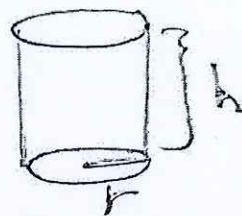


Ch 6 Applications of Definite Integrals:

6.1 Volume Using Cross-Sections:

The volume of the cylindrical solid is always defined to be its base area times its height.

$$\text{Volume} = \pi r^2 \cdot h$$



Volume by Slicing:

Def: The volume of a solid of integrable cross-sectional area $A(x)$ from $x=a$ to $x=b$

is
$$V = \int_a^b A(x) dx.$$

Calculating the volume of a solid:

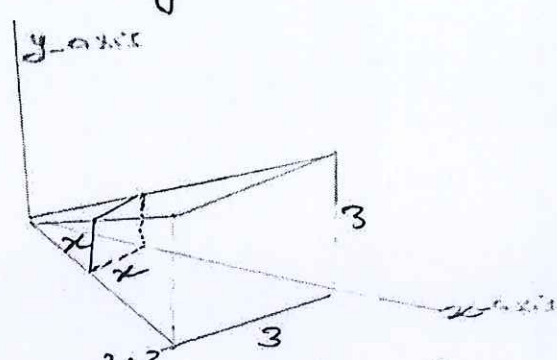
1. Sketch the solid and a typical cross-section.
2. Find a formula of $A(x)$, (Area of cross-section).
3. Find the limits of integration.
4. Integrating $A(x)$ using FTC.

Example: A pyramid 3 m high has a square base that is 3 m on a side. The cross-section of the pyramid perpendicular to the altitude x m down from the vertex is a square x m on a side. Find the volume of the pyramid.

Cross-sections are squares:

$\Rightarrow A(x) = x^2$

Volume = $\int_0^3 A(x) dx = \int_0^3 x^2 dx = \frac{x^3}{3} \Big|_0^3 = \boxed{9} \text{ m}^3$



Note: If the cross-section \perp x -axis, then we integrate w.r.t x .

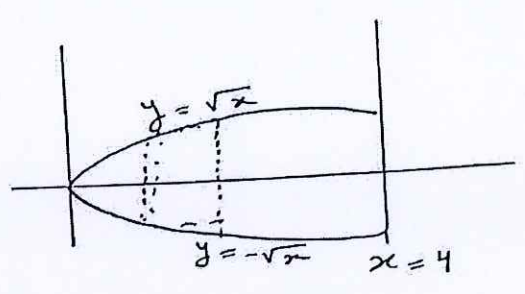
If the cross-section \perp y -axis, then we integrate w.r.t y .

Example: Find the Volume of a solid whose cross-sections are squares perpendicular to x -axis between $x = 0$ & $x = 4$. whose side of each square runs from

$$y = \sqrt{x} \quad \& \quad y = -\sqrt{x}$$

$$V = \int_a^b A(x) dx$$

$$= \int_0^4 (2\sqrt{x})^2 dx = 32$$

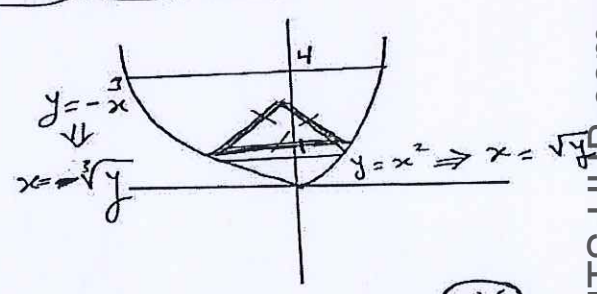


Example: Find the Volume of a solid whose cross-sections are equilateral triangles perpendicular to y -axis between $y = 1$ and $y = 4$ whose side of each triangle runs from $y = -x^3$ & $y = x^2$.

$$V = \int_c^d A(y) dy, \text{ where } A = \frac{1}{2} a \cdot a \sin \frac{\pi}{3}$$

$$= \int_1^4 \frac{\sqrt{3}}{4} a^2 dy$$

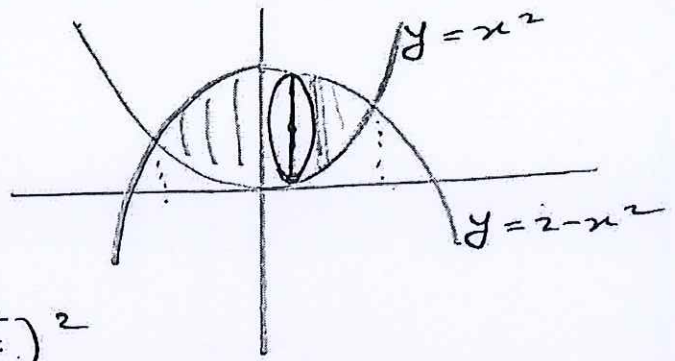
$$= \int_1^4 \frac{\sqrt{3}}{4} (\sqrt{y} + \sqrt[3]{y})^2 dy =$$



Q2) The solid lies between planes perpendicular to the x -axis at $x = -1$ and $x = 1$.

The cross-sections perpendicular to the x -axis are circular disks whose diameters run from the parabola $y = x^2$ to the parabola $y = 2 - x^2$.

القطر
 Diameter = $(2 - x^2) - x^2$
 $= 2 - 2x^2$.



$$A(x) = \pi r^2 = \pi \left(\frac{2 - 2x^2}{2} \right)^2$$

$$= \pi (1 - x^2)^2$$

To find the Limits of Integration:

$$2 - x^2 = x^2 \iff 2x^2 = 2 \iff x = \pm 1$$

$$\text{Volume} = \int_{-1}^1 A(x) dx = \int_{-1}^1 \pi (1 - x^2)^2 dx$$

$$= \int_{-1}^1 \pi (1 - 2x^2 + x^4) dx = \frac{16}{15}$$

Solids of Revolution: The Disk Method

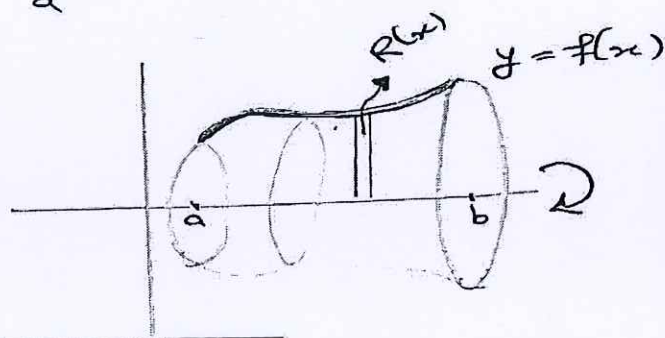
The area of cross-section $A(x)$ is the area of a disk of radius $R(x)$.

