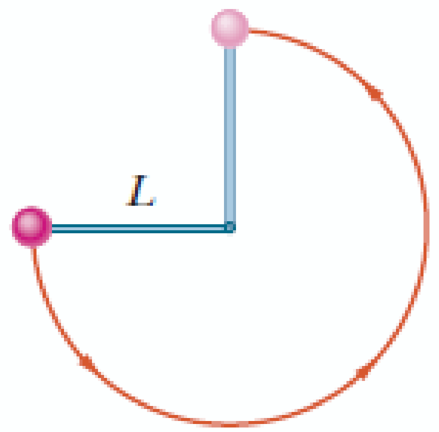
$U_B = 3.4\times10^5$ J

$U_A = 6.8\times10^5$ J

The change in the gravitational potential energy of the system between A & B increased as the mass is doubled.

P-4 The below figure shows a ball with mass m = 0.341 Kg attached to the end of a thin rod with length L = 0.452 m and negligible mass. The other end of the rod is pivoted so that the ball can move in a vertical circle. The rod is held horizontally as shown and then given enough of a downward push to cause the ball to swing down and around and just reach the vertically up position, with zero speed there. How much work is done on the ball by the gravitational force from the initial point to (a) the lowest point, (b) the highest point, and (c) the point on the right level with the initial point? If the gravitational potential energy of ball-Earth system is taken to be zero at the initial point, what is it when the ball reaches (d) the lowest point, (e) the height point, and (f) the point on the right level with the initial point? (g) suppose the rod were pushed harder so that the ball passed through the heighest point with a nonzero speed. Would ΔU_g from the lowest point to the height point then be greater than, less than, or the same as it was when the ball stopped at the heighest point?

The gravitational force is conservative force:
. The work & change in potential energy only depend on the initial and final position.



(a) W_g from the initial point to the lowest point

$W = mgL$

$(0.341 Kg)(9.8 m/s^2)(0.452 m)$

$\boxed{W = 1.51 J}$

b) Initial point ⟶ Highest point
($\phi = 180°$ between the displacement and the gravitational force)

$\boxed{W = -mgL = -1.51 J}$

(c) $W = $ zero

$(\phi = 90°) \Rightarrow$ the displacement in the direction of the gravitational force is zero

The potential is zero at the initial point

(d) $\boxed{U_{\text{Lowest point}} = mg(-L) = -mgL = -1.51\ J}$

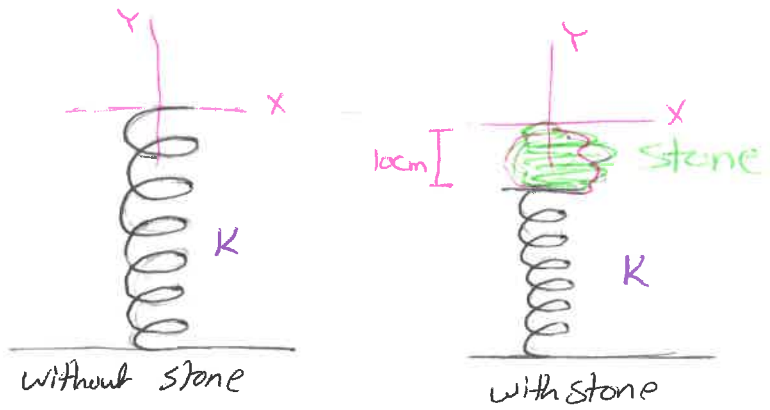
(e) $\boxed{U_{\text{Highest point}} = mgL = 1.51\ J}$

f) $U = 0\ J$ (at the right point at the same level with the initial point)

g) the change in the gravitational potential energy depends on the initial and final positions of the ball, not on its speed anywhere.

