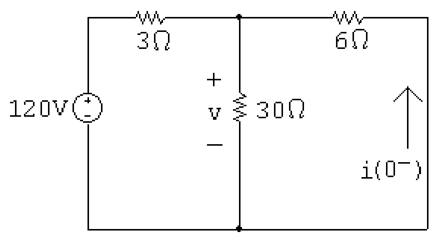
7

Response of First-Order RL and RC Circuits

Assessment Problems

AP 7.1 **[a]** The circuit for $t < 0$ is shown below. Note that the inductor behaves like a short circuit, effectively eliminating the $2\,\Omega$ resistor from the circuit.



First combine the $30\,\Omega$ and $6\,\Omega$ resistors in parallel:
$30\|6 = 5\,\Omega$
Use voltage division to find the voltage drop across the parallel resistors:
$v = \frac{5}{5+3}(120) = 75\,\text{V}$
Now find the current using Ohm's law:
$i(0^-) = -\frac{v}{6} = -\frac{75}{6} = -12.5\,\text{A}$

[b] $w(0) = \frac{1}{2}Li^2(0) = \frac{1}{2}(8\times 10^{-3})(12.5)^2 = 625\,\text{mJ}$

[c] To find the time constant, we need to find the equivalent resistance seen by the inductor for $t > 0$. When the switch opens, only the $2\,\Omega$ resistor remains connected to the inductor. Thus,
$\tau = \frac{L}{R} = \frac{8\times 10^{-3}}{2} = 4\,\text{ms}$

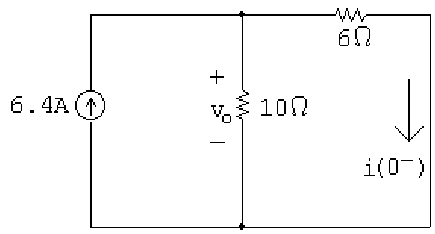
[d] $i(t) = i(0^-)e^{t/\tau} = -12.5e^{-t/0.004} = -12.5e^{-250t}\,\text{A}, \qquad t \geq 0$

[e] $i(5\,\text{ms}) = -12.5e^{-250(0.005)} = -12.5e^{-1.25} = -3.58\,\text{A}$
So $\quad w\,(5\,\text{ms}) = \frac{1}{2}Li^2(5\,\text{ms}) = \frac{1}{2}(8)\times 10^{-3}(3.58)^2 = 51.3\,\text{mJ}$

$w\,(\text{dis}) = 625 - 51.3 = 573.7\,\text{mJ}$

$\%\ \text{dissipated} = \left(\frac{573.7}{625}\right)100 = 91.8\%$

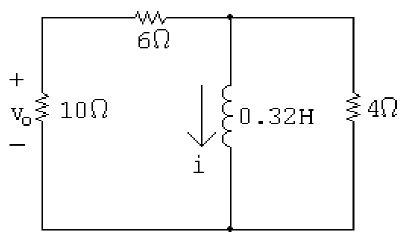
AP 7.2 **[a]** First, use the circuit for $t < 0$ to find the initial current in the inductor:



Using current division,

$i(0^-) = \frac{10}{10+6}(6.4) = 4\,\text{A}$

Now use the circuit for $t > 0$ to find the equivalent resistance seen by the inductor, and use this value to find the time constant:



$R_{\text{eq}} = 4\|(6+10) = 3.2\,\Omega, \quad \therefore \quad \tau = \frac{L}{R_{\text{eq}}} = \frac{0.32}{3.2} = 0.1\,\text{s}$

Use the initial inductor current and the time constant to find the current in the inductor:

$i(t) = i(0^-)e^{-t/\tau} = 4e^{-t/0.1} = 4e^{-10t}\,\text{A}, \quad t \geq 0$

Use current division to find the current in the $10\,\Omega$ resistor:

$$i_o(t) = \frac{4}{4+10+6}(-i) = \frac{4}{20}(-4e^{-10t}) = -0.8e^{-10t}\,\text{A}, \quad t \geq 0^+$$

Finally, use Ohm's law to find the voltage drop across the $10\,\Omega$ resistor:

$v_o(t) = 10i_o = 10(-0.8e^{-10t}) = -8e^{-10t}\,\text{V}, \quad t \geq 0^+$

[b] The initial energy stored in the inductor is

$$w(0) = \frac{1}{2}Li^2(0^-) = \frac{1}{2}(0.32)(4)^2 = 2.56\,\text{J}$$

Find the energy dissipated in the $4\,\Omega$ resistor by integrating the power over all time:

$$v_{4\Omega}(t) = L\frac{di}{dt} = 0.32(-10)(4e^{-10t}) = -12.8e^{-10t}\,\text{V}, \qquad t \geq 0^+$$

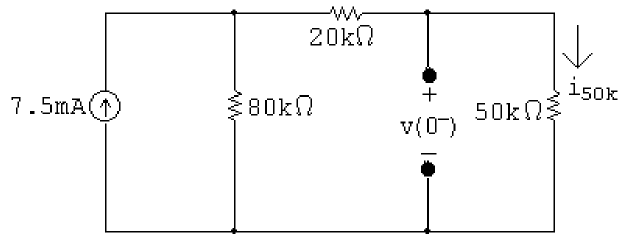
$$p_{4\Omega}(t) = \frac{v_{4\Omega}^2}{4} = 40.96e^{-20t}\,\text{W}, \qquad t \geq 0^+$$

$$w_{4\Omega}(t) = \int_0^\infty 40.96e^{-20t}dt = 2.048\,\text{J}$$

Find the percentage of the initial energy in the inductor dissipated in the $4\,\Omega$ resistor:

$$\%\text{ dissipated} = \left(\frac{2.048}{2.56}\right)100 = 80\%$$

AP 7.3 **[a]** The circuit for $t < 0$ is shown below. Note that the capacitor behaves like an open circuit.



Find the voltage drop across the open circuit by finding the voltage drop across the $50\,\text{k}\Omega$ resistor. First use current division to find the current through the $50\,\text{k}\Omega$ resistor:

$$i_{50\text{k}} = \frac{80 \times 10^3}{80 \times 10^3 + 20 \times 10^3 + 50 \times 10^3}(7.5 \times 10^{-3}) = 4\,\text{mA}$$

Use Ohm's law to find the voltage drop:

$v(0^-) = (50 \times 10^3)i_{50\text{k}} = (50 \times 10^3)(0.004) = 200\,\text{V}$

[b] To find the time constant, we need to find the equivalent resistance seen by the capacitor for $t > 0$. When the switch opens, only the $50\,\text{k}\Omega$ resistor remains connected to the capacitor. Thus,

$\tau = RC = (50 \times 10^3)(0.4 \times 10^{-6}) = 20\,\text{ms}$

[c] $v(t) = v(0^-)e^{-t/\tau} = 200e^{-t/0.02} = 200e^{-50t}\,\text{V}, \quad t \geq 0$

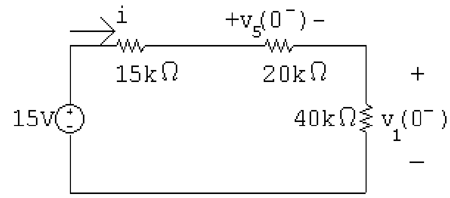
[d] $w(0) = \frac{1}{2}Cv^2 = \frac{1}{2}(0.4 \times 10^{-6})(200)^2 = 8\,\text{mJ}$

[e] $w(t) = \frac{1}{2}Cv^2(t) = \frac{1}{2}(0.4 \times 10^{-6})(200e^{-50t})^2 = 8e^{-100t}\,\text{mJ}$

The initial energy is 8 mJ, so when 75% is dissipated, 2 mJ remains:

$$8 \times 10^{-3}e^{-100t} = 2 \times 10^{-3}, \qquad e^{100t} = 4, \qquad t = (\ln 4)/100 = 13.86\,\text{ms}$$

AP 7.4 **[a]** This circuit is actually two *RC* circuits in series, and the requested voltage, v_o, is the sum of the voltage drops for the two *RC* circuits. The circuit for $t < 0$ is shown below:



Find the current in the loop and use it to find the initial voltage drops across the two RC circuits:

$$i = \frac{15}{75{,}000} = 0.2\,\text{mA}, \qquad v_5(0^-) = 4\,\text{V}, \qquad v_1(0^-) = 8\,\text{V}$$

There are two time constants in the circuit, one for each RC subcircuit. τ_5 is the time constant for the $5\,\mu$F – $20\,\text{k}\Omega$ subcircuit, and τ_1 is the time constant for the $1\,\mu$F – $40\,\text{k}\Omega$ subcircuit:

$\tau_5 = (20 \times 10^3)(5 \times 10^{-6}) = 100\,\text{ms}; \qquad \tau_1 = (40 \times 10^3)(1 \times 10^{-6}) = 40\,\text{ms}$

Therefore,

$v_5(t) = v_5(0^-)e^{-t/\tau_5} = 4e^{-t/0.1} = 4e^{-10t}\,\text{V}, \quad t \geq 0$

$v_1(t) = v_1(0^-)e^{-t/\tau_1} = 8e^{-t/0.04} = 8e^{-25t}\,\text{V}, \quad t \geq 0$

Finally,

$v_o(t) = v_1(t) + v_5(t) = [8e^{-25t} + 4e^{-10t}]\,\text{V}, \qquad t \geq 0$

[b] Find the value of the voltage at 60 ms for each subcircuit and use the voltage to find the energy at 60 ms:

$v_1(60\,\text{ms}) = 8e^{-25(0.06)} \cong 1.79\,\text{V}, \qquad v_5(60\,\text{ms}) = 4e^{-10(0.06)} \cong 2.20\,\text{V}$

$w_1(60\,\text{ms}) = \frac{1}{2}Cv_1^2(60\,\text{ms}) = \frac{1}{2}(1 \times 10^{-6})(1.79)^2 \cong 1.59\,\mu\text{J}$

$w_5(60\,\text{ms}) = \frac{1}{2}Cv_5^2(60\,\text{ms}) = \frac{1}{2}(5 \times 10^{-6})(2.20)^2 \cong 12.05\,\mu\text{J}$

$w(60\,\text{ms}) = 1.59 + 12.05 = 13.64\,\mu\text{J}$

Find the initial energy from the initial voltage:

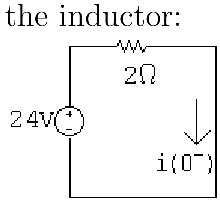
$w(0) = w_1(0) + w_2(0) = \frac{1}{2}(1 \times 10^{-6})(8)^2 + \frac{1}{2}(5 \times 10^{-6})(4)^2 = 72\,\mu\text{J}$

Now calculate the energy dissipated at 60 ms and compare it to the initial energy:

$w_{\text{diss}} = w(0) - w(60\,\text{ms}) = 72 - 13.64 = 58.36\,\mu\text{J}$

$\%\ \text{dissipated} = (58.36 \times 10^{-6}/72 \times 10^{-6})(100) = 81.05\,\%$

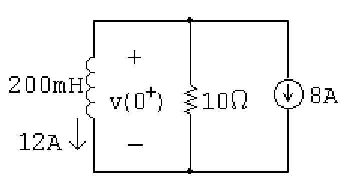
AP 7.5 **[a]** Use the circuit at $t < 0$, shown below, to calculate the initial current in the inductor:



$i(0^-) = 24/2 = 12\,\text{A} = i(0^+)$

Note that $i(0^-) = i(0^+)$ because the current in an inductor is continuous.

[b] Use the circuit at $t = 0^+$, shown below, to calculate the voltage drop across the inductor at 0^+. Note that this is the same as the voltage drop across the $10\,\Omega$ resistor, which has current from two sources — 8 A from the current source and 12 A from the initial current through the inductor.

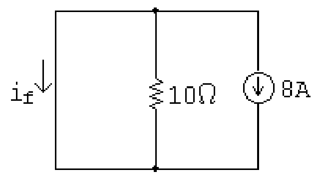


$v(0^+) = -10(8 + 12) = -200\,\text{V}$

[c] To calculate the time constant we need the equivalent resistance seen by the inductor for $t > 0$. Only the $10\,\Omega$ resistor is connected to the inductor for $t > 0$. Thus,

$\tau = L/R = (200 \times 10^{-3}/10) = 20\,\text{ms}$

[d] To find $i(t)$, we need to find the final value of the current in the inductor. When the switch has been in position a for a long time, the circuit reduces to the one below:



Note that the inductor behaves as a short circuit and all of the current from the 8 A source flows through the short circuit. Thus,

$i_f = -8\,\text{A}$

Now,

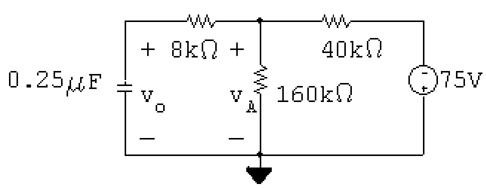
$i(t) = i_f + [i(0^+) - i_f]e^{-t/\tau} = -8 + [12 - (-8)]e^{-t/0.02}$

$\quad = -8 + 20e^{-50t}\,\text{A}, \quad t \geq 0$

[e] To find $v(t)$, use the relationship between voltage and current for an inductor:

$$v(t) = L\frac{di(t)}{dt} = (200 \times 10^{-3})(-50)(20e^{-50t}) = -200e^{-50t}\,\text{V}, \qquad t \geq 0^+$$

AP 7.6 **[a]**



From Example 7.6,

$v_o(t) = -60 + 90e^{-100t}\,\text{V}$

Write a KCL equation at the top node and use it to find the relationship between v_o and v_A:

$$\frac{v_A - v_o}{8000} + \frac{v_A}{160{,}000} + \frac{v_A + 75}{40{,}000} = 0$$

$20v_A - 20v_o + v_A + 4v_A + 300 = 0$

$25v_A = 20v_o - 300$

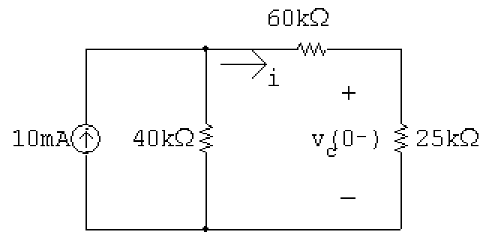
$v_A = 0.8v_o - 12$

Use the above equation for v_A in terms of v_o to find the expression for v_A:

$v_A(t) = 0.8(-60 + 90e^{-100t}) - 12 = -60 + 72e^{-100t}\,\text{V}, \qquad t \geq 0^+$

[b] $t \geq 0^+$, since there is no requirement that the voltage be continuous in a resistor.

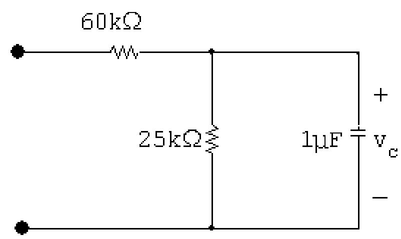
AP 7.7 **[a]** Use the circuit shown below, for $t < 0$, to calculate the initial voltage drop across the capacitor:



$$i = \left(\frac{40 \times 10^3}{125 \times 10^3}\right)(10 \times 10^{-3}) = 3.2\,\text{mA}$$

$$v_c(0^-) = (3.2 \times 10^{-3})(25 \times 10^3) = 80\,\text{V} \quad \text{so} \quad v_c(0^+) = 80\,\text{V}$$

Now use the next circuit, valid for $0 \leq t \leq 10\,\text{ms}$, to calculate $v_c(t)$ for that interval:

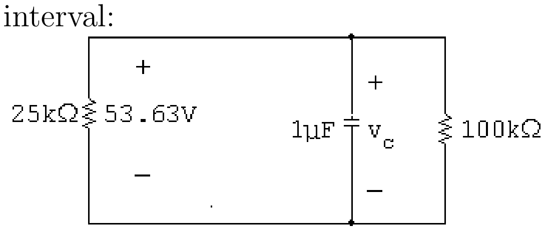


For $\quad 0 \leq t \leq 100\,\text{ms}$:

$$\tau = RC = (25 \times 10^3)(1 \times 10^{-6}) = 25\,\text{ms}$$

$$v_c(t) = v_c(0^-)e^{t/\tau} = 80e^{-40t}\,\text{V} \quad 0 \leq t \leq 10\,\text{ms}$$

[b] Calculate the starting capacitor voltage in the interval $t \geq 10\,\text{ms}$, using the capacitor voltage from the previous interval:
$v_c(0.01) = 80e^{-40(0.01)} = 53.63\,\text{V}$
Now use the next circuit, valid for $t \geq 10\,\text{ms}$, to calculate $v_c(t)$ for that interval:



For $\quad t \geq 10\,\text{ms}$:

$$R_{\text{eq}} = 25\,\text{k}\Omega \| 100\,\text{k}\Omega = 20\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (20 \times 10^3)(1 \times 10^{-6}) = 0.02\,\text{s}$$

Therefore $\quad v_c(t) = v_c(0.01^+)e^{-(t-0.01)/\tau} = 53.63e^{-50(t-0.01)}\,\text{V}, \qquad t \geq 0.01\,\text{s}$

[c] To calculate the energy dissipated in the 25 kΩ resistor, integrate the power absorbed by the resistor over all time. Use the expression $p = v^2/R$ to calculate the power absorbed by the resistor.

$$w_{25\,\mathrm{k}} = \int_0^{0.01} \frac{[80e^{-40t}]^2}{25{,}000}dt + \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{25{,}000}dt = 2.91\,\mathrm{mJ}$$

[d] Repeat the process in part (c), but recognize that the voltage across this resistor is non-zero only for the second interval:

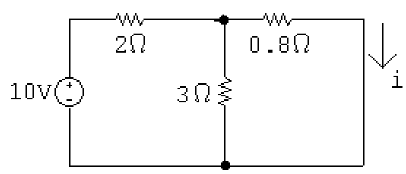
$$w_{100\,\mathrm{k}\Omega} = \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{100{,}000}dt = 0.29\,\mathrm{mJ}$$

We can check our answers by calculating the initial energy stored in the capacitor. All of this energy must eventually be dissipated by the 25 kΩ resistor and the 100 kΩ resistor.

Check: $w_{\mathrm{stored}} = (1/2)(1 \times 10^{-6})(80)^2 = 3.2\,\mathrm{mJ}$

$$w_{\mathrm{diss}} = 2.91 + 0.29 = 3.2\,\mathrm{mJ}$$

AP 7.8 **[a]** Prior to switch a closing at $t = 0$, there are no sources connected to the inductor; thus, $i(0^-) = 0$.
At the instant A is closed, $i(0^+) = 0$.
For $0 \le t \le 1\,\mathrm{s}$,



The equivalent resistance seen by the 10 V source is $2 + (3\|0.8)$. The current leaving the 10 V source is

$$\frac{10}{2 + (3\|0.8)} = 3.8\,\mathrm{A}$$

The final current in the inductor, which is equal to the current in the 0.8 Ω resistor is

$$I_{\mathrm{F}} = \frac{3}{3 + 0.8}(3.8) = 3\,\mathrm{A}$$

The resistance seen by the inductor is calculated to find the time constant:

$$[(2\|3) + 0.8]\|3\|6 = 1\,\Omega \qquad \tau = \frac{L}{R} = \frac{2}{1} = 2\,\mathrm{s}$$

Therefore,

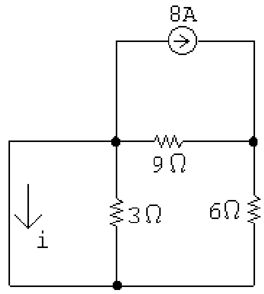
$$i = i_{\mathrm{F}} + [i(0^+) - i_{\mathrm{F}}]e^{-t/\tau} = 3 - 3e^{-0.5t}\,\mathrm{A}, \quad 0 \le t \le 1\,\mathrm{s}$$

For part (b) we need the value of $i(t)$ at $t = 1$ s:

$i(1) = 3 - 3e^{-0.5} = 1.18\,\text{A}$

.

