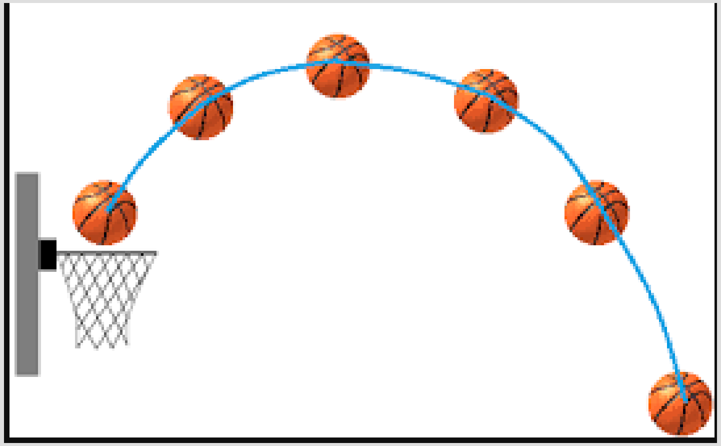
Chapter 4

Motion in Two and Three Dimensions



4-1 Position and Displacement

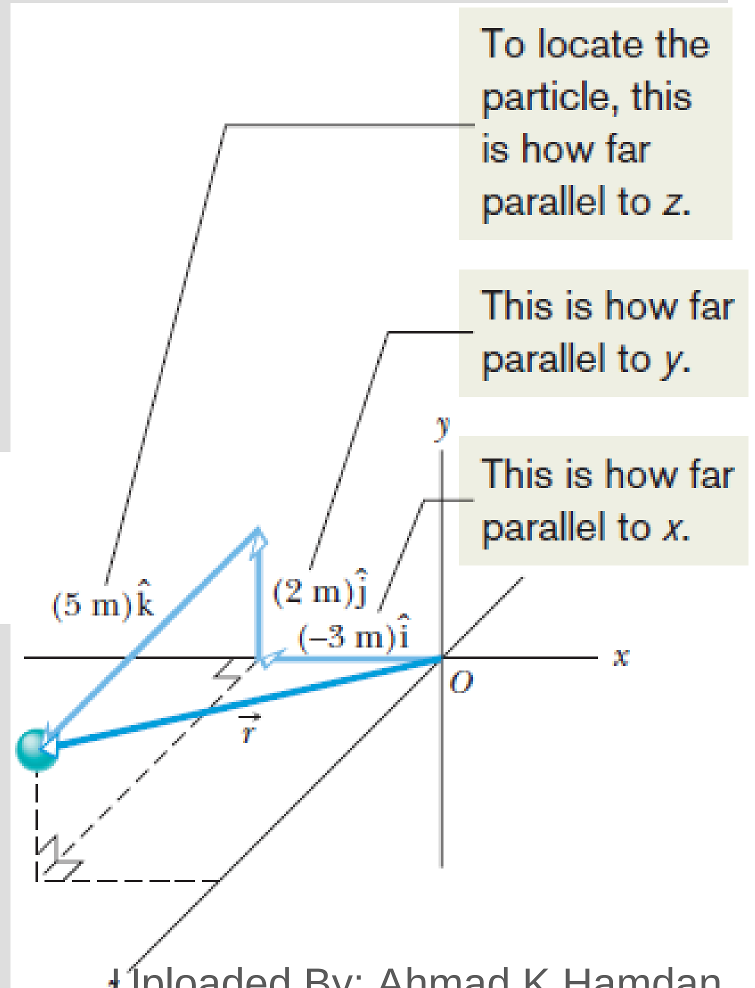
- A **position vector** locates a particle in space
  - Extends from a reference point (origin) to the particle

$$\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}},$$

Example

- Position vector (-3m, 2m, 5m)

$$\vec{r} = (-3\ \mathrm{m})\hat{\mathrm{i}} + (2\ \mathrm{m})\hat{\mathrm{j}} + (5\ \mathrm{m})\hat{\mathrm{k}}$$



- Change in position vector is a **Displacement**

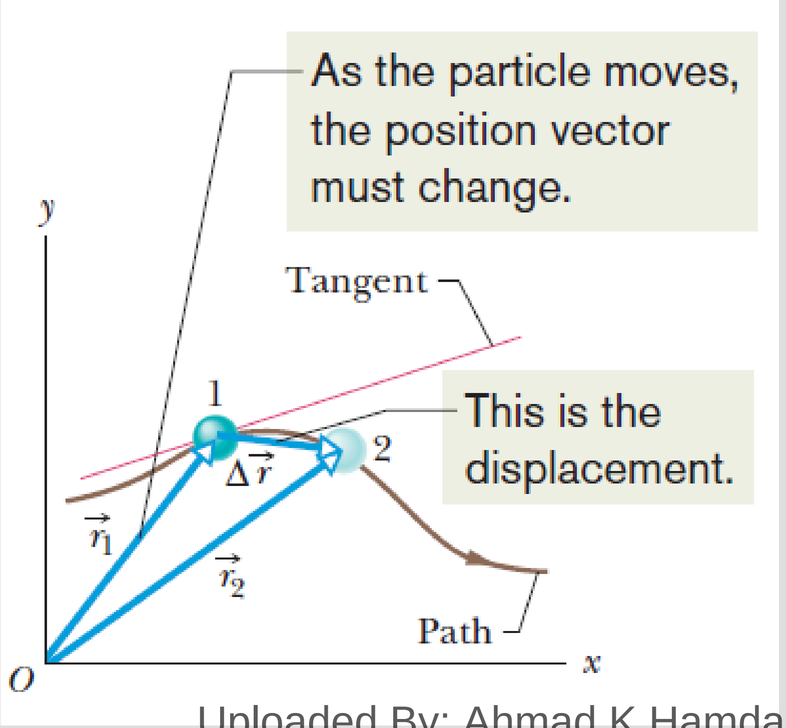
$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1.$$

- We can rewrite this as:

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k},$$

- Or express it in terms of changes in each coordinate:

$$\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}.$$



4-2 Average Velocity and Instantaneous Velocity

- **Average velocity** is
  - A displacement divided by its time interval

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}.$$

- We can write this in component form:

$$\vec{v}_{avg} = \frac{\Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}.$$

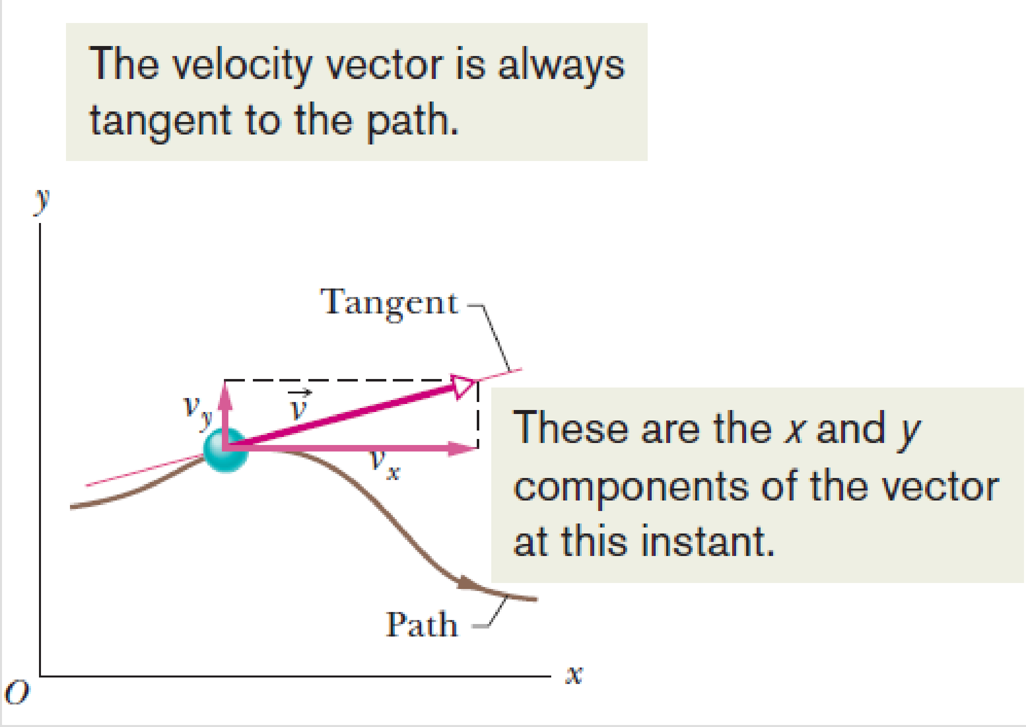
Example

- A particle moves through displacement (12 m)i + (3.0 m)k in 2.0 s:

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{(12\text{ m})\hat{i} + (3.0\text{ m})\hat{k}}{2.0\text{ s}} = (6.0\text{ m/s})\hat{i} + (1.5\text{ m/s})\hat{k}.$$

- **Instantaneous velocity** is
  - The velocity of a particle at a single point in time
  - The limit of avg. velocity as the time interval shrinks to 0

$$\vec{v} = \frac{d\vec{r}}{dt}.$$



Note: A velocity vector does not extend one point to another. Only shows direction and magnitude.