$$A(x) = \pi (R(x))^2.$$

Def: Volume by Disks for Rotation about x -axis:

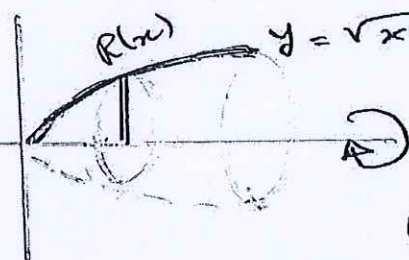
$$V = \int_a^b A(x) dx = \int_a^b \pi (R(x))^2 dx.$$

Cross-section \perp axis of Revolution



Example: The region between the curve $y = \sqrt{x}$, $0 \leq x \leq 4$ and the x -axis is revolved about the x -axis to generate a solid. Find its Volume.

$$\begin{aligned} V &= \int_0^4 \pi (R(x))^2 dx = \int_0^4 \pi (\sqrt{x})^2 dx \\ &= \int_0^4 \pi x dx = \left. \frac{\pi x^2}{2} \right|_0^4 = \boxed{8\pi} \end{aligned}$$



(78)

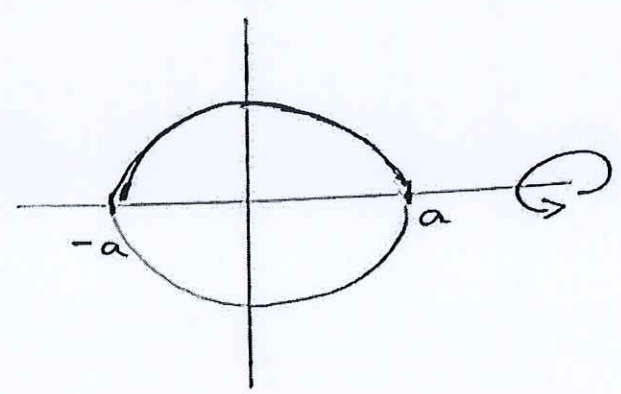
Example: The Circle $x^2 + y^2 = a^2$ is rotated about the x -axis to generate a sphere.

Find its Volume.

$$x^2 + y^2 = a^2 \Leftrightarrow y^2 = a^2 - x^2$$

$$\Rightarrow y = +\sqrt{a^2 - x^2}$$

$$\Rightarrow R(x) = \sqrt{a^2 - x^2}$$



$$\text{Volume} = \int_{-a}^a \pi (\sqrt{a^2 - x^2})^2 dx = \int_{-a}^a \pi (a^2 - x^2) dx$$

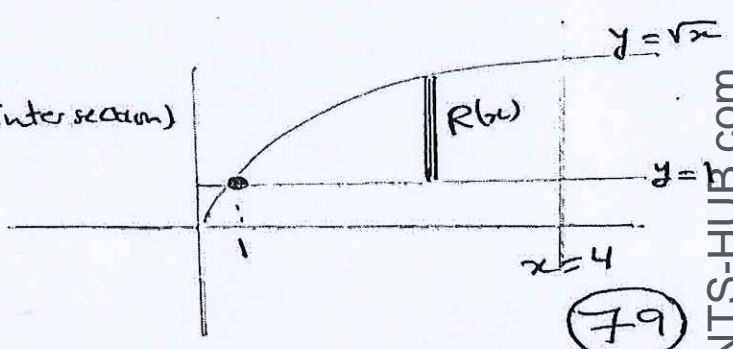
$$= \pi \left[a^2 x - \frac{x^3}{3} \right]_{-a}^a = \pi \left[\left(a^3 - \frac{a^3}{3} \right) - \left(-a^3 + \frac{a^3}{3} \right) \right]$$

$$= \pi \left(\frac{2}{3} a^3 - \frac{2}{3} a^3 \right) = \frac{4}{3} \pi a^3 \quad (\text{Volume of Sphere})$$

Example: Find the Volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$, $y = 1$ & $x = 4$ about $y = 1$?

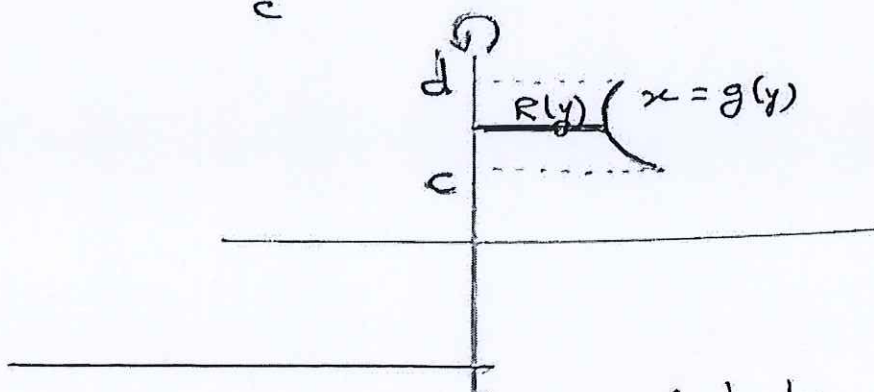
$$1 = \sqrt{x} \Leftrightarrow x = 1 \quad (\text{point of Intersection})$$

$$V = \int_1^4 \pi (\sqrt{x} - 1)^2 dx = \frac{7\pi}{6}$$



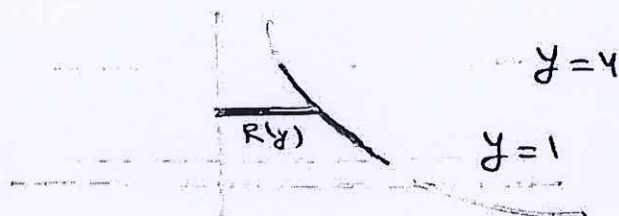
Def: Volume by Disks for Rotation about y -axis

$$V = \int_c^d A(y) dy = \int_c^d \pi (R(y))^2 dy.$$



Example: Find the Volume of the solid generated by revolving the region between the y -axis and the curve $x = \frac{2}{y}$, $1 \leq y \leq 4$ about y -axis.

$$V = \int_1^4 \pi \left(\frac{2}{y}\right)^2 dy = 3\pi.$$

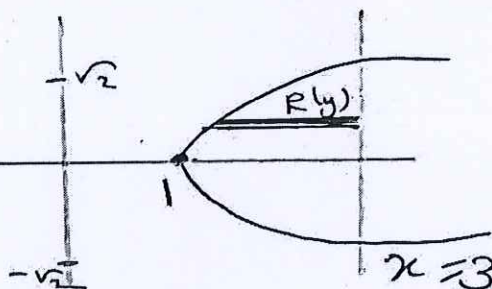


Example: Find the Volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about $x = 3$.