[b] For $t > 1$ s



Use current division to find the final value of the current:

$$i = \frac{9}{9+6}(-8) = -4.8\,\text{A}$$

The equivalent resistance seen by the inductor is used to calculate the time constant:

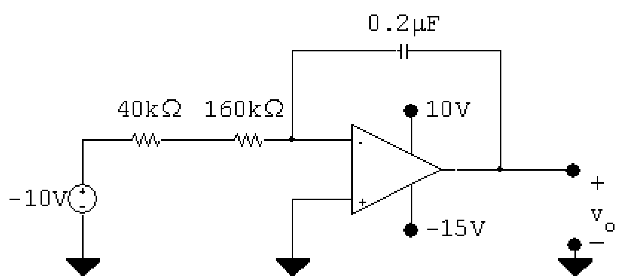
$$3\|(9+6) = 2.5\,\Omega \qquad \tau = \frac{L}{R} = \frac{2}{2.5} = 0.8\,\text{s}$$

Therefore,

$$i = i_{\text{F}} + [i(1^+) - i_{\text{F}}]e^{-(t-1)/\tau}$$

$$= -4.8 + 5.98e^{-1.25(t-1)}\,\text{A}, \quad t \geq 1\,\text{s}$$

AP 7.9 $0 \leq t \leq 32\,\text{ms}$:

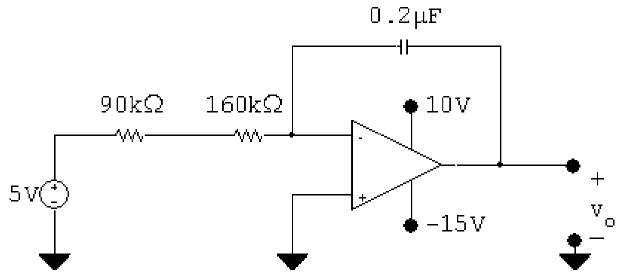


$$v_o = -\frac{1}{RC_f}\int_0^{32\times10^{-3}} -10\,dt + 0 = -\frac{1}{RC_f}(-10t)\,\Big|_0^{32\times10^{-3}} = -\frac{1}{RC_f}(-320\times10^{-3})$$

$$RC_f = (200\times10^3)(0.2\times10^{-6}) = 40\times10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 25$$

$$v_o = -25(-320\times10^{-3}) = 8\,\text{V}$$

$t \geq 32\,\text{ms}$:



$$v_o = -\frac{1}{RC_f}\int_{32\times10^{-3}}^{t} 5\,dy + 8 = -\frac{1}{RC_f}(5y)\Big|_{32\times10^{-3}}^{t} + 8 = -\frac{1}{RC_f}5(t - 32\times10^{-3}) + 8$$

$$RC_f = (250\times10^3)(0.2\times10^{-6}) = 50\times10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 20$$

$$v_o = -20(5)(t - 32\times10^{-3}) + 8 = -100t + 11.2$$

The output will saturate at the negative power supply value:

$$-15 = -100t + 11.2 \qquad \therefore \qquad t = 262\,\text{ms}$$

AP 7.10 **[a]** Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (0+2)e^{-t/\tau}$$

$$\tau = (160\times10^3)(10\times10^{-9}) = 10^{-3}; \qquad 1/\tau = 625$$

$$v_p = -2 + 2e^{-625t}\,\text{V}; \qquad v_n = v_p$$

Write a KVL equation at the inverting input, and use it to determine v_o:

$$\frac{v_n}{10{,}000} + \frac{v_n - v_o}{40{,}000} = 0$$

$$\therefore \quad v_o = 5v_n = 5v_p = -10 + 10e^{-625t}\,\text{V}$$

The output will saturate at the negative power supply value:

$$-10 + 10e^{-625t} = -5; \quad e^{-625t} = 1/2; \quad t = \ln 2/625 = 1.11\,\text{ms}$$

[b] Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (1+2)e^{-625t} = -2 + 3e^{-625t}\,\text{V}$$

The analysis for v_o is the same as in part (a):

$$v_o = 5v_p = -10 + 15e^{-625t}\,\text{V}$$

The output will saturate at the negative power supply value:

$$-10 + 15e^{-625t} = -5; \qquad e^{-625t} = 1/3; \quad t = \ln 3/625 = 1.76\,\text{ms}$$

Problems

P 7.1 [a] $i_o(0) = \dfrac{20}{16+12+4+8} = \dfrac{20}{40} = 0.5\,\text{A}$

$i_o(\infty) = 0\,\text{A}$

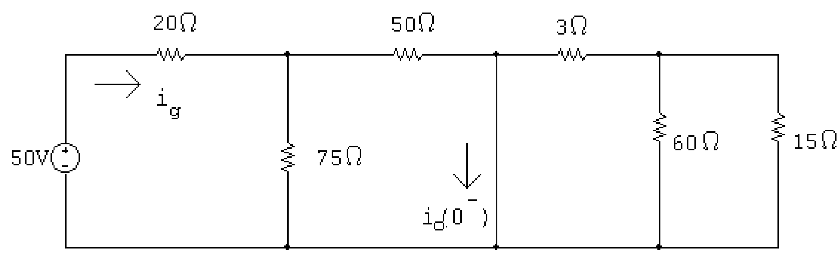
[b] $i_o = 0.5e^{-t/\tau}; \qquad \tau = \dfrac{L}{R} = \dfrac{80\times 10^{-3}}{12+8} = 4\,\text{ms}$

$i_o = 0.5e^{-250t}\,\text{A}, \quad t \geq 0$

[c] $0.5e^{-250t} = 0.1$

$e^{250t} = 5 \qquad \therefore \quad t = 6.44\,\text{ms}$

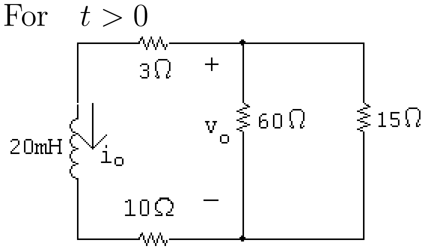
P 7.2 [a] For $t < 0$



$$i_g = \frac{50}{20 + (75\|50)} = \frac{50}{50} = 1\,\text{A}$$

$$i_o(0^-) = \frac{75\|50}{50}(1) = 0.6\,\text{A} = i_o(0^+)$$

For $t > 0$



$$i_o(t) = i_o(0^+)e^{-t/\tau}\,\text{A}, \qquad t \geq 0$$

$$\tau = \frac{L}{R} = \frac{0.02}{3 + 60\|15} = 1.33\,\text{ms}; \qquad \frac{1}{\tau} = 750$$

$$i_o(t) = 0.6e^{-750t}\,\text{A}, \qquad t \geq 0$$

[b] $v_{\text{L}} = L\dfrac{di_o}{dt} = 0.02(-750)(0.6e^{-750t}) = -9e^{-750t}\,\text{V}$

$$v_o = \frac{60\|15}{3+60\|15}v_{\text{L}} = \frac{12}{15}(-9e^{-750t}) = -7.2e^{-750t}\,\text{V} \qquad t \geq 0^+$$

P 7.3 **[a]** $i(0) = \dfrac{60}{120} = 0.5\,\text{A}$

[b] $\tau = \dfrac{L}{R} = \dfrac{0.32}{160} = 2\,\text{ms}$

[c] $i = 0.5e^{-500t}\,\text{A}, \qquad t \geq 0$

$$v_1 = L\frac{d}{dt}(0.5e^{-500t}) = -80e^{-500t}\,\text{V} \qquad t \geq 0^+$$

$$v_2 = -70i = -35e^{-500t}\,\text{V} \qquad t \geq 0^+$$

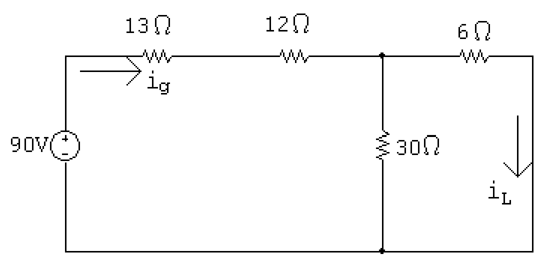
[d] $w(0) = \dfrac{1}{2}(0.32)(0.5)^2 = 40\,\text{mJ}$

$$w_{90\Omega} = \int_0^t 90(0.25e^{-1000x})\,dx = 22.5\frac{e^{-1000x}}{-1000}\Bigg|_0^t = 22.5(1 - e^{-1000t})\,\text{mJ}$$

$$w_{90\Omega}(1\,\text{ms}) = 0.0225(1 - e^{-1}) = 14.22\,\text{mJ}$$

$$\%\ \text{dissipated} = \frac{14.22}{40}(100) = 35.6\%$$

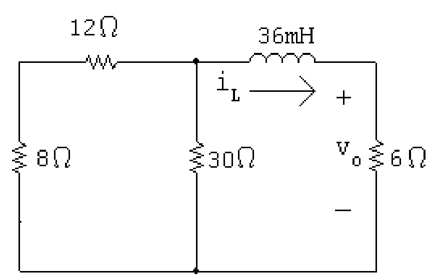
P 7.4 $t < 0$:



$$i_g = \frac{90}{13 + 12 + 6\|30} = 3\,\text{A}$$

$$i_L(0^-) = \frac{30}{36}(3) = 2.5\,\text{A}$$

$t > 0$:



$$R_e = 6 + 30\|(8 + 12) = 6 + 12 = 18\,\Omega$$

$$\tau = \frac{L}{R_e} = \frac{36 \times 10^{-3}}{18} = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$\therefore \quad i_L = 2.5e^{-500t}\,\text{A}$$

$$v_o = 6i_o = 15e^{-500t}\,\text{V}, \quad t \geq 0^+$$

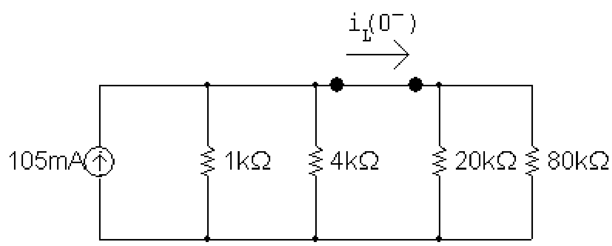
P 7.5 $$p_{6\Omega} = \frac{v_o^2}{6} = \frac{(15)^2}{6}e^{-1000t} = 37.5e^{-1000t}\,\text{W}$$

$$w_{6\Omega} = \int_0^\infty 37.5e^{-1000t}\,dt = 37.5\frac{e^{-1000t}}{-1000}\Big|_0^\infty = 37.5\,\text{mJ}$$

$$w(0) = \frac{1}{2}(36 \times 10^{-3})(2.5)^2 = 112.5\,\text{mJ}$$

$$\%\ \text{diss} = \frac{37.5}{112.5}(100) = 33.33\%$$

P 7.6 **[a]** $t < 0$

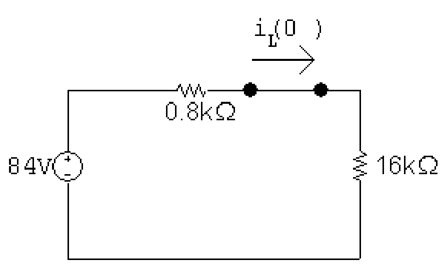


Simplify this circuit by creating a Thévenin equivalant to the left of the inductor and an equivalent resistance to the right of the inductor:

$$1\,\text{k}\Omega \| 4\,\text{k}\Omega = 0.8\,\text{k}\Omega$$

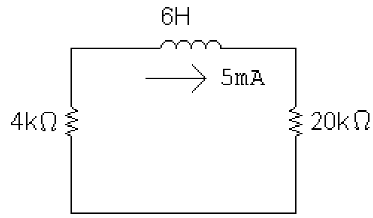
$$20\,\text{k}\Omega \| 80\,\text{k}\Omega = 16\,\text{k}\Omega$$

$$(105 \times 10^{-3})(0.8 \times 10^3) = 84\,\text{V}$$



$$i_L(0^-) = \frac{84}{16{,}800} = 5\,\text{mA}$$

$t > 0$



$$\tau = \frac{L}{R} = \frac{6}{24} \times 10^{-3} = 250\,\mu\text{s}; \qquad \frac{1}{\tau} = 4000$$

$$i_L(t) = 5e^{-4000t}\,\text{mA}, \qquad t \geq 0$$

$$p_{4\text{k}} = 25 \times 10^{-6} e^{-8000t}(4000) = 0.10e^{-8000t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^t 0.10e^{-8000x}\,dx = 12.5 \times 10^{-6}[1 - e^{-8000t}]\,\text{J}$$

$$w(0) = \frac{1}{2}(6)(25 \times 10^{-6}) = 75\,\mu\text{J}$$

$$0.10w(0) = 7.5\,\mu\text{J}$$

$$12.5(1 - e^{-8000t}) = 7.5; \qquad \therefore \quad e^{8000t} = 2.5$$

$$t = \frac{\ln 2.5}{8000} = 114.54\,\mu\text{s}$$

[b] $w_{\text{diss}}(\text{total}) = 75(1 - e^{-8000t})\,\mu\text{J}$

$$w_{\text{diss}}(114.54\,\mu\text{s}) = 45\,\mu\text{J}$$

$$\% = (45/75)(100) = 60\%$$

P 7.7 **[a]** $v_o(t) = v_o(0^+)e^{-t/\tau}$

$$\therefore \quad v_o(0^+)e^{-10^{-3}/\tau} = 0.5v_o(0^+)$$

$$\therefore \quad e^{10^{-3}/\tau} = 2$$

$$\therefore \quad \tau = \frac{L}{R} = \frac{10^{-3}}{\ln 2}$$

$$\therefore \quad L = \frac{10 \times 10^{-3}}{\ln 2} = 14.43\,\text{mH}$$

[b] $v_o(0^+) = -10i_L(0^+) = -10(1/10)(30 \times 10^{-3}) = -30\,\text{mV}$

$$v_o(t) = -0.03e^{-t/\tau}\,\text{V}$$

$$p_{10\Omega} = \frac{v_o^2}{10} = 9 \times 10^{-5} e^{-2t/\tau}$$

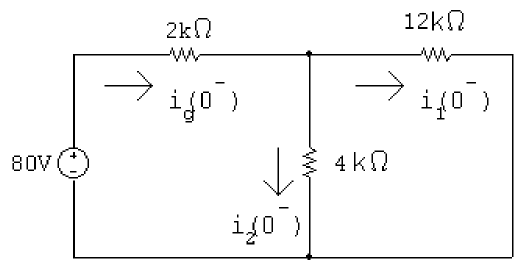
$$w_{10\Omega} = \int_0^{10^{-3}} 9 \times 10^{-5} e^{-2t/\tau}\,dt = 4.5\tau \times 10^{-5}(1 - e^{-2\times10^{-3}/\tau})$$

$$\tau = \frac{1}{1000 \ln 2} \qquad \therefore \qquad w_{10\Omega} = 48.69\,\text{nJ}$$

$$w_L(0) = \frac{1}{2}Li_L^2(0) = \frac{1}{2}(14.43 \times 10^{-3})(3 \times 10^{-3})^2 = 64.92\,\text{nJ}$$

$$\%\ \text{diss in 1 ms} = \frac{48.69}{64.92} \times 100 = 75\%$$

P 7.8 **[a]** $t < 0$



$$4\,\text{k}\Omega \| 12\,\text{k}\Omega = 3\,\text{k}\Omega$$

Find the current from the voltage source by combining the resistors in series and parallel and using Ohm's law:

$$i_g(0^-) = \frac{80}{(2000 + 3000)} = 16\,\text{mA}$$

Find the branch currents using current division:

$$i_1(0^-) = \frac{3000}{12{,}000}(0.016) = 4\,\text{mA}$$

$$i_2(0^-) = \frac{3000}{4000}(0.016) = 12\,\text{mA}$$

[b] The current in an inductor is continuous. Therefore,

$$i_1(0^+) = i_1(0^-) = 4\,\text{mA}$$

$$i_2(0^+) = -i_1(0^+) = -4\,\text{mA} \qquad \text{(when switch is open)}$$

[c] $\tau = \dfrac{L}{R} = \dfrac{0.64 \times 10^{-3}}{16 \times 10^3} = 4 \times 10^{-5}\,\text{s}; \qquad \dfrac{1}{\tau} = 25{,}000$

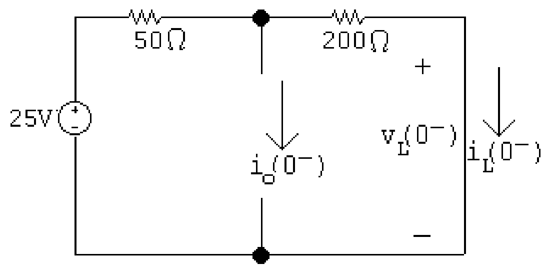
$$i_1(t) = i_1(0^+)e^{-t/\tau} = 4e^{-25{,}000t}\,\text{mA}, \qquad t \geq 0$$

[d] $i_2(t) = -i_1(t) \qquad \text{when} \quad t \geq 0^+$

$$\therefore \quad i_2(t) = -4e^{-25{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

[**e**] The current in a resistor can change instantaneously. The switching operation forces $i_2(0^-)$ to equal 12 mA and $i_2(0^+) = -4$ mA.

P 7.9 [**a**] For $t = 0^-$ the circuit is:

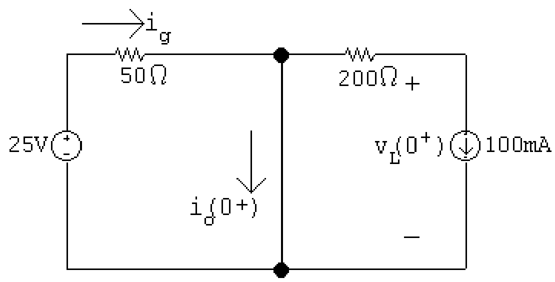


$$i_o(0^-) = 0 \quad \text{since the switch is open}$$

$$i_L(0^-) = \frac{25}{250} = 0.1 = 100\,\text{mA}$$

$$v_L(0^-) = 0 \quad \text{since the inductor behaves like a short circuit}$$

[**b**] For $t = 0^+$ the circuit is:



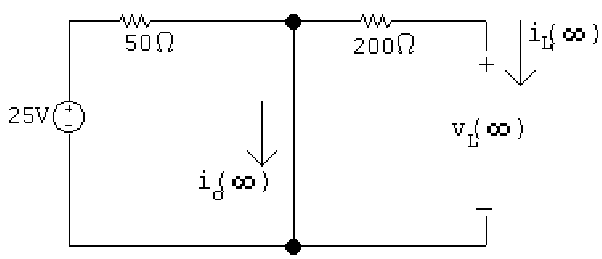
$$i_L(0^+) = i_L(0^-) = 100\,\text{mA}$$

$$i_g = \frac{25}{50} = 0.5 = 500\,\text{mA}$$

$$i_o(0^+) = i_g - i_L(0^+) = 500 - 100 = 400\,\text{mA}$$

$$200i_L(0^+) + v_L(0^+) = 0 \qquad \therefore \qquad v_L(0^+) = -200i_L(0^+) = -20\,\text{V}$$

[**c**] As $t \to \infty$ the circuit is:



$$i_L(\infty) = 0; \qquad v_L(\infty) = 0$$

$$i_o(\infty) = \frac{25}{50} = 500\,\text{mA}$$

[d] $\tau = \frac{L}{R} = \frac{0.05}{200} = 0.25\,\text{ms}$

$$i_{\text{L}}(t) = 0 + (0.1 - 0)e^{-4000t} = 0.1e^{-4000t}\,\text{A}$$

[e] $i_o(t) = i_{\text{g}} - i_{\text{L}} = 0.5 - 0.1e^{-4000t}\,\text{A}$

[f] $v_{\text{L}}(t) = L\frac{di_{\text{L}}}{dt} = 0.05(-4000)(0.1)e^{-4000t} = -20e^{-4000t}\,\text{V}$

P 7.10 $w(0) = \frac{1}{2}(10 \times 10^{-3})(5)^2 = 125\,\text{mJ}$

$$0.9w(0) = 112.5\,\text{mJ}$$

$$w(t) = \frac{1}{2}(10 \times 10^{-3})i(t)^2, \qquad i(t) = 5e^{-t/\tau}\,\text{A}$$

$$\therefore \quad w(t) = 0.005(25e^{-2t/\tau}) = 125e^{-2t/\tau})\,\text{mJ}$$

$$w(10\,\mu\text{s}) = 125e^{-20\times 10^{-6}/\tau}\,\text{mJ}$$

$$\therefore \quad 125e^{-20\times 10^{-6}/\tau} = 112.5 \qquad \text{so} \qquad e^{20\times 10^{-6}/\tau} = \frac{10}{9}$$

$$\tau = \frac{20 \times 10^{-6}}{\ln(10/9)} = \frac{L}{R}$$

$$R = \frac{10 \times 10^{-3}\ln(10/9)}{20 \times 10^{-6}} = 52.68\,\Omega$$

P 7.11 **[a]** $w(0) = \frac{1}{2}LI_g^2$

$$w_{\text{diss}} = \int_0^{t_o} I_g^2 R e^{-2t/\tau}\,dt = I_g^2 R \frac{e^{-2t/\tau}}{(-2/\tau)}\Bigg|_0^{t_o}$$
$$= \frac{1}{2}I_g^2 R\tau(1 - e^{-2t_o/\tau}) = \frac{1}{2}I_g^2 L(1 - e^{-2t_o/\tau})$$

$$w_{\text{diss}} = \sigma w(0)$$

$$\therefore \quad \frac{1}{2}LI_g^2(1 - e^{-2t_o/\tau}) = \sigma\left(\frac{1}{2}LI_g^2\right)$$

$$1 - e^{-2t_o/\tau} = \sigma; \qquad e^{2t_o/\tau} = \frac{1}{(1-\sigma)}$$

$$\frac{2t_o}{\tau} = \ln\left[\frac{1}{(1-\sigma)}\right]; \qquad \frac{R(2t_o)}{L} = \ln[1/(1-\sigma)]$$

$$R = \frac{L\ \ln[1/(1-\sigma)]}{2t_o}$$

[b] $R = \dfrac{(10 \times 10^{-3})\ln[1/0.9]}{20 \times 10^{-6}}$

$$R = 52.68\,\Omega$$

P 7.12 **[a]** $R = \dfrac{v}{i} = 25\,\Omega$

[b] $\tau = \dfrac{1}{10} = 100\,\text{ms}$

[c] $\tau = \dfrac{L}{R} = 0.1$

$$L = (0.1)(25) = 2.5\,\text{H}$$

[d] $w(0) = \dfrac{1}{2}L[i(0)]^2 = \dfrac{1}{2}(2.5)(6.4)^2 = 51.2\,\text{J}$

[e] $w_{\text{diss}} = \displaystyle\int_0^t 1024e^{-20x}\,dx = 1024\frac{e^{-20x}}{-20}\Bigg|_0^t = 51.2(1 - e^{-20t})\,\text{J}$

$$\%\,\text{dissipated} = \frac{51.2(1 - e^{-20t})}{51.2}(100) = 100(1 - e^{-20t})$$

$$\therefore \quad 100(1 - e^{-20t}) = 60 \qquad \text{so} \qquad e^{-20t} = 0.4$$

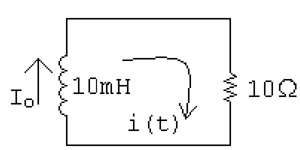
$$\text{Therefore} \quad t = \frac{1}{20}\ln 2.5 = 45.81\,\text{ms}$$

P 7.13 **[a]** Note that there are several different possible solutions to this problem, and the answer to part (c) depends on the value of inductance chosen.

$$R = \frac{L}{\tau}$$

Choose a 10 mH inductor from Appendix H. Then,

$R = \dfrac{0.01}{0.001} = 10\,\Omega$ which is a resistor value from Appendix H.