$\Rightarrow \Delta U_g$ is the same since the initial and final positions are the same

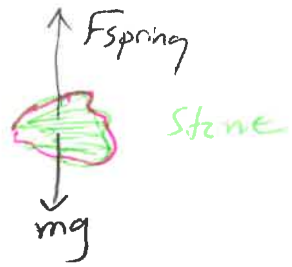
P-19 An 8.00 kg stone at rest on a spring. The spring is compressed 10.0cm by the stone. (a) What is the spring constant? (b) The stone is pushed down an additional 30.0 cm and released. What is the elastic potential energy of the compressed spring just before that release? (c) What is the change in the gravitational potential energy of the stone - Earth system when the stone moves from the release point to its maximum height? (d) What is that maximum height, measured from the release point?



$y_0 = -10\text{ cm} = -0.1\text{ m}$

initial compression

• Relaxed position of the spring at origin point



(a) Stone at rest $\Rightarrow a = \text{zero},\ F_{net} = 0$

$$F_{net} = \text{zero}$$

$$F_{spring} - mg = 0$$

$$-Ky - mg = 0 \rightarrow -K(-0.1\text{m}) = mg$$

$$K = \frac{8\text{ Kg}(9.8\text{ m/s}^2)}{0.1\text{ m}}$$

$$\boxed{K = 784\ N/m}$$

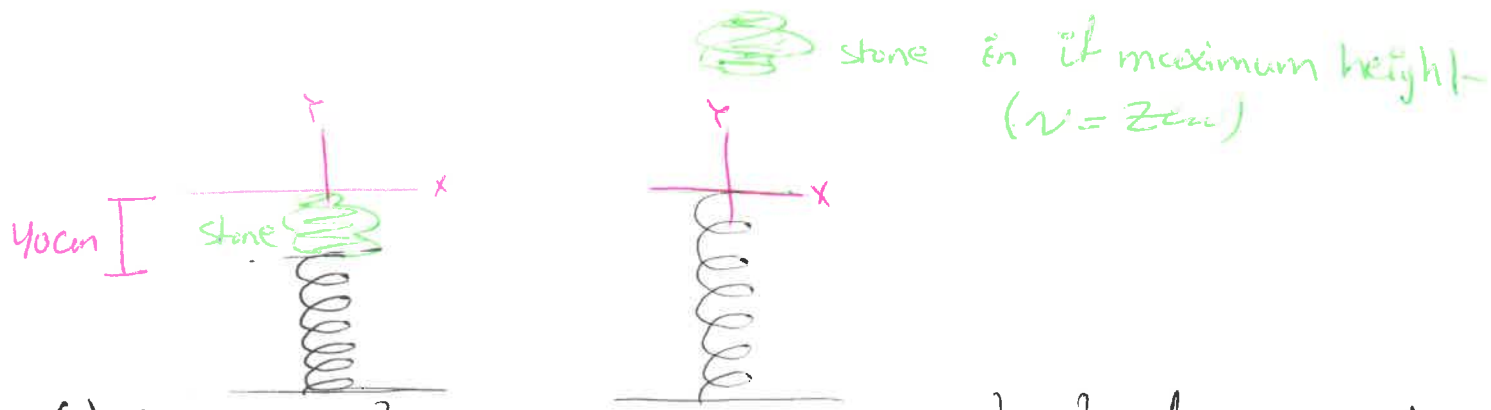
(b) $U = \frac{1}{2}Ky^2 = \frac{1}{2}\left(784\frac{N}{m}\right)(-0.4\text{ m})^2$

$$\boxed{U = 62.7\text{ J}}$$

(c) Mechanical Energy is conserved

$$K_1 + U_1 = K_2 + U_2$$

$$0 + \frac{1}{2} K (-0.4)^2 = 0 + mgh$$



(1) Stone compressed the spring by 40 cm

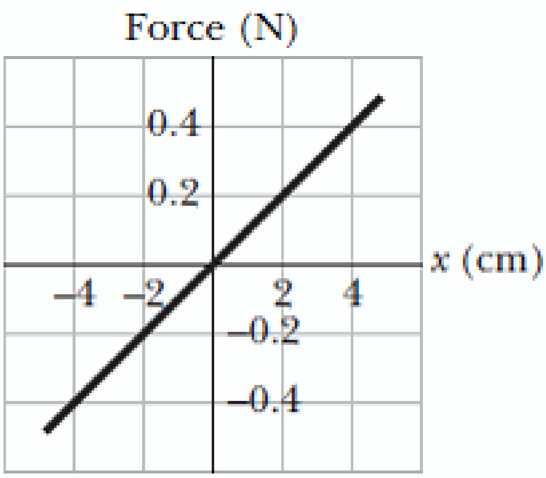
(2) stone is released and in its maximum height