$x = y^2 + 1$ and the line $x = 3$ about $x = 3$.

$$y^2 + 1 = 3 \Leftrightarrow y = \pm\sqrt{2}.$$

$$V = \int_{-\sqrt{2}}^{\sqrt{2}} \pi (3 - (y^2 + 1))^2 dy = \dots$$

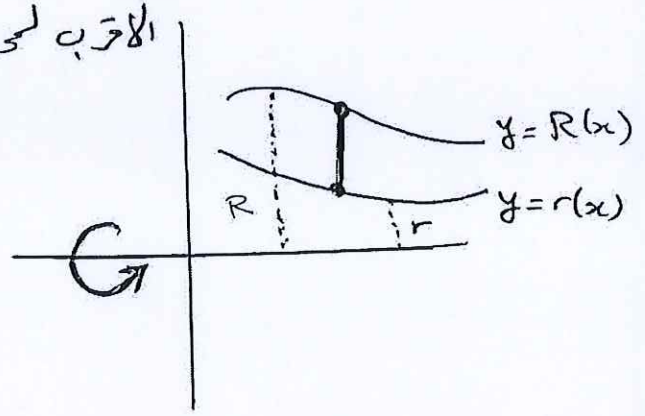


Solids of Revolution : The Washer Method :

Notice the following graph.

$R(x)$: Outer radius - الأبعد من المحور الدوراني

$r(x)$: Inner radius - الأقرب للمحور الدوراني



In Disk Method : $r(x) = 0$

Def: Volume by Washers for Rotation about x-axis:

$$V = \int_a^b A(x) dx = \int_a^b \pi (R(x)^2 - r(x)^2) dx.$$

Example: The region bounded by the Curve

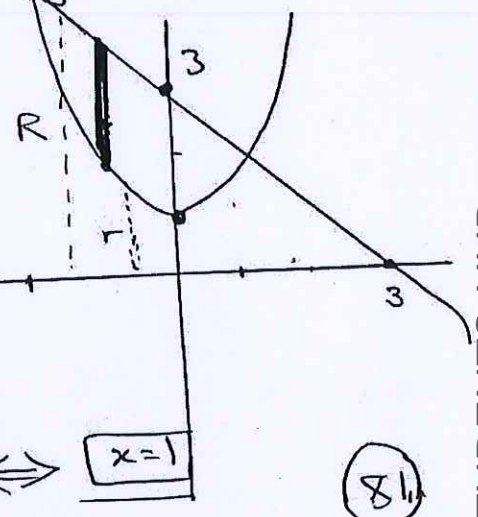
$y = x^2 + 1$ and the Line $y = -x + 3$

is revolved about the x-axis to generate a solid.

Find its volume.

$R(x) = -x + 3, r(x) = x^2 + 1$

$$V = \int_{-2}^1 \pi ((-x+3)^2 - (x^2+1)^2) dx = \boxed{\frac{117\pi}{5}}$$



Points of Intersection : $x^2 + 1 = -x + 3 \iff \boxed{x=1}$ (8/1)

Example: The region bounded by the parabola

$y = x^2$ and the line $y = 2x$ in the ^{دول} first

quadrant is revolved about y -axis to generate

a solid. Find the volume of the solid.

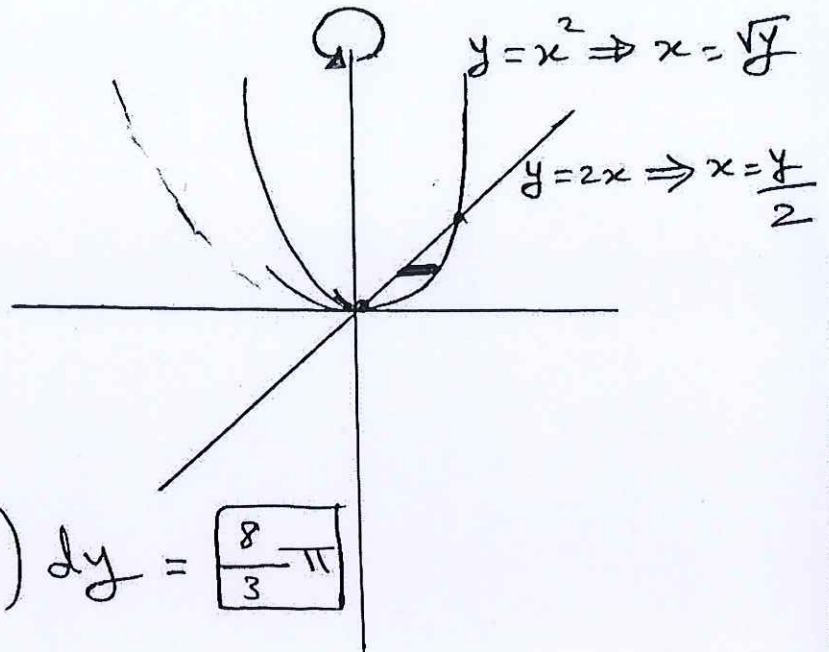
points of Intersection:

$$x^2 = 2x$$

$$\Leftrightarrow x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$\Rightarrow x = 0 \text{ \& } x = 2$$



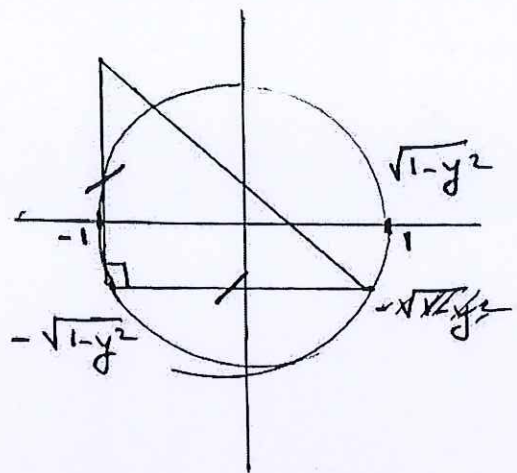
$$V = \int_0^2 \pi \left((\sqrt{y})^2 - \left(\frac{y}{2}\right)^2 \right) dy = \frac{8}{3} \pi$$