[b] $i(t) = I_o e^{-t/\tau} = 10e^{-1000t}\,\text{mA}, \qquad t \geq 0$

[**c**] $w(0) = \frac{1}{2}LI_o^2 = \frac{1}{2}(0.01)(0.01)^2 = 0.5\,\mu\text{J}$

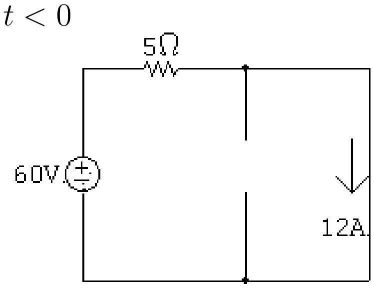
$$w(t) = \frac{1}{2}(0.01)(0.01e^{-1000t})^2 = 0.5 \times 10^{-6}e^{-2000t}$$

So $0.5 \times 10^{-6}e^{-2000t} = \frac{1}{2}w(0) = 0.25 \times 10^{-6}$

$e^{-2000t} = 0.5$ then $e^{2000t} = 2$

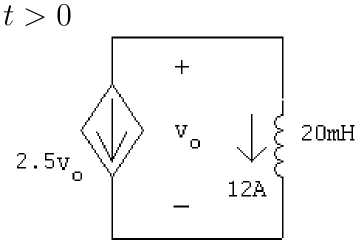
$\therefore \quad t = \frac{\ln 2}{2000} = 346.57\,\mu\text{s}$ (for a 10 mH inductor)

P 7.14 $t < 0$

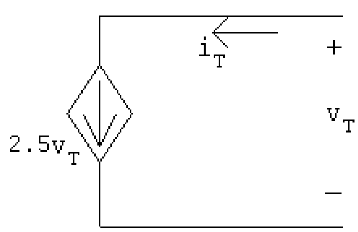


$i_L(0^-) = i_L(0^+) = 12\,\text{A}$

$t > 0$

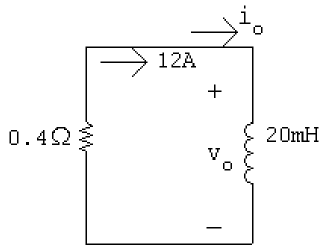


Find Thévenin resistance seen by inductor:



$i_T = 2.5v_T; \qquad \frac{v_T}{i_T} = R_{\text{Th}} = \frac{1}{2.5} = 0.4\,\Omega$

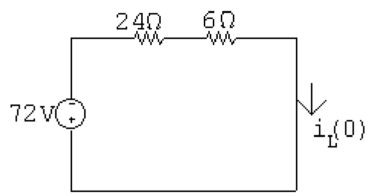
$\tau = \frac{L}{R} = \frac{20 \times 10^{-3}}{0.4} = 50\,\text{ms}; \qquad 1/\tau = 20$



$$i_o = 12e^{-20t}\,\text{A}, \qquad t \geq 0$$

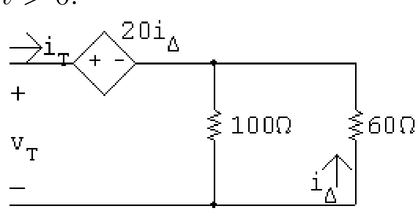
$$v_o = L\frac{di_o}{dt} = (20 \times 10^{-3})(-240e^{-20t}) = -4.8e^{-20t}\,\text{V}, \quad t \geq 0^+$$

P 7.15 **[a]** $t < 0$:



$$i_L(0) = -\frac{72}{24+6} = -2.4\,\text{A}$$

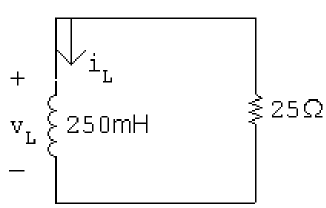
$t > 0$:



$$i_\Delta = -\frac{100}{160}i_T = -\frac{5}{8}i_T$$

$$v_T = 20i_\Delta + i_T\frac{(100)(60)}{160} = -12.5i_T + 37.5i_T$$

$$\frac{v_T}{i_T} = R_{\text{Th}} = -12.5 + 37.5 = 25\,\Omega$$



$$\tau = \frac{L}{R} = \frac{250 \times 10^{-3}}{25} \qquad \frac{1}{\tau} = 100$$

$$i_L = -2.4e^{-100t}\,\text{A}, \qquad t \geq 0$$

[b] $v_L = 250 \times 10^{-3}(240e^{-100t}) = 60e^{-100t}\,\text{V}, \quad t \geq 0^+$

[c] $i_\Delta = 0.625 i_L = -1.5e^{-100t}\,\text{A} \qquad t \geq 0^+$

P 7.16 $w(0) = \frac{1}{2}(250 \times 10^{-3})(-2.4)^2 = 720\,\text{mJ}$

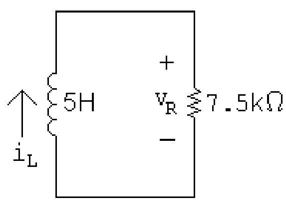
$$p_{60\Omega} = 60(-1.5e^{-100t})^2 = 135e^{-200t}\,\text{W}$$

$$w_{60\Omega} = \int_0^\infty 135e^{-200t}\,dt = 135\frac{e^{-200t}}{-200}\Big|_0^\infty = 675\,\text{mJ}$$

$$\%\text{ dissipated } = \frac{675}{720}(100) = 93.75\%$$

P 7.17 **[a]** $t > 0$:

$$L_{\text{eq}} = 1.25 + \frac{60}{16} = 5\,\text{H}$$



$$i_L(t) = i_L(0)e^{-t/\tau}\,\text{mA}; \qquad i_L(0) = 2\,\text{A}; \qquad \frac{1}{\tau} = \frac{R}{L} = \frac{7500}{5} = 1500$$

$$i_L(t) = 2e^{-1500t}\,\text{A}, \qquad t \geq 0$$

$$v_R(t) = Ri_L(t) = (7500)(2e^{-1500t}) = 15{,}000e^{-1500t}\,\text{V}, \qquad t \geq 0^+$$

$$v_o = -3.75\frac{di_L}{dt} = 11{,}250e^{-1500t}\,\text{V}, \qquad t \geq 0^+$$

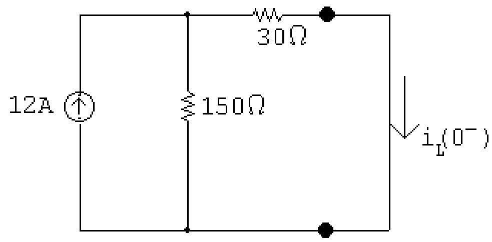
[b] $i_o = \frac{-1}{6}\int_0^t 11{,}250e^{-1500x}\,dx + 0 = 1.25e^{-1500t} - 1.25\,\text{A}$

P 7.18 **[a]** From the solution to Problem 7.17,

$$w(0) = \frac{1}{2}L_{\text{eq}}[i_L(0)]^2 = \frac{1}{2}(5)(2)^2 = 10\,\text{J}$$

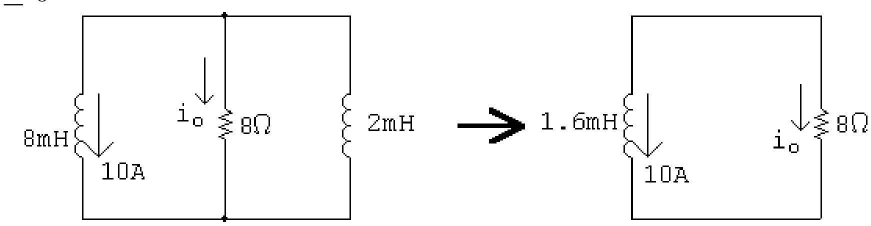
[b] $w_{\text{trapped}} = \frac{1}{2}(10)(1.25)^2 + \frac{1}{2}(6)(1.25)^2 = 12.5\,\text{J}$

P 7.19 **[a]** $t < 0$



$$i_L(0^-) = \frac{150}{180}(12) = 10\,\text{A}$$

$t \geq 0$



$$\tau = \frac{1.6 \times 10^{-3}}{8} = 200 \times 10^{-6}; \qquad 1/\tau = 5000$$

$$i_o = -10e^{-5000t}\,\text{A} \quad t \geq 0$$

[b] $w_{\text{del}} = \dfrac{1}{2}(1.6 \times 10^{-3})(10)^2 = 80\,\text{mJ}$

[c] $0.95w_{\text{del}} = 76\,\text{mJ}$

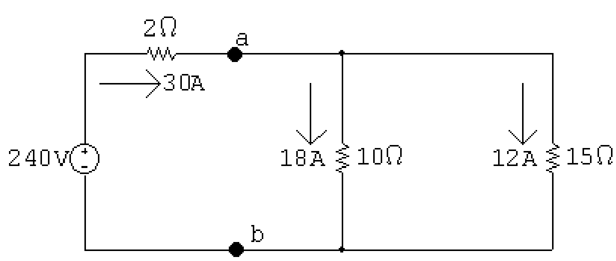
$$\therefore \quad 76 \times 10^{-3} = \int_0^{t_o} 8(100e^{-10{,}000t})\,dt$$

$$\therefore \quad 76 \times 10^{-3} = -80 \times 10^{-3}e^{-10{,}000t}\Big|_0^{t_o} = 80 \times 10^{-3}(1 - e^{-10{,}000t_o})$$

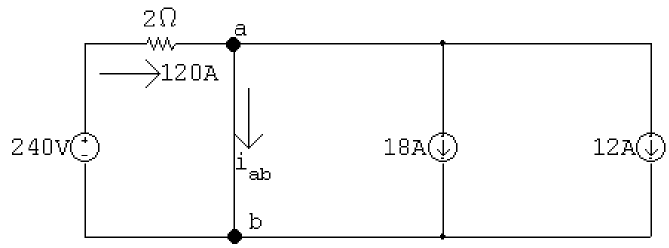
$$\therefore \quad e^{-10{,}000t_o} = 0.05 \quad \text{so} \quad t_o = 299.57\,\mu\text{s}$$

$$\therefore \quad \frac{t_o}{\tau} = \frac{299.57 \times 10^{-6}}{200 \times 10^{-6}} = 1.498 \quad \text{so} \quad t_o \approx 1.498\tau$$

P 7.20 **[a]** $t < 0$:

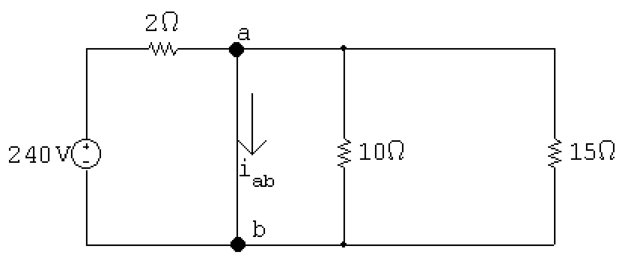


$t = 0^+$:

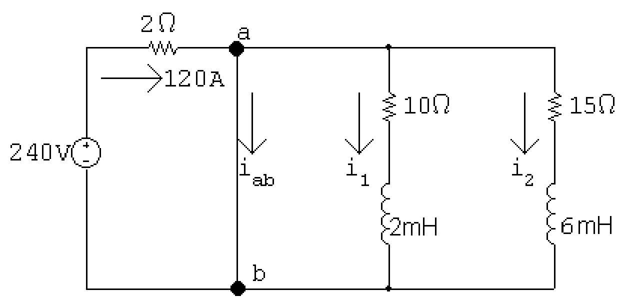


$120 = i_{ab} + 18 + 12, \qquad i_{ab} = 90\,\text{A}, \quad t = 0^+$

[b] At $\quad t = \infty$:



$i_{ab} = 240/2 = 120\,\text{A}, \quad t = \infty$



[c] $i_1(0) = 18, \qquad \tau_1 = \dfrac{2 \times 10^{-3}}{10} = 0.2\,\text{ms}$

$i_2(0) = 12, \qquad \tau_2 = \dfrac{6 \times 10^{-3}}{15} = 0.4\,\text{ms}$

$i_1(t) = 18e^{-5000t}\,\text{A}, \quad t \geq 0$

$i_2(t) = 12e^{-2500t}\,\text{A}, \quad t \geq 0$

$i_{ab} = 120 - 18e^{-5000t} - 12e^{-2500t}\,\text{A}, \quad t \geq 0$

$120 - 18e^{-5000t} - 12e^{-2500t} = 114$

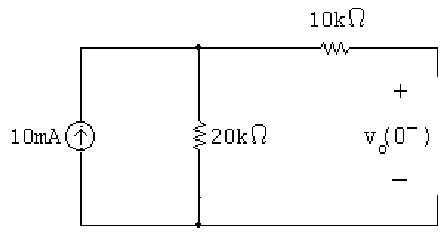
$6 = 18e^{-5000t} + 12e^{-2500t}$

Let $\quad x = e^{-2500t} \qquad$ so $\qquad 6 = 18x^2 + 12x$

Solving $\quad x = \dfrac{1}{3} = e^{-2500t}$

$$\therefore \quad e^{2500t} = 3 \qquad \text{and} \qquad t = \frac{\ln 3}{2500} = 439.44\,\mu\text{s}$$

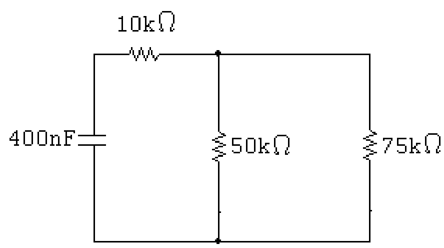
P 7.21 **[a]** For $t < 0$:



$$v(0) = 20{,}000(0.01) = 200\,\text{V}$$

[b] $w(0) = \frac{1}{2}Cv(0)^2 = \frac{1}{2}(400 \times 10^{-9})(200)^2 = 8\,\text{mJ}$

[c] For $t > 0$:

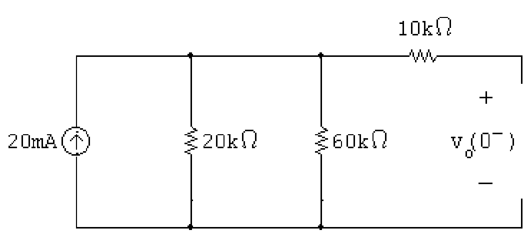


$$R_{\text{eq}} = 10{,}000 + 50{,}000\|75{,}000 = 40\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (40{,}000)(400 \times 10^{-9}) = 16\,\text{ms}$$

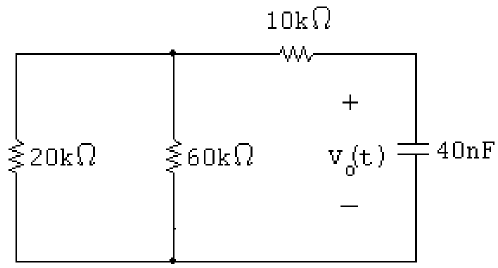
[d] $v(t) = v(0)e^{-t/\tau} = 200e^{-62.5t}\,\text{V} \qquad t \geq 0$

P 7.22 For $t < 0$:



$$V_o = (20{,}000\|60{,}000)(20 \times 10^{-3}) = 300\,\text{V}$$

For $t \geq 0$:

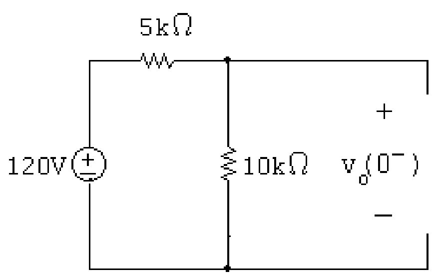


$$R_{\text{eq}} = 10{,}000 + (20{,}000 \| 60{,}000) = 25\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (25{,}000)(40 \times 10^{-9}) = 1\,\text{ms}$$

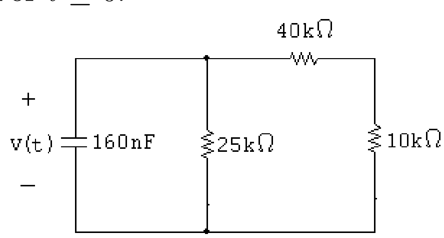
$$v(t) = V_o e^{-t/\tau} = 300e^{-1000t}\,\text{V} \qquad t \geq 0$$

P 7.23 **[a]** For $t < 0$:



$$V_o = \frac{10{,}000}{15{,}000}(120) = 80\,\text{V}$$

For $t \geq 0$:

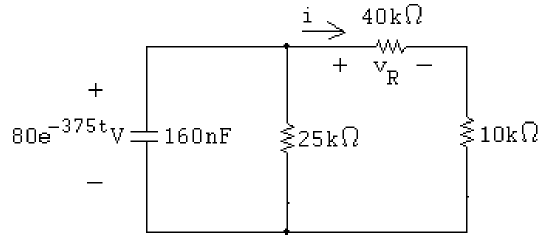


$$R_{\text{eq}} = 25{,}000 \| (40{,}000 + 10{,}000) = 16.67\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (16{,}666/67)(160 \times 10^{-9}) = 2.67\,\text{ms}$$

$$v(t) = V_o e^{-t/\tau} = 80e^{-375t}\,\text{V} \qquad t \geq 0$$

[**b**] For $t \geq 0$:



$$v_R(t) = \frac{40{,}000}{50{,}000}(80e^{-375t}) = 64e^{-375t}\,\text{V}$$

$$i(t) = \frac{v_R}{40{,}000} = 1.6e^{-375t}\,\text{mA}, \quad t \geq 0^+$$

P 7.24 Using the results of Problem 7.23:

$$w(0) = \frac{1}{2}CV_o^2 = \frac{1}{2}(160 \times 10^{-9})(80)^2 = 512\,\mu\text{J}$$

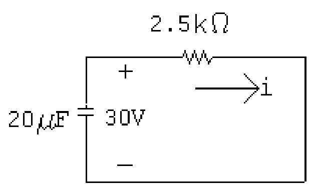
$$p_{40\text{k}} = Ri^2 = (40{,}000)(1.6 \times 10^{-3}e^{-375t})^2 = 0.1024e^{-750t}$$

$$w_{40\text{k}} = \int_0^\infty p_{40k}\,dt = \int_0^\infty 0.1024e^{-750t}\,dt = \frac{0.1024e^{-750t}}{-750}\Big|_0^\infty = 136.53\,\mu\text{J}$$

$$\text{percent} = \frac{136.53}{512}(100) = 26.67\%$$

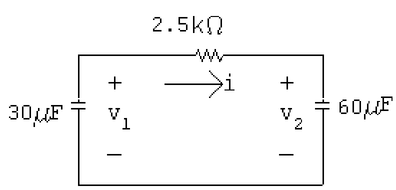
P 7.25 [**a**] $v_1(0^-) = v_1(0^+) = (0.006)(5000) = 30\,\text{V}$ $\qquad v_2(0^+) = 0$

$$C_{\text{eq}} = (30)(40)/90 = 20\,\mu\text{F}$$



$$\tau = (2.5 \times 10^3)(20 \times 10^{-6}) = 50\text{ms}; \qquad \frac{1}{\tau} = 20$$

$$i = \frac{30}{2500}e^{-20t} = 12e^{-20t}\,\text{mA}, \qquad t \geq 0^+$$



$$v_1 = \frac{-1}{30^{-6}}\int_0^t 12 \times 10^{-3}e^{-20x}\,dx + 30 = 20e^{-20t} + 10\,\text{V}, \qquad t \geq 0$$

$$v_2 = \frac{1}{60 \times 10^{-6}} \int_0^t 12 \times 10^{-3} e^{-20x}\, dx + 0 = -10e^{-20t} + 10\,\text{V}, \qquad t \geq 0$$

[b] $w(0) = \frac{1}{2}(30 \times 10^{-6})(30)^2 = 13.5\,\text{mJ}$

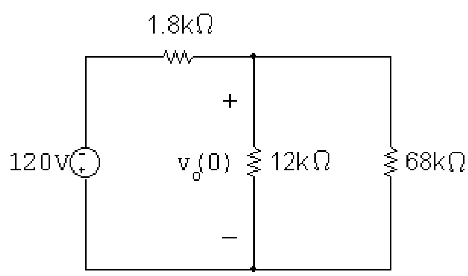
[c] $w_{\text{trapped}} = \frac{1}{2}(30 \times 10^{-6})(10)^2 + \frac{1}{2}(60 \times 10^{-6})(10)^2 = 4.5\,\text{mJ}.$

The energy dissipated by the 2.5 kΩ resistor is equal to the energy dissipated by the two capacitors; it is easier to calculate the energy dissipated by the capacitors:

$$w_{\text{diss}} = \frac{1}{2}(20 \times 10^{-6})(30)^2 = 9\,\text{mJ}.$$

Check: $w_{\text{trapped}} + w_{\text{diss}} = 4.5 + 9 = 13.5\,\text{mJ}; \qquad w(0) = 13.5\,\text{mJ}.$

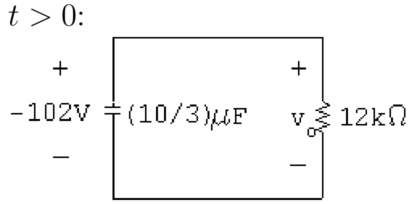
P 7.26 **[a]** $t < 0$:



$$R_{\text{eq}} = 12\,\text{k} \| 8\,\text{k} = 10.2\,\text{k}\Omega$$

$$v_o(0) = \frac{10{,}200}{10{,}200 + 1800}(-120) = -102\,\text{V}$$

$t > 0$:



$$\tau = [(10/3) \times 10^{-6})(12{,}000) = 40\,\text{ms}; \qquad \frac{1}{\tau} = 25$$

$$v_o = -102e^{-25t}\,\text{V}, \quad t \geq 0$$

$$p = \frac{v_o^2}{12{,}000} = 867 \times 10^{-3} e^{-50t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^{12\times10^{-3}} 867 \times 10^{-3} e^{-50t}\, dt$$
$$= 17.34 \times 10^{-3}(1 - e^{-50(12\times10^{-3})}) = 7824\,\mu\text{J}$$

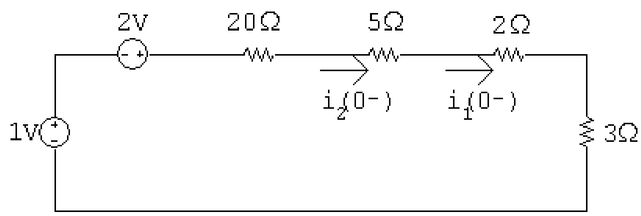
[b] $w(0) = \left(\frac{1}{2}\right)\left(\frac{10}{3}\right)(102)^2 \times 10^{-6} = 17.34\,\text{mJ}$

$$0.75w(0) = 13\,\text{mJ}$$

$$\int_0^{t_o} 867 \times 10^{-3} e^{-50x}\,dx = 13 \times 10^{-3}$$

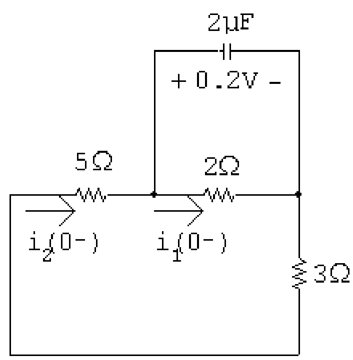
$$\therefore \quad 1 - e^{-50t_o} = 0.75; \qquad e^{50t_o} = 4; \quad \text{so} \quad t_o = 27.73\,\text{ms}$$

P 7.27 **[a]** $t < 0$:



$$i_1(0^-) = i_2(0^-) = \frac{3}{30} = 100\,\text{mA}$$

[b] $t > 0$:



$$i_1(0^+) = \frac{0.2}{2} = 100\,\text{mA}$$

$$i_2(0^+) = \frac{-0.2}{8} = -25\,\text{mA}$$

[c] Capacitor voltage cannot change instantaneously, therefore,

$$i_1(0^-) = i_1(0^+) = 100\,\text{mA}$$

[d] Switching can cause an instantaneous change in the current in a resistive branch. In this circuit

$$i_2(0^-) = 100\,\text{mA} \quad \text{and} \quad i_2(0^+) = 25\,\text{mA}$$

[e] $v_c = 0.2e^{-t/\tau}\,\text{V}, \qquad t \geq 0$

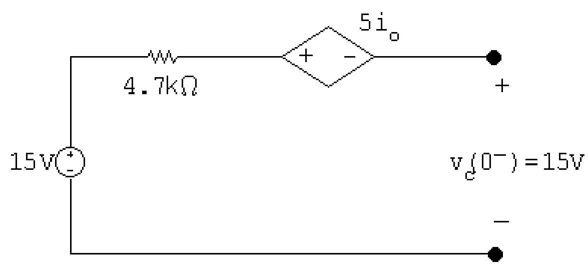
$$\tau = R_eC = 1.6(2 \times 10^{-6}) = 3.2\,\mu\text{s}; \qquad \frac{1}{\tau} = 312{,}500$$

$$v_c = 0.2e^{-312{,}000t}\,\text{V}, \qquad t \geq 0$$

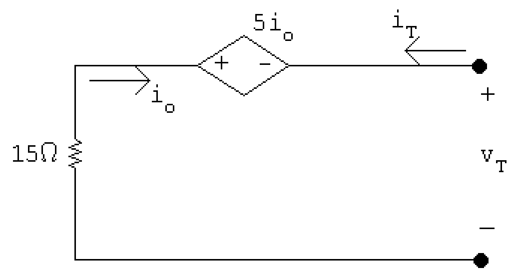
$$i_1 = \frac{v_c}{2} = 0.1e^{-312{,}000t}\,\text{A}, \qquad t \geq 0$$

[f] $i_2 = \dfrac{-v_c}{8} = -25e^{-312{,}000t}\,\text{mA}, \qquad t \geq 0^+$

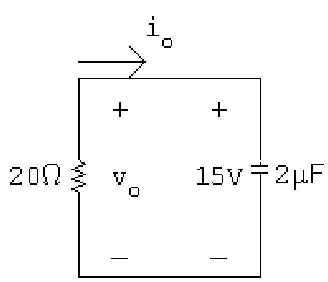
P 7.28 $t < 0$



$t > 0$



$$v_T = -5i_o - 15i_o = -20i_o = 20i_T \qquad \therefore \qquad R_{\text{Th}} = \frac{v_T}{i_T} = 20\,\Omega$$



$$\tau = RC = 40\,\mu\text{s}; \qquad \frac{1}{\tau} = 25{,}000$$

$$v_o = 15e^{-25{,}000t}\,\text{V}, \qquad t \geq 0$$

$$i_o = -\frac{v_o}{20} = -0.75e^{-25{,}000t}\,\text{A}, \qquad t \geq 0^+$$

P 7.29 **[a]** $R = \dfrac{v}{i} = 8\,\text{k}\Omega$

[b] $\dfrac{1}{\tau} = \dfrac{1}{RC} = 500; \qquad C = \dfrac{1}{(500)(8000)} = 0.25\,\mu\text{F}$

[c] $\tau = \dfrac{1}{500} = 2\,\text{ms}$

[d] $w(0) = \dfrac{1}{2}(0.25 \times 10^{-6})(72)^2 = 648\,\mu\text{J}$

[e] $w_{\text{diss}} = \displaystyle\int_0^{t_o} \frac{(72)^2 e^{-1000t}}{(800)} dt$

$$= 0.648 \frac{e^{-1000t}}{-1000} \Big|_0^{t_o} = 648(1 - e^{-1000t_o})\,\mu\text{J}$$

$\%\text{diss} = 100(1 - e^{-1000t_o}) = 68 \qquad \text{so} \qquad e^{1000t_o} = 3.125$

$\therefore \quad t = \dfrac{\ln 3.125}{1000} = 1139\,\mu\text{s}$

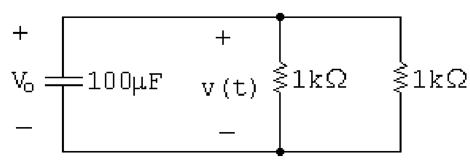
P 7.30 **[a]** Note that there are many different possible correct solutions to this problem.

