ساره بجار

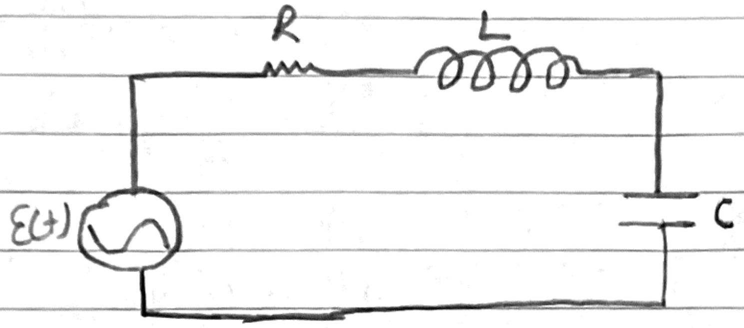
Experiment 8

Impedance and Reactance

* Impedance (Z) is a measure of opposing overall components to electric flow (AC or DC)

* Reactance (X) is a measure of opposing by individual components like inductor and capacitor to electric flow when AC passing through the circuit.

* RLC circuit



* Impedance = Rsistance + j Reactance ⇒ complep number

$$Z = R + jX \quad , \text{ where } j = \sqrt{-1}$$

* Resistive impedance $Z_R \Rightarrow Z_R = R$

Capacitive impedance $Z_C \Rightarrow Z_C = \frac{-j}{\omega C}$

Inductive impedance $Z_L \Rightarrow Z_L = j\omega L$

a The unit of impedance is the ohm (Ω), $j = \sqrt{-1}$ hence the impedance Z is defined as a complex number

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* Capacitive reactance $(X_c) \Rightarrow X_c = \frac{-1}{wC}$

Inductive reactance $(X_L) \Rightarrow X_L = wL$

* equivilent impedance (Z_{eq})

$$\Rightarrow \quad Z_{eq} = Z_R + Z_L + Z_C$$

$$= R + jwL - \frac{j}{wC}$$

$$Z_{eq} = R + j\left(wL - \frac{1}{wC}\right)$$

$$Z_{eq} = R + j\,(X_L + X_c)$$

قيمة $Z_{eq} = \sqrt{R^2 + (X_L + X_c)^2}$

$$Z_{eq} = \sqrt{R^2 + \left(wL - \frac{1}{wC}\right)^2}$$

* The current passing through The RLC circuit is given by:

$$I(t) = \frac{\mathcal{E}(t)}{Z_{eq}} \quad \text{(Generalized Ohm's law)}$$

but $I(t) = I_o \cos(\omega t + \phi)$, $\mathcal{E}(t) = \mathcal{E}_o \cos(\omega t + \phi)$

$$\Rightarrow \quad I_o \cancel{\cos(\omega t + \phi)} = \frac{\mathcal{E}_o \cancel{\cos(\omega t + \phi)}}{Z_{eq}}$$

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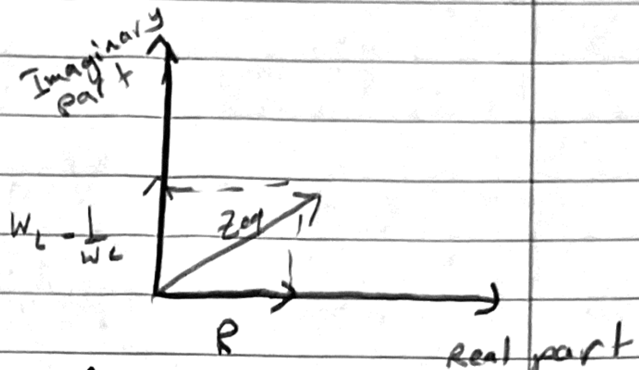
$$I_0 = \frac{\mathcal{E}_0}{Z_{eq}} \Rightarrow I_0 = \frac{\mathcal{E}_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