The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle's position.

- In unit-vector form, we write:

$$\vec{v} = \frac{d}{dt}(x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}) = \frac{dx}{dt}\hat{\mathrm{i}} + \frac{dy}{dt}\hat{\mathrm{j}} + \frac{dz}{dt}\hat{\mathrm{k}}.$$

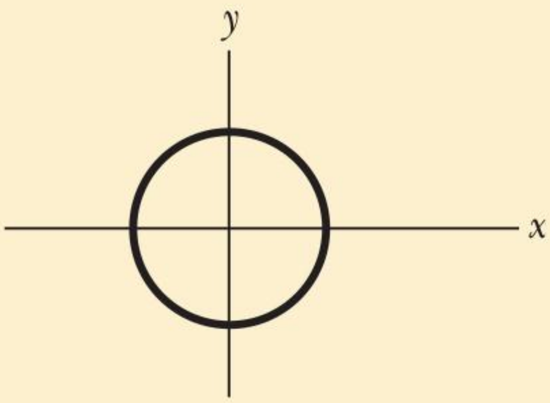
- Which can also be written:

$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}},$$

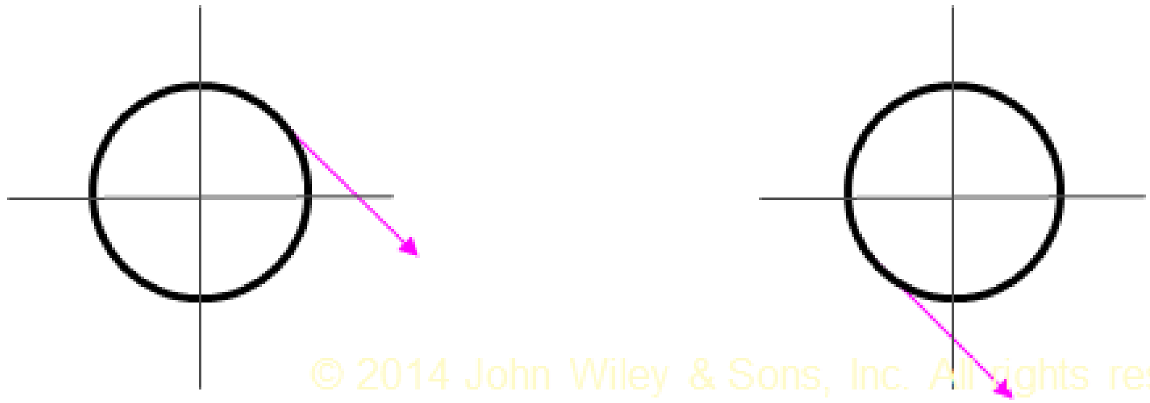
$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad \text{and} \quad v_z = \frac{dz}{dt}.$$

Checkpoint 1

The figure shows a circular path taken by a particle. If the instantaneous velocity of the particle is $\vec{v} = (2 \text{ m/s})\hat{i} - (2 \text{ m/s})\hat{j}$, through which quadrant is the particle moving at that instant if it is traveling (a) clockwise and (b) counterclockwise around the circle? For both cases, draw $\vec{v}$ on the figure.



Answer: (a) Quadrant I (b) Quadrant III



4-3 Average Acceleration and Instantaneous Acceleration

- **Average acceleration** is
  - A change in velocity divided by its time interval

$$\vec{a}_{avg} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}.$$

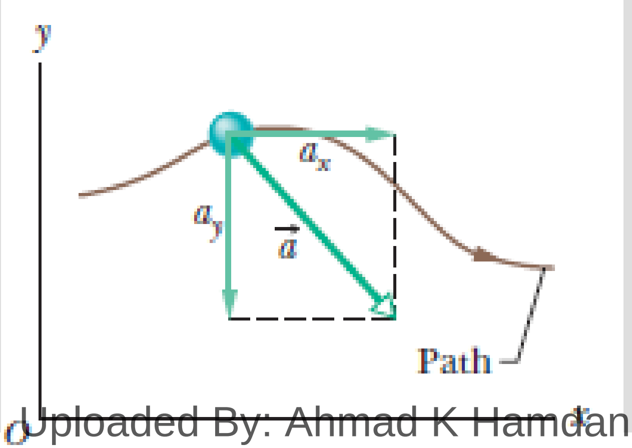
- **Instantaneous acceleration** is again the limit $t \rightarrow 0$:

$$\vec{a} = \frac{d\vec{v}}{dt}.$$

$$\vec{a} = \frac{d}{dt}(v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}}) = \frac{dv_x}{dt}\hat{\mathrm{i}} + \frac{dv_y}{dt}\hat{\mathrm{j}} + \frac{dv_z}{dt}\hat{\mathrm{k}}.$$

$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}},$$

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad \text{and} \quad a_z = \frac{dv_z}{dt}.$$



Checkpoint 2

Here are four descriptions of the position (in meters) of a puck as it moves in an xy plane:

(1) $x = -3t^2 + 4t - 2$ and $y = 6t^2 - 4t$ (3) $\vec{r} = 2t^2\hat{i} - (4t + 3)\hat{j}$

(2) $x = -3t^3 - 4t$ and $y = -5t^2 + 6$ (4) $\vec{r} = (4t^3 - 2t)\hat{i} + 3\hat{j}$

Are the x and y acceleration components constant? Is acceleration $\vec{a}$ constant?

1) a_x: Yes, a_y: Yes, $\vec{a}$: Yes
2) a_x: No, a_y: Yes, $\vec{a}$: No
3) a_x: Yes, a_y: Yes, $\vec{a}$: Yes
4) a_x: No, a_y: Yes, $\vec{a}$: No

3) $\vec{a} = a_x\hat{i} + a_y\hat{j} = (4\ m/s^2)\ \hat{i}$

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} = (4\ m/s^2)$$

$$a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt^2} = 0$$

Solve:
Sample problem 4.01, 4.02 and 4.03 (Rabbit)

* Find the avergge velocity of the rabit between $t = 1s$ to $t = 15s$?

$$\vec{v}_{avg} = \frac{\text{Displacement}}{\Delta t} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{\Delta t}$$

* $\vec{r}_f = \vec{r}(t = 15\,s) = x(t = 15s)\hat{i} + y(t = 15s)\hat{j}$

use $x(t) = -0.31t^2 + 7.2t + 28$

$y(t) = 0.22t^2 - 9.1t + 30$

$x(t = 1s) = -0.31(1)^2 + 7.2(1) + 28 = 34.89\,m$

$x(t = 15\,s) = 66.25\,m$

$y(t = 1\,s) = 21.12\,m$

$y(t = 15\,s) = -57\,m$

$\vec{r}_f = \vec{r}(t = 15s) = (66.25\,m)\hat{i} - (57.0\,m)\hat{j}$

$\vec{r}_i = \vec{r}(t = 1s) = (34.89\,m)\hat{i} + (21.12\,m)\hat{j}$

$\Delta \vec{r} = \vec{r}_f - \vec{r}_i = (66.25 - 34.89)\,m\,\hat{i} + (-57 - 21.12)\,m\,\hat{j}$

$= (31.36\,m)\hat{i} + (78.12\,m)\hat{j}$

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{(31.36\,m)\hat{i} + (78.12\,m)\hat{j}}{14\,s}$$

$$\boxed{\vec{v}_{avg} = (2.24\,m/s)\hat{i} + (5.58\,m/s)\hat{j}}$$

4-4 Projectile Motion

- A **projectile** is
  - A particle moving in the vertical plane
  - With some initial velocity
  - Whose acceleration is always free-fall acceleration (g)
- The motion of a projectile is **projectile motion**
- Launched with an initial velocity v_0

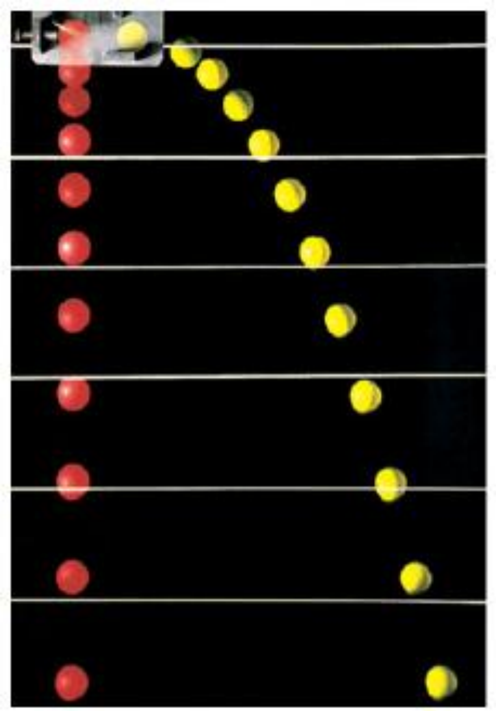
$$\vec{v}_0 = v_{0x}\hat{\mathrm{i}} + v_{0y}\hat{\mathrm{j}}.$$

$$v_{0x} = v_0 \cos\theta_0 \quad \text{and} \quad v_{0y} = v_0 \sin\theta_0.$$

In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.