$R = \dfrac{\tau}{C}$

Choose a $100\,\mu$F capacitor from Appendix H. Then,

$R = \dfrac{0.05}{100 \times 10^{-6}} = 500\,\Omega$

Construct a $500\,\Omega$ resistor by combining two $1\,\text{k}\Omega$ resistors in parallel:

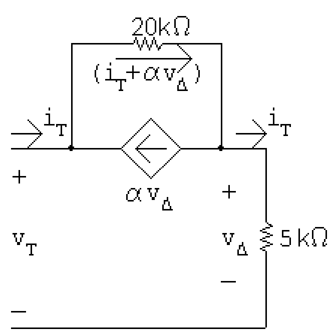


[b] $v(t) = V_o e^{-t/\tau} = 50e^{-20t}\,\text{V}, \qquad t \geq 0$

[c] $50e^{-20t} = 10 \qquad \text{so} \qquad e^{20t} = 5$

$\therefore \quad t = \dfrac{\ln 5}{20} = 80.47\,\text{ms}$

P 7.31 **[a]**



$v_T = 20 \times 10^3(i_T + \alpha v_\Delta) + 5 \times 10^3 i_T$

$v_\Delta = 5 \times 10^3 i_T$

$v_T = 25 \times 10^3 i_T + 20 \times 10^3 \alpha (5 \times 10^3 i_T)$

$R_{\text{Th}} = 25{,}000 + 100 \times 10^6 \alpha$

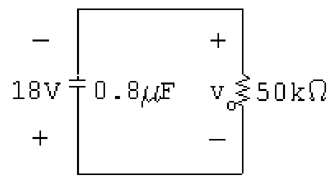
$\tau = R_{\text{Th}} C = 40 \times 10^{-3} = R_{\text{Th}}(0.8 \times 10^{-6})$

$R_{\text{Th}} = 50\,\text{k}\Omega = 25{,}000 + 100 \times 10^6 \alpha$

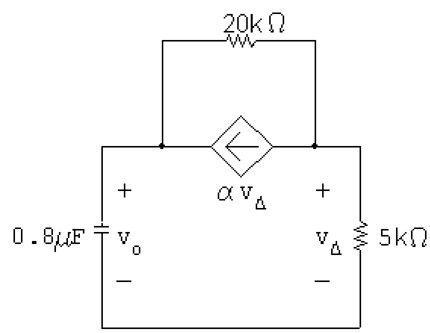
$\alpha = \dfrac{25{,}000}{100 \times 10^6} = 2.5 \times 10^{-4}\,\text{A/V}$

[b] $v_o(0) = (-5 \times 10^{-3})(3600) = -18\,\text{V} \qquad t < 0$

$t > 0$:



$v_o = -18e^{-25t}\,\text{V}, \quad t \geq 0$

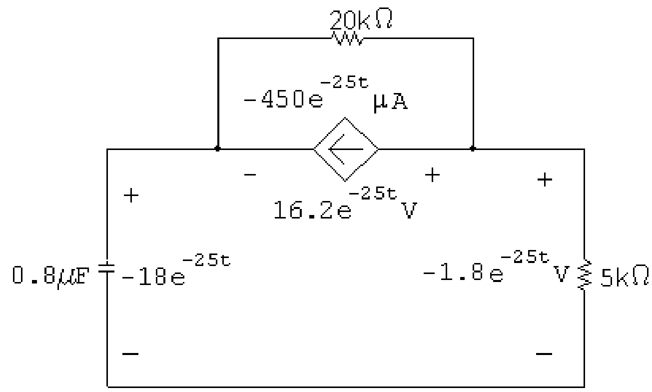


$\dfrac{v_\Delta}{5000} + \dfrac{v_\Delta - v_o}{20{,}000} + 2.5 \times 10^{-4} v_\Delta = 0$

$4v_\Delta + v_\Delta - v_o + 5v_\Delta = 0$

$\therefore \quad v_\Delta = \dfrac{v_o}{10} = -1.8e^{-25t}\,\text{V}, \quad t \geq 0^+$

P 7.32 [**a**]



$$p_{ds} = (16.2e^{-25t})(-450 \times 10^{-6}e^{-25t}) = -7290 \times 10^{-6}e^{-50t}\,\text{W}$$

$$w_{ds} = \int_0^\infty p_{ds}\,dt = -145.8\,\mu\text{J}.$$

$\therefore$ dependent source is delivering $145.8\,\mu$J.

[**b**] $$w_{5\text{k}} = \int_0^\infty (5000)(0.36 \times 10^{-3}e^{-25t})^2\,dt = 648 \times 10^{-6}\int_0^\infty e^{-50t}\,dt = 12.96\,\mu\text{J}$$

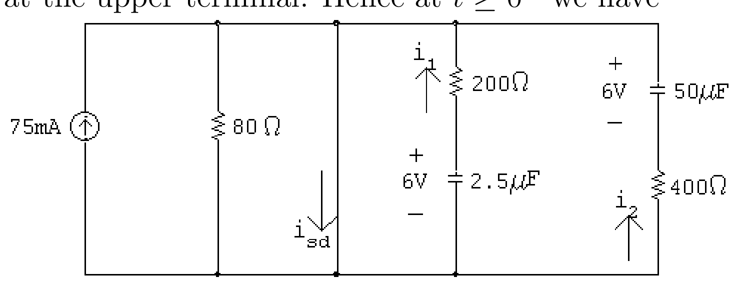
$$w_{20\text{k}} = \int_0^\infty \frac{(16.2e^{-25t})^2}{20{,}000}\,dt = 13{,}122 \times 10^{-6}\int_0^\infty e^{-50t}\,dt = 262.44\,\mu\text{J}$$

$$w_c(0) = \frac{1}{2}(0.8 \times 10^{-6})(18)^2 = 129.6\,\mu\text{J}$$

$$\sum w_{\text{diss}} = 12.96 + 262.44 = 275.4\,\mu\text{J}$$

$$\sum w_{\text{dev}} = 145.8 + 129.6 = 275.4\,\mu\text{J}.$$

P 7.33 [**a**] At $t = 0^-$ the voltage on each capacitor will be 6 V (0.075×80), positive at the upper terminal. Hence at $t \geq 0^+$ we have



$$\therefore \quad i_{sd}(0^+) = 0.075 + \frac{6}{200} + \frac{6}{400} = 120\,\text{mA}$$

At $t = \infty$, both capacitors will have completely discharged.

$$\therefore \quad i_{sd}(\infty) = 75\,\text{mA}$$

[b] $i_{sd}(t) = 0.075 + i_1(t) + i_2(t)$

$\tau_1 = 200(25 \times 10^{-6}) = 5\,\text{ms}$

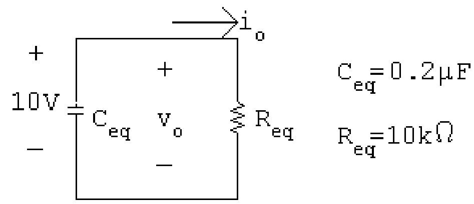
$\tau_2 = 400(50 \times 10^{-6}) = 20\,\text{ms}$

$\therefore \quad i_1(t) = 30e^{-200t}\,\text{mA}, \qquad t \geq 0^+$

$i_2(t) = 15e^{-50t}\,\text{mA}, \qquad t \geq 0$

$\therefore \quad i_{sd} = 75 + 30e^{-200t} + 15e^{-50t}\,\text{mA}, \qquad t \geq 0^+$

P 7.34 **[a]** The equivalent circuit for $t > 0$:



$\tau = 2\,\text{ms}; \qquad 1/\tau = 500$

$v_o = 10e^{-500t}\,\text{V}, \qquad t \geq 0$

$i_o = e^{-500t}\,\text{mA}, \qquad t \geq 0^+$

$i_{24\text{k}\Omega} = e^{-500t}\left(\frac{16}{40}\right) = 0.4e^{-500t}\,\text{mA}, \qquad t \geq 0^+$

$p_{24\text{k}\Omega} = (0.16 \times 10^{-6}e^{-1000t})(24{,}000) = 3.84e^{-1000t}\,\text{mW}$

$w_{24\text{k}\Omega} = \int_0^\infty 3.84 \times 10^{-3}e^{-1000t}\,dt = -3.84 \times 10^{-6}(0-1) = 3.84\,\mu\text{J}$

$w(0) = \frac{1}{2}(0.25 \times 10^{-6})(40)^2 + \frac{1}{2}(1 \times 10^{-6})(50)^2 = 1.45\,\text{mJ}$

$\%\ \text{diss}\,(24\,\text{k}\Omega) = \frac{3.84 \times 10^{-6}}{1.45 \times 10^{-3}} \times 100 = 0.26\%$

[b] $p_{400\Omega} = 400(1 \times 10^{-3}e^{-500t})^2 = 0.4 \times 10^{-3}e^{-1000t}$

$w_{400\Omega} = \int_0^\infty p_{400}\,dt = 0.40\,\mu\text{J}$

$\%\ \text{diss}\,(400\,\Omega) = \frac{0.4 \times 10^{-6}}{1.45 \times 10^{-3}} \times 100 = 0.03\%$

$i_{16k\Omega} = e^{-500t}\left(\frac{24}{40}\right) = 0.6e^{-500t}\,\text{mA}, \quad t \geq 0^+$

$p_{16k\Omega} = (0.6 \times 10^{-3}e^{-500t})^2(16{,}000) = 5.76 \times 10^{-3}e^{-1000t}\,\text{W}$

$w_{16k\Omega} = \int_0^\infty 5.76 \times 10^{-3}e^{-1000t}\,dt = 5.76\,\mu\text{J}$

$\%\ \text{diss}\,(16\,\text{k}\Omega) = 0.4\%$

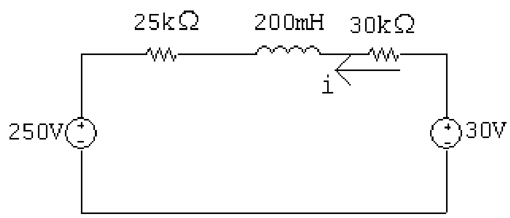
[c] $\sum w_{\text{diss}} = 3.84 + 5.76 + 0.4 = 10\,\mu\text{J}$

$$w_{\text{trapped}} = w(0) - \sum w_{\text{diss}} = 1.45 \times 10^{-3} - 10 \times 10^{-6} = 1.44\,\text{mJ}$$

$$\%\ \text{trapped} = \frac{1.44}{1.45} \times 100 = 99.31\%$$

Check: $0.26 + 0.03 + 0.4 + 99.31 = 100\%$

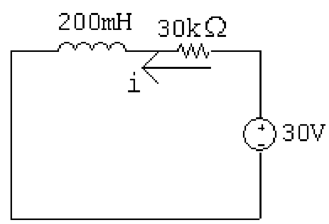
P 7.35 **[a]** For $t < 0$, calculate the Thévenin equivalent for the circuit to the left and right of the 200 mH inductor. We get



$$i(0^-) = \frac{30 - 250}{25\,\text{k} + 30\,\text{k}} = -4\,\text{mA}$$

$$i(0^-) = i(0^+) = -4\,\text{mA}$$

[b] For $t > 0$, the circuit reduces to



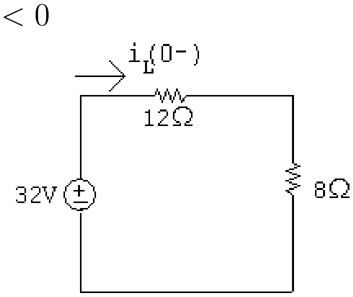
Therefore $i(\infty) = 30/30{,}000 = 1\,\text{mA}$

[c] $\tau = \dfrac{L}{R} = \dfrac{200 \times 10^{-3}}{30{,}000} = 6.67\,\mu\text{s}$

[d] $i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$

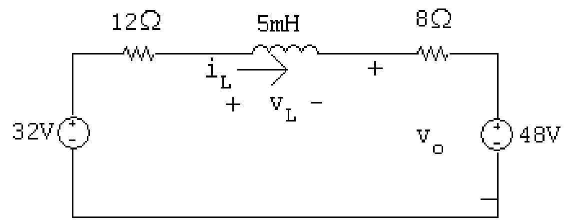
$$= 0.001 + [-0.004 - 0.001]e^{-150{,}000t} = 1 - 5e^{-150{,}000t}\,\text{mA}, \qquad t \geq 0$$

P 7.36 **[a]** $t < 0$



$$i_L(0^-) = \frac{32}{20} = 1.6\,\text{A}$$

$t > 0$



$$i_L(\infty) = \frac{32 - 48}{12 + 8} = -0.8\,\text{A}$$

$$\tau = \frac{L}{R} = \frac{5 \times 10^{-3}}{12 + 8} = 250\,\mu\text{s}; \qquad \frac{1}{\tau} = 4000$$

$$i_L = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$$

$$= -0.8 + (1.6 + 0.8)e^{-4000t} = -0.8 + 2.4e^{-4000t}\,\text{A}, \qquad t \geq 0$$

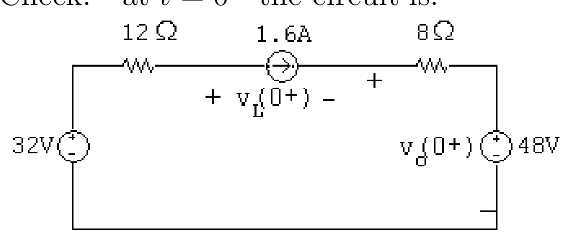
$$v_o = 8i_L + 48 = 8(-0.8 + 2.4e^{-4000t}) + 48 = 41.6 + 19.2e^{-4000t}\,\text{V}, \qquad t \geq 0$$

[b] $v_L = L\dfrac{di_L}{dt} = 5 \times 10^{-3}(-4000)[2.4e^{-4000t}] = -48e^{-4000t}\,\text{V}, \qquad t \geq 0^+$

$v_L(0^+) = -48\,\text{V}$

From part (a) $\quad v_o(0^+) = 0\,\text{V}$

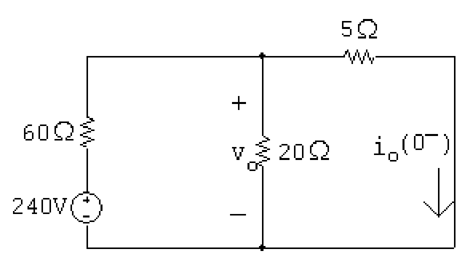
Check: at $t = 0^+$ the circuit is:



$v_o(0^+) = 48 + (8\,\Omega)(1.6\,\text{A}) = 60.8\,\text{V}; \qquad v_\text{L}(0^+) + v_o(0^+) = 12(-1.6) + 32$

$\therefore \qquad v_\text{L}(0^+) = -19.2 + 32 - 60.8 = -48\,\text{V}$

P 7.37 **[a]** $t < 0$



KVL equation at the top node:

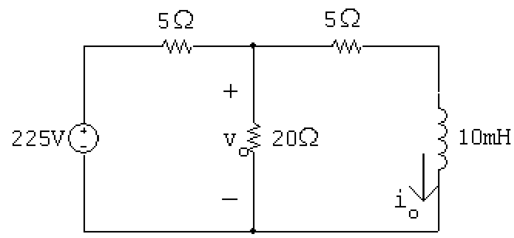
$$\frac{v_o - 240}{60} + \frac{v_o}{20} + \frac{v_o}{5} = 0$$

Multiply by 60 and solve:

$$240 = (3 + 1 + 12)v_o; \qquad v_o = 15\,\text{V}$$

$$\therefore \quad i_o(0^-) = \frac{v_o}{5} = 15/5 = 3\,\text{A}$$

$t > 0$



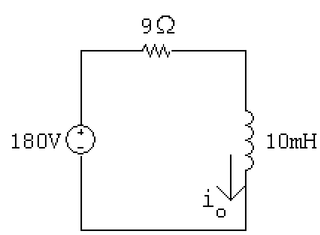
Use voltage division to find the Thévenin voltage:

$$V_{\text{Th}} = v_o = \frac{20}{20+5}(225) = 180\,\text{V}$$

Remove the voltage source and make series and parallel combinations of resistors to find the equivalent resistance:

$$R_{\text{Th}} = 5 + 20\|5 = 5 + 4 = 9\,\Omega$$

The simplified circuit is:



$$\tau = \frac{L}{R} = \frac{10 \times 10^{-3}}{9} = 1.11\,\text{ms}; \qquad \frac{1}{\tau} = 900$$

$$i_o(\infty) = \frac{190}{9} = 20\,\text{A}$$

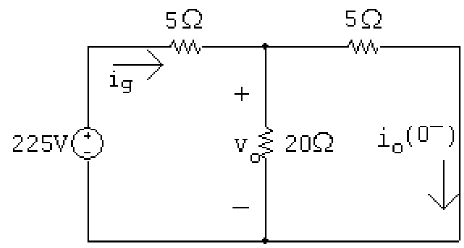
$$\therefore \quad i_o = i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau}$$

$$= 20 + (3 - 20)e^{-900t} = 20 - 17e^{-900t}\,\text{A}, \qquad t \geq 0$$

[b]

$$\begin{aligned} v_o &= 5i_o + L\frac{di_o}{dt} \\ &= 5(20 - 17e^{-900t}) + 0.01(-900)(17e^{-900t}) \\ &= 100 - 85e^{-900t} + 153e^{-900t} \\ v_o &= 100 + 68e^{-900t}\,\text{V}, \qquad t \geq 0^+ \end{aligned}$$

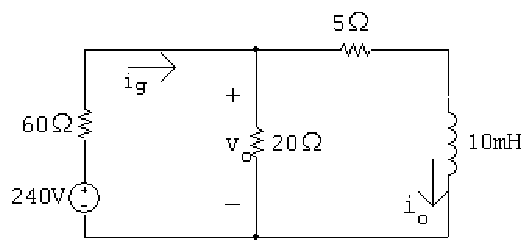
P 7.38 **[a]** $t < 0$



$$i_g = \frac{225}{5 + 20\|5} = \frac{225}{9} = 25\,\text{A}$$

$$\therefore \quad i_o(0^-) = \frac{20\|5}{5}(25) = 20\,\text{A}$$

$t > 0$



$$i_g(\infty) = \frac{240}{60 + 20\|5} = \frac{240}{64} = 3.75\,\text{A}$$

$$i_o(\infty) = \frac{20\|5}{5} i_g(\infty) = 3\,\text{A}$$

$$R_{\text{eq}} = 5 + 20\|60 = 3 + 15 = 20\,\Omega$$

$$\tau = \frac{L}{R_{\text{eq}}} = \frac{10 \times 10^{-3}}{20} = 0.5\,\text{ms}; \qquad \frac{1}{\tau} = 2000$$

$$\therefore \quad i_o = i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau}$$

$$= 3 + (20 - 3)e^{-2000t} = 3 + 17e^{-2000t}\,\text{A}, \qquad t \geq 0$$

[b]

$$\begin{aligned} v_o &= 5i_o + L\frac{di_o}{dt} \\ &= 5(3 + 17e^{-2000t}) + 0.01(-2000)(17e^{-2000t}) \\ &= 15 + 65e^{-2000t} - 340e^{-2000t} \\ v_o &= 15 - 255e^{-2000t}\,\text{V}, \qquad t \geq 0^+ \end{aligned}$$

P 7.39 **[a]** From Eqs. (7.35) and (7.42)

$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right) e^{-(R/L)t}$$

$$v = (V_s - I_oR)e^{-(R/L)t}$$

$$\therefore \quad \frac{V_s}{R} = 4; \qquad I_o - \frac{V_s}{R} = 4$$

$$V_s - I_oR = -80; \qquad \frac{R}{L} = 40$$

$$\therefore \quad I_o = 4 + \frac{V_s}{R} = 8\,\text{A}$$

Now since $V_s = 4R$ we have

$$4R - 8R = -80; \qquad R = 20\,\Omega$$

$$V_s = 80\,\text{V}; \qquad L = \frac{R}{40} = 0.5\,\text{H}$$

[b] $i = 4 + 4e^{-40t}; \qquad i^2 = 16 + 32e^{-40t} + 16e^{-80t}$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.5)[16 + 32e^{-40t} + 16e^{-80t}] = 4 + 8e^{-40t} + 4e^{-80t}$$

$$\therefore \quad 4 + 8e^{-40t} + 4e^{-80t} = 9 \quad \text{or} \quad e^{-80t} + 2e^{-40t} - 1.25 = 0$$

Let $x = e^{-40t}$:

$x^2 + 2x - 1.25 = 0;$ Solving, $x = 0.5;$ $x = -2.5$

But $x \geq 0$ for all t. Thus,

$$e^{-40t} = 0.5; \qquad e^{40t} = 2; \qquad t = 25\ln 2 = 17.33\,\text{ms}$$

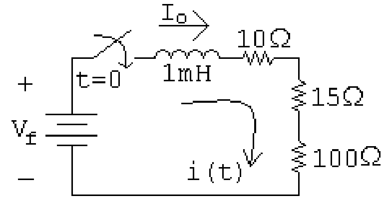
P 7.40 **[a]** Note that there are many different possible solutions to this problem.

$$R = \frac{L}{\tau}$$

Choose a 1 mH inductor from Appendix H. Then,

$$R = \frac{0.001}{8 \times 10^{-6}} = 125\,\Omega$$

Construct the resistance needed by combining $100\,\Omega$, $10\,\Omega$, and $15\,\Omega$ resistors in series:



[b] $i(t) = I_f + (I_o - I_f)e^{-t/\tau}$

$$I_o = 0\,\text{A}; \qquad I_f = \frac{V_f}{R} = \frac{25}{125} = 200\,\text{mA}$$

$$\therefore \quad i(t) = 200 + (0 - 200)e^{-125{,}000t}\,\text{mA} = 200 - 200e^{-125{,}000t}\,\text{mA}, \qquad t \geq 0$$

[c] $i(t) = 0.2 - 0.2e^{-125{,}000t} = (0.75)(0.2) = 0.15$

$$e^{-125{,}000t} = 0.25 \qquad \text{so} \qquad e^{125{,}000t} = 4$$

$$\therefore \quad t = \frac{\ln 4}{125{,}000} = 11.09\,\mu\text{s}$$

P 7.41 **[a]** $v_o(0^+) = -I_g R_2; \qquad \tau = \dfrac{L}{R_1 + R_2}$

$$v_o(\infty) = 0$$

$$v_o(t) = -I_g R_2 e^{-[(R_1+R_2)/L]t}\,\text{V}, \qquad t \geq 0^+$$

[b] $v_o(0^+) \to \infty$, and the duration of $v_o(t) \to$ zero

[c] $v_{sw} = R_2 i_o; \qquad \tau = \dfrac{L}{R_1 + R_2}$

$$i_o(0^+) = I_g; \qquad i_o(\infty) = I_g \frac{R_1}{R_1 + R_2}$$

Therefore $\quad i_o(t) = \frac{I_g R_1}{R_1+R_2} + \left[I_g - \frac{I_g R_1}{R_1+R_2}\right] e^{-[(R_1+R_2)/L]t}$

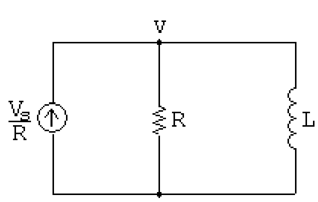
$$i_o(t) = \frac{R_1 I_g}{(R_1+R_2)} + \frac{R_2 I_g}{(R_1+R_2)} e^{-[(R_1+R_2)/L]t}$$

Therefore $\quad v_{sw} = \frac{R_1 I_g}{(1+R_1/R_2)} + \frac{R_2 I_g}{(1+R_1/R_2)} e^{-[(R_1+R_2)/L]t}, \qquad t \geq 0^+$

[d] $|v_{sw}(0^+)| \to \infty; \qquad$ duration $\to 0$

P 7.42 Opening the inductive circuit causes a very large voltage to be induced across the inductor L. This voltage also appears across the switch (part [d] of Problem 7.41), causing the switch to arc over. At the same time, the large voltage across L damages the meter movement.