Lecture Problems:

Q10) Find the Volume of the solid whose base is the disk $x^2 + y^2 \leq 1$. The cross-sections by planes perpendicular to the y-axis between $y = -1$ and $y = 1$ are ^{مساوي الساقين} isosceles right triangles ^{مساوية الساقين} with one leg in the disk

$$x^2 + y^2 = 1 \Leftrightarrow x = \pm \sqrt{1 - y^2}$$

$$\begin{aligned} A(y) &= \frac{1}{2} (\text{base})(\text{height}) \\ &= \frac{1}{2} \left[\sqrt{1 - y^2} + \sqrt{1 - y^2} \right]^2 \\ &= \frac{1}{2} \left(2 \sqrt{1 - y^2} \right)^2 \\ &= \frac{4}{2} (1 - y^2) = 2(1 - y^2) \end{aligned}$$



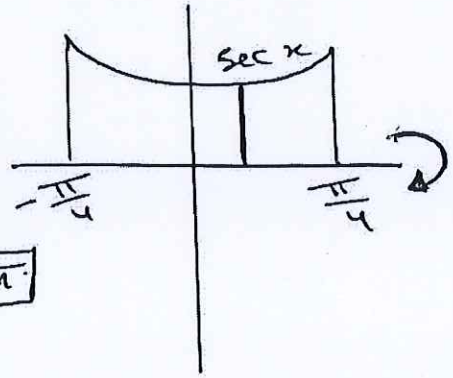
$$\begin{aligned} V &= \int_{-1}^1 2(1 - y^2) dy = \int_{-1}^1 (2 - 2y^2) dy \\ &= \left[2y - \frac{2y^3}{3} \right]_{-1}^1 = \boxed{\frac{8}{3}} \end{aligned}$$

Q24) Find the volume of the solid for:

$y = \sec x$, $y = 0$, $x = -\frac{\pi}{4}$, $x = \frac{\pi}{2}$ when revolved about x -axis.

$$V = \int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \pi [\sec^2 x] dx$$

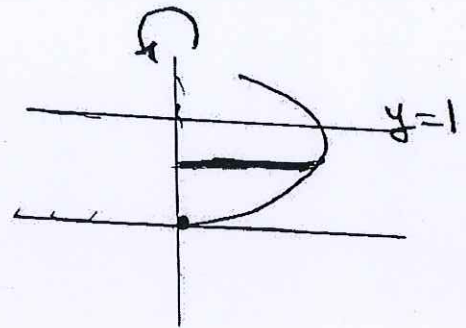
$$= \pi \tan x \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{2}} = \pi [1 - (-1)] = \boxed{2\pi}$$



32) $x = \frac{\sqrt{2y}}{y^2 + 1}$, $x = 0$, $y = 1$ about y -axis.

$$V = \int_0^1 \pi \left(\frac{\sqrt{2y}}{y^2 + 1} \right)^2 dy$$

$$= \int_0^1 \pi \frac{(2y)}{(y^2 + 1)^2} dy$$



Using substitution: Let $u = y^2 + 1 \Rightarrow du = 2y dy$

when $y = 0$, then $u = 1$

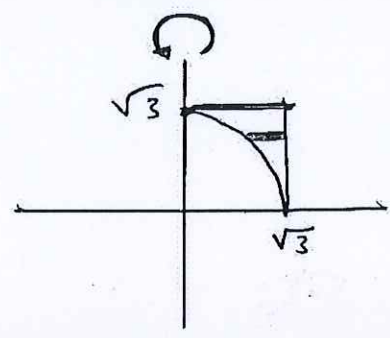
when $y = 1$, then $u = 2$

$$\therefore = \int_1^2 \pi \frac{du}{u^2} = \left[\frac{-1}{u} \right]_1^2 = \boxed{\frac{\pi}{2}}$$

(84)

Q44) The region in the first quadrant bounded on the left by $x^2 + y^2 = 3$, on the right by $x = \sqrt{3}$ and above by $y = \sqrt{3}$ is revolved about y-axis. Find the Volume.

Using Washer Method,
 Since there is a gap between the region and the axis of Revolution.



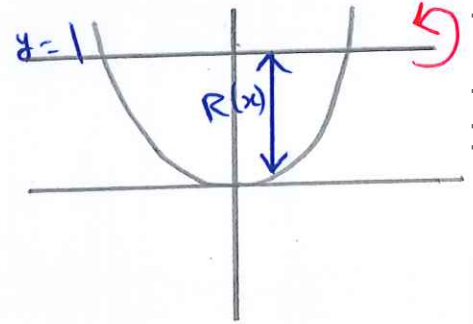
$$\Rightarrow V = \int_0^{\sqrt{3}} \pi \left((\sqrt{3})^2 - (\sqrt{3-y^2})^2 \right) dy$$

$$= \boxed{\pi (\sqrt{3})}$$

Q 49 Find the Volume of the Solid generated by revolving the region bounded by $y = x^2$, $y = 1$ about :

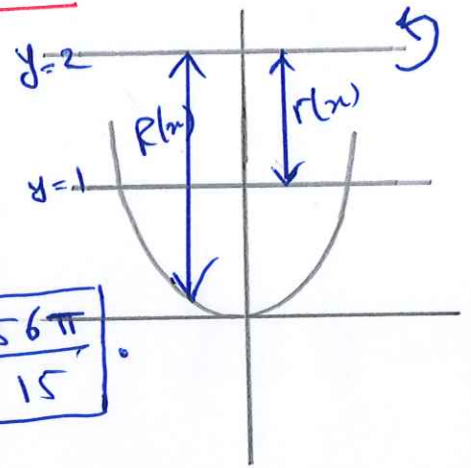
a) $y = 1$.