$$mgh = 62.7\ J$$

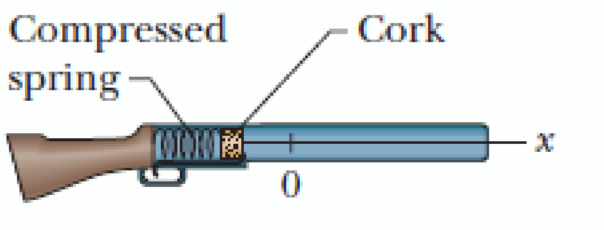
(d) $h = \dfrac{62.7\ J}{mg} = \dfrac{62.7\ J}{(8)(9.8)N} = 0.7997\,m$

$$\boxed{h = 80\ cm}$$

P-28 Figure (a) applies to the spring in a cork gun (Figure b); it shows the spring force as a function of the stretch or compression of the spring. The spring is compressed by 5.5 cm and used to propel a 3.8 g cork from the gun. (a) What is the speed of the cork if it released as the spring passes through its relaxed position? (b) Suppose, Instead, that the cork sticks to the spring and stretches it 1.5 cm before separation occurs. What now is the speed of the cork at the time of release?



(a)



(b)

Spring Constant $\Rightarrow K = \frac{\Delta F}{\Delta x} = \frac{0.1\,N}{cm} = 10\,N/m$

(a) $\Delta E_{mec} = $ Zero $\Longleftrightarrow \frac{1}{2} K x^2 = \frac{1}{2} m v^2$

$$v = x\sqrt{\frac{K}{m}} = 5.5\times10^{-2}\,m\sqrt{\frac{10\,N/m}{3.8\times10^{-3}\,Kg}}$$

$$\boxed{v = 2.8\,m/s}$$

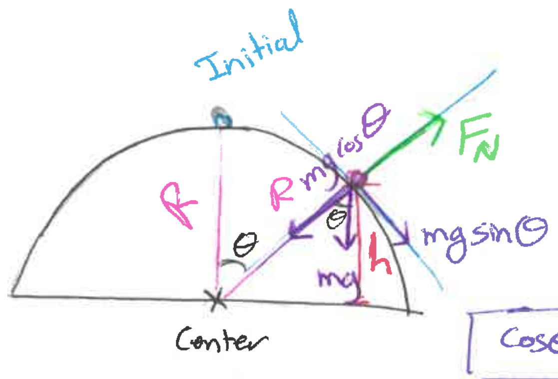
(b) $\Delta E_{mec} = $ Zero $\Longleftrightarrow \frac{1}{2} K x^2 = \frac{1}{2} m v^2 + \frac{1}{2} K x_{stretching}$

$$v^2 = \frac{K}{m}\left(x^2 - x^2_{stretching}\right)$$

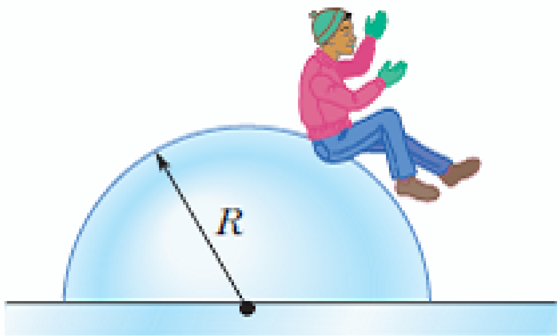
$$v^2 = \frac{10\,N/m}{0.0038\,Kg}\left((5.5\times10^{-2})^2 - (1.5\times10^{-2})^2\right)$$

$$\boxed{v = 2.7\,m/s}$$

P-34 A boy is initially seated on the top of a hemispherical ice mouned of radius $R = 13.8$ m. He begins to slide down the ice, with a negligible initial speed. Approximate the ice as being frictionless. At what height does the boy lose contact with the ice?



