12.5 Lecture Problems

10 Find parametric equations for the line through $P(2,3,0)$ and $\perp$ to the vectors $\vec{u} = \vec{i} + 2\vec{j} + 3\vec{k}$ and
$\vec{v} = 3\vec{i} + 4\vec{j} + 5\vec{k}$

- we need point $P(\underset{}{\overset{x_0}{2}},\overset{y_0}{3},\overset{z_0}{0})$ on the line ✓
- we need vector $\vec{w}$ // Line

$$\vec{w} = \vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{vmatrix} = \underset{w_1}{-2}\,\vec{i} + \underset{w_2}{4}\vec{j} \underset{w_3}{-2}\vec{k}$$

- The line is

$$x = x_0 + t\,w_1 = 2 - 2t$$

$$y = y_0 + t\,w_2 = 3 + 4t$$

$$z = z_0 + t\,w_3 = 0 - 2t$$

31 Find the equation of the plane M through $P_0(2,1,-1)$ and $\perp$ to the line of intersection of the planes 2

$M_1: \quad 2x + y - z = 3 \Rightarrow \vec{n}_1 = 2\vec{i} + \vec{j} - \vec{k}$

$M_2: \quad x + 2y + z = 2 \Rightarrow \vec{n}_2 = \vec{i} + 2\vec{j} + \vec{k}$

- We need point on the plane $M \Rightarrow P_0(\underset{x_0}{2}, \underset{y_0}{1}, \underset{z_0}{-1})$

- we need normal vector $\vec{n} \perp M$

$$\vec{n} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = \underset{A}{3}\vec{i} \underset{B}{-3}\vec{j} + \underset{C}{3}\vec{k}$$

- Note that $\vec{n} \parallel$ to the line intersection of the planes

- The equation of the plane M is

$$Ax + By + Cz = Ax_0 + By_0 + Cz_0$$

$$3x - 3y + 3z = (3)(2) + (-3)(1) + (3)(-1)$$

$$x - y + z = 0$$

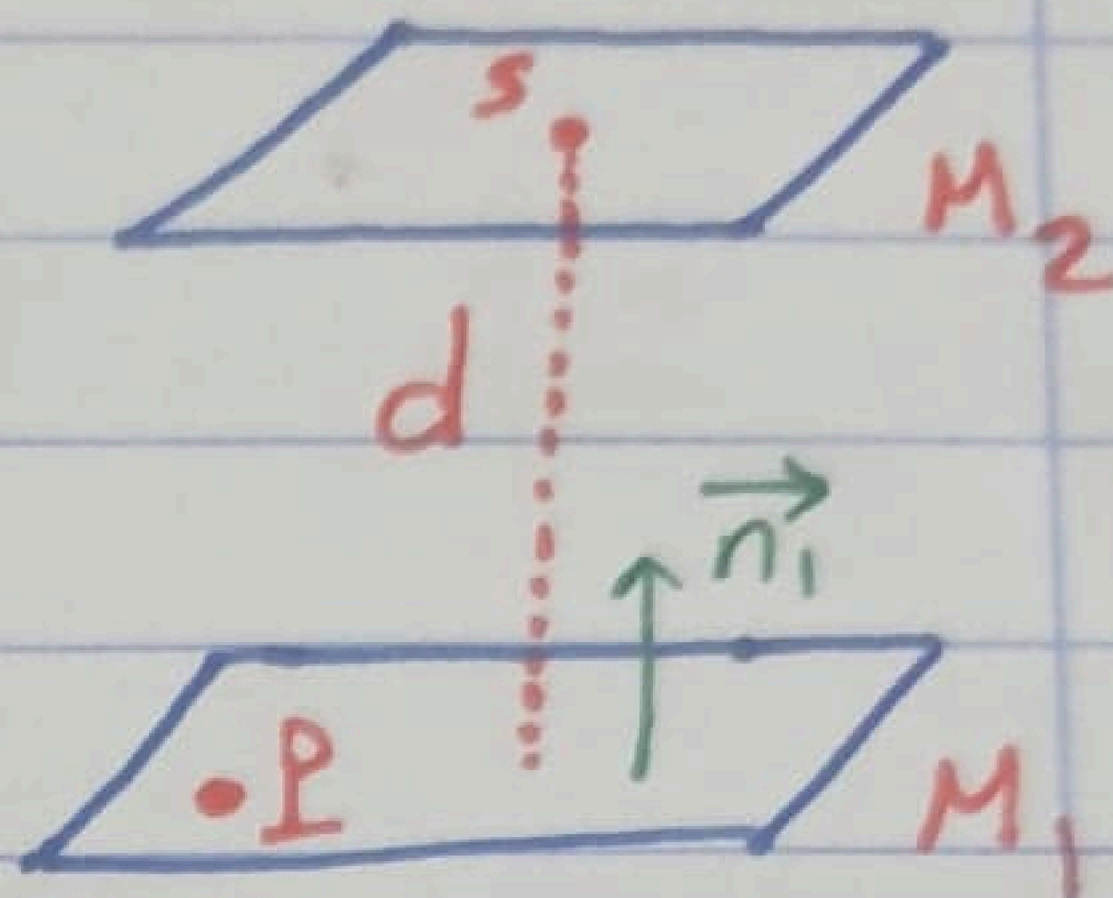
$\boxed{45}$ Find distance from the plane $\boxed{3}$

M_1: $x + 2y + 6z = 1$ to the plane

M_2: $x + 2y + 6z = 10$

$P = (1, 0, 0)$ on M_1

$S = (10, 0, 0)$ on M_2



$\vec{PS} = 9\vec{i}$

$\vec{n}_1 = \vec{i} + 2\vec{j} + 6\vec{k} \perp M_1$

The distance from S to M_1 is

$$d = \left| \frac{\vec{PS} \cdot \vec{n}_1}{|\vec{n}_1|} \right|$$

$|\vec{n}_1| = \sqrt{1 + 4 + 36} = \sqrt{41}$

$\vec{PS} \cdot \vec{n}_1 = 9$

$$= \left| \frac{9}{\sqrt{41}} \right|$$

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