$$V = \int_{-1}^1 \pi (1 - x^2)^2 dx = \boxed{\frac{16\pi}{15}}$$



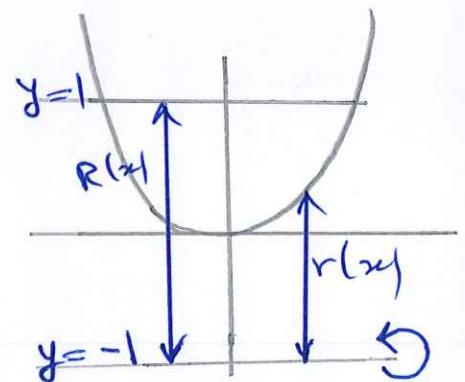
b) $y = 2$

$$V = \int_{-1}^1 \pi ((2 - x^2)^2 - (1)^2) dx = \boxed{\frac{56\pi}{15}}$$



c) $y = -1$

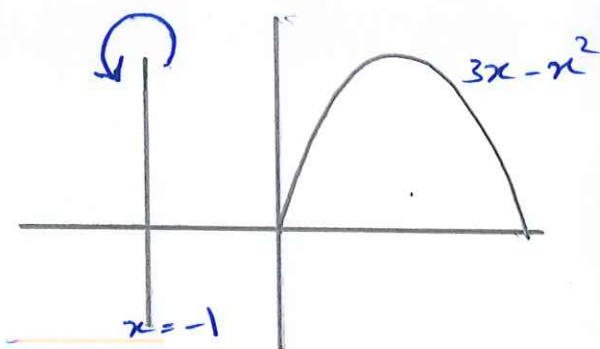
$$V = \int_{-1}^1 \pi ((2)^2 - (x^2 + 1)^2) dx = \boxed{\frac{64\pi}{15}}$$



6.2 Volumes Using Cylindrical shells:

Example: The region enclosed by the x -axis and $y = 3x - x^2$ is revolved about $x = -1$ to generate a solid. Find the Volume of the solid.

Sol: If we want to find the volume using Disk or Washer then we should write $y = f(x)$ as $x = f(y)$ which is not easy.



Def: Shell Formula for Revolution about $x = L$ vertical line

$$V = \int_a^b 2\pi (\text{shell radius}) (\text{shell height}) dx$$

where: shell radius = Distance between cross-section and axis of Revolution.

shell height = length of the Cross-section.

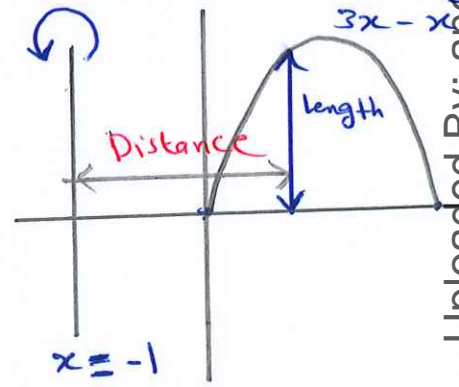
Note: Cross-section // axis of Revolution

Example Consider the previous example:

$$3x - x^2 = 0 \Leftrightarrow x = 0, 3$$

$$V = \int_0^3 2\pi (x+1)(3x-x^2) dx$$

$$= \boxed{\frac{45\pi}{2}}$$



Example: The region bounded by the curve

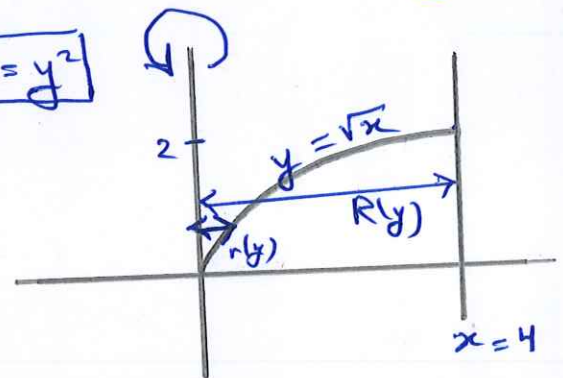
$y = \sqrt{x}$ and the x -axis and the line

$x = 4$ is revolved about the y -axis ^{vertical line} to generate a solid. Find the Volume of solid

Using Washer Method: $y = \sqrt{x} \Leftrightarrow \boxed{x = y^2}$

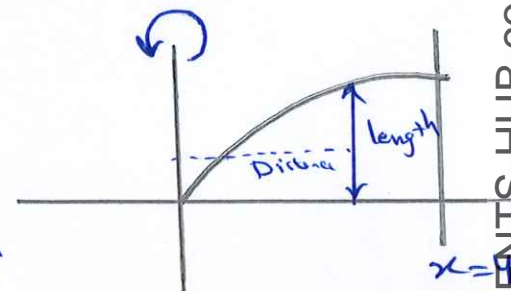
$$V = \int_0^2 \pi \left((4)^2 - (y^2)^2 \right) dy$$

$$= \boxed{\frac{128\pi}{5}}$$



Using Shell Method:

$$V = \int_0^4 2\pi (x)(\sqrt{x}) dx = \boxed{\frac{128\pi}{5}}$$



Def: shell formula for Revolution about $y=L$

Horizontal Line

$$V = \int_c^d 2\pi (\text{shell radius}) (\text{shell height}) dy$$

Example: The region bounded by the Curve

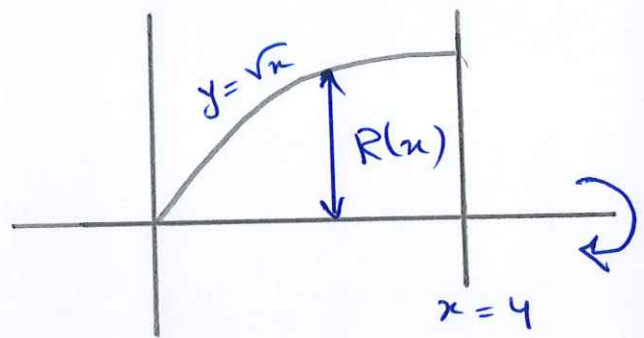
$y = \sqrt{x}$ and x -axis and the line $x = 4$

is revolved about the x -axis to generate a solid. Find the Volume of the solid.

sol:

Using Disk Method:

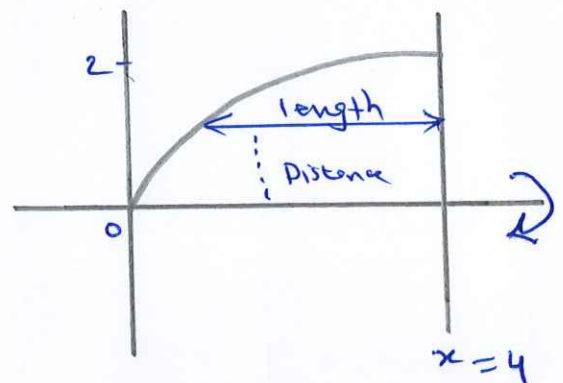
$$V = \int_0^4 \pi (\sqrt{x})^2 dx = \boxed{8\pi}$$



Using shell Method:

$$y = \sqrt{x} \Leftrightarrow x = y^2$$

$$V = \int_0^2 2\pi (y) (4 - y^2) dy = \boxed{8\pi}$$



Q9 (6.2) Use the shell method to find

the Volume of the solid generated by revolving the region bounded by the curves $y = x^2$,

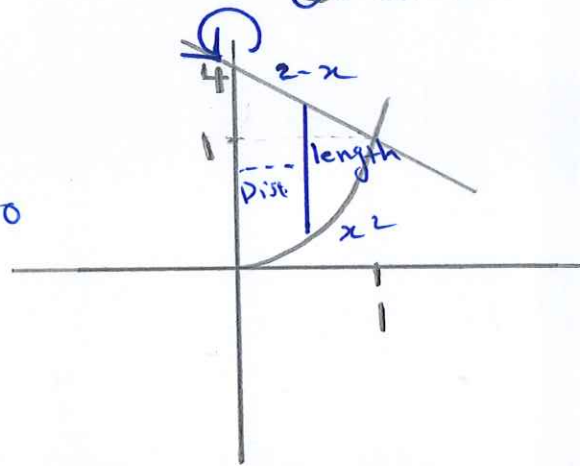
$y = 2 - x$, $x = 0$, $x \geq 0$ about y -axis

sol: points of Intersection:

$$x^2 = 2 - x \Leftrightarrow x^2 + x - 2 = 0$$

$$\Leftrightarrow x = -2 \text{ \& } x = 1$$

rejected



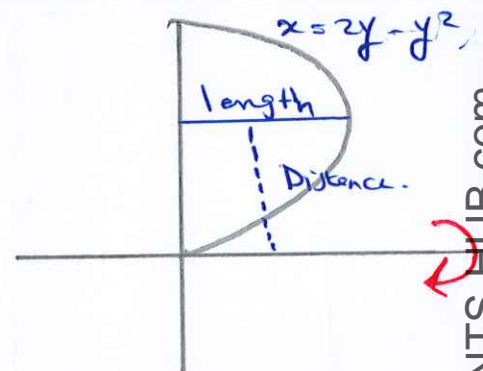
$$\Rightarrow V = \int_0^1 2\pi(x) \left((2-x) - x^2 \right) dx = \boxed{\frac{5\pi}{6}}$$

17 Use shell method to find the Volume of the solid generated by Revolving $x = 2y - y^2$ $x = 0$, about x -axis.

$$2y - y^2 = 0 \Leftrightarrow y = 0, 2$$

$$V = \int_0^2 2\pi(y) (2y - y^2) dy$$

$$= \boxed{\frac{8\pi}{3}}$$

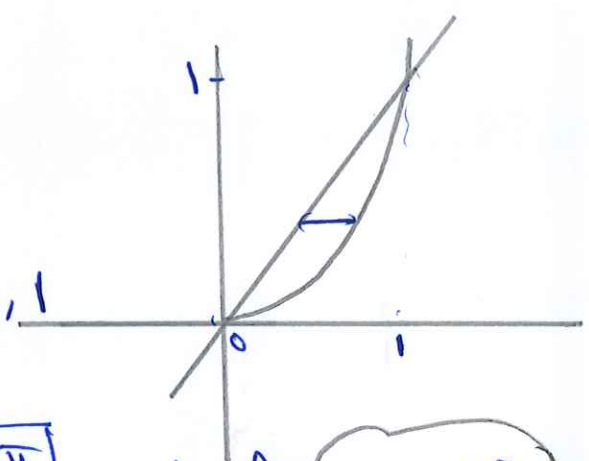


Q29 (6.2) Compute the Volume of the Solid generated by Revolving the region bounded by $y = x$ and $y = x^2$ about each coordinate axis

(a) Using shell Method.

① $y = x^2 \Rightarrow x = \sqrt{y}$
 $y = x \Rightarrow x = y$

$\Rightarrow y = \sqrt{y} \Leftrightarrow y^2 - y = 0 \Leftrightarrow y = 0, 1$



$V = \int_0^1 2\pi (y)(\sqrt{y} - y) dy = \boxed{\frac{2\pi}{15}}$ about x-axis

② $V = \int_0^1 2\pi (x)(x - x^2) dx = \boxed{\frac{\pi}{6}}$ about y-axis

(b) Using Washer Method.

① about x-axis:

$V = \int_0^1 \pi ((x)^2 - (x^2)^2) dx = \boxed{\frac{2\pi}{15}}$

② about y-axis:

$V = \int_0^1 \pi ((\sqrt{y})^2 - (y)^2) dy = \boxed{\frac{\pi}{6}}$

(39) The Region is Revolved about x-axis:

(6.2)

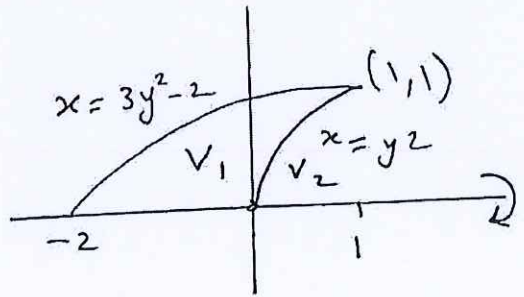
Disk:

$$V = V_1 - V_2$$

$$V_1 = \int_{-2}^1 \pi \left[\sqrt{\frac{x+2}{3}} \right]^2 dx = 1.5\pi$$

$$V_2 = \int_0^1 \pi [\sqrt{x}]^2 dx = 0.5\pi$$

$$V = 1.5\pi - 0.5\pi = \pi$$



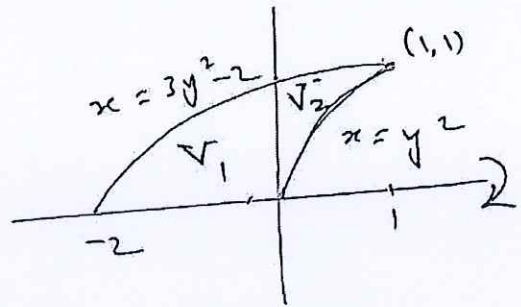
Washer:

$$V = V_1 + V_2$$

$$V_1 = \int_{-2}^0 \pi \left(\left[\sqrt{\frac{x+2}{3}} \right]^2 - [0]^2 \right) dx = \frac{2}{3}\pi$$