- Therefore we can decompose two-dimensional motion into 2 one-dimensional problems

Projectile Motion = Horizontal Motion + Vertical Motion



Projectile Motion (2D motion)

The initial velocity

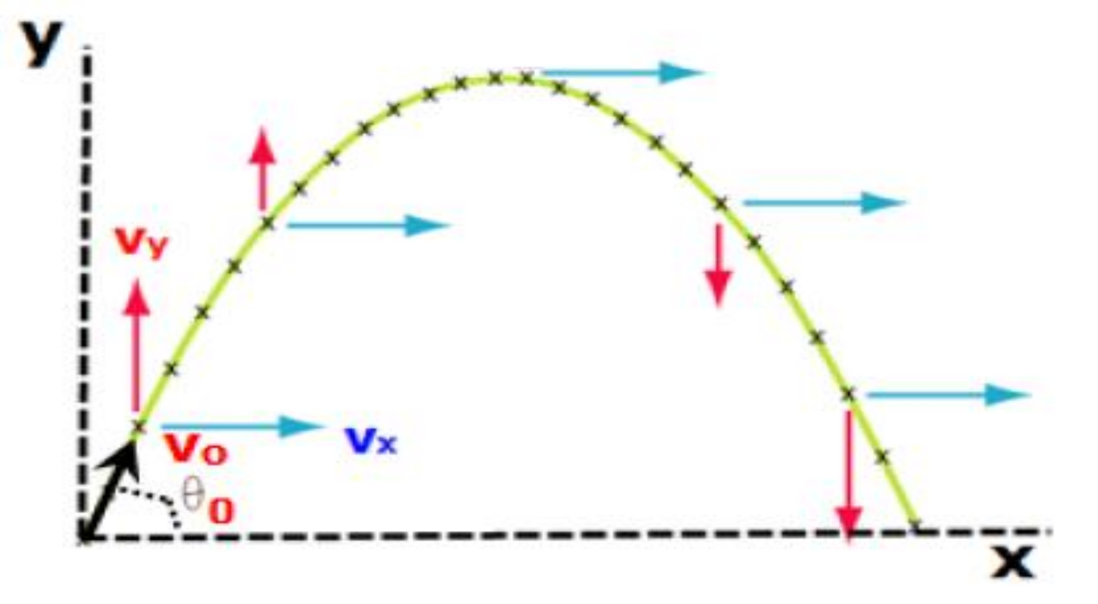
$$\vec{v}_{o} = v_{ox}\hat{\imath} + v_{oy}\hat{\jmath}$$

$$v_{0x} = v_0 \cos(\theta_o)$$

$$v_{0y} = v_0 \sin(\theta_o)$$

Acceleration

$$a_x = 0,$$
$$a_y = -g.$$



Velocity

$$v_x = v_{0x}$$

$$v_y = v_{0y} - gt$$

$$v = \sqrt{v_x^2 + v_y^2}$$

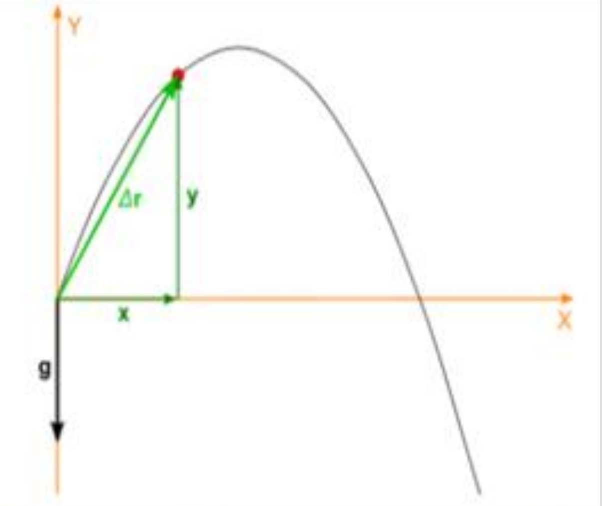
Displacement

$$x = v_0 t \cos(\theta_0)$$

$$y = v_0 t \sin(\theta_0) - \frac{1}{2} g t^2.$$

The magnitude of the displacement:

$$\Delta r = \sqrt{x^2 + y^2}.$$



Displacement and coordinates of parabolic throwing

Projectile Motion

- Horizontal motion:
  - No acceleration, so velocity is constant ($a_x = 0$):

$$x - x_0 = v_{0x} t.$$

$$x - x_0 = (v_0 \cos \theta_0) t.$$

- Vertical motion:
  - Acceleration is always –g ($a_y = -g$):

$$\begin{aligned} y - y_0 &= v_{0y} t - \tfrac{1}{2} g t^2 \\ &= (v_0 \sin \theta_0) t - \tfrac{1}{2} g t^2, \end{aligned}$$

$$v_y = v_0 \sin \theta_0 - g t$$

$$v_y^2 = (v_0 \sin \theta_0)^2 - 2g(y - y_0).$$

- The projectile's **trajectory** is
  - Its path through space (traces a parabola)

$$y = (\tan \theta_0)x - \frac{gx^2}{2(v_0 \cos \theta_0)^2}$$

- The **Horizontal Range** is:
  - The distance the projectile travels in *x* by the time it returns to its initial height

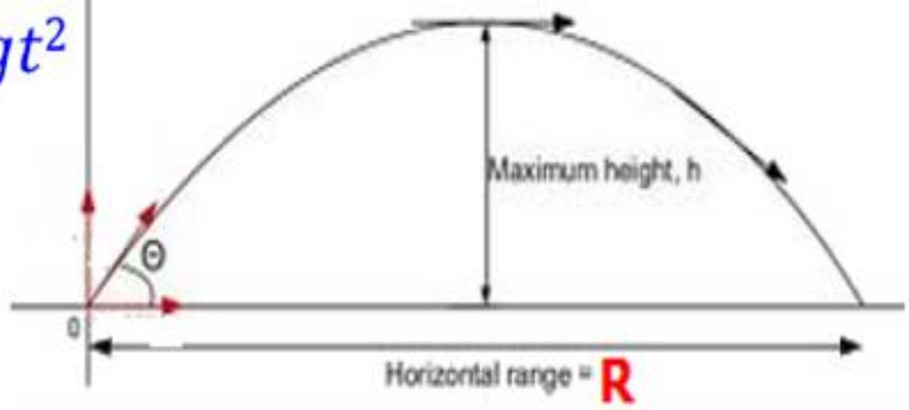
$$R = v_{0x} t_f$$

$$0 - 0 = v_{0y} t - \tfrac{1}{2} g t^2$$

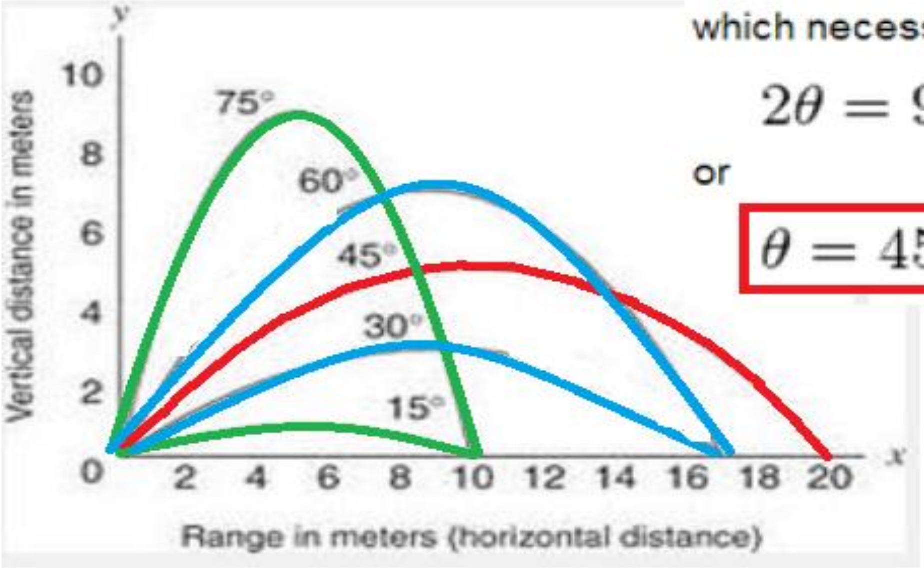
$$\boxed{t_f = 2 v_{0y} / g}$$

$$R = 2 v_{0x} v_{0y} / g$$

$$\boxed{R = \frac{2v^2{}_0 \cos(\theta_o) \sin(\theta_o)}{g} = \frac{v^2{}_0 \sin(2\theta_o)}{g}}$$