$\boxed{\cos\Theta = h/R}$



$$F_c = \frac{mv^2}{R} = mg\cos\Theta - F_N$$

When the boy lose contact with ice $\Rightarrow F_N = 0$

$$mg\cos\Theta = \frac{mv^2}{R}$$

$$g\cos\Theta = \frac{v^2}{R} \longrightarrow \boxed{v^2 = Rg\cos\Theta}$$

- Conservation of mechanical energy

$$\Delta E_{mec} = 0 \leftrightarrow E_{mec, top} = E_{mec, \text{lose the contact with ice}}$$

$$K_i + U_i = K_f + U_f$$

$$0 + mgR = \frac{1}{2}mv^2 + mgh \; ; \; h = R\cos\Theta$$

$$gR = \frac{v^2}{2} + gR\cos\Theta$$

$$gR = \frac{Rg\cos\Theta}{2} + Rg\cos\Theta$$

$$1 = \frac{3}{2}\cos\Theta$$

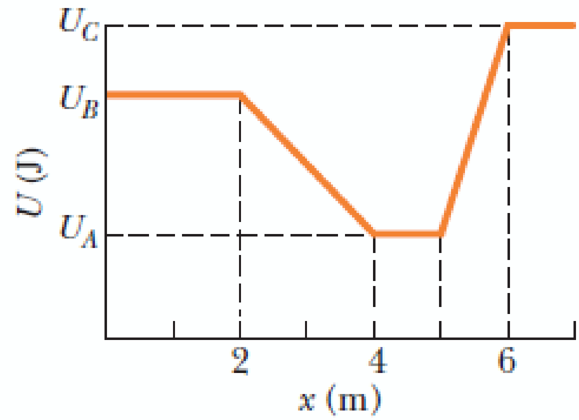
$$\cos\Theta = \frac{2}{3} \; , \; \boxed{\Theta = 48.2^\circ}$$

$$h = R\cos\Theta$$

$$h = 13.8\cos(48.2^\circ)$$

$$\boxed{h = 9.2 \text{ m}}$$

P-39 The figure shows a plot of potential energy U versus position x of a 0.90 Kg particle that can travel only along an x axis. (Nonconservative forces are not involved.) Three values are $U_A = 15.0\,J$, $U_B = 35.0\,J$, and $U_C = 45.0\,J$. The particle is released at $x = 4.5\,m$ with an initial speed of $7.0\,m/s$, headed in the negative x direction. (a) If the particle can reach $x = 1.0\,m$, what is its speed there, and if it cannot, what is its turning point? What are the (b) magnitude and (c) direction of the force on the particle as it begins to move to the left of $x = 4.0\,m$? Suppose, Instead, the particle is headed in the positive x direction when it is released at $x = 4.5\,m$ at speed $7.0\,m/s$ (d) If the particle can reach $x = 7.0\,m$, what is its speed there, and if it cannot, what is its turning point? What are the (e) magnitude and (f) direction of of the force on the particle as it begins to move to the right of $x = 5.0\,m$?



• At $x = 4.5\,m$, $U = 15.0\,J$, $v = 7.0\,\frac{m}{s}$

$$E_{mec} = U + K = 15 + \left[\tfrac{1}{2}(0.9)(7.0)^2\right]$$

$$\boxed{E_{mec} = 37.05\,J}$$

(a) At $x = 1.0\,m$, $U = 35.0\,J$

$$E_{mec} = K + U \Rightarrow K = 2.0\,J > 0$$

So the particle can reach $x = 1.0\,m$ with $\boxed{v = 2.11\,m/s}$

$$K = 2\,J = \tfrac{1}{2} m v^2$$

b) $$F(x) = -\frac{dU(x)}{dx} = -\frac{35\,J - 15\,J}{2\,m - 4\,m} = +10.0\,N$$

c) $F_x > 0$; The force points in $+x$-direction

d) At $x = 7.0\,m \rightarrow U = 45\,J$; the particle can never reach there.