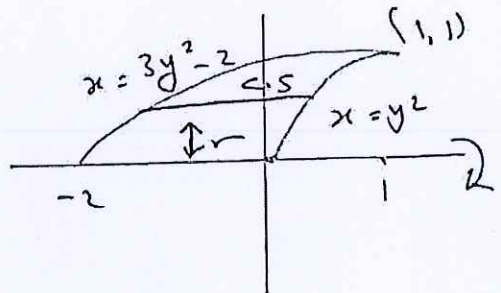
$$V_2 = \int_0^1 \pi \left(\left[\sqrt{\frac{x+2}{3}} \right]^2 - [\sqrt{x}]^2 \right) dx = \frac{1}{3}\pi$$

$$V_1 + V_2 = \frac{2}{3}\pi + \frac{1}{3}\pi = \pi$$



Shell

$$V = \int_0^1 2\pi (y) [y^2 - (3y^2 - 2)] dy = \pi$$



6.3 Arc length.

Length of a Curve $y = f(x)$:

Def: If f is continuous on $[a, b]$, then the length (arc length) of the curve $y = f(x)$ from $A = (a, f(a))$ to the point $B = (b, f(b))$ is the value of the Integral:

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

Example: Find the length of the Curve:

$$y = \frac{4\sqrt{2}}{3} x^{3/2} - 1, \quad 0 \leq x \leq 1$$

$$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_0^1 \sqrt{1 + (2\sqrt{2} x^{1/2})^2} dx$$

$$= \int_0^1 \sqrt{1 + 8x} dx = \frac{13}{6}$$

Dealing with Discontinuities in dy/dx

At some point where dy/dx fails to exist, dx/dy may exist.

Def: If g' is continuous on $[c, d]$, the length of the

Curve $x = g(y)$ from $A = (g(c), c)$ to $B = (g(d), d)$.

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy = \int_c^d \sqrt{1 + [g'(y)]^2} dy.$$

Example: Find the length of the Curve $y = \left(\frac{x}{2}\right)^{2/3}$

from $x = 0$ to $x = 2$.

$$\frac{dy}{dx} = \frac{1}{3} \left(\frac{2}{x}\right)^{1/3} \quad \text{which is not defined at } x=0$$

Therefore: $x = 2y^{3/2}$ & $\frac{dx}{dy} = 3y^{1/2}$

when $x=0 \Rightarrow y=0$, when $x=2 \Rightarrow y=1$

$$\Rightarrow L = \int_0^1 \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy = \int_0^1 \sqrt{1 + 9y} dy$$

$u = 1 + 9y$
 $du = 9dy$

$$= \int_1^{10} \frac{\sqrt{u}}{9} du = \frac{2}{27} (10\sqrt{10} - 1)$$

The Differential formula for Arc Length.

The arc length function of the curve $y = f(x)$ starting from the point $P_0(a, f(a))$ is defined by:

$$S(x) = \int_a^x \sqrt{1 + [f'(t)]^2} dt.$$

Example: Find the arc length function for $f(x) = \frac{x^3}{12} + \frac{1}{x}$ starting from $A = (1, 13/12)$.

$$f'(x) = \frac{x^2}{4} - \frac{1}{x^2} \Rightarrow (f'(x))^2 = \frac{x^4}{16} - \frac{1}{2} + \frac{1}{x^4}$$

$$1 + (f'(x))^2 = \frac{x^4}{16} + \frac{1}{2} + \frac{1}{x^4} = \left(\frac{x^2}{4} + \frac{1}{x^2}\right)^2$$

$$\Rightarrow S(x) = \int_1^x \sqrt{\left(\frac{t^2}{4} + \frac{1}{t^2}\right)^2} dt = \int_1^x \left(\frac{t^2}{4} + \frac{1}{t^2}\right) dt$$

$$= \left[\frac{t^3}{12} - \frac{1}{t} \right]_1^x = \frac{x^3}{12} - \frac{1}{x} + \frac{11}{12}$$

If we want to compute the arc length from $A = (1, 13/12)$ to $B = (4, 67/12)$, we simply substitute

$$S(4) = 6$$

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6.3 Q4 Find the length of the Curve :

$$x = \frac{y^{\frac{3}{2}}}{3} - y^{\frac{1}{2}} \quad \text{from } y=1 \quad \text{to } y=9$$

$$\frac{dx}{dy} = \frac{1}{2} y^{\frac{1}{2}} - \frac{1}{2} y^{-\frac{1}{2}} \Rightarrow \left(\frac{dx}{dy}\right)^2 = \frac{1}{4} \left(y - 2 + \frac{1}{y}\right)$$

$$\Rightarrow L = \int_1^9 \sqrt{1 + \frac{1}{4} \left(y - 2 + \frac{1}{y}\right)} dy = \int_1^9 \sqrt{\frac{1}{4}y + \frac{1}{2} + \frac{1}{4y}} dy$$

$$= \int_1^9 \sqrt{\left(\frac{1}{2}\sqrt{y} + \frac{1}{2\sqrt{y}}\right)\left(\frac{1}{2}\sqrt{y} + \frac{1}{2\sqrt{y}}\right)} dy = \int_1^9 \frac{1}{2} \left(\sqrt{y} + \frac{1}{\sqrt{y}}\right) dy = \boxed{\frac{32}{3}}$$

Lecture

Q19 (a) Find a Curve through (1,1) whose length integral is $L = \int_1^4 \sqrt{1 + \frac{1}{4x}} dx$

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{4x} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \quad , \text{ then}$$

$$y = \sqrt{x} + C.$$

Now (1,1) $\Rightarrow 1 = 1 + C \Rightarrow C = 0$

then $y = \sqrt{x}$ from (1,1) to (4,2)

(b) How many such Curves are there?
only one.