Maximum Range



R has its maximum value when

$$\sin 2\theta = 1,$$

which necessarily corresponds to

$$2\theta = 90°,$$

or

$$\theta = 45°.$$

Same Range with two different angles

$$R_1 = R_2$$

$$\sin 2\theta_1 = \sin 2\theta_2$$

$$2\theta_1 + 2\theta_2 = \pi$$

$$\theta_1 + \theta_2 = \pi/2$$

The maximum height of projectile

The highest height which the object will reach is known as the peak of the object's motion. The increase of the height will last, until $v_y = 0$, that is,

$$0 = v_0 \sin(\theta) - g t_h.$$

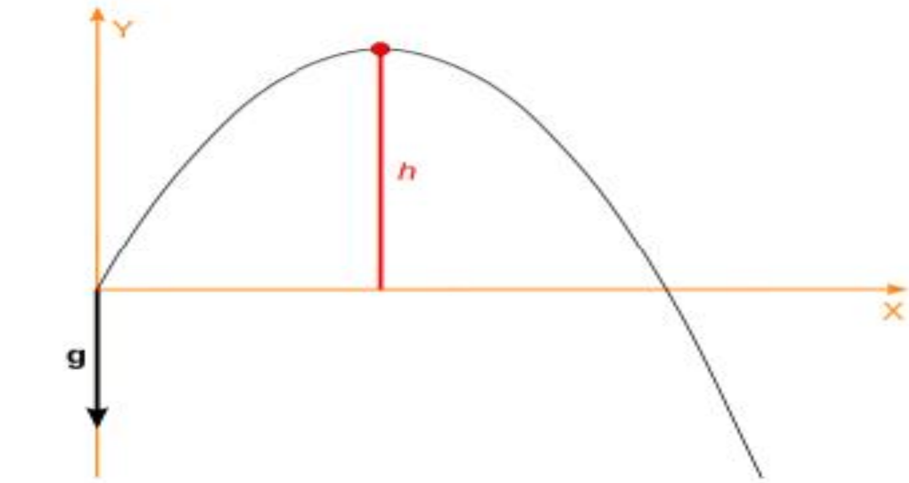
Time to reach the maximum height:

$$t_h = \frac{v_0 \sin(\theta)}{g}.$$

From the vertical displacement

$$h = v_0 t_h \sin(\theta) - \frac{1}{2} g t_h^2$$

$$h = \frac{v_0^2 \sin^2(\theta)}{2g}.$$



Maximum height of projectile

The relation between the range (R) on the horizontal plane and the maximum height (h) is:

$$h = \frac{v_0^2 \sin^2(\theta)}{2g}. \qquad \mathbf{R} = \frac{v_0^2 \sin\ (2\theta)}{g}.$$

$$h = \frac{R \tan\theta}{4}$$

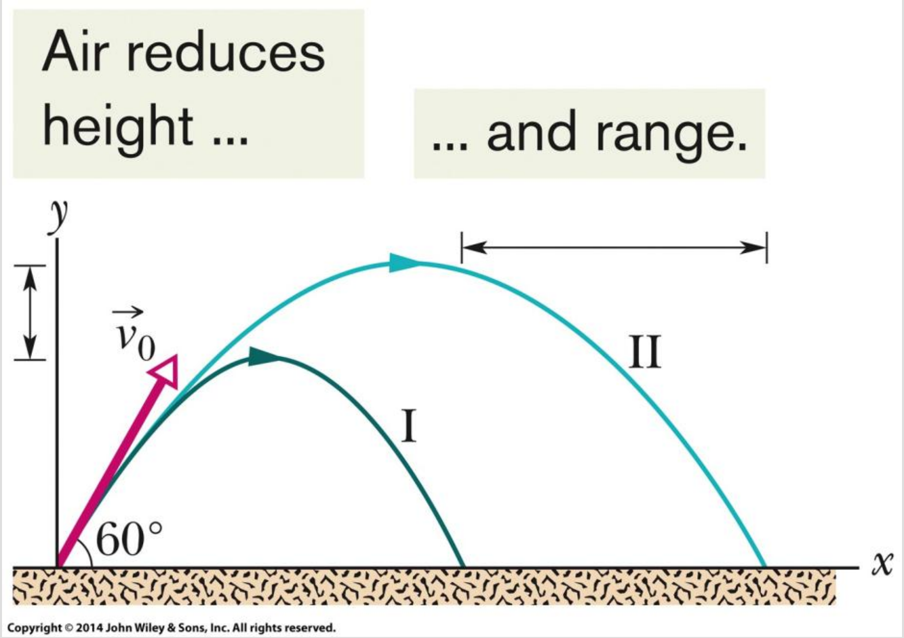
Air- resistance effects on Projectile Motion

- In these calculations we assume air resistance is negligible
- In many situations this is a poor assumption:

Table 4-1 Two Fly Balls[a]

	Path I (Air)	Path II (Vacuum)
Range	98.5 m	177 m
Maximum height	53.0 m	76.8 m
Time of flight	6.6 s	7.9 s

[a]See Fig. 4-13. The launch angle is 60° and the launch speed is 44.7 m/s.

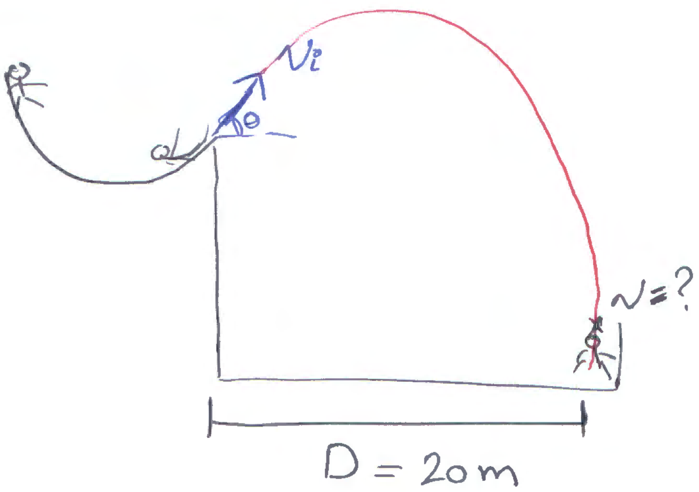


Checkpoint 3

At a certain instant, a fly ball has velocity $\vec{v} = 25\hat{i} - 4.9\hat{j}$ (the x axis is horizontal, the y axis is upward, and $\vec{v}$ is in meters per second). Has the ball passed its highest point?

Answer: Yes. The y-velocity is negative, so the ball is now falling.

Example Water Land (sample problem 4.05)



$\theta = 40°$

$t = 2.5$ sec (flight time)

1) $v_0 = ?$

Horizonal displacement $= D = \Delta x = v_{ix} t$

$20m = v_{ix} (2.5 s)$

$$v_{ix} = 8 m/s$$

Use $v_{ix} = v_i \cos\theta$

$8 \frac{m}{s} = v_i \cos(40°) \Rightarrow v_i = 10.44 \, m/s$

2) v (when he Land in water) $= ?$

$v_x = 8 m/s$ (constant)

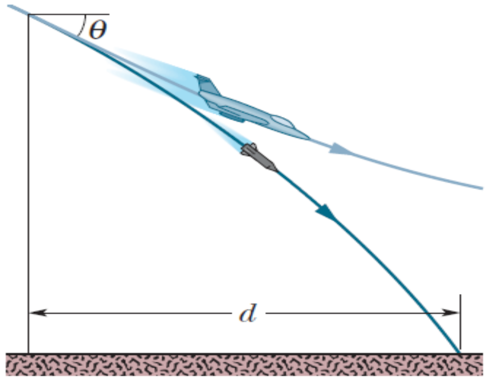
$v_{fy} = v_{iy} - gt$

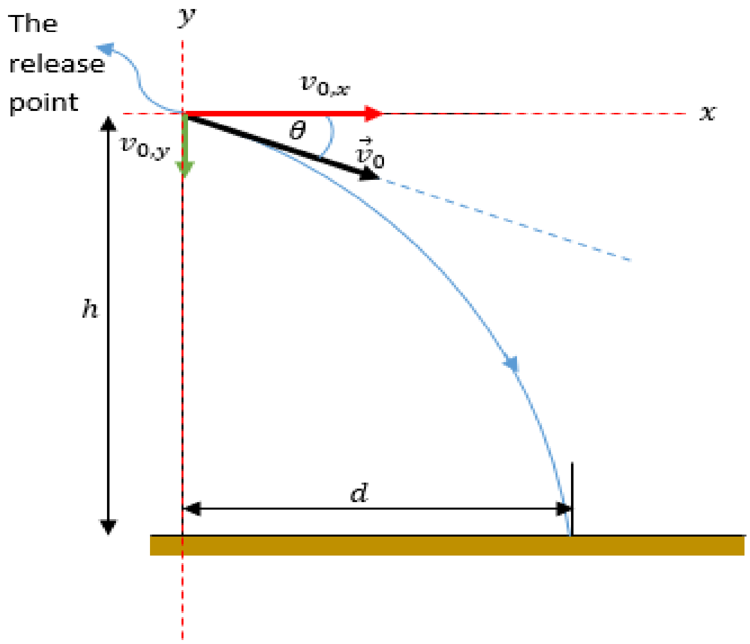
$= v_i \sin\theta - gt = 10.44 \sin(40°) - (10)(2.5)$

$$v_{fy} = -18.3 \, m/s$$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{8^2 + (-18.3)^2} = 20.0 \, m/s$

4-27) A certain airplane has a speed of 290.0 km/h and is diving at an angle of θ = 30.0° below the horizontal when the pilot releases a radar decoy. The horizontal distance between the release point and the point where the decoy strikes the ground is d =700 m. (a) How long is the decoy in the air? (b) How high was the release point?