I is max when $\omega L = \frac{1}{\omega C}$

$$2\pi f L = \frac{1}{2\pi f C}$$

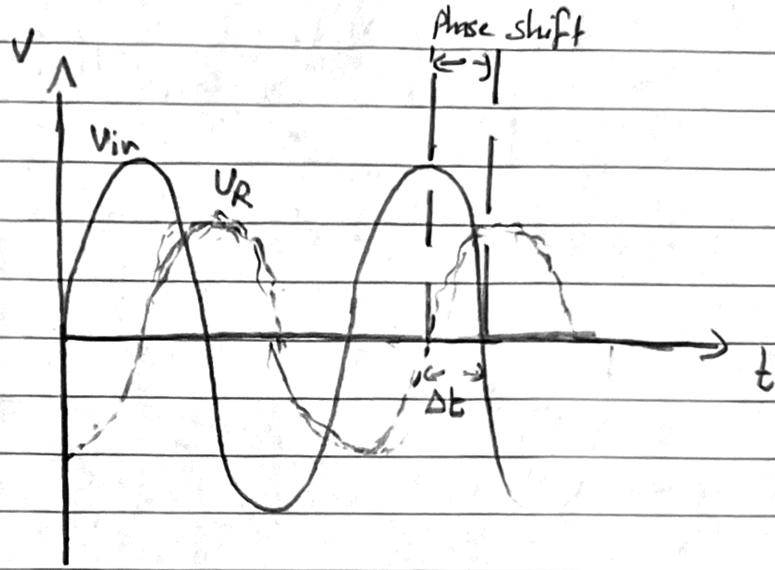
resonance f $= 1/2\pi\sqrt{LC}$

$$\phi = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right)$$



Where is ϕ is the phase shift between the voltage $\mathcal{E}(t)$ and current $I(t)$

$\phi = 2\pi f \Delta t$



It is obvious from the fig and equation $I(t) = I_0 \cos(\omega t + \phi)$

that the current leads or lags the voltage by a time interval that is dependent on the frequency of cosine function

⇒ In other words, there exists a phase shift

$\phi = \omega \Delta t$ between them

$\phi = 2\pi f t$

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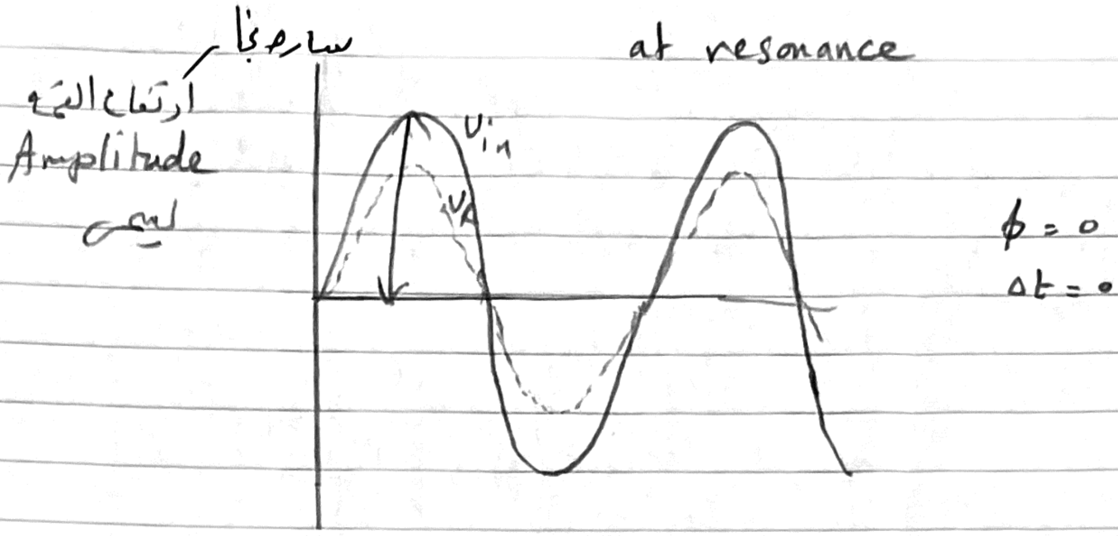

$\ast$ $V_L = L \frac{dI(t)}{dt} = L \frac{d}{dt}(I_0 \cos(\omega t + \phi))$

$$\boxed{V_L = -\omega L I_0 \sin(\omega t + \phi)}$$

$\ast$ $V_R = RI(t) = R I_0 \cos(\omega t + \phi)$

but $\cos\theta = \sin\theta + \frac{\pi}{2}$ متطابقة

$\Rightarrow$ $\boxed{V_R = R I_0 \sin(\omega t + \phi + \frac{\pi}{2})}$

$\ast$ $V_C = \frac{Q}{C}$, $I = \frac{dQ}{dt} \Rightarrow dQ = I dt \Rightarrow Q = \int I dt$