6.4 Areas of Surfaces of Revolution:

Def: (Revolution about the x -axis) If the function $f(x) \geq 0$ is continuously differentiable on $[a, b]$, the area of the surface generated by revolving the graph of $y = f(x)$ about the x -axis is

$$S = \int_a^b \underbrace{2\pi y}_{\substack{\text{radius} \\ \text{arc length}}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

Example: Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \leq x \leq 2$ about x -axis

$$S = \int_1^2 2\pi (2\sqrt{x}) \sqrt{1 + \left(\frac{1}{\sqrt{x}}\right)^2} dx =$$

$$= \int_1^2 2\pi (2\sqrt{x}) \cdot \frac{\sqrt{x+1}}{\sqrt{x}} dx = \int_1^2 4\pi \sqrt{x+1} dx$$

$$= 4\pi \cdot \frac{2}{3} (x+1)^{3/2} \Big|_1^2 = \frac{8\pi}{3} (3\sqrt{3} - 2\sqrt{2}).$$

Def: Revolution about the y -axis:

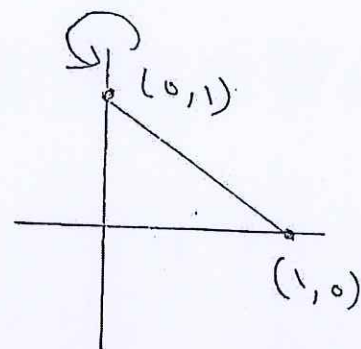
If $x = g(y) \geq 0$ is continuously differentiable on $[c, d]$, the area of the surface generated by revolving the graph of $x = g(y)$ about the y -axis is:

$$S = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy.$$

Example: The line segment $x = 1 - y$ is revolved about y -axis to generate a Cone. Find the lateral surface area.

$$S = \int_0^1 2\pi (1-y) \sqrt{1 + (-1)^2} dy$$

$$= \int_0^1 2\pi (1-y) \sqrt{2} dy = \pi \sqrt{2}.$$



6.4 (15) Find the area of the surface generated

by revolving the curve $y = \sqrt{2x - x^2}$ about

x -axis, $0.5 \leq x \leq 1.5$

$$\text{sol: } \frac{dy}{dx} = \frac{1}{2} \frac{(2-2x)}{\sqrt{2x-x^2}} = \frac{1-x}{\sqrt{2x-x^2}}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{(1-x)^2}{2x-x^2}$$

$$S = \int_{0.5}^{1.5} 2\pi \left(\sqrt{2x-x^2}\right) \left(\sqrt{1 + \frac{(1-x)^2}{2x-x^2}}\right) dx = 2\pi \int_{0.5}^{1.5} dx = \boxed{2\pi}$$

(24) Write an Integral for the area of the surface generated by revolving the curve $y = \cos x$

$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ about the x -axis.

$$\frac{dy}{dx} = -\sin x \Rightarrow \left(\frac{dy}{dx}\right)^2 = \sin^2 x$$

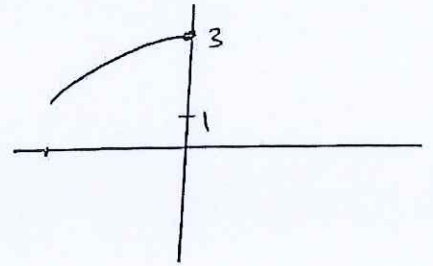
$$S = 2\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos x) \sqrt{1 + \sin^2 x} dx$$

6.4 ^{Lecture} (18)

Find the area of the surface generated by revolving the curve

$$x = \frac{1}{3}y^{\frac{3}{2}} - y^{\frac{1}{2}}, \quad 1 \leq y \leq 3 \quad \text{about } y\text{-axis}$$

Note: to get positive area, we take



$$x = -\left(\frac{1}{3}y^{\frac{3}{2}} - y^{\frac{1}{2}}\right)$$

$$\Rightarrow \frac{dx}{dy} = -\frac{1}{2}\left(y^{\frac{1}{2}} - y^{-\frac{1}{2}}\right) \Rightarrow \left(\frac{dx}{dy}\right)^2 = \frac{1}{4}(y+2+y^{-1})$$

$$\Rightarrow S = \int_1^3 2\pi \left(\frac{1}{3}y^{\frac{3}{2}} - y^{\frac{1}{2}}\right) \sqrt{1 + \frac{1}{4}(y+2+y^{-1})} dy$$

$$= -2\pi \int_1^3 \left(\frac{1}{3}y^{\frac{3}{2}} - y^{\frac{1}{2}}\right) \sqrt{\frac{1}{4}(y+2+y^{-1})} dy$$

$$= -2\pi \int_1^3 \left(\frac{1}{3}y^{\frac{3}{2}} - y^{\frac{1}{2}}\right) \cdot \frac{1}{2} \sqrt{(y^{\frac{1}{2}} + y^{-\frac{1}{2}})^2} dy$$

$$= \frac{-2\pi}{2} \int_1^3 \left(\frac{1}{3}y^{\frac{3}{2}} - y^{\frac{1}{2}}\right) \cdot (y^{\frac{1}{2}} + y^{-\frac{1}{2}}) dy$$

$$= -\pi \int_1^3 y^{\frac{1}{2}} \left[\frac{1}{3}y - 1\right] \left[y^{\frac{1}{2}} + y^{-\frac{1}{2}}\right] dy$$

$$= -\pi \int_1^3 \left(\frac{1}{3}y - 1\right) (y+1) dy \Rightarrow$$

$$= \pi \int_1^3 \left(\frac{1}{3}y^2 - \frac{2}{3}y - 1\right) dy$$

$$\boxed{\frac{16\pi}{3}}$$

(100)

6.4 (20) Find the area of the surface generated by revolving the curve $x = \sqrt{2y-1}$, $\frac{5}{8} \leq y \leq 1$, y -axis

$$\frac{dx}{dy} = \frac{1(2)}{2\sqrt{2y-1}} \Rightarrow \left(\frac{dx}{dy}\right)^2 = \frac{1}{2y-1}$$

$$S = \int_{\frac{5}{8}}^1 2\pi \sqrt{2y-1} \cdot \sqrt{1 + \frac{1}{2y-1}} dy$$

$$= \int_{\frac{5}{8}}^1 2\pi \sqrt{2y-1} \cdot \frac{\sqrt{2y}}{\sqrt{2y-1}} dy$$

$$= \int_{\frac{5}{8}}^1 2\pi \cdot \sqrt{2} \cdot y^{\frac{1}{2}} dy = \frac{\pi}{12} (16\sqrt{2} - 5\sqrt{2})$$