$\theta = 30^{\circ}$

$v_0 = 290 \text{ Km/h} = 80.6 \text{ m/s}$

a) How long is the decoy in the air ?

$$d = v_{0,x} t = v_0 \cos\theta_0 \, t$$

$$t = \frac{d}{v_0 \cos\theta_0} = \frac{700}{80.6 \cos(30^{\circ})} = 10 \text{ sec}$$

b) How high was the release point ?

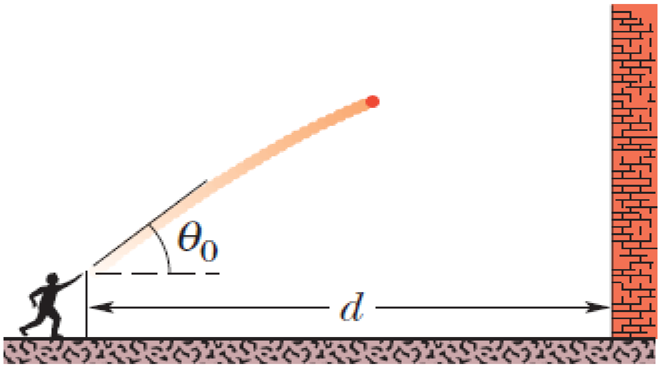
$$y_f - y_0 = v_{0,y} t + \frac{1}{2} g t^2$$

$$-h - 0 = -v_0 \sin\theta_0 \, t + \frac{1}{2} g t^2$$

$$-h = -80.6 * (\sin 30^{\circ}) * (10) + \frac{1}{2}(-10) * (10)^2$$

$$h = 903 \text{ m}$$

p-32) You throw a ball toward a wall at speed 25.0 m/s and at angle $\theta_\circ = 40.0°$ above the horizontal. The wall is distance d = 22.0 m from the release point of the ball. (a) How far above the release point does the ball hit the wall? What are the (b) horizontal and (c) vertical components of its velocity as it hits the wall? (d) When it hits, has it passed the highest point on its trajectory?



$v_{0x} = 25.0\cos(40.0°)\frac{m}{s}$, $v_{0y} = 25.0\sin(40.0°)\frac{m}{s}$

d = 22.0 m is the horizontal range of the ball

$d = v_{0x}t = v_0\cos(\theta_0)\,t$

$$t = \frac{d}{v_0\cos(\theta_0)} = \frac{22.0\,m}{25.0\cos(40.0°)\,m/s} = 1.15\,s$$

$t = 1.15\,s\ is\ the\ ball\ flight\ time$

(a) $y_f - y_0 = v_{0,y}t + \frac{1}{2}gt^2$

$$\Delta y\ = 25.0\sin(40.0°)\,(1.15) + \frac{1}{2}(-9.8)(1.15)^2 = 12.0\,m$$

(b)The horizontal component of ball's velocity when it hits the wall equals the initial horizontal component of the velocity $(a_x = zero)$

$$v_x = 25.0\cos(40.0°)\,m/s = 19.15\,m/s$$

(c) The vertical component of the ball's velocity when it hits the wall: $v_y = v_{0y} + gt = 25.0\sin(40.0°) - 9.8\,(1.15) = 4.8\,m/s$

The ball's velocity when it hits the wall is $(19.15\,m/s\,\hat{\imath} + 4.8\ m/s\,\hat{\jmath})$
The ball's speed when it hits the wall is $19.74\ m/s$

(d) The vertical component of the ball's velocity when it hits the wall is positive ($v_y > 0$), so it has not reached the highest point yet.

Example: A bullet is fired horizontally from a gun that's 1.5 m from the ground. The bullet travels at 1000 m/s and strikes a tree 150 m away. How far up the tree from the ground does the bullet hit? (neglect air-resistance)

$v_{ix} = 1000\ m/s = v_{fx} \quad (a_x = 0)$

$\Delta x = v_{ix} t \Rightarrow 150m = (1000 \frac{m}{s}) t$

$t = 0.15\ s$ Flight time

$v_{fy} = v_{iy} + gt \quad , v_{iy} = 0$

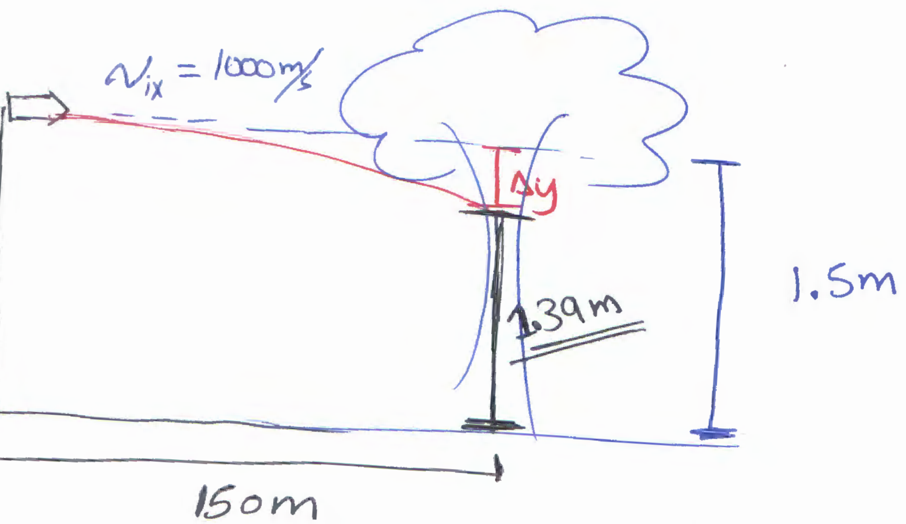
$= 0 - 10(0.15) \longrightarrow v_{fy} = -1.5\ m/s$

$\Delta y_{bullet} = v_{iy} t + \frac{1}{2} g t^2$

$= 0 + \frac{1}{2}(-10)(0.15)^2 = -0.1125\ m$

[the bullet deflected down]

$\Rightarrow 1.5\ m - 0.1125\ m = \underline{1.39\ m}$



↓ $g = 10\ m/s^2$ down

Example: To Catch a Thief

police officer chases a master jewel thief across city rooftops. They are both running when they come to a gap between buildings that is 4.0 m wide and has a drop of 3.0 m. The thief having studied a little physics, leaps at 5.0 m/s at an angle of 45° above the horizontal and clears the gap easily. The police officer did not study physics and thinks he should maximize his horizontal velocity, so he leaps horizontally at 5.0 m/s.

(a) Does he clear the gap? No

(b) By how much does the thief clear the gap? 31 cm

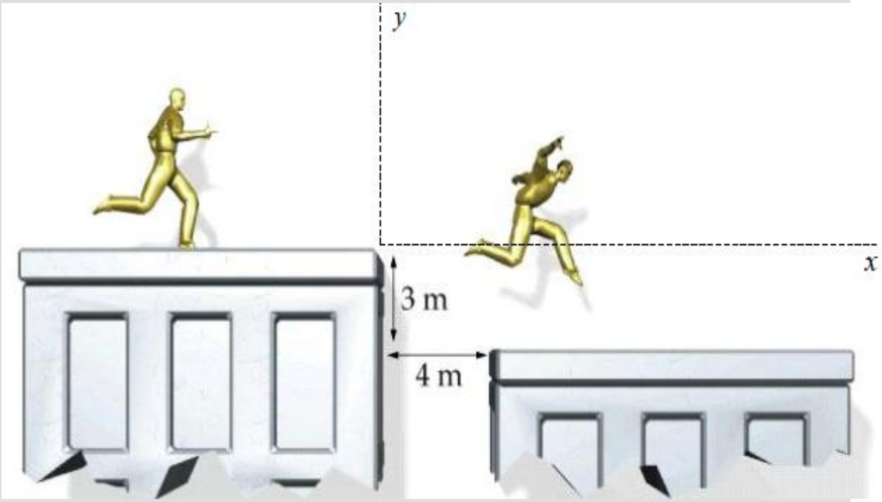
Police officer

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$-3.0\ \text{m} = 0 + 0 - \frac{1}{2}(9.81\ \text{m/s}^2)t^2$$