P 7.43 **[a]**



$$-\frac{V_s}{R} + \frac{v}{R} + \frac{1}{L}\int_0^t v\,dt + I_o = 0$$

Differentiating both sides,

$$\frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$

$$\therefore \qquad \frac{dv}{dt} + \frac{R}{L}v = 0$$

[b] $\dfrac{dv}{dt} = -\dfrac{R}{L}v$

$$\frac{dv}{dt}dt = -\frac{R}{L}v\,dt \qquad \text{so} \qquad dv = -\frac{R}{L}v\,dt$$

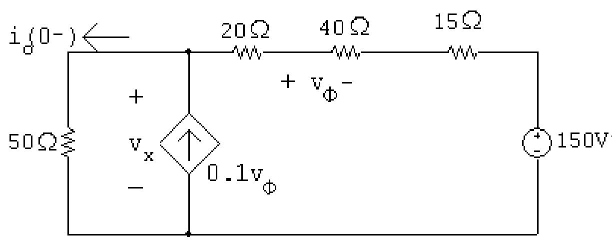
$$\frac{dv}{v} = -\frac{R}{L}dt$$

$$\int_{V_o}^{v(t)}\frac{dx}{x} = -\frac{R}{L}\int_0^t dy$$

$$\ln\frac{v(t)}{V_o} = -\frac{R}{L}t$$

$$\therefore \qquad v(t) = V_o e^{-(R/L)t} = (V_\text{s} - RI_o)e^{-(R/L)t}$$

P 7.44 For $t < 0$



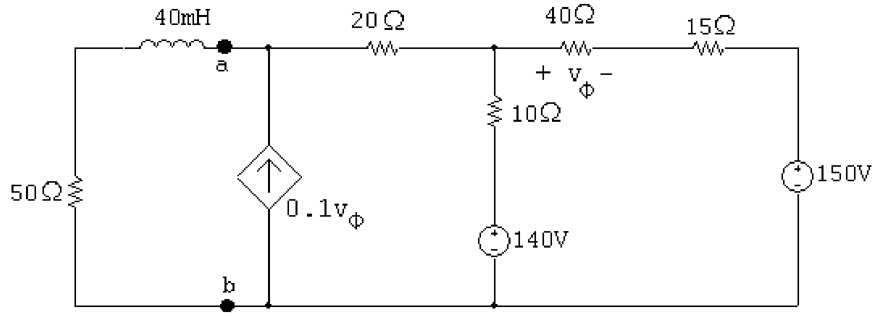
$$\frac{v_x}{50} - 0.1v_\phi + \frac{v_x - 150}{75} = 0$$

$$v_\phi = \frac{40}{75}(v_x - 150)$$

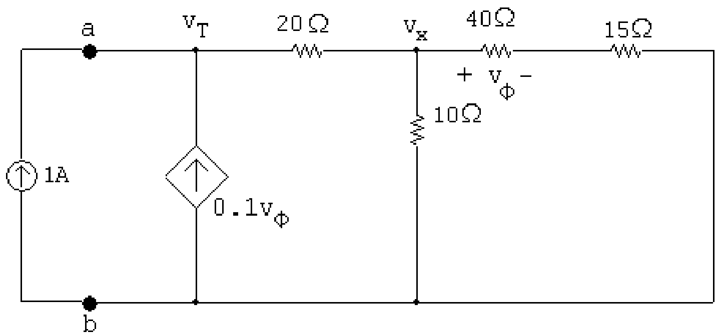
Solving,

$$v_x = 300\,\text{V}; \qquad i_o(0^-) = \frac{v_x}{50} = 6\,\text{A}$$

$t > 0$



Find Thévenin equivalent with respect to a, b. Use a test source to find the Thévenin equivalent resistance:



$$-1 - 0.1v_\phi + \frac{v_\mathrm{T} - v_x}{20} = 0$$

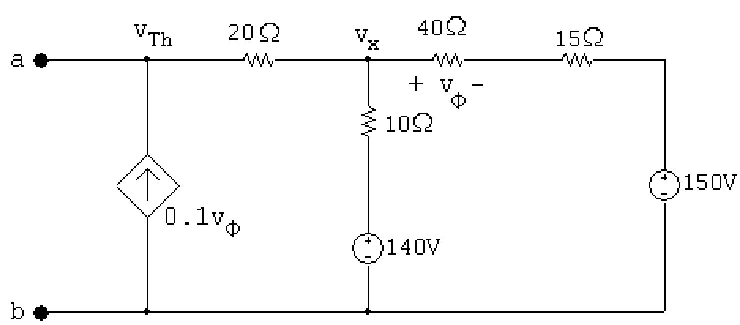
$$\frac{v_x - v_\mathrm{T}}{20} + \frac{v_x}{10} + \frac{v_x}{55} = 0$$

$$v_\phi = \frac{40}{55}v_x$$

Solving,

$$v_\mathrm{T} = 74\,\mathrm{V} \qquad \text{so} \qquad R_\mathrm{Th} = \frac{v_\mathrm{T}}{1\,\mathrm{A}} = 74\,\Omega$$

Find the open circuit voltage with respect to a, b:



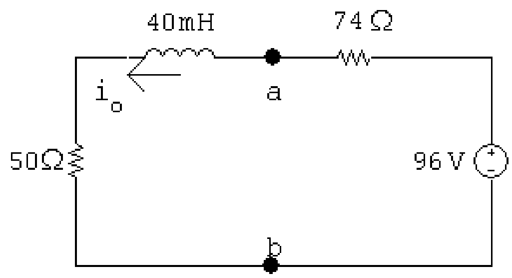
$$-0.1v_\phi + \frac{v_\mathrm{Th} - v_x}{20} = 0$$

$$\frac{v_x - v_{\text{Th}}}{20} + \frac{v_x - 140}{10} + \frac{v_x - 150}{55} = 0$$

$$v_\phi = \frac{40}{55}(v_x - 150)$$

Solving,

$$v_{\text{Th}} = 96\,\text{V}$$

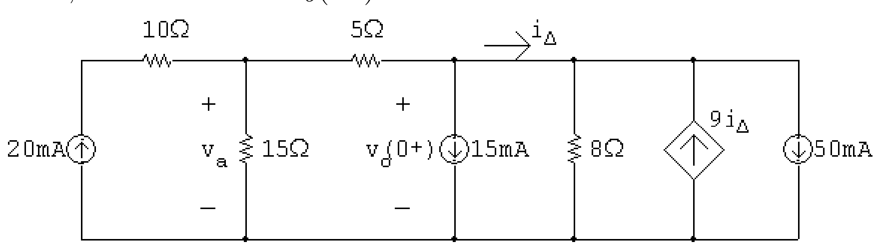


$$i_o(\infty) = 96/124 = 0.774\,\text{A}$$

$$\tau = \frac{40 \times 10^{-3}}{124} = 0.3226\,\text{ms}; \qquad 1/\tau = 3100$$

$$i_o = 0.774 + (6 - 0.774)e^{-3100t} = 0.774 + 5.226e^{-3100t}\,\text{A}, \qquad t \geq 0$$

P 7.45 $t > 0$; calculate $v_o(0^+)$



$$\frac{v_\text{a}}{15} + \frac{v_\text{a} - v_o(0^+)}{5} = 20 \times 10^{-3}$$

$$\therefore \quad v_\text{a} = 0.75v_o(0^+) + 75 \times 10^{-3}$$

$$15 \times 10^{-3} + \frac{v_o(0^+) - v_\text{a}}{5} + \frac{v_o(0^+)}{8} - 9i_\Delta + 50 \times 10^{-3} = 0$$

$$13v_o(0^+) - 8v_\text{a} - 360i_\Delta = -2600 \times 10^{-3}$$

$$i_\Delta = \frac{v_o(0^+)}{8} - 9i_\Delta + 50 \times 10^{-3}$$

$$\therefore \quad i_\Delta = \frac{v_o(0^+)}{80} + 5 \times 10^{-3}$$

$$\therefore \quad 360i_\Delta = 4.5v_o(0^+) + 1800 \times 10^{-3}$$

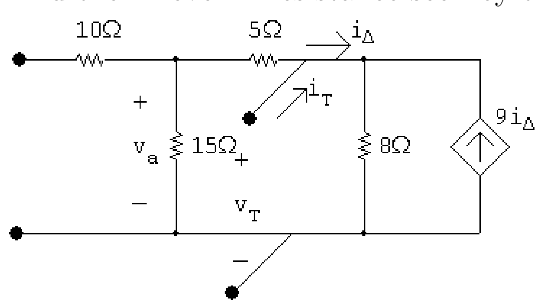
$$8v_a = 6v_o(0^+) + 600 \times 10^{-3}$$

$$\therefore \quad 13v_o(0^+) - 6v_o(0^+) - 600 \times 10^{-3} - 4.5v_o(0^+) -$$
$$1800 \times 10^{-3} = -2600 \times 10^{-3}$$

$$2.5v_o(0^+) = -200 \times 10^{-3}; \qquad v_o(0^+) = -80\,\text{mV}$$

$$v_o(\infty) = 0$$

Find the Thévenin resistance seen by the 4 mH inductor:



$$i_T = \frac{v_T}{20} + \frac{v_T}{8} - 9i_\Delta$$

$$i_\Delta = \frac{v_T}{8} - 9i_\Delta \qquad \therefore \quad 10i_\Delta = \frac{v_T}{8}; \qquad i_\Delta = \frac{v_T}{80}$$

$$i_T = \frac{v_T}{20} + \frac{10v_T}{80} - \frac{9v_T}{80}$$

$$\frac{i_T}{v_T} = \frac{1}{20} + \frac{1}{80} = \frac{5}{80} = \frac{1}{16}\,\text{S}$$

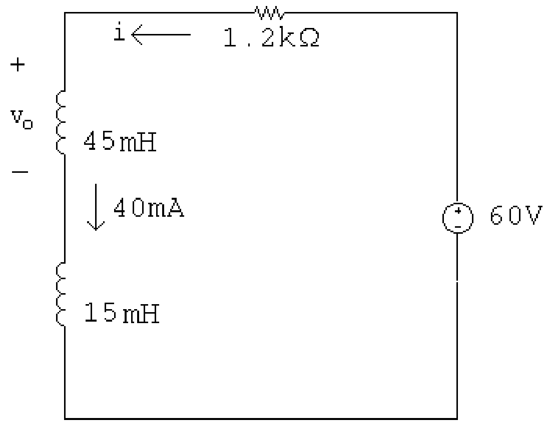
$$\therefore \quad R_{\text{Th}} = 16\Omega$$

$$\tau = \frac{4 \times 10^{-3}}{16} = 0.25\,\text{ms}; \qquad 1/\tau = 4000$$

$$\therefore \quad v_o = 0 + (-80 - 0)e^{-4000t} = -80e^{-4000t}\,\text{mV}, \qquad t \geq 0^+$$

P 7.46 For $t < 0$, $\quad i_{45\text{mH}}(0) = 80\,\text{V}/2000\,\Omega = 40\,\text{mA}$

For $t > 0$, after making a Thévenin equivalent of the circuit to the right of the inductors we have



$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-t/\tau}$$

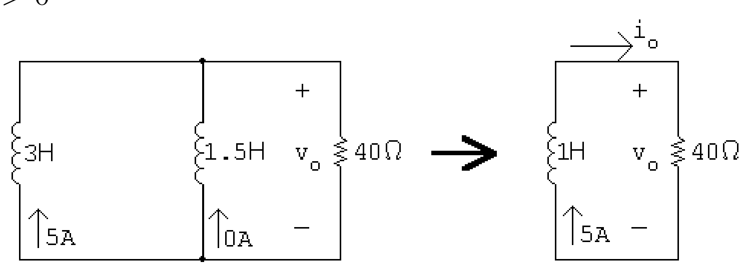
$$\frac{1}{\tau} = \frac{R}{L} = \frac{1200}{60 \times 10^{-3}} = 20{,}000$$

$$I_o = 40\,\text{mA}; \qquad I_f = \frac{V_s}{R} = \frac{60}{1200} = 50\,\text{mA}$$

$$i = 0.05 + (0.04 - 0.05)e^{-20{,}000t} = 50 - 10e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0$$

$$v_o = 0.045\frac{di}{dt} = 0.045(-0.01)(-20{,}000e^{-20{,}000t}) = 9e^{-20{,}000t}\,\text{V}, \qquad t \geq 0^+$$

P 7.47 $t > 0$



$$\tau = \frac{1}{40}$$

$$i_o = 5e^{-40t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 40i_o = 200e^{-40t}\,\text{V}, \qquad t > 0^+$$

$$200e^{-40t} = 100; \qquad e^{40t} = 2$$

$$\therefore \quad t = \frac{1}{40}\ln 2 = 17.33\,\text{ms}$$

P 7.48 **[a]** $w_{\text{diss}} = \frac{1}{2}L_e i^2(0) = \frac{1}{2}(1)(5)^2 = 12.5\,\text{J}$

[b] $i_{3H} = \frac{1}{3}\int_0^t (200)e^{-40x}\,dx - 5$

$$= 1.67(1 - e^{-40t}) - 5 = -1.67e^{-40t} - 3.33\,\text{A}$$

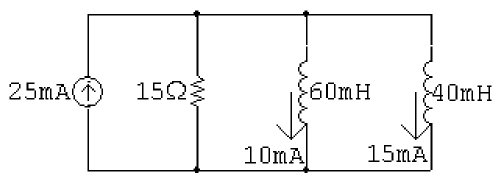
$$i_{1.5H} = \frac{1}{1.5}\int_0^t (200)e^{-40x}\,dx + 0$$

$$= -3.33e^{-40t} + 3.33\,\text{A}$$

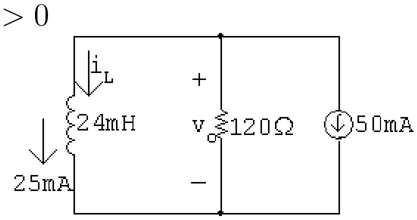
$$w_{\text{trapped}} = \frac{1}{2}(4.5)(3.33)^2 = 25\,\text{J}$$

[c] $w(0) = \frac{1}{2}(3)(5)^2 = 37.5\,\text{J}$

P 7.49 **[a]** $t < 0$



$t > 0$



$$i_L(0^-) = i_L(0^+) = 25\,\text{mA}; \qquad \tau = \frac{24\times10^{-3}}{120} = 0.2\,\text{ms}; \qquad \frac{1}{\tau} = 5000$$

$$i_L(\infty) = -50\,\text{mA}$$

$$i_L = -50 + (25+50)e^{-5000t} = -50 + 75e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

$$v_o = -120[75\times10^{-3}e^{-5000t}] = -9e^{-5000t}\,\text{V}, \qquad t \geq 0^+$$

[b] $i_1 = \frac{1}{60\times10^{-3}}\int_0^t -9e^{-5000x}\,dx + 10\times10^{-3} = (30e^{-5000t} - 20)\,\text{mA}, \qquad t \geq 0$

[c] $i_2 = \frac{1}{40\times10^{-3}}\int_0^t -9e^{-5000x}\,dx + 15\times10^{-3} = (45e^{-5000t} - 30)\,\text{mA}, \qquad t \geq 0$

P 7.50 **[a]** Let v be the voltage drop across the parallel branches, positive at the top node, then

$$-I_g + \frac{v}{R_g} + \frac{1}{L_1}\int_0^t v\,dx + \frac{1}{L_2}\int_0^t v\,dx = 0$$

$$\frac{v}{R_g} + \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\int_0^t v\,dx = I_g$$

$$\frac{v}{R_g} + \frac{1}{L_e}\int_0^t v\,dx = I_g$$

$$\frac{1}{R_g}\frac{dv}{dt} + \frac{v}{L_e} = 0$$

$$\frac{dv}{dt} + \frac{R_g}{L_e}v = 0$$

Therefore $v = I_g R_g e^{-t/\tau}; \qquad \tau = L_e/R_g$
Thus

$$i_1 = \frac{1}{L_1}\int_0^t I_g R_g e^{-x/\tau}\,dx = \frac{I_g R_g}{L_1}\frac{e^{-x/\tau}}{(-1/\tau)}\bigg|_0^t = \frac{I_g L_e}{L_1}(1 - e^{-t/\tau})$$

$$i_1 = \frac{I_g L_2}{L_1 + L_2}(1 - e^{-t/\tau}) \quad \text{and} \quad i_2 = \frac{I_g L_1}{L_1 + L_2}(1 - e^{-t/\tau})$$

[b] $i_1(\infty) = \frac{L_2}{L_1 + L_2}I_g; \qquad i_2(\infty) = \frac{L_1}{L_1 + L_2}I_g$

P 7.51 **[a]** $v_c(0^+) = -120\,\text{V}$

[b] Use voltage division to find the final value of voltage:

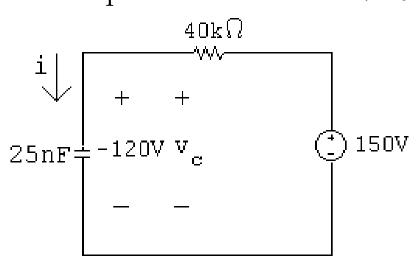
$$v_c(\infty) = \frac{150{,}000}{200{,}000}(200) = 150\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$V_{\text{Th}} = 150\,\text{V}, \qquad R_{\text{Th}} = 2500 + 150\,\text{k}\|50\,\text{k} = 40\,\text{k}\Omega,$

Therefore $\tau = R_{\text{eq}}C = (40{,}000)(25 \times 10^{-9}) = 1\,\text{ms}$

The simplified circuit for $t > 0$ is:



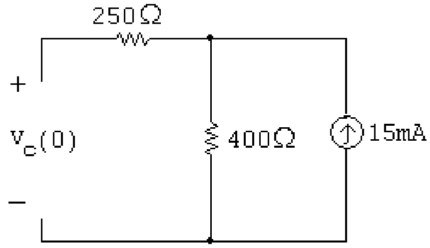
[d] $i(0^+) = \frac{150 - (-120)}{40{,}000} = 6.75\,\text{mA}$

[e] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= 150 + (-120 - 150)e^{-t/\tau} = 150 - 270e^{-1000t}\,\text{V}, \qquad t \geq 0$$

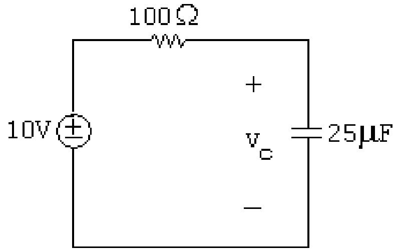
[f] $i = C\frac{dv_c}{dt} = (25 \times 10^{-9})(-1000)(-270e^{-1000t}) = 6.75e^{-1000t}\,\text{mA}, \qquad t \geq 0^+$

P 7.52 **[a]** for $t < 0$:



$v_c(0) = 400(0.015) = 6\,\text{V}$

For $t \geq 0$:

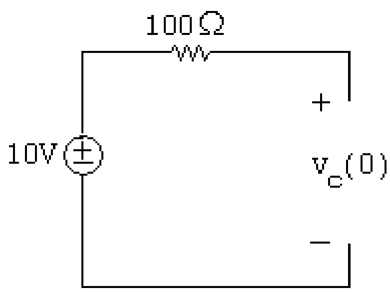


$v_c(\infty) = 10\,\text{V}$

$R_{\text{eq}} = 20\,\Omega \qquad \text{so} \qquad \tau = R_{\text{eq}}C = 250(25 \times 10^{-6}) = 6.25\,\text{ms}$

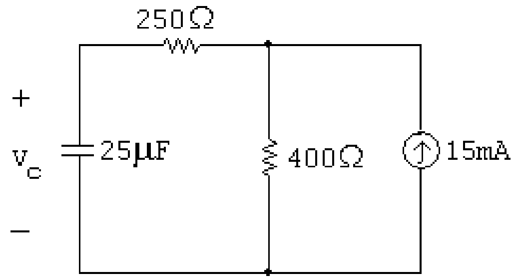
$v_c(t) = v_c(\infty) + (v_c(0) - v_c(\infty))e^{-t/\tau} = 10 + (6 - 10)e^{-160t} = 10 - 4e^{-160t}\,\text{V}$

[b] For $t < 0$:



$v_c(0) = 10\,\text{V}$

For $t \geq 0$:



$v_c(\infty) = 400(0.015) = 6\,\text{V}$

$R_{\text{eq}} = 100 + 400 = 500\,\Omega \qquad \text{so} \qquad \tau = R_{\text{eq}}C = 500(25 \times 10^{-6}) = 12.5\,\text{ms}$

$v_c(t) = v_c(\infty) + (v_c(0) - v_c(\infty))e^{-t/\tau} = 6 + (10 - 6)e^{-80t} = 6 + 4e^{-80t}\,\text{V}$

P 7.53 **[a]** Use voltage division to find the initial value of the voltage:

$$v_c(0^+) = v_{9k} = \frac{9\,\text{k}}{9\,\text{k} + 3\,\text{k}}(120) = 90\,\text{V}$$