$V_C = \frac{\int I dt}{C} = \int \frac{I_0 \cos(\omega t + \phi)}{C}$

$V_C = \frac{I_0}{C} \frac{1}{\omega} \sin(\omega t + \phi)$

$\Rightarrow$ $\boxed{V_C = \frac{I_0}{\omega C} \sin(\omega t + \phi)}$

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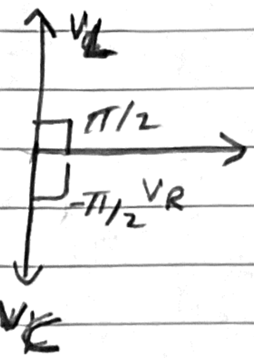
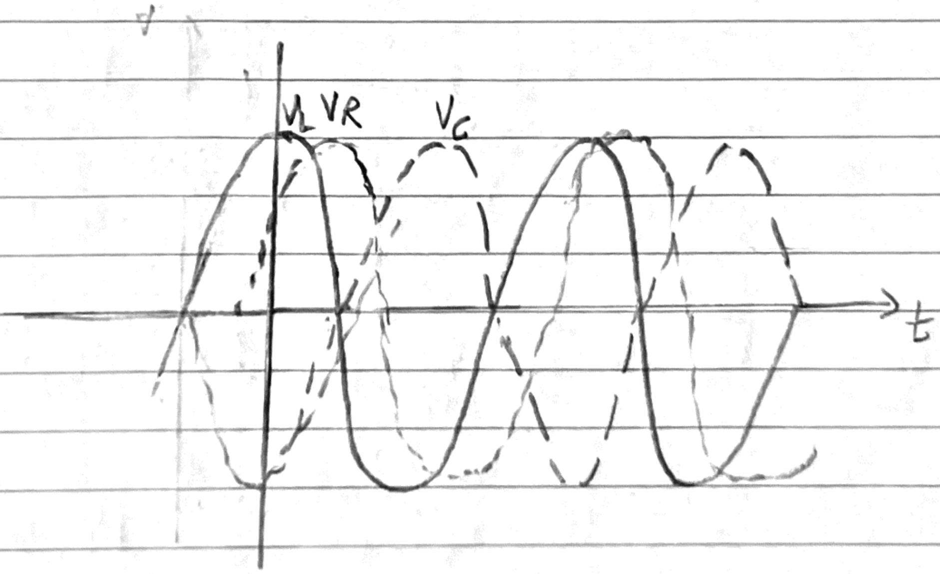
$$V_L = -\omega L I_0 \sin(\omega t + \phi)$$

$$V_R = R I_0 \sin(\omega t + \phi + \frac{\pi}{2})$$

$$V_C = \frac{I_0}{\omega C} \sin(\omega t + \phi)$$



- phase shift btw V_L and V_R $\frac{\pi}{2}$
- phase shift btw V_R and V_C $-\frac{\pi}{2}$
- Not that $V_R = RI_0 \cos(\omega t + \phi)$ also which means that V_R in phase ($\phi = 0$) with $I(t)$
- phase shift btw V_L and V_C is π (out of phase)