$$t = \sqrt{6.0\ \text{m} / 9.81\ \text{m/s}^2} = 0.782\ \text{s}$$

$$x = x_0 + v_{0x}t = 0 + (5.0\ \text{m/s})\ 0.78\ \text{s} = 3.91\ \text{m}$$



Thief

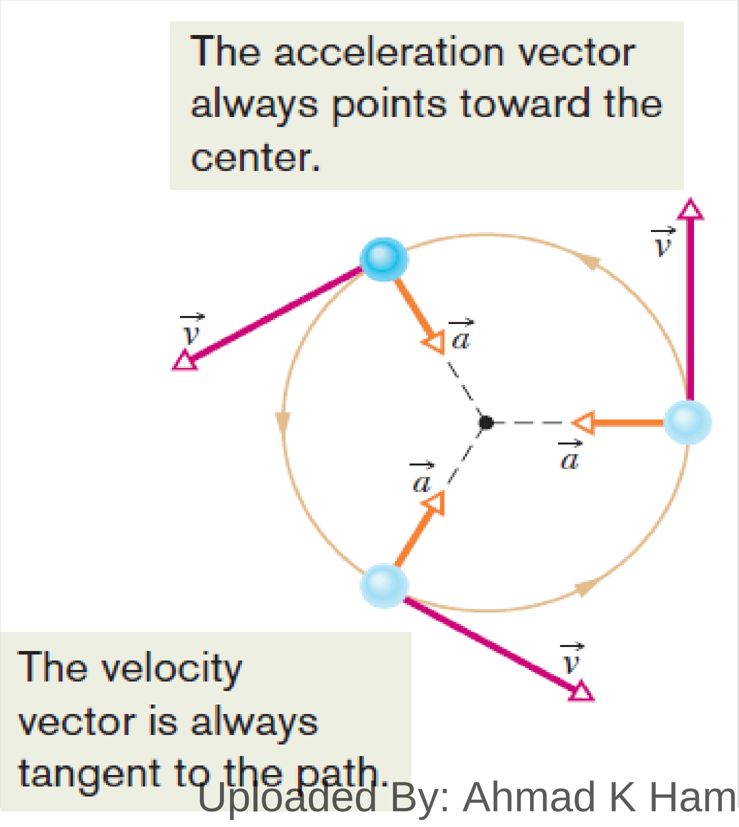
$$-3.0\ \text{m} = 0 + (5.0\ \text{m/s})\sin 45° t - \frac{1}{2}(9.81\ \text{m/s}^2)t^2$$

$$t = -0.50\ \text{s} \quad \text{or} \quad t = 1.22\ \text{s}$$

$$x = x_0 + v_{0x}t = 0 + (5.0\ \text{m/s})(1.22\ \text{s})\cos 45° = 4.31\ \text{m}$$

4-5 Uniform Circular Motion

- A particle is in **uniform circular motion** if
  - It travels around a circle or circular arc
  - At a constant speed
- Since the velocity changes, the particle is accelerating!
- Velocity and acceleration have:
  - Constant magnitude
  - Changing direction



- Acceleration is called **centripetal acceleration**
  - Means “center seeking”
  - Directed radially inward

$$a = \frac{v^2}{r}$$

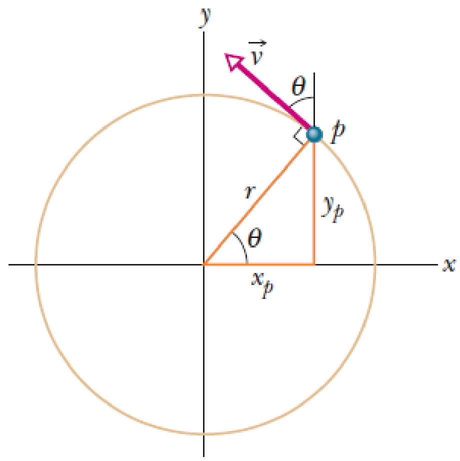
- The **period of revolution** is:
  - The time it takes for the particle go around the closed path exactly once

$$T = \frac{2\pi r}{v}$$

Uniform Circular Motion:

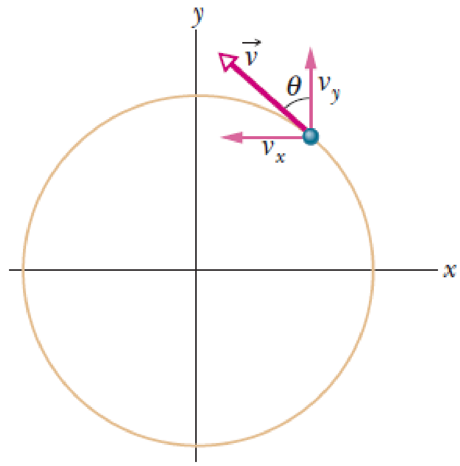
$$a_c = \frac{v^2}{r} \quad (Radially\ Inward)$$

Proof:



$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} = (-v \sin\theta)\hat{\mathrm{i}} + (v \cos\theta)\hat{\mathrm{j}}.$$

$$\vec{v} = \left(-\frac{vy_p}{r}\right)\hat{\mathrm{i}} + \left(\frac{vx_p}{r}\right)\hat{\mathrm{j}}.$$



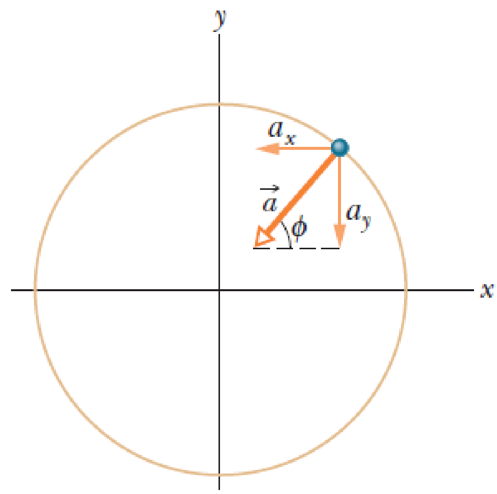
$$\vec{a} = \frac{d\vec{v}}{dt} = \left(-\frac{v}{r}\frac{dy_p}{dt}\right)\hat{\mathrm{i}} + \left(\frac{v}{r}\frac{dx_p}{dt}\right)\hat{\mathrm{j}}.$$

$$\vec{a} = \left(-\frac{v^2}{r}\cos\theta\right)\hat{\mathrm{i}} + \left(-\frac{v^2}{r}\sin\theta\right)\hat{\mathrm{j}}$$

$$a = \sqrt{a_x^2 + a_y^2} = \frac{v^2}{r}\sqrt{(\cos\theta)^2 + (\sin\theta)^2} = \frac{v^2}{r}\sqrt{1} = \frac{v^2}{r}$$

$$\tan\phi = \frac{a_y}{a_x} = \frac{-(v^2/r)\sin\theta}{-(v^2/r)\cos\theta} = \tan\theta.$$

Thus, $\phi = \theta$, which means that $\vec{a}$ is directed along the radius r toward the circle's center, as we wanted to prove.

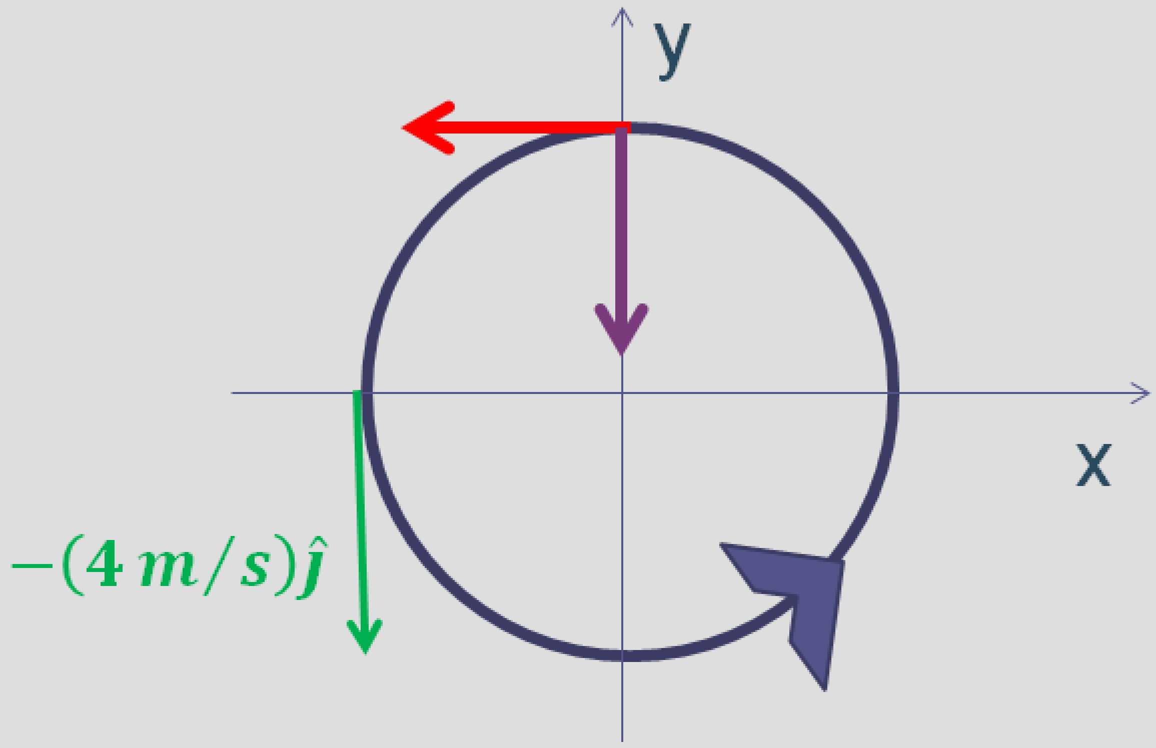


Checkpoint 5

An object moves at constant speed along a circular path in a horizontal *xy* plane, with the center at the origin. When the object is at $x = -2$ m, its velocity is $-(4\ \mathrm{m/s})\hat{\mathrm{j}}$. Give the object's (a) velocity and (b) acceleration at $y = 2$ m.