[b] Use Ohm's law to find the final value of voltage:

$$v_c(\infty) = v_{40k} = -(1.5 \times 10^{-3})(40 \times 10^3) = -60\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$V_{\text{Th}} = -60\,\text{V}, \qquad R_{\text{Th}} = 10\,\text{k} + 40\,\text{k} = 50\,\text{k}\Omega$$

$$\tau = R_{\text{Th}}C = 1\,\text{ms} \;= 1000\,\mu\text{s}$$

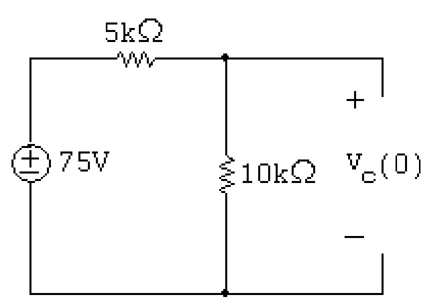
[d] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= -60 + (90 + 60)e^{-1000t} = -60 + 150e^{-1000t}\,\text{V}, \quad t \geq 0$$

We want $v_c = -60 + 150e^{-1000t} = 0$:

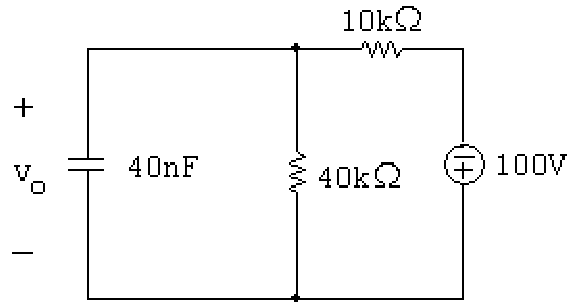
Therefore $\quad t = \dfrac{\ln(150/60)}{1000} = 916.3\,\mu\text{s}$

P 7.54 **[a]** For $t < 0$:



$$v_o(0) = \frac{10{,}000}{15{,}000}(75) = 50\,\text{V}$$

For $t \geq 0$:



$$v_o(\infty) = \frac{40{,}000}{50{,}000}(-100) = -80\,\text{V}$$

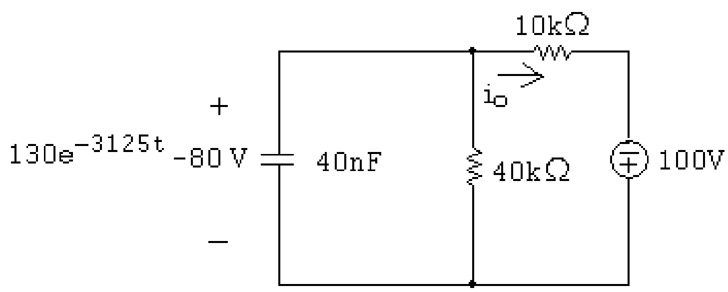
$$R_{\text{eq}} = 40\,\text{k}\|10\,\text{k} = 8\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (8000)(40 \times 10^{-9}) = 0.32\,\text{ms}$$

$$v_o(t) = v_o(\infty) + (v_o(0) - v_o(\infty))e^{-t/\tau} = -80 + (50 + 80)e^{-3125t}$$

$$= -80 + 130e^{-3125t}\,\text{V}$$

[b] For $t \geq 0$:

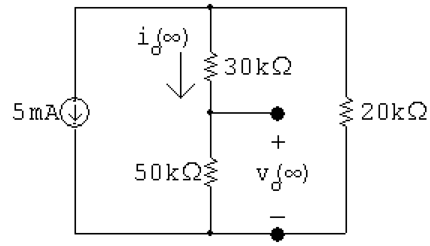


$$i_o = \frac{130e^{-3125t} - 80 + 100}{10{,}000} = 13e^{-3125t} + 2\,\text{mA}$$

P 7.55 $t < 0$:

$$i_o(0^-) = \frac{20}{100}(10 \times 10^{-3}) = 2\,\text{mA}; \qquad v_o(0^-) = (2 \times 10^{-3})(50{,}000) = 100\,\text{V}$$

$t = \infty$:

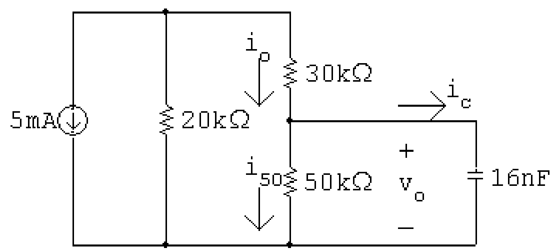


$$i_o(\infty) = -5 \times 10^{-3}\left(\frac{20}{100}\right) = -1\,\text{mA}; \qquad v_o(\infty) = i_o(\infty)(50{,}000) = -50\,\text{V}$$

$R_{\rm Th} = 50\,{\rm k}\Omega \| 50\,{\rm k}\Omega = 25\,{\rm k}\Omega; \qquad C = 16\,{\rm nF}$

$\tau = (25{,}000)(16 \times 10^{-9}) = 0.4\,{\rm ms}; \qquad \dfrac{1}{\tau} = 2500$

$\therefore \quad v_o(t) = -50 + 150e^{-2500t}\,{\rm V}, \qquad t \geq 0$

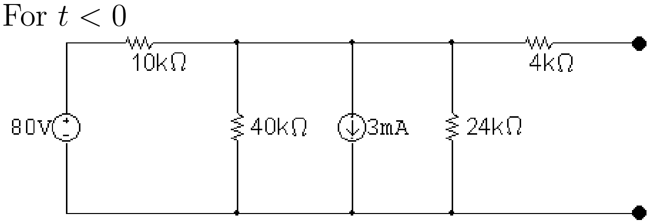


$i_c = C\dfrac{dv_o}{dt} = -6e^{-2500t}\,{\rm mA}, \qquad t \geq 0^+$

$i_{50k} = \dfrac{v_o}{50{,}000} = -1 + 3e^{-2500t}\,{\rm mA}, \qquad t \geq 0^+$

$i_o = i_c + i_{50k} = -(1 + 3e^{-2500t})\,{\rm mA}, \qquad t \geq 0^+$

P 7.56 For $t < 0$



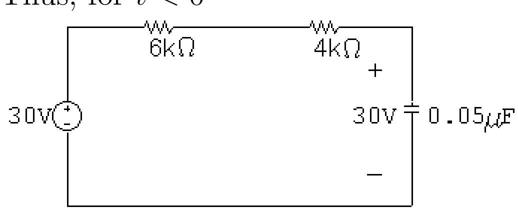
Simplify the circuit:

$80/10{,}000 = 8\,{\rm mA}, \qquad 10\,{\rm k}\Omega \| 40\,{\rm k}\Omega \| 24\,{\rm k}\Omega = 6\,{\rm k}\Omega$

$8\,{\rm mA} - 3\,{\rm mA} = 5\,{\rm mA}$

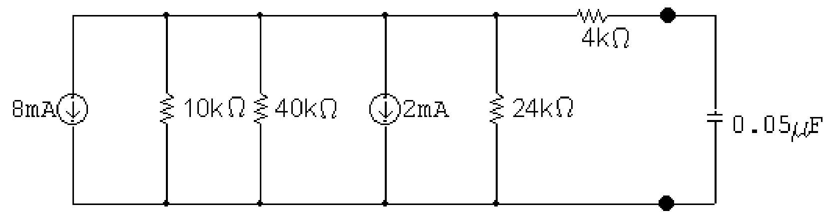
$5\,{\rm mA} \times 6\,{\rm k}\Omega = 30\,{\rm V}$

Thus, for $t < 0$



$\therefore \quad v_o(0^-) = v_o(0^+) = 30\,{\rm V}$

$t > 0$



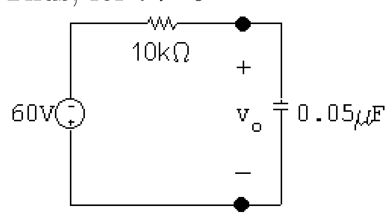
Simplify the circuit:

$8\,\text{mA} + 2\,\text{mA} = 10\,\text{mA}$

$10\,\text{k}\|40\,\text{k}\|24\,\text{k} = 6\,\text{k}\Omega$

$(10\,\text{mA})(6\,\text{k}\Omega) = 60\,\text{V}$

Thus, for $t > 0$



$v_o(\infty) = -10 \times 10^{-3}(6 \times 10^3) = -60\,\text{V}$

$\tau = RC = (10\,\text{k})(0.05\,\mu) = 0.5\,\text{ms}; \qquad \dfrac{1}{\tau} = 2000$

$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau} = -60 + [30 - (-60)]e^{-2000t}$

$\qquad = -60 + 90e^{-2000t}\,\text{V} \qquad t \geq 0$

P 7.57 Use voltage division to find the initial voltage:

$v_o(0) = \dfrac{60}{40 + 60}(50) = 30\,\text{V}$

Use Ohm's law to find the final value of voltage:

$v_o(\infty) = (-5\,\text{mA})(20\,\text{k}\Omega) = -100\,\text{V}$

$\tau = RC = (20 \times 10^3)(250 \times 10^{-9}) = 5\,\text{ms}; \qquad \dfrac{1}{\tau} = 200$

$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau}$

$\qquad = -100 + (30 + 100)e^{-200t} = -100 + 130e^{-200t}\,\text{V}, \qquad t \geq 0$

P 7.58 **[a]** $v = I_sR + (V_o - I_sR)e^{-t/RC} \qquad i = \left(I_s - \frac{V_o}{R}\right)e^{-t/RC}$

$\therefore \quad I_sR = 40, \qquad V_o - I_sR = -24$

$\therefore \quad V_o = 16\,\text{V}$

$I_s - \frac{V_o}{R} = 3 \times 10^{-3}; \qquad I_s - \frac{16}{R} = 3 \times 10^{-3}; \qquad R = \frac{40}{I_s}$

$\therefore \quad I_s - 0.4I_s = 3 \times 10^{-3}; \qquad I_s = 5\,\text{mA}$

$R = \frac{40}{5} \times 10^3 = 8\,\text{k}\Omega$

$\frac{1}{RC} = 2500; \quad C = \frac{1}{2500R} = \frac{10^{-3}}{20 \times 10^3} = 50\,\text{nF}; \quad \tau = RC = \frac{1}{2500} = 400\,\mu\text{s}$

[b] $v(\infty) = 40\,\text{V}$

$w(\infty) = \frac{1}{2}(50 \times 10^{-9})(1600) = 40\,\mu\text{J}$

$0.81w(\infty) = 32.4\,\mu\text{J}$

$v^2(t_o) = \frac{32.4 \times 10^{-6}}{25 \times 10^{-9}} = 1296; \qquad v(t_o) = 36\,\text{V}$

$40 - 24e^{-2500t_o} = 36; \qquad e^{2500t_o} = 6; \qquad \therefore \quad t_o = 716.70\,\mu\text{s}$

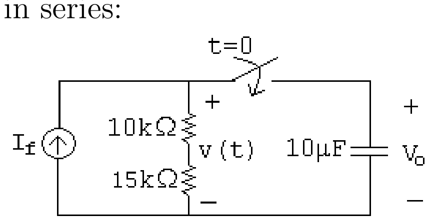
P 7.59 **[a]** Note that there are many different possible solutions to this problem.

$R = \frac{\tau}{C}$

Choose a $10\,\mu$H capacitor from Appendix H. Then,

$R = \frac{0.25}{10 \times 10^{-6}} = 25\,\text{k}\Omega$

Construct the resistance needed by combining $10\,\text{k}\Omega$ and $15\,\text{k}\Omega$ resistors in series:



[b] $v(t) = V_f + (V_o - V_f)e^{-t/\tau}$

$V_o = 100\,\text{V}; \qquad V_f = (I_f)(R) = (1 \times 10^{-3})(25 \times 10^3) = 25\,\text{V}$

$\therefore \quad v(t) = 25 + (100 - 25)e^{-4t}\,\text{V} = 25 + 75e^{-4t}\,\text{V}, \qquad t \geq 0$

[c] $v(t) = 25 + 75e^{-4t} = 50 \qquad \text{so} \qquad e^{-4t} = \frac{1}{3}$

$\therefore \quad t = \frac{\ln 3}{4} = 274.65\,\text{ms}$

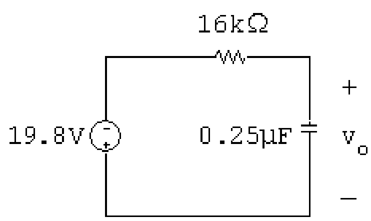
P 7.60 For $t > 0$

$V_{\text{Th}} = (-25)(16{,}000)i_{\text{b}} = -400 \times 10^3 i_{\text{b}}$

$i_{\text{b}} = \frac{33{,}000}{80{,}000}(120 \times 10^{-6}) = 49.5\,\mu\text{A}$

$V_{\text{Th}} = -400 \times 10^3(49.5 \times 10^{-6}) = -19.8\,\text{V}$

$R_{\text{Th}} = 16\,\text{k}\Omega$



$v_o(\infty) = -19.8\,\text{V}; \qquad v_o(0^+) = 0$

$\tau = (16{,}000)(0.25 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/\tau = 250$

$v_o = -19.8 + 19.8e^{-250t}\,\text{V}, \qquad t \geq 0$

$w(t) = \frac{1}{2}(0.25 \times 10^{-6})v_o^2 = w(\infty)(1 - e^{-250t})^2\,\text{J}$

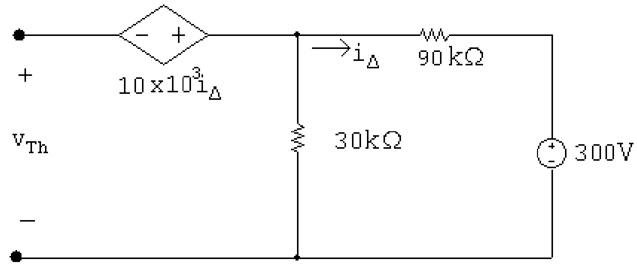
$(1 - e^{-250t})^2 = \frac{0.36w(\infty)}{w(\infty)} = 0.36$

$1 - e^{-250t} = 0.6$

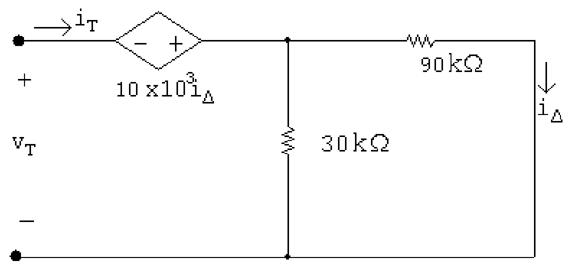
$e^{-250t} = 0.4 \quad \therefore \qquad t = 3.67\,\text{ms}$

P 7.61 For $t < 0$, $v_o(0) = 90\,\text{V}$

$t > 0$:



$$v_{\text{Th}} = -10\times10^3 i_\Delta + (30/120)(300) = -10\times10^3\left(\frac{-300}{120\times10^3}\right) + 75 = 100\,\text{V}$$

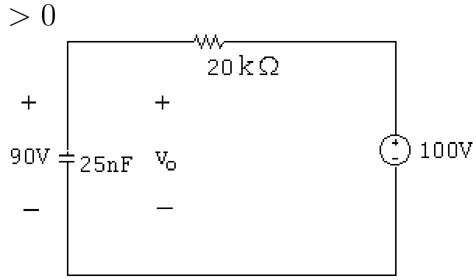


$$v_T = -10\times10^3 i_\Delta + 22.5\times10^3 i_T = -10\times10^3(30/120)i_T + 22.5\times10^3 i_T$$

$$= 20\times10^3 i_T$$

$$R_{\text{Th}} = \frac{v_T}{i_T} = 20\,\text{k}\Omega$$

$t > 0$



$$v_o = 100 + (90-100)e^{-t/\tau}$$

$$\tau = RC = (20\times10^3)(25\times10^{-9}) = 500\times10^{-6}; \qquad \frac{1}{\tau} = 2000$$

$$v_o = 100 - 10e^{-2000t}\,\text{V}, \quad t \ge 0$$

P 7.62 From Problem 7.61,

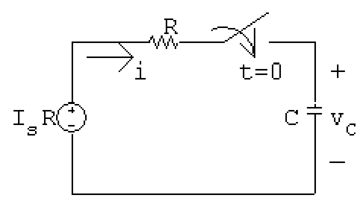
$$v_o(0) = 100\,\text{V}; \qquad v_o(\infty) = 90\,\text{V}$$

$$R_{\text{Th}} = 40\,\text{k}\Omega$$

$$\tau = (40)(25 \times 10^{-6}) = 10^{-3}; \qquad \frac{1}{\tau} = 1000$$

$$v = 90 + (100 - 90)e^{-1000t} = 90 + 10e^{-1000t}\,\text{V}, \quad t \geq 0$$

P 7.63 **[a]**



$$I_sR = Ri + \frac{1}{C}\int_{0^+}^{t} i\,dx + V_o$$

$$0 = R\frac{di}{dt} + \frac{i}{C} + 0$$

$$\therefore \quad \frac{di}{dt} + \frac{i}{RC} = 0$$

[b] $\frac{di}{dt} = -\frac{i}{RC}; \qquad \frac{di}{i} = -\frac{dt}{RC}$

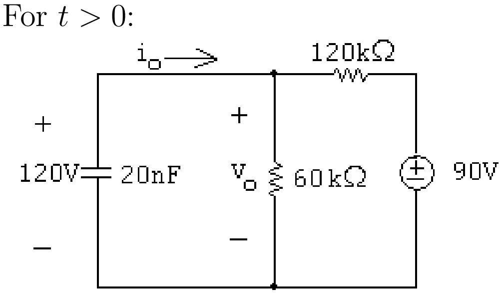
$$\int_{i(0^+)}^{i(t)} \frac{dy}{y} = -\frac{1}{RC}\int_{0^+}^{t} dx$$

$$\ln\frac{i(t)}{i(0^+)} = \frac{-t}{RC}$$

$$i(t) = i(0^+)e^{-t/RC}; \qquad i(0^+) = \frac{I_sR - V_o}{R} = \left(I_s - \frac{V_o}{R}\right)$$

$$\therefore \quad i(t) = \left(I_s - \frac{V_o}{R}\right)e^{-t/RC}$$

P 7.64 **[a]** For $t > 0$:



$$v(\infty) = \frac{60}{180}(90) = 30\,\text{V}$$

$$R_{\text{eq}} = 60\,\text{k}\|120\,\text{k} = 40\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (40\times10^3)(20\times10^{-9}) = 0.8\,\text{ms}; \qquad \frac{1}{\tau} = 1250$$

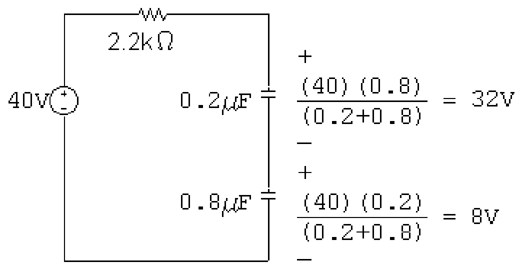
$$v_o = 30 + (120-30)e^{-1250t} = 30 + 90e^{-1250t}\,\text{V}, \qquad t \geq 0^+$$

[b] $$i_o = \frac{v_o}{60{,}000}\frac{v_o}{-}90120{,}000 = \frac{30+90e^{-1250t}}{60{,}000} + \frac{30+90e^{-1250t}-90}{120{,}000}$$

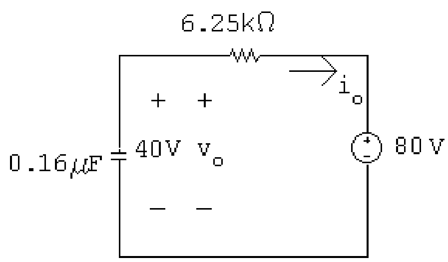
$$= 2.25e^{-1250t}\,\text{mA}$$

$$v_1 = \frac{1}{60\times10^{-9}} \times 2.25\times10^{-3}\int_0^t e^{-1250x}\,dx = -30e^{-1250t} + 30\,\text{V}, \quad t \geq 0$$

P 7.65 **[a]** $t < 0$



$t > 0$



$$v_o(0^-) = v_o(0^+) = 40\,\text{V}$$

$$v_o(\infty) = 80\,\text{V}$$

$$\tau = (0.16\times10^{-6})(6.25\times10^3) = 1\,\text{ms}; \qquad 1/\tau = 1000$$

$$v_o = 80 - 40e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[b] $$i_o = -C\frac{dv_o}{dt} = -0.16\times10^{-6}[40{,}000e^{-1000t}]$$

$$= -6.4e^{-1000t}\,\text{mA}; \qquad t \geq 0^+$$

[c] $$v_1 = \frac{-1}{0.2\times10^{-6}}\int_0^t -6.4\times10^{-3}e^{-1000x}\,dx + 32$$

$$= 64 - 32e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[d] $$v_2 = \frac{-1}{0.8 \times 10^{-6}} \int_0^t -6.4 \times 10^{-3} e^{-1000x}\, dx + 8$$

$$= 16 - 8e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[e] $$w_{\text{trapped}} = \frac{1}{2}(0.2 \times 10^{-6})(64)^2 + \frac{1}{2}(0.8 \times 10^{-6})(16)^2 = 512\,\mu\text{J}.$$

P 7.66 **[a]** Let i be the current in the clockwise direction around the circuit. Then

$$V_g = iR_g + \frac{1}{C_1}\int_0^t i\,dx + \frac{1}{C_2}\int_0^t i\,dx$$

$$= iR_g + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_0^t i\,dx = iR_g + \frac{1}{C_e}\int_0^t i\,dx$$

Now differentiate the equation

$$0 = R_g\frac{di}{dt} + \frac{i}{C_e} \quad \text{or} \quad \frac{di}{dt} + \frac{1}{R_gC_e}i = 0$$

Therefore $$i = \frac{V_g}{R_g}e^{-t/R_gC_e} = \frac{V_g}{R_g}e^{-t/\tau}; \qquad \tau = R_gC_e$$

$$v_1(t) = \frac{1}{C_1}\int_0^t \frac{V_g}{R_g}e^{-x/\tau}\,dx = \frac{V_g}{R_gC_1}\,\frac{e^{-x/\tau}}{-1/\tau}\Big|_0^t = -\frac{V_gC_e}{C_1}(e^{-t/\tau} - 1)$$

$$v_1(t) = \frac{V_gC_2}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_gC_e$$

$$v_2(t) = \frac{V_gC_1}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_gC_e$$

[b] $$v_1(\infty) = \frac{C_2}{C_1 + C_2}V_g; \qquad v_2(\infty) = \frac{C_1}{C_1 + C_2}V_g$$

P 7.67 **[a]** $$L_{\text{eq}} = \frac{(3)(15)}{3 + 15} = 2.5\,\text{H}$$

$$\tau = \frac{L_{\text{eq}}}{R} = \frac{2.5}{7.5} = \frac{1}{3}\,\text{s}$$

$$i_o(0) = 0; \qquad i_o(\infty) = \frac{120}{7.5} = 16\,\text{A}$$

$$\therefore \quad i_o = 16 - 16e^{-3t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 120 - 7.5i_o = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

$$i_1 = \frac{1}{3}\int_0^t 120e^{-3x}\,dx = \frac{40}{3} - \frac{40}{3}e^{-3t}\,\text{A}, \qquad t \geq 0$$

$$i_2 = i_o - i_1 = \frac{8}{3} - \frac{8}{3}e^{-3t}\,\text{A}, \qquad t \geq 0$$

[b] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with initial conditions.
$v_o(0^+) = 120$ V, consistent with $i_o(0) = 0$.

$$v_o = 3\frac{di_1}{dt} = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

or

$$v_o = 15\frac{di_2}{dt} = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

The voltage solution is consistent with the current solutions.

$$\lambda_1 = 3i_1 = 40 - 40e^{-3t}\text{ Wb-turns}$$

$$\lambda_2 = 15i_2 = 40 - 40e^{-3t}\text{ Wb-turns}$$

$\therefore\quad \lambda_1 = \lambda_2$ as it must, since

$$v_o = \frac{d\lambda_1}{dt} = \frac{d\lambda_2}{dt}$$

$$\lambda_1(\infty) = \lambda_2(\infty) = 40\text{ Wb-turns}$$

$$\lambda_1(\infty) = 3i_1(\infty) = 3(40/3) = 40\text{ Wb-turns}$$

$$\lambda_2(\infty) = 15i_2(\infty) = 15(8/3) = 40\text{ Wb-turns}$$

$\therefore\quad i_1(\infty)$ and $i_2(\infty)$ are consistent with $\lambda_1(\infty)$ and $\lambda_2(\infty)$.

