Two-Port Review

The **two-port network** in Fig. C.1a is very useful for modeling the behavior of amplifiers in complex systems. We can use the two-port to provide a relatively simple representation of a much more complicated circuit. Thus, the two-port helps us hide or encapsulate the complexity of the circuit, so we can more easily manage the overall analysis and design. One important limitation must be remembered, however. The two-ports we use are linear network models, and are valid under small-signal conditions that are fully discussed in Chapter 13.

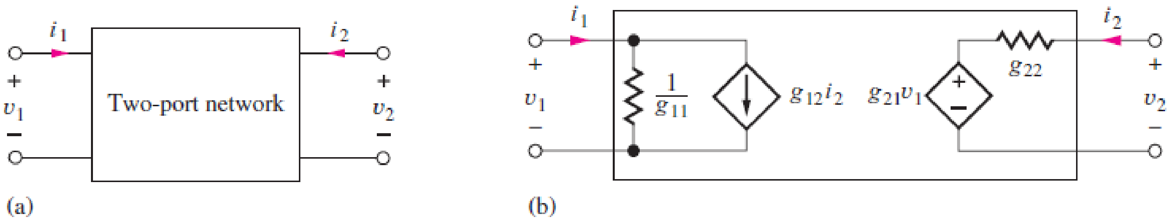

Figure C.1 (a) Two-port network representation. (b) Two port g-parameter representation.

From network theory, we know that two-port networks can be represented in terms of **two-port parameters.** Four of these sets are often used as models for amplifiers: the g-, h-, y-, and z-parameters; the s- and $abcd$-parameters are not required here. Note in these two-port representations that (v_1, i_1) and (v_2, i_2) represent the signal components of the voltages and currents at the two ports of the network.

C.1 THE *g*-PARAMETERS

The **g-parameter** description is one of the most commonly used representations for a voltage amplifier:

$$\begin{aligned} \mathbf{i}_1 &= g_{11}\mathbf{v}_1 + g_{12}\mathbf{i}_2 \\ \mathbf{v}_2 &= g_{21}\mathbf{v}_1 + g_{22}\mathbf{i}_2 \end{aligned} \tag{C.1}$$

Figure C.1b is a network representation of these equations.

The g-parameters are determined from a given network using a combination of **open-circuit** $(i = 0)$ and **short-circuit** $(v = 0)$ **termination** conditions by applying these parameter definitions:

$$\begin{aligned} g_{11} &= \left.\frac{\mathbf{i}_1}{\mathbf{v}_1}\right|_{\mathbf{i}_2=0} = \textbf{open-circuit input conductance} \\ g_{12} &= \left.\frac{\mathbf{i}_1}{\mathbf{i}_2}\right|_{\mathbf{v}_1=0} = \textbf{reverse short-circuit current gain} \\ g_{21} &= \left.\frac{\mathbf{v}_2}{\mathbf{v}_1}\right|_{\mathbf{i}_2=0} = \textbf{forward open-circuit voltage gain} \\ g_{22} &= \left.\frac{\mathbf{v}_2}{\mathbf{i}_2}\right|_{\mathbf{v}_1=0} = \textbf{short-circuit output resistance} \end{aligned} \tag{C.2}$$

C.2 THE HYBRID OR *h*-PARAMETERS

The ***h*-parameter** description is also widely used in electronic circuits and is one convenient model for a current amplifier:

$$\begin{aligned} \mathbf{v}_1 &= h_{11}\mathbf{i}_1 + h_{12}\mathbf{v}_2 \\ \mathbf{i}_2 &= h_{21}\mathbf{i}_1 + h_{22}\mathbf{v}_2 \end{aligned} \tag{C.3}$$

Figure C.2 is the network representation of these equations.

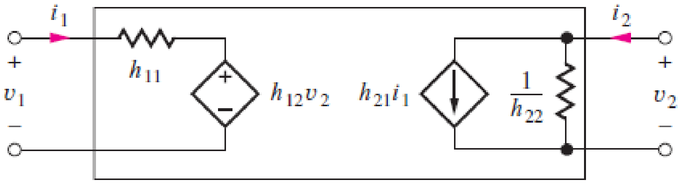

Figure C.2 Two-port *h*-parameter representation.