$$\vec{v} = -(4\,m/s)\hat{\imath}$$

$$\vec{a} = -(8\,m/s)\hat{\jmath}$$



P-58) A rotating fan completes 1200 revolutions every minute. Consider the tip of a blade, at a radius of 0.15 m. (a) Through what distance does the tip move in one revolution? What are (b) the tip's speed and (c) the magnitude of its acceleration? (d) What is the period of the motion?

a) One revolution equals one circumference

$$C = 2\pi r = 2\pi(0.15m) = 0.942\ m$$

b) 1200 revolutions every minute:

$$v = \frac{1200\ revolution}{1\ min}\left(\frac{0.942\ m}{1\ revolution}\right)\left(\frac{1\ min}{60\ s}\right) = 18.84\ m/s$$

c) $a = \frac{v^2}{r} = \frac{(18.84\ m/s)^2}{0.15\ m} = 2.37 \times 10^3 m/s^2$

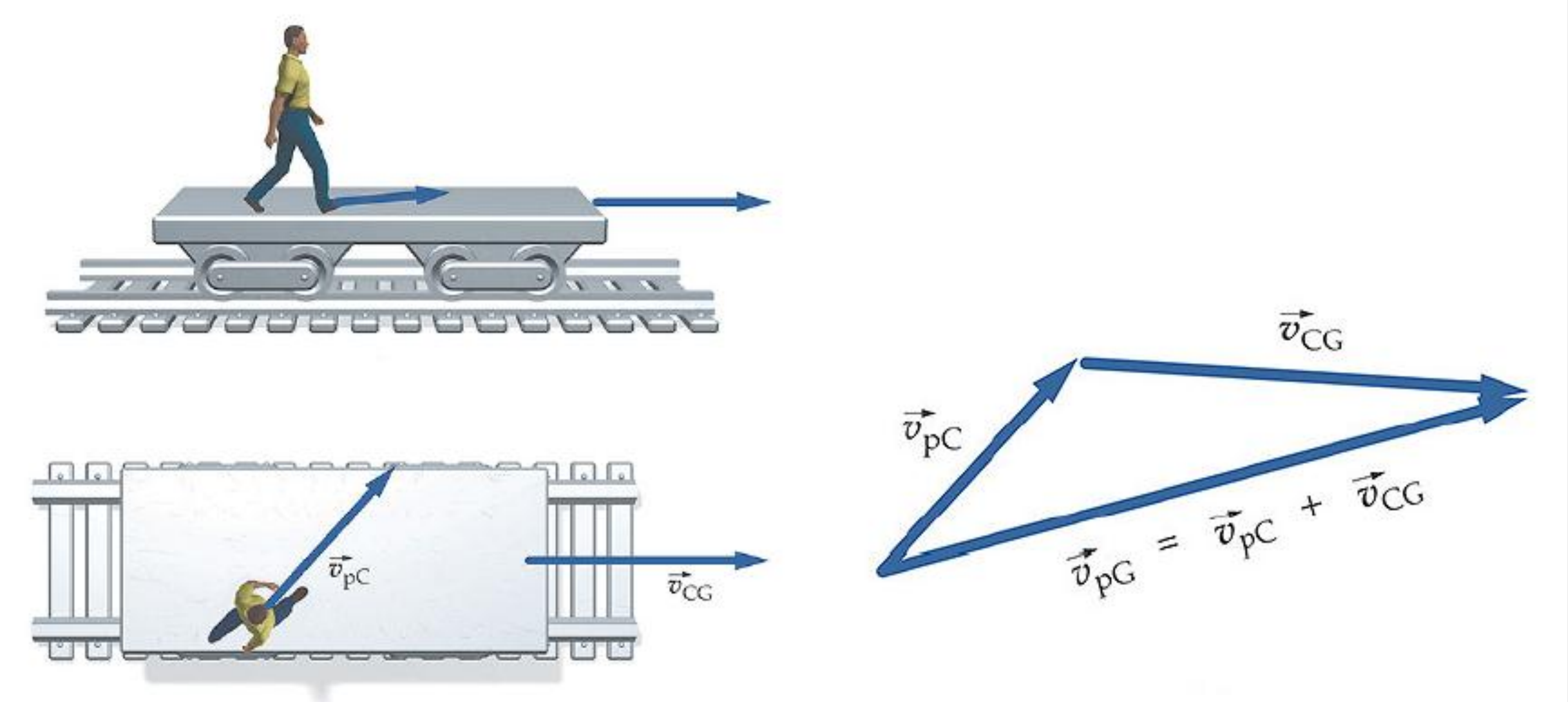
d) $T = \frac{2\pi r}{v} = \frac{0.942\ m}{18.84\ m/s} = 0.05\ s$

$$1200\ revolution \rightarrow 60\ s$$

$$1\ revolution \rightarrow ?\ s$$

$$\left(\frac{60}{1200}\right) = 0.05\ s$$

4-6 Relative Motion in One Dimension



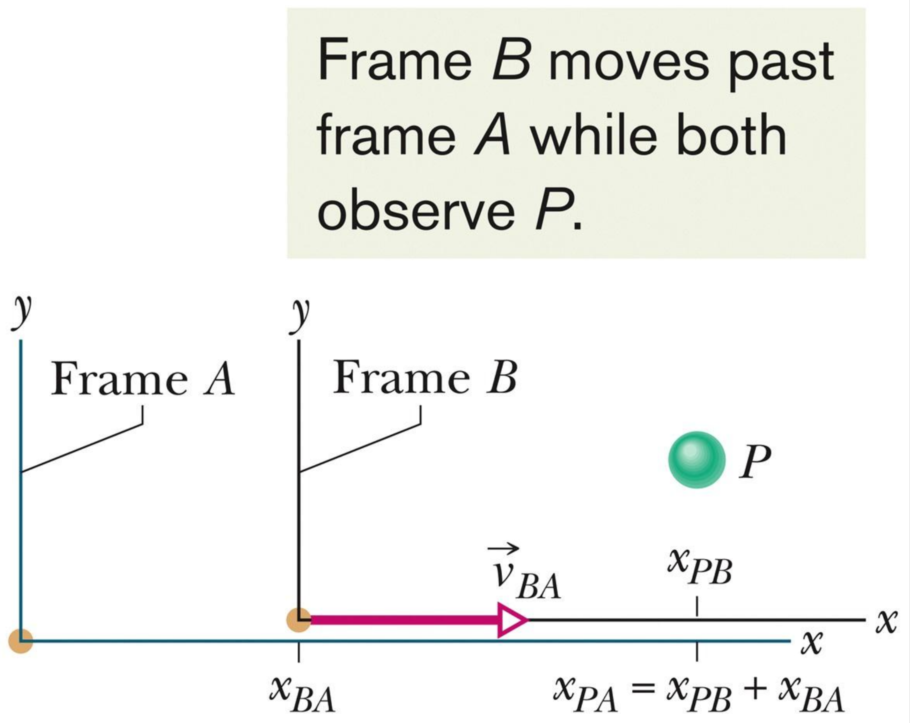
4-6 Relative Motion in One Dimension

- Measures of position and velocity depend on the **reference frame** of the measurer
  - How is the observer moving?
  - Our usual reference frame is that of the ground

- Read subscripts:

"PA" → P as measured by A
"PB" → P as measured by B
"BA" → B as measured by A

Frames A and B are each watching the movement of object P



- Positions in different frames are related by:

$$x_{PA} = x_{PB} + x_{BA}.$$

- Taking the derivative, velocities are related by:

$$\frac{d}{dt}(x_{PA}) = \frac{d}{dt}(x_{PB}) + \frac{d}{dt}(x_{BA}).$$

$$v_{PA} = v_{PB} + v_{BA}.$$

- But accelerations (for non-accelerating reference frames, $a_{BA} = 0$) are related by

$$\frac{d}{dt}(v_{PA}) = \frac{d}{dt}(v_{PB}) + \frac{d}{dt}(v_{BA}).$$

$$a_{PA} = a_{PB}.$$

Observers on different frames of reference that move at constant velocity relative to each other will measure the same acceleration for a moving particle.