P 7.68 **[a]** From Example 7.10,

$$L_{\text{eq}} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{0.125 - 0.0625}{0.75 + 0.5} = 50\,\text{mH}$$

$$\tau = \frac{L}{R} = \frac{1}{5000}; \qquad \frac{1}{\tau} = 5000$$

$$\therefore\quad i_o(t) = 40 - 40e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[b] $v_o = 10 - 250i_o = 10 - 250(0.04 + 0.04e^{-5000t} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+$

[c] $v_o = 0.5\dfrac{di_1}{dt} - 0.25\dfrac{di_2}{dt} = 10e^{-5000t}\,\text{V}$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = 200e^{-5000t}\text{ A/s}$$

$$\therefore\quad \frac{di_2}{dt} = 200e^{-5000t} - \frac{di_1}{dt}$$

$$\therefore\quad 10e^{-5000t} = 0.5\frac{di_1}{dt} - 50e^{-5000t} + 0.25\frac{di_1}{dt}$$

$$\therefore \quad 0.75\frac{di_1}{dt} = 60e^{-5000t}; \qquad di_1 = 80e^{-5000t}\,dt$$

$$\int_0^{t_1} dx = \int_0^t 80e^{-5000y}\,dy$$

$$i_1 = \frac{80}{-5000}e^{-5000y}\Big|_0^t = 16 - 16e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[d] $i_2 = i_o - i_1 = 40 - 40e^{-5000t} - 16 + 16e^{-5000t}$

$$= 24 - 24e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[e] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with zero initial stored energy.

$$v_o = L_{\text{eq}}\frac{di_o}{dt} = (0.05)(200)e^{-5000t} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

Also,

$$v_o = 0.5\frac{di_1}{dt} - 0.25\frac{di_2}{dt} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$$v_o = 0.25\frac{di_2}{dt} - 0.25\frac{di_1}{dt} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$v_o(0^+) = 10\,\text{V}$, which agrees with $i_o(0^+) = 0\,\text{A}$

$i_o(\infty) = 40\,\text{mA}; \qquad i_o(\infty)L_{\text{eq}} = (0.04)(0.05) = 2$ mWb-turns

$i_1(\infty)L_1 + i_2(\infty)M = (16\,\text{m})(500) + (24\,\text{m})(-250) = 2$ mWb-turns (ok)

$i_2(\infty)L_2 + i_1(\infty)M = (24\,\text{m})(250) + (16\,\text{m})(-250) = 2$ mWb-turns (ok)

Therefore, the final values of i_o, i_1, and i_2 are consistent with conservation of flux linkage. Hence, the answers make sense in terms of known circuit behavior.

P 7.69 **[a]** $L_{\text{eq}} = 0.02 + 0.04 + 2(0.015) = 0.09 = 90\,\text{mH}$

$$\tau = \frac{L}{R} = \frac{0.09}{4500} = 20\,\mu\text{s}; \qquad \frac{1}{\tau} = 50{,}000$$

$$i = 20 - 20e^{-50{,}000t}\,\text{mA}, \quad t \geq 0$$

[b] $v_1(t) = 0.02\frac{di}{dt} + 0.015\frac{di}{dt} = 0.035\frac{di}{dt} = 0.035(1000e^{-50{,}000t}) = 35e^{-50{,}000t}\,\text{V}, \quad t \geq 0^+$

[c] $v_2(t) = 0.04\frac{di}{dt} + 0.015\frac{di}{dt} = 0.055\frac{di}{dt} = 0.055(1000e^{-50{,}000t}) = 55e^{-50{,}000t}\,\text{V}, \quad t \geq 0^+$

[d] $i(0) = 0.02 - 0.02 = 0$, which agrees with initial conditions.

$$90 = 4500i + v_1 + v_2 = 4500(0.02 - 0.02e^{-50{,}000t}) + 35e^{-50{,}000t} + 55e^{-50{,}000t} = 90\,\text{V}$$

Therefore, Kirchhoff's voltage law is satisfied for all values of $t \geq 0$. Thus, the answers make sense in terms of known circuit behavior.

P 7.70 **[a]** $L_{\text{eq}} = 0.02 + 0.04 - 2(0.015) = 0.03 = 30\,\text{mH}$

$$\tau = \frac{L}{R} = \frac{0.03}{4500} = 6.67\,\mu\text{s}; \qquad \frac{1}{\tau} = 150{,}000$$

$$i = 0.02 - 0.02e^{-150{,}000t}\,\text{A}, \quad t \geq 0$$

[b] $v_1(t) = 0.02\frac{di}{dt} - 0.015\frac{di}{dt} = 0.005\frac{di}{dt} = 0.005(3000e^{-150{,}000t})$

$$= 15e^{-150{,}000t}\,\text{V}, \quad t \geq 0^+$$

[c] $v_2(t) = 0.04\frac{di}{dt} - 0.015\frac{di}{dt} = 0.025\frac{di}{dt} = 0.025(3000e^{-150{,}000t})$

$$= 75e^{-150{,}000t}\,\text{V}, \quad t \geq 0^+$$

[d] $i(0) = 0,$ which agrees with initial conditions.

$$90 = 4500i_1 + v_1 + v_2 = 4500(0.02 - 0.02e^{-150{,}000t}) + 15e^{-150{,}000t}$$

$$+ 75e^{-150{,}000t} = 90\,\text{V}$$

Therefore, Kirchhoff's voltage law is satisfied for all values of $t \geq 0$. Thus, the answers make sense in terms of known circuit behavior.

P 7.71 **[a]** From Example 7.10,

$$L_{\text{eq}} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{50 - 25}{15 + 10} = 1\,\text{H}$$

$$\tau = \frac{L}{R} = \frac{1}{20}; \qquad \frac{1}{\tau} = 20$$

$$\therefore \quad i_o(t) = 4 - 4e^{-20t}\,\text{A}, \qquad t \geq 0$$

[b] $v_o = 80 - 20i_o = 80 - 80 + 80e^{-20t} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+$

[c] $v_o = 5\frac{di_1}{dt} - 5\frac{di_2}{dt} = 80e^{-20t}\,\text{V}$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = 80e^{-20t}\ \text{A/s}$$

$$\therefore \quad \frac{di_2}{dt} = 80e^{-20t} - \frac{di_1}{dt}$$

$$\therefore \quad 80e^{-20t} = 5\frac{di_1}{dt} - 400e^{-20t} + 5\frac{di_1}{dt}$$

$$\therefore \quad 10\frac{di_1}{dt} = 480e^{-20t}; \qquad di_1 = 48e^{-20t}\,dt$$

$$\int_0^{t_1} dx = \int_0^t 48e^{-20y}\, dy$$

$$i_1 = \frac{48}{-20}e^{-20y}\Big|_0^t = 2.4 - 2.4e^{-20t}\,\text{A}, \qquad t \geq 0$$

[d] $i_2 = i_o - i_1 = 4 - 4e^{-20t} - 2.4 + 2.4e^{-20t}$

$$= 1.6 - 1.6e^{-20t}\,\text{A}, \qquad t \geq 0$$

[e] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with zero initial stored energy.

$$v_o = L_{\text{eq}}\frac{di_o}{dt} = 1(80)e^{-20t} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

Also,

$$v_o = 5\frac{di_1}{dt} - 5\frac{di_2}{dt} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$$v_o = 10\frac{di_2}{dt} - 5\frac{di_1}{dt} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$v_o(0^+) = 80\,\text{V}$, which agrees with $i_o(0^+) = 0\,\text{A}$

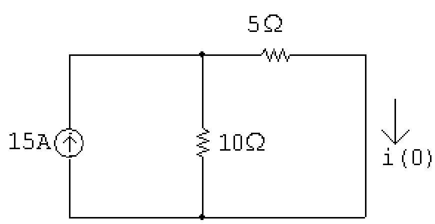
$i_o(\infty) = 4\,\text{A}; \qquad i_o(\infty)L_{\text{eq}} = (4)(1) = 4$ Wb-turns

$i_1(\infty)L_1 + i_2(\infty)M = (2.4)(5) + (1.6)(-5) = 4$ Wb-turns (ok)

$i_2(\infty)L_2 + i_1(\infty)M = (1.6)(10) + (2.4)(-5) = 4$ Wb-turns (ok)

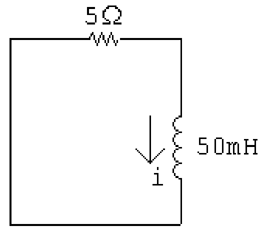
Therefore, the final values of i_o, i_1, and i_2 are consistent with conservation of flux linkage. Hence, the answers make sense in terms of known circuit behavior.

P 7.72 For $t < 0$:



$$i(0) = \frac{10}{15}(15) = 10\,\text{A}$$

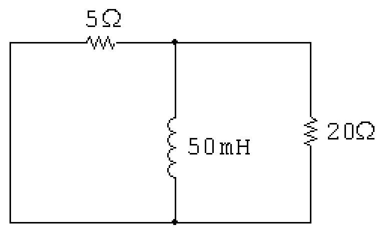
$0 \le t \le 10\,\text{ms}$:



$$i = 10e^{-100t}\,\text{A}$$

$$i(10\,\text{ms}) = 10e^{-1} = 3.68\,\text{A}$$

$10\,\text{ms} \le t \le 20\,\text{ms}$:



$$R_{\text{eq}} = \frac{(5)(20)}{25} = 4\,\Omega$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{4}{50 \times 10^{-3}} = 80$$

$$i = 3.68e^{-80(t-0.01)}\,\text{A}$$

$20\,\text{ms} \le t < \infty$:

$$i(20\,\text{ms}) = 3.68e^{-80(0.02-0.01)} = 1.65\,\text{A}$$

$$i = 1.65e^{-100(t-0.02)}\,\text{A}$$

$$v_o = L\frac{di}{dt}; \qquad L = 50\,\text{mH}$$

$$\frac{di}{dt} = 1.65(-100)e^{-100(t-0.02)} = -165e^{-100(t-0.02)}$$

$$v_o = (50 \times 10^{-3})(-165)e^{-100(t-0.02)}$$

$$= -8.26e^{-100(t-0.02)}\,\text{V}, \qquad t > 20^{+}\,\text{ms}$$

$$v_o(25\,\text{ms}) = -8.26e^{-100(0.025-0.02)} = -5.013\,\text{V}$$

P 7.73 From the solution to Problem 7.72, the initial energy is

$$w(0) = \frac{1}{2}(50\,\text{mH})(10\,\text{A})^2 = 2.5\,\text{J}$$

$$0.04w(0) = 0.1\,\text{J}$$

$$\therefore \quad \frac{1}{2}(50 \times 10^{-3})i_L^2 = 0.1 \quad \text{so} \quad i_L = 2\,\text{A}$$

Again, from the solution to Problem 7.73, t must be between 10 ms and 20 ms since

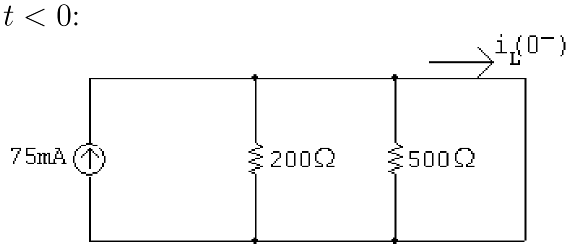
$$i(10\,\text{ms}) = 3.68\,\text{A} \quad \text{and} \quad i(20\,\text{ms}) = 1.65\,\text{A}$$

For $10\,\text{ms} \leq t \leq 20\,\text{ms}$:

$$i = 3.68e^{-80(t-0.01)} = 2$$

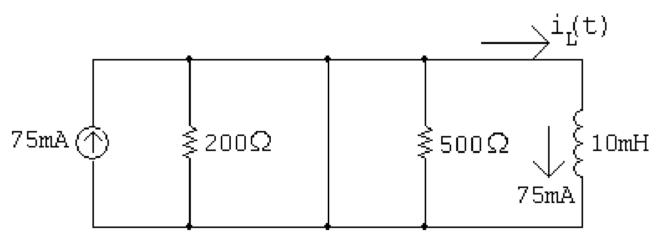
$$e^{80(t-0.01)} = \frac{3.68}{2} \quad \text{so} \quad t - 0.01 = 0.0076 \quad \therefore \quad t = 17.6\,\text{ms}$$

P 7.74 $t < 0$:



$$i_L(0^-) = 75\,\text{mA} = i_L(0^+)$$

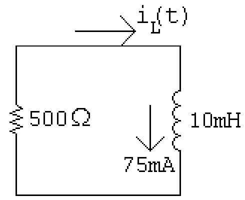
$0 \leq t \leq 25\,\text{ms}$:



$$\tau = 0.01/0 = \infty$$

$$i_L(t) = 0.075e^{-t/\infty} = 0.075e^{-0} = 75\,\text{mA}$$

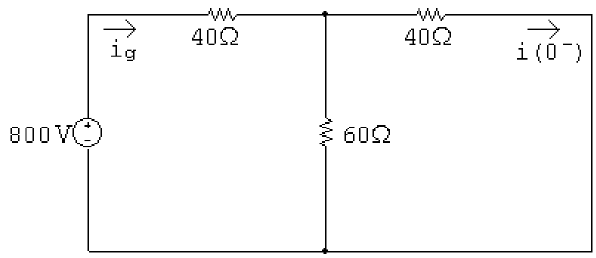
$25\,\text{ms} \leq t$:



$$\tau = \frac{0.01}{500} = 20\,\mu\text{s}; \qquad 1/\tau = 50{,}000$$

$$i_L(t) = 75e^{-50{,}000(t-0.025)}\,\text{mA}, \quad t \geq 25\,\text{ms}$$

P 7.75 **[a]** $t < 0$:



Using Ohm's law,

$$i_g = \frac{800}{40 + 60\|40} = 12.5\,\text{A}$$

Using current division,

$$i(0^-) = \frac{60}{60+40}(12.5) = 7.5\,\text{A} = i(0^+)$$

[b] $0 \leq t \leq 1\,\text{ms}$:

$$i = i(0^+)e^{-t/\tau} = 7.5e^{-t/\tau}$$

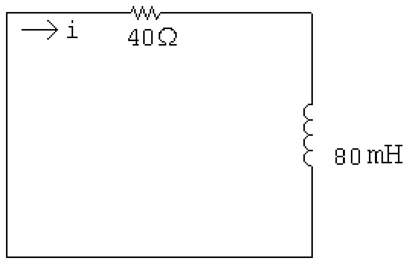
$$\frac{1}{\tau} = \frac{R}{L} = \frac{40 + 120\|60}{80 \times 10^{-3}} = 1000$$

$$i = 7.5e^{-1000t}$$

$$i(200\mu\text{s}) = 7.5e^{-10^3(200\times10^{-6})} = 7.5e^{-0.2} = 6.14\,\text{A}$$

[c] $i(1\,\text{ms}) = 7.5e^{-1} = 2.7591\,\text{A}$

$1\,\text{ms} \le t$:



$$\frac{1}{\tau} = \frac{R}{L} = \frac{40}{80 \times 10^{-3}} = 500$$

$$i = i(1\,\text{ms})e^{-(t-1\,ms)/\tau} = 2.7591e^{-500(t-0.001)}\,\text{A}$$

$$i(6\text{ms}) = 2.7591e^{-500(0.005)} = 2.7591e^{-2.5} = 226.48\,\text{mA}$$

[d] $0 \le t \le 1\,\text{ms}$:

$$i = 7.5e^{-1000t}$$

$$v = L\frac{di}{dt} = (80 \times 10^{-3})(-1000)(7.5e^{-1000t}) = -600e^{-1000t}\,\text{V}$$

$$v(1^{-}\text{ms}) = -600e^{-1} = -220.73\,\text{V}$$

[e] $1\,\text{ms} \le t \le \infty$:

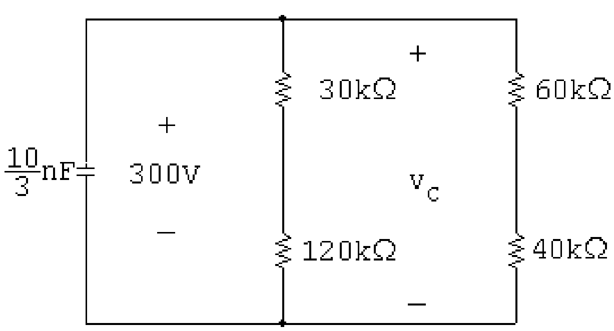
$$i = 2.759e^{-500(t-0.001)}$$

$$v = L\frac{di}{dt} = (80 \times 10^{-3})(-500)(2.759e^{-500(t-0.001)})$$

$$= -110.4e^{-500(t-0.001)}\,\text{V}$$

$$v(1^{+}\text{ms}) = -110.4\,\text{V}$$

P 7.76 $0 \le t \le 200\,\mu\text{s}$;

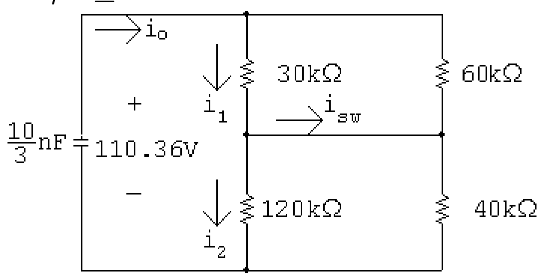


$$R_e = 150\|100 = 60\,\text{k}\Omega; \qquad \tau = \left(\frac{10}{3} \times 10^{-9}\right)(60{,}000) = 200\,\mu\text{s}$$

$v_c = 300e^{-5000t}\,\text{V}$

$v_c(200\,\mu\text{s}) = 300e^{-1} = 110.36\,\text{V}$

$200\,\mu\text{s} \le t < \infty$:



$R_e = 30\|60 + 120\|40 = 20 + 30 = 50\,\text{k}\Omega$

$\tau = \left(\frac{10}{3} \times 10^{-9}\right)(50{,}000) = 166.67\,\mu\text{s}; \qquad \frac{1}{\tau} = 6000$

$v_c = 110.36e^{-6000(t-200\,\mu s)}\,\text{V}$

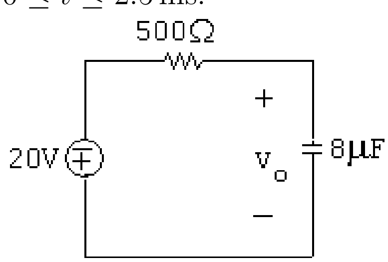
$v_c(300\,\mu\text{s}) = 110.36e^{-6000(100\,\mu\text{S})} = 60.57\,\text{V}$

$i_o(300\,\mu\text{s}) = \frac{60.57}{50{,}000} = 1.21\,\text{mA}$

$i_1 = \frac{60}{90}i_o = \frac{2}{3}i_o; \qquad i_2 = \frac{40}{160}i_o = \frac{1}{4}i_o$

$i_{\text{sw}} = i_1 - i_2 = \frac{2}{3}i_o - \frac{1}{4}i_o = \frac{5}{12}i_o = \frac{5}{12}(1.21 \times 10^{-3}) = 0.50\,\text{mA}$

P 7.77 $\quad 0 \le t \le 2.5\,\text{ms}$:

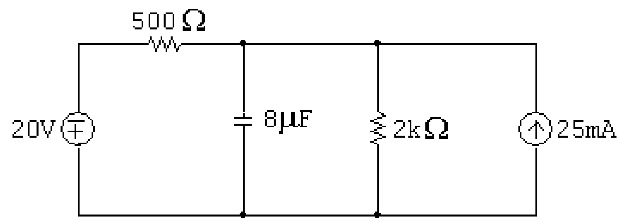


$\tau = RC = (500)(8 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/\tau = 250$

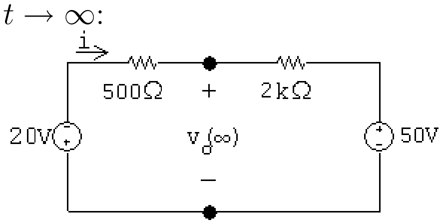
$v_o(0) = 0\,\text{V}; \qquad v_o(\infty) = -20\,\text{V}$

$v_o = -20 + 20e^{-250t}\,\text{V} \qquad 0 \le t \le 2.5\,\text{ms}$

$2.5\,\text{ms} \le t$:



$t \to \infty$:



$$i = \frac{-70\,\text{V}}{2.5\,\text{k}\Omega} = -28\,\text{mA}$$

$$v_o(\infty) = (-28 \times 10^{-3})(2000) + 50 = -6\,\text{V}$$

$$v_o(0.0025) = -20 + 20e^{-0.625} = -9.29\,\text{V}$$

$$v_o = -6 + (-9.29 + 6)e^{-(t-0.0025)/\tau}$$

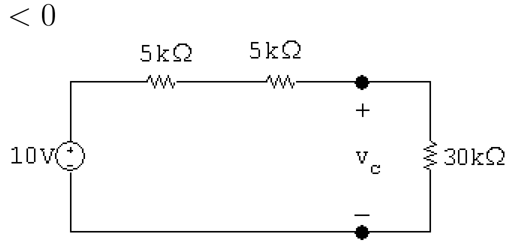
$$R_{\text{Th}} = 2000\|500 = 400\,\Omega$$

$$\tau = (400)(8 \times 10^{-6}) = 3.2\,\text{ms}; \qquad 1/\tau = 312.5$$

$$v_o = -6 - 3.29e^{-312.5(t-0.0025)} \qquad 2.5\,\text{ms} \le t$$

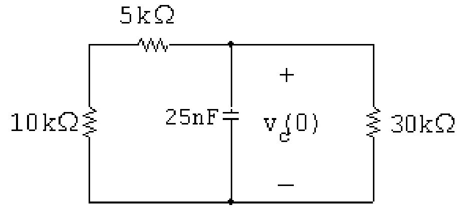
P 7.78 Note that for $t > 0$, $v_o = (10/15)v_c$, where v_c is the voltage across the 25 nF capacitor. Thus we will find v_c first.