As with the *g*-parameters, the *h*-parameters are determined from a given network using a combination of open- and short-circuit measurement conditions:

$$h_{11} = \left.\frac{\mathbf{v}_1}{\mathbf{i}_1}\right|_{\mathbf{v}_2=0} = \textbf{short-circuit input resistance}$$

$$\begin{aligned} h_{12} &= \left.\frac{\mathbf{v}_1}{\mathbf{v}_2}\right|_{\mathbf{i}_1=0} = \textbf{reverse open-circuit voltage gain} \\ h_{21} &= \left.\frac{\mathbf{i}_2}{\mathbf{i}_1}\right|_{\mathbf{v}_2=0} = \textbf{forward short-circuit current gain} \\ h_{22} &= \left.\frac{\mathbf{i}_2}{\mathbf{v}_2}\right|_{\mathbf{i}_1=0} = \textbf{open-circuit output conductance} \end{aligned} \tag{C.4}$$

C.3 THE ADMITTANCE OR *y*-PARAMETERS

The admittance, or ***y*-parameter,** description is useful in modeling transconductance amplifiers.

$$\begin{aligned}\mathbf{i}_1 &= y_{11}\mathbf{v}_1 + y_{12}\mathbf{v}_2 \\ \mathbf{i}_2 &= y_{21}\mathbf{v}_1 + y_{22}\mathbf{v}_2\end{aligned} \tag{C.5}$$

Figure C.3 is a network representation of these equations.

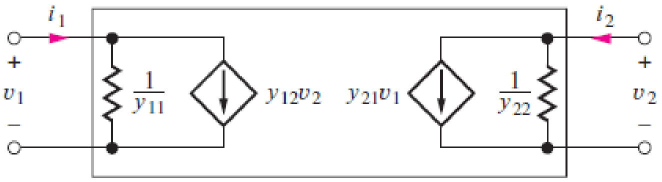

Figure C.3 Two-port *y*-parameter representation.

The *y*-parameters are often referred to as the short-circuit parameters because they are determined from a given network using only short-circuit terminations:

$$\begin{aligned}
y_{11} &= \left.\frac{\mathbf{i}_1}{\mathbf{v}_1}\right|_{\mathbf{v}_2=0} = \textbf{short-circuit input conductance} \\
y_{12} &= \left.\frac{\mathbf{i}_1}{\mathbf{v}_2}\right|_{\mathbf{v}_1=0} = \textbf{reverse short-circuit transconductance} \\
y_{21} &= \left.\frac{\mathbf{i}_2}{\mathbf{v}_1}\right|_{\mathbf{v}_2=0} = \textbf{forward short-circuit transconductance} \\
y_{22} &= \left.\frac{\mathbf{i}_2}{\mathbf{v}_2}\right|_{\mathbf{v}_1=0} = \textbf{short-circuit output conductance}
\end{aligned} \tag{C.6}$$

C.4 THE IMPEDANCE OR *z*-PARAMETERS

The impedance, or ***z*-parameters,** can also be used for modeling voltage amplifiers.

$$\begin{aligned}\mathbf{v}_1 &= z_{11}\mathbf{i}_1 + z_{12}\mathbf{i}_2 \\ \mathbf{v}_2 &= z_{21}\mathbf{i}_1 + z_{22}\mathbf{i}_2\end{aligned} \tag{C.7}$$

Figure C.4 is a network representation of Eq. (C.7).

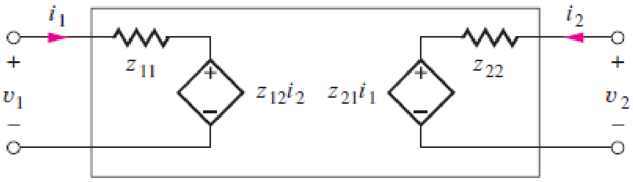

Figure C.4 Two-port *z*-parameter representation.

The *z*-parameters are determined from a given network using open-circuit measurement conditions and are often referred to as the open-circuit parameters:

$$z_{11} = \left.\frac{\mathbf{v}_1}{\mathbf{i}_1}\right|_{\mathbf{i}_2=0} = \textbf{open-circuit input resistance}$$

$$\begin{aligned} z_{12} &= \left.\frac{\mathbf{v}_1}{\mathbf{i}_2}\right|_{\mathbf{i}_1=0} = \textbf{reverse open-circuit transresistance} \\ z_{21} &= \left.\frac{\mathbf{v}_2}{\mathbf{i}_1}\right|_{\mathbf{i}_2=0} = \textbf{forward open-circuit transresistance} \\ z_{22} &= \left.\frac{\mathbf{v}_2}{\mathbf{i}_2}\right|_{\mathbf{i}_1=0} = \textbf{open-circuit output resistance}\end{aligned} \tag{C.8}$$