Internal mode : x-axis time
y-axis voltage

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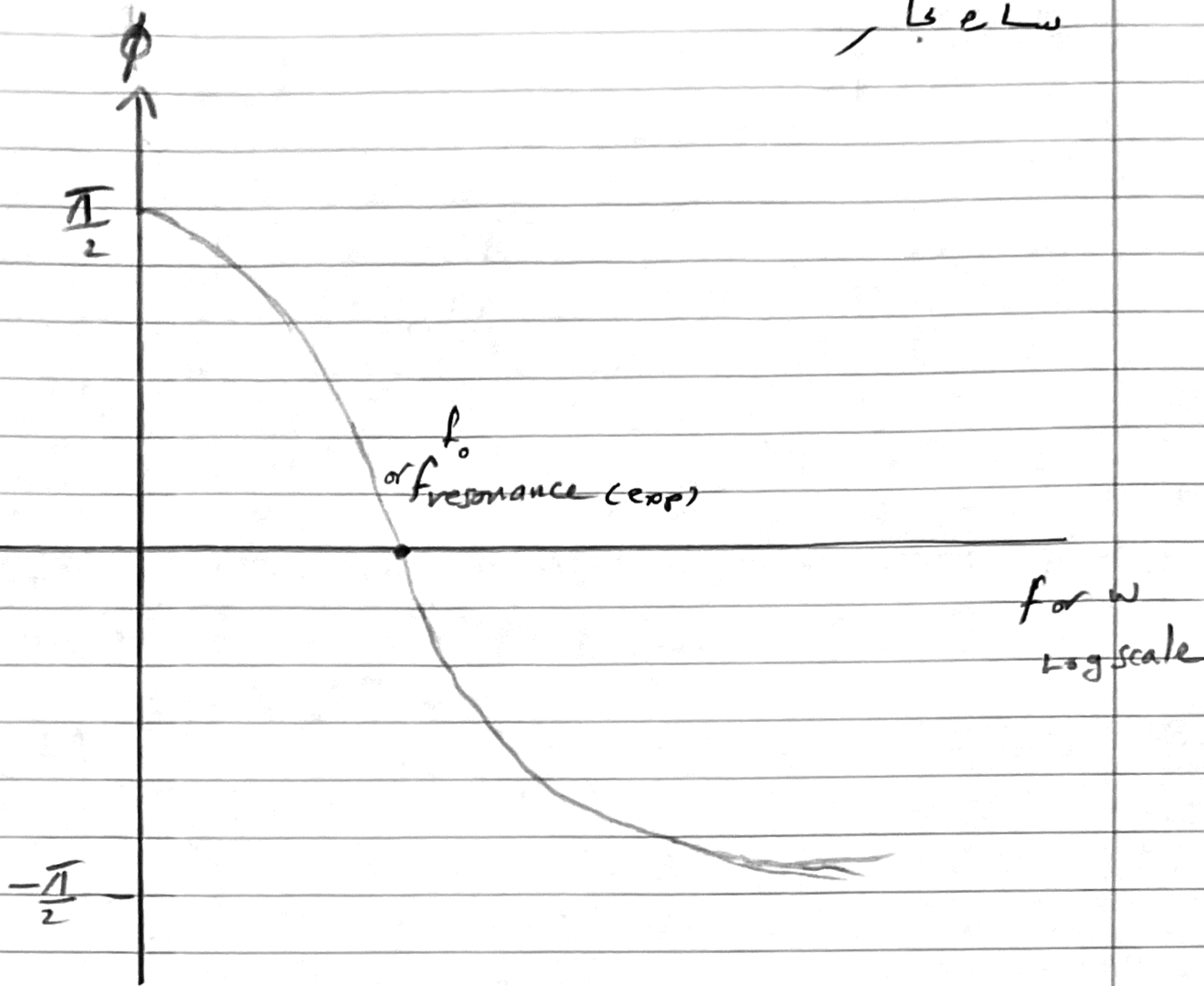

f_0 exp from the graph

$f_{0\,theo} = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$

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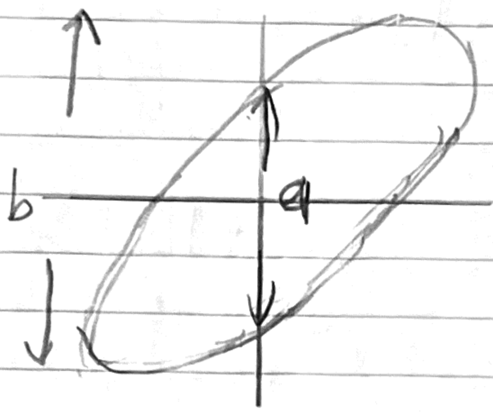
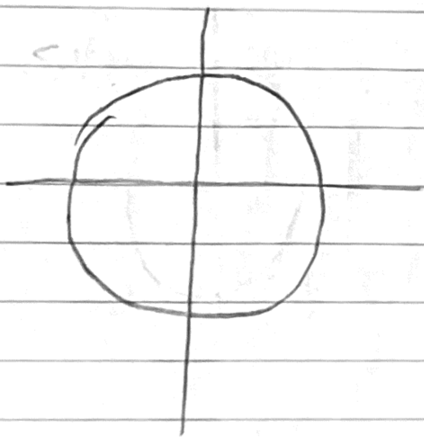
external Mode

- No time لا زمن
- Just voltag in one axis

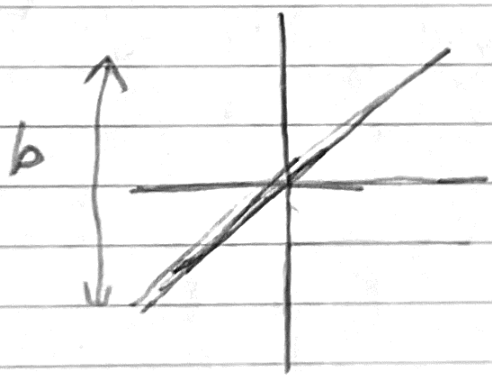


$$\phi = \sin^{-1}\left(\frac{a}{b}\right)$$



$a = b$

$$\phi = \sin^{-1}\frac{a}{b} = \sin^{-1}(1) = \frac{\pi}{2}$$



$a = 0$

b موجود

$$\phi = \sin^{-1}\left(\frac{0}{b}\right) = \sin^{-1} 0 = 0$$

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