$t < 0$



$$v_c(0) = \frac{30}{40}(10) = 7.5\,\text{V}$$

$0 \leq t \leq 0.2\,\text{ms}$:



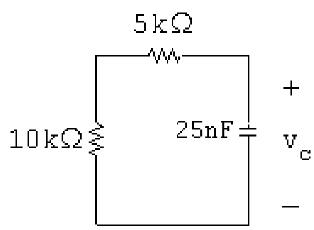
$\tau = R_eC, \qquad R_e = 15{,}000\|30{,}000 = 10\,\text{k}\Omega$

$\tau = (10 \times 10^3)(25 \times 10^{-9}) = 0.25\,\text{ms}, \qquad \dfrac{1}{\tau} = 4000$

$v_c = 7.5e^{-4000t}\,\text{V}, \qquad t \geq 0$

$v_c(0.2\,\text{ms}) = 7.5e^{-0.8} = 3.37\,\text{V}$

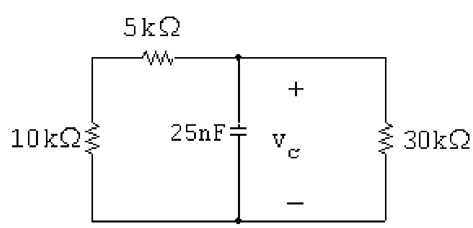
$0.2\,\text{ms} \leq t \leq 0.8\,\text{ms}$:



$\tau = (15 \times 10^3)(2.5 \times 10^{-9}) = 375\,\mu\text{s}, \qquad \dfrac{1}{\tau} = 2666.67$

$v_c = 3.37e^{-2666.67(t-200\times10^{-6})}\,\text{V}$

$0.8\,\text{ms} \leq t <$:



$\tau = 0.25\,\text{ms}, \qquad \dfrac{1}{\tau} = 4000$

$v_c(0.8\,\text{ms}) = 3.37e^{-2666.67(800-200)10^{-6}} = 3.37e^{-1.6} = 0.68\,\text{V}$

$v_c = 0.68e^{-4000(t-0.8\times10^{-3})}\,\text{V}$

$v_c(1\,\text{ms}) = 0.68e^{-4000(1-0.8)10^{-3}} = 0.68e^{-0.8} = 0.306\,\text{V}$

$v_o = (10/15)(0.306) = 0.204\,\text{V}$

P 7.79 $w(0) = \frac{1}{2}(25 \times 10^{-9})(7.5)^2 = 703.125\,\text{nJ}$

$0 \le t \le 200\,\mu\text{s}$:

$v_c = 7.5e^{-4000t}; \qquad v_c^2 = 56.25e^{-8000t}$

$p_{30k} = 1.875e^{-8000t}\,\text{mW}$

$$w_{30k} = \int_0^{200\times10^{-6}} 1.875 \times 10^{-3} e^{-8000t}\,dt$$
$$= 1.875 \times 10^{-3} \frac{e^{-8000t}}{-8000}\Big|_0^{200\times10^{-6}}$$
$$= -234.375 \times 10^{-9}(e^{-1.6} - 1) = 187.1\,\text{nJ}$$

$0.8\,\text{ms} \le t$:

$v_c = 0.68e^{-4000(t-0.8\times10^{-3})}\,\text{V}; \qquad v_c^2 = 0.46e^{-8000(t-0.8\times10^{-3})}$

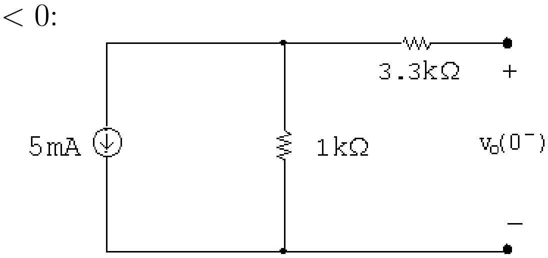
$p_{30k} = 15.33e^{-8000(t-0.8\times10^{-3})}\,\mu\text{W}$

$$w_{30k} = \int_{0.8\times10^{-3}}^{\infty} 15.33 \times 10^{-6} e^{-8000(t-0.8\times10^{-3})}\,dt$$
$$= 15.33 \times 10^{-6} \frac{e^{-8000(t-0.8\times10^{-3})}}{-8000}\Big|_{0.8\times10^{-3}}^{\infty}$$
$$= -1.9 \times 10^{-9}(0 - 1) = 1.9\,\text{nJ}$$

$w_{30k} = 187.1 + 1.9 = 189\,\text{nJ}$

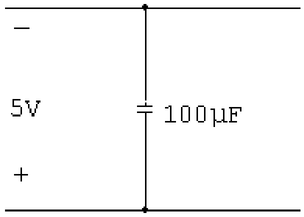
$\% = \frac{189}{703.125}(100) = 26.88\%$

P 7.80 $t < 0$:



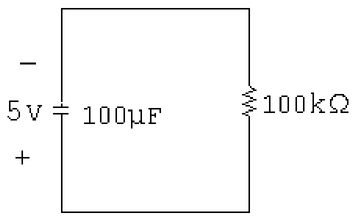
$v_c(0^-) = -(5)(1000) \times 10^{-3} = -5\,\text{V} = v_c(0^+)$

$0 \le t \le 5\,\text{s}$:



$\tau = \infty; \qquad 1/\tau = 0; \qquad v_o = -5e^{-0} = -5\,\text{V}$

$5\,\text{s} \le t < \infty$:



$\tau = (100)(0.1) = 10\,\text{s}; \qquad 1/\tau = 0.1; \qquad v_o = -5e^{-0.1(t-5)}\,\text{V}$

Summary:

$v_o = -5\,\text{V}, \qquad 0 \le t \le 5\,\text{s}$

$v_o = -5e^{-0.1(t-5)}\,\text{V}, \qquad 5\,\text{s} \le t < \infty$

P 7.81 **[a]** $i_o(0) = 0; \qquad i_o(\infty) = 50\,\text{mA}$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{3000}{75} \times 10^3 = 40{,}000$$

$$i_o = (50 - 50e^{-40{,}000t})\,\text{mA}, \qquad 0 \le t \le 25\,\mu\text{s}$$

$$v_o = 0.075\frac{di_o}{dt} = 150e^{-40{,}000t}\,\text{V}, \qquad 0 \le t \le 25\,\mu s$$

$25\,\mu\text{s} \le t$:

$$i_o(25\mu\text{s}) = 50 - 50e^{-1} = 31.6\,\text{mA}; \qquad i_o(\infty) = 0$$

$$i_o = 31.6e^{-40{,}000(t-25\times10^{-6})}\,\text{mA}$$

$$v_o = 0.075\frac{di_o}{dt} = -94.82e^{-40{,}000(t-25\mu s)}$$

$$\begin{aligned} \therefore\quad t < 0: &\qquad v_o = 0 \\ 0 \le t \le 25\,\mu\text{s}: &\qquad v_o = 150e^{-40{,}000t}\,\text{V} \\ 25\,\mu\text{s} \le t: &\qquad v_o = -94.82e^{-40{,}000(t-25\mu s)}\,\text{V} \end{aligned}$$

[b] $v_o(25^-\mu\text{s}) = 150e^{-1} = 55.18\,\text{V}$

$v_o(25^+\mu\text{s}) = -94.82\,\text{V}$

[c] $i_o(25^-\mu\text{s}) = i_o(25^+\mu\text{s}) = 31.6\,\text{mA}$

P 7.82 **[a]** $0 \leq t \leq 2.5\,\text{ms}$

$v_o(0^+) = 80\,\text{V}; \qquad v_o(\infty) = 0$

$\tau = \dfrac{L}{R} = 2\,\text{ms}; \qquad 1/\tau = 500$

$v_o(t) = 80e^{-500t}\,\text{V}, \qquad 0^+ \leq t \leq 2.5^-\,\text{ms}$

$v_o(2.5^-\,\text{ms}) = 80e^{-1.25} = 22.92\,\text{V}$

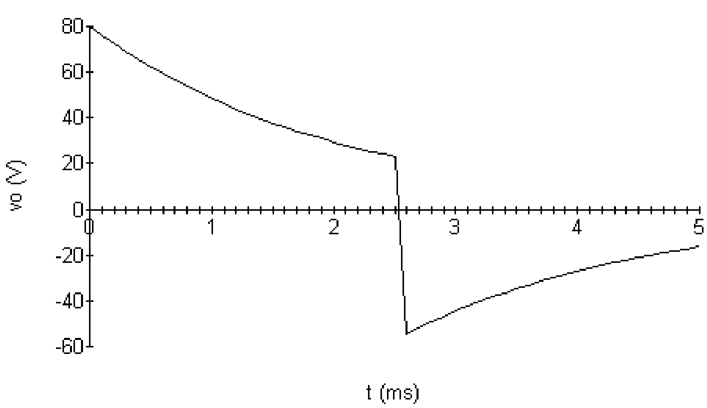
$i_o(2.5^-\,\text{ms}) = \dfrac{(80 - 22.92)}{20} = 2.85\,\text{A}$

$v_o(2.5^+\,\text{ms}) = -20(2.85) = -57.08\,\text{V}$

$v_o(\infty) = 0; \qquad \tau = 2\,\text{ms}; \qquad 1/\tau = 500$

$v_o = -57.08e^{-500(t-0.0025)}\,\text{V} \qquad t \geq 2.5^+\,\text{ms}$

[b]



[c] $v_o(5\,\text{ms}) = -16.35\,\text{V}$

$i_o = \dfrac{+16.35}{20} = 817.68\,\text{mA}$

P 7.83 **[a]** $t < 0; \qquad v_o = 0$

$0 \leq t \leq 25\,\mu\text{s}$:

$\tau = (4000)(50 \times 10^{-9}) = 0.2\,\text{ms}; \qquad 1/\tau = 5000$

$v_o = 10 - 10e^{-5000t}\,\text{V}, \qquad 0 \leq t \leq 25\,\mu\text{s}$

$v_o(25\,\mu\text{s}) = 10(1 - e^{-0.125}) = 1.175\,\text{V}$

$25\,\mu\text{s} \le t \le 50\,\mu\text{s}$:

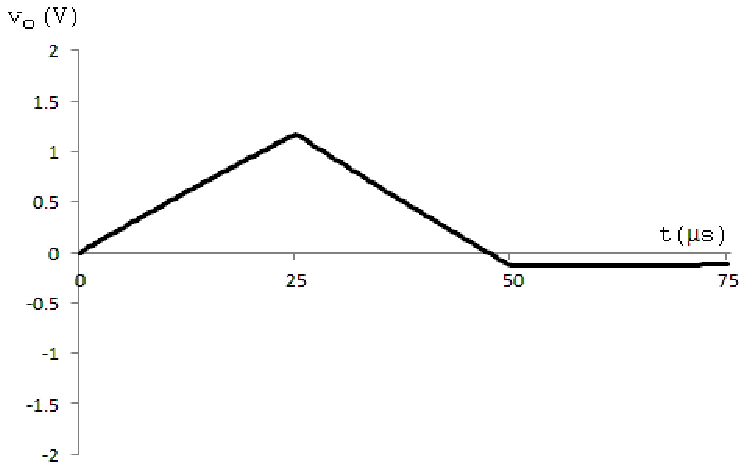
$v_o = -10 + 11.175e^{-5000(t-25\times10^{-6})}\,\text{V}, \quad 25\,\mu\text{s} \le t \le 50\,\mu\text{s}$

$v_o(50\,\mu\text{s}) = -10 + 11.175e^{-0.125} = -0.138\,\text{V}$

$t \ge 50\,\mu\text{s}$:

$v_o = -0.138e^{-5000(t-50\times10^{-6})}\,\text{V}, \quad t \ge 50\,\mu\text{s}$

[**b**]



[**c**] $t \le 0$: $v_o = 0$

$0 \le t \le 25\,\mu\text{s}$:

$\tau = (800)(50 \times 10^{-9}) = 40\,\mu\text{s} \quad 1/\tau = 25{,}000$

$v_o = 10 - 10e^{-25{,}000t}\,\text{V}, \qquad 0 \le t \le 25\,\mu\text{s}$

$v_o(25\,\mu\text{s}) = 10 - 10e^{-0.625} = 4.65\,\text{V}$

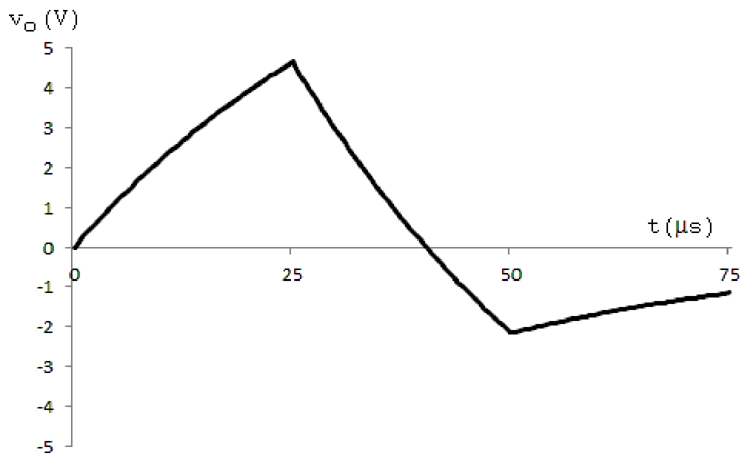
$25\,\mu\text{s} \le t \le 50\,\mu\text{s}$:

$v_o = -10 + 14.65e^{-25{,}000(t-25\times10^{-6})}\,\text{V}, \qquad 25\,\mu\text{s} \le t \le 50\,\mu\text{s}$

$v_o(50\,\mu\text{s}) = -10 + 14.65e^{-0.625} = -2.16\,\text{V}$

$t \ge 50\,\mu\text{s}$:

$v_o = -2.16e^{-25{,}000(t-50\times10^{-6})}\,\text{V}, \quad t \ge 50\,\mu\text{s}$



P 7.84 **[a]** $0 \le t \le 1\,\text{ms}$:

$$v_c(0^+) = 0; \qquad v_c(\infty) = 50\,\text{V};$$

$$RC = 400 \times 10^3(0.01 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/RC = 250$$

$$v_c = 50 - 50e^{-250t}$$

$$v_o = 50 - 50 + 50e^{-250t} = 50e^{-250t}\,\text{V}, \qquad 0 \le t \le 1\,\text{ms}$$

$1\,\text{ms} \le t < \infty$:

$$v_c(1\,\text{ms}) = 50 - 50e^{-0.25} = 11.06\,\text{V}$$

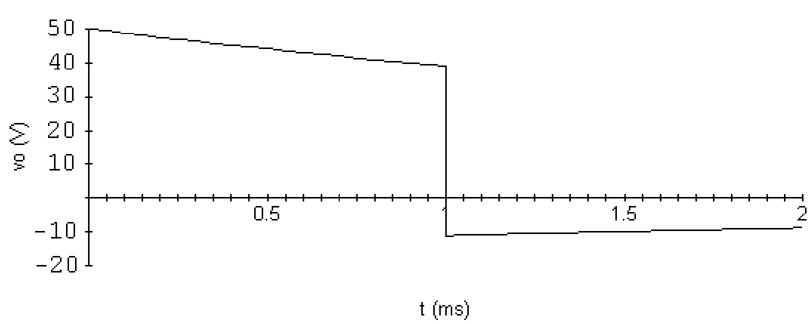
$$v_c(\infty) = 0\,\text{V}$$

$$\tau = 4\,\text{ms}; \qquad 1/\tau = 250$$

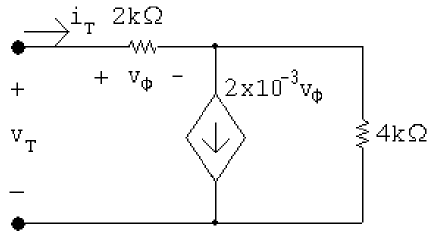
$$v_c = 11.06e^{-250(t\,-\,0.001)}\,\text{V}$$

$$v_o = -v_c = -11.06e^{-250(t\,-\,0.001)}\,\text{V}, \qquad t \ge 1\,\text{ms}$$

[b]



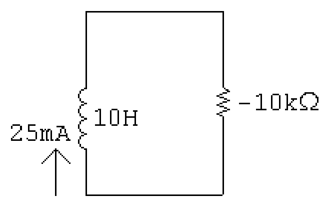
P 7.85



$$v_T = 2000i_T + 4000(i_T - 2\times10^{-3}v_\phi) = 6000i_T - 8v_\phi$$

$$= 6000i_T - 8(2000i_T)$$

$$\frac{v_T}{i_T} = -10{,}000$$

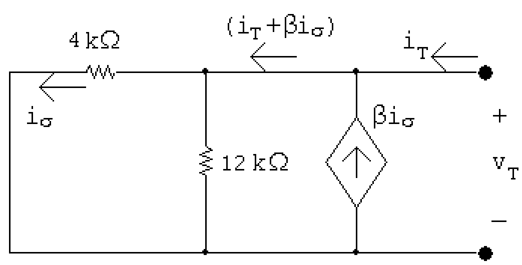


$$\tau = \frac{10}{-10{,}000} = -1\,\text{ms}; \qquad 1/\tau = -1000$$

$$i = 25e^{1000t}\,\text{mA}$$

$$\therefore \quad 25e^{1000t}\times10^{-3} = 5; \qquad t = \frac{\ln 200}{1000} = 5.3\,\text{ms}$$

P 7.86 **[a]**



Using Ohm's law,

$$v_T = 4000i_\sigma$$

Using current division,

$$i_\sigma = \frac{12{,}000}{12{,}000+4000}(i_T+\beta i_\sigma) = 0.75i_T + 0.75\beta i_\sigma$$

Solve for i_σ:

$i_\sigma(1 - 0.75\beta) = 0.75 i_T$

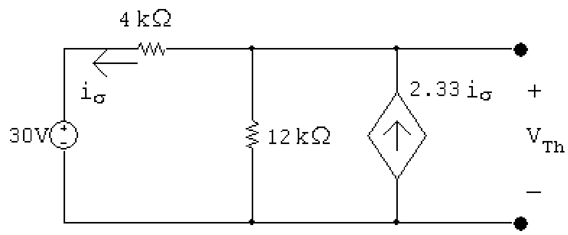
$$i_\sigma = \frac{0.75 i_T}{1 - 0.75\beta}; \qquad v_T = 4000 i_\sigma = \frac{3000 i_T}{(1 - 0.75\beta)}$$

Find β such that $R_{\text{Th}} = -4\,\text{k}\Omega$:

$$R_{\text{Th}} = \frac{v_T}{i_T} = \frac{3000}{1 - 0.75\beta} = -4000$$

$1 - 0.75\beta = -0.75 \qquad \therefore \quad \beta = 2.33$

[b] Find V_{Th};



Write a KCL equation at the top node:

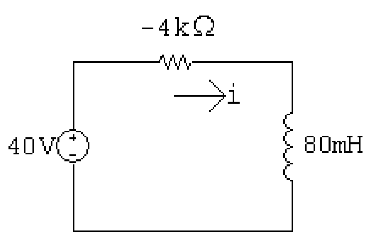
$$\frac{V_{\text{Th}} - 30}{4000} + \frac{V_{\text{Th}}}{12{,}000} - 2.33 i_\sigma = 0$$

The constraint equation is:

$$i_\sigma = \frac{(V_{\text{Th}} - 30)}{4000}$$

Solving,

$V_{\text{Th}} = 40\,\text{V}$



Write a KVL equation around the loop:

$$40 = -4000i + 0.08\frac{di}{dt}$$

Rearranging:

$$\frac{di}{dt} = 500 + 50{,}000i = 25{,}000(i + 0.01)$$

Separate the variables and integrate to find i;

$$\frac{di}{i + 0.01} = 50{,}000\,dt$$

$$\int_0^i \frac{dx}{x+0.01} = \int_0^t 50{,}000\,dx$$

$\therefore \quad i = -10 + 10e^{50{,}000t}\,\text{mA}$

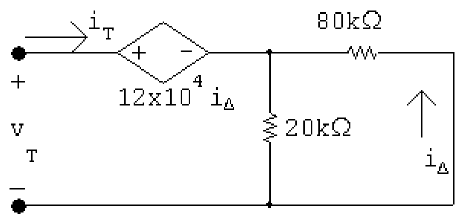
$$\frac{di}{dt} = (10\times 10^{-3})(50{,}000)e^{50{,}000t} = 500e^{50{,}000t}$$

Solve for the arc time:

$$v = 0.08\frac{di}{dt} = 40e^{50{,}000t} = 30{,}000; \qquad e^{50{,}000t} = 750$$

$$\therefore \quad t = \frac{\ln 750}{50{,}000} = 132.4\,\mu\text{s}$$

P 7.87 $t > 0$:



$$v_T = 12\times 10^4 i_\Delta + 16\times 10^3 i_T$$

$$i_\Delta = -\frac{20}{100}i_T = -0.2i_T$$

$$\therefore \quad v_T = -24\times 10^3 i_T + 16\times 10^3 i_T$$

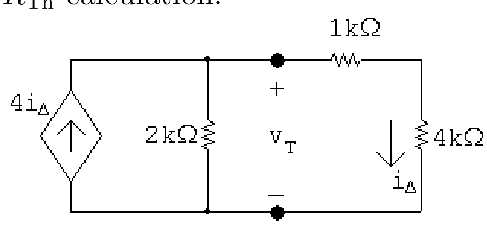
$$R_{\text{Th}} = \frac{v_T}{i_T} = -8\,\text{k}\Omega$$

$$\tau = RC = (-8\times 10^3)(2.5\times 10^{-6}) = -0.02 \quad 1/\tau = -50$$

$$v_c = 20e^{50t}\,\text{V}; \qquad 20e^{50t} = 20{,}000$$

$$50t = \ln 1000 \quad \therefore \quad t = 138.16\,\text{ms}$$

P 7.88 Find the Thévenin equivalent with respect to the terminals of the capacitor. R_{Th} calculation:

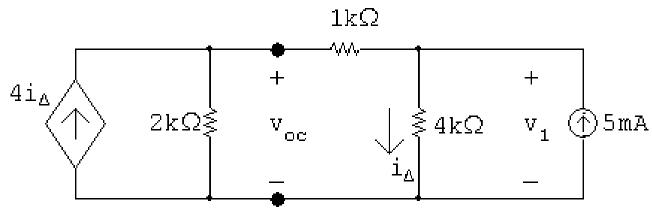


$$i_T = \frac{v_T}{2000} + \frac{v_T}{5000} - 4\frac{v_T}{5000}$$

$$\therefore \quad \frac{i_T}{v_T} = \frac{5+2-8}{10{,}000} = -\frac{1}{10{,}000}$$

$$\frac{v_T}{i_T} = -\frac{10{,}000}{1} = -10\,\text{k}\Omega$$

Open circuit voltage calculation:



The node voltage equations:

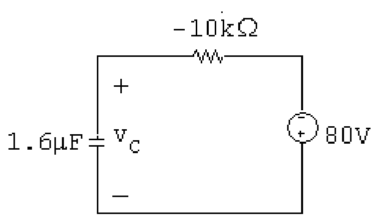
$$\frac{v_{\text{oc}}}{2000} + \frac{v_{\text{oc}} - v_1}{1000} - 4i_\Delta = 0$$

$$\frac{v_1 - v_{\text{oc}}}{1000} + \frac{v