Turning points $\Rightarrow K = $ zero "between $x = 5.0\,m$ and $6.0\,m$"

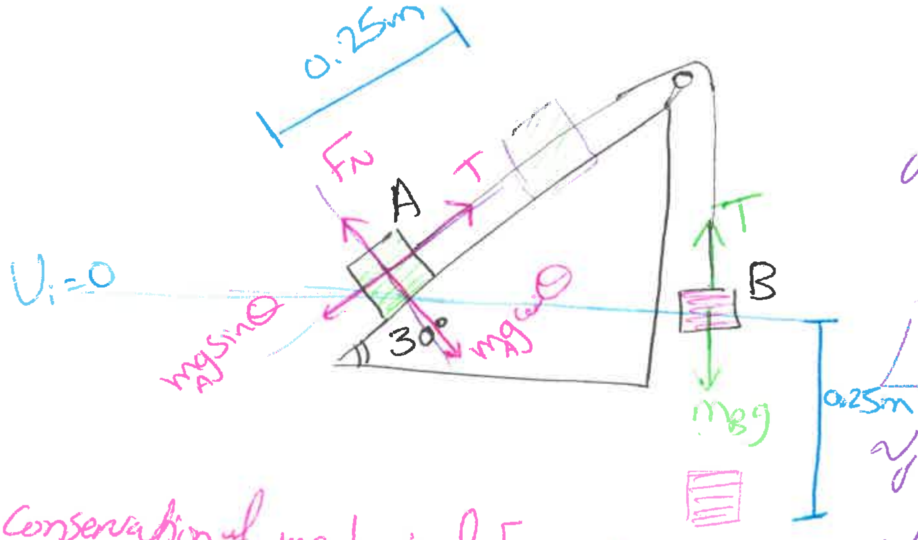
The slope of a straight line is constant $\Rightarrow (5, 15), (x, 37), (6, 45)$

$$\frac{45-15}{6-5} = 30 = \frac{37-15}{x-5} \Longrightarrow \boxed{\underset{\text{turning point}}{x = 5.7\,m} \text{ " } U = 37\,J,\ K = 0 \text{ "}}$$

e) At $x = 5.0\,m$, $$F_x = -\frac{\Delta U}{\Delta x} = -\frac{(45-15)\,J}{(6-5)\,m} = -30\,N$$

f) $F_x < 0$, the force points in the negative x direction

P-69 The pulley has negligible mass, and both it and the inclined plane are frictionless. Block A has a mass of 1.0 Kg, Block B has a mass of 2.0 Kg, and angle θ is 30°. If the blocks are released from rest with the connecting cord taut, what is their total kinetic energy when block B has fallen 25 cm?



$$m_B g - m_A g\sin\theta = (m_A + m_B)a$$

$$a = \frac{m_B g - m_A g\sin\theta}{m_B + m_A}$$

$$\boxed{a = 4.9\ m/s^2}$$

$$v_f^2 = v_i^2 + 2a(\Delta y);\ v_i = 0$$

$$v_f^2 = 2.45\,(m/s)^2$$

$$K_{System} = \frac{1}{2}(m_A + m_B)v^2$$

$$\boxed{K = 3.675 = 3.7\ J}$$

By Conservation of mechanical Energy:

$$K_i + U_i = K_f + U_f$$

$$0 + 0 = K_f + U_{fA} + U_{fB}$$

$$U_{fB} = m_B g(\bar{0}.25m) = -4.9\ J$$

$$U_{fA} = m_A g(0.25\sin 30°) = 1.225$$

$$\Rightarrow K_f = -(U_{fA} + U_{fB})$$

$$K_f = -(1.225 - 4.9)$$

$$K_f = +3.675\ J$$

$$\boxed{K = 3.7\ J}$$