Example

Frame A: $x = 2$ m, $v = 4$ m/s

Frame B: $x = 3$ m, $v = -2$ m/s

P as measured by A: $x_{PA} = 5$ m, $v_{PA} = 2$ m/s, $a = 1$ m/s^2

So P as measured by B:

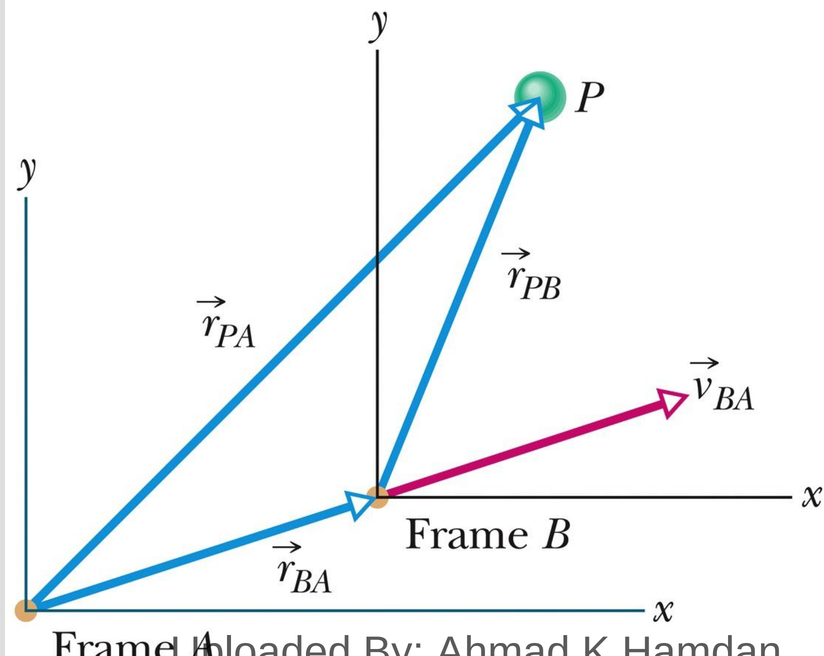
- $x_{PB} = x_{PA} + x_{AB} = 5$ m + (2m – 3m) = 4 m
- $v_{PB} = v_{PA} + v_{AB} = 2$ m/s + (4 m/s – -2m/s) = 8 m/s
- $a = 1$ m/s^2

4-7 Relative Motion in Two Dimensions

- The same as in one dimension, but now with vectors:
- Positions in different frames are related by: $\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}.$
- Velocities: $\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}.$
- Accelerations (for non-accelerating reference frames):

$$\vec{a}_{PA} = \vec{a}_{PB}.$$

- Again, observers in different frames will see the same acceleration



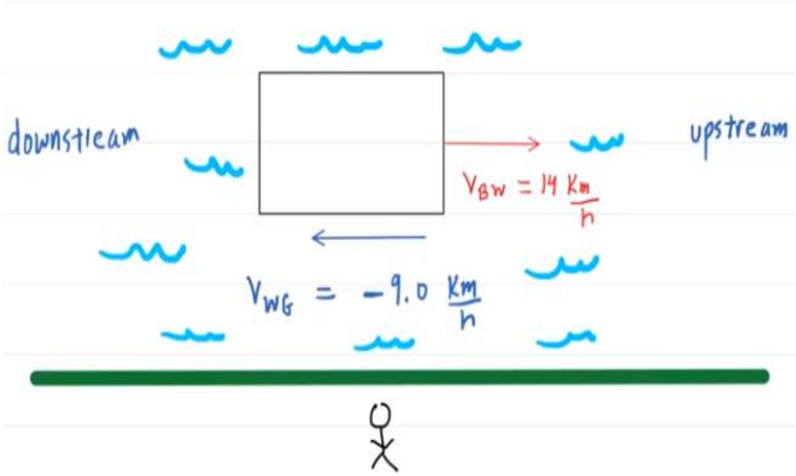
•70 A boat is traveling upstream in the positive direction of an x axis at 14 km/h with respect to the water of a river. The water is flowing at 9.0 km/h with respect to the ground. What are the (a) magnitude and (b) direction of the boat's velocity with respect to the ground? A child on the boat walks from front to rear at 6.0 km/h with respect to the boat. What are the (c) magnitude and (d) direction of the child's velocity with respect to the ground?

a)

$$\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WD}$$

$$\vec{v}_{BG} = \left(14\frac{km}{h}\hat{\imath}\right) + \left(-9\frac{km}{h}\hat{\imath}\right)$$

$$\vec{v}_{BG} = 5\frac{km}{h}\hat{\imath}$$

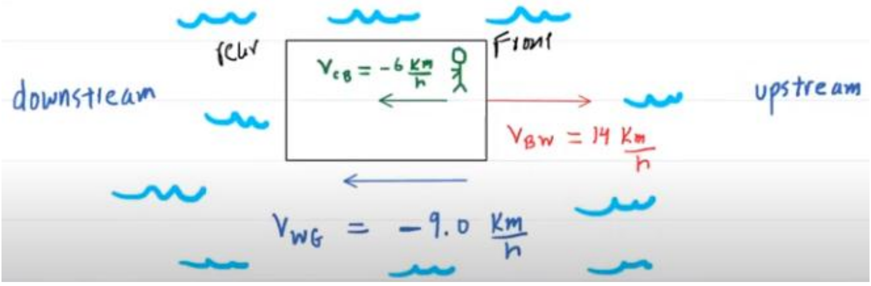


b) Upstream (+ x-axis)

c)

$$\vec{v}_{CG} = \vec{v}_{CB} + \vec{v}_{BG}$$

$$\vec{v}_{CG} = \left(-6\frac{km}{h}\hat{\imath}\right) + \left(5\frac{km}{h}\hat{\imath}\right)$$



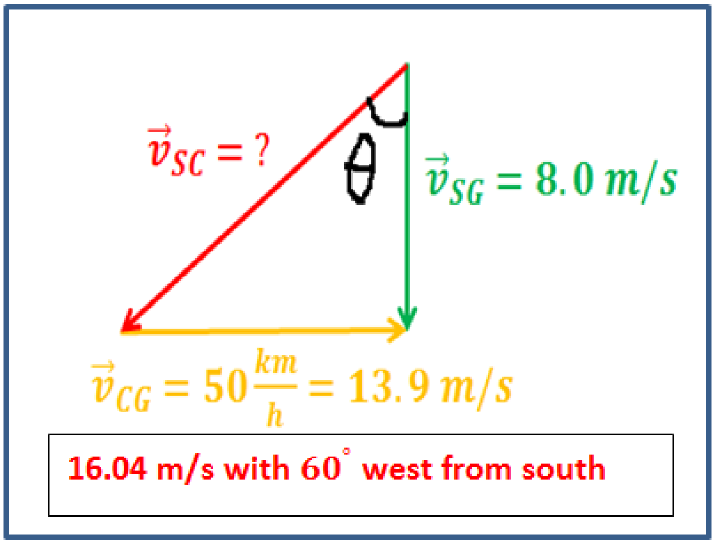
$$\vec{v}_{CG} = \left(-1\frac{km}{h}\hat{\imath}\right)$$

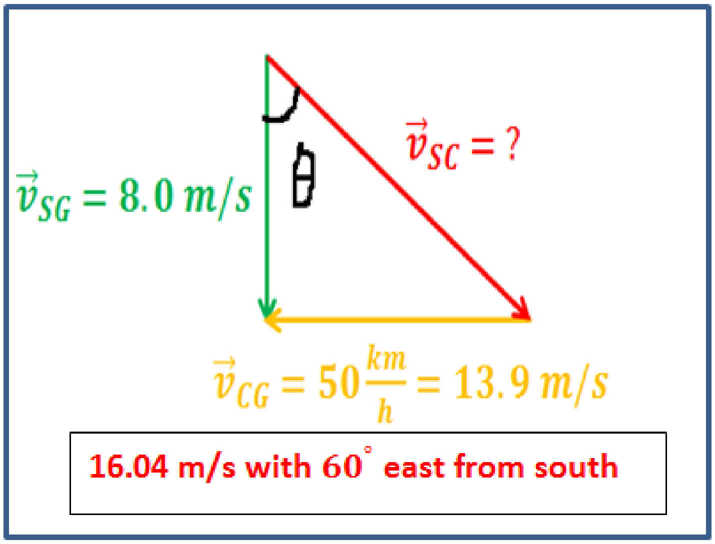
$$v_{CG} = 1\frac{km}{h}$$

d) Downstream

••77 SSM Snow is falling vertically at a constant speed of 8.0 m/s. At what angle from the vertical do the snowflakes appear to be falling as viewed by the driver of a car traveling on a straight, level road with a speed of 50 km/h?

$$\vec{v}_{SG} = \vec{v}_{SC} + \vec{v}_{CG}$$





$$\vec{v}_{SC} = \sqrt{(8.0\ m/s)^2 + (13.9\ m/s)^2} = 16.04\ m/s$$

$$\tan\theta = \frac{v_{CG}}{v_{SG}} = \frac{13.9}{8} \rightarrow\rightarrow\rightarrow \theta = 60^{\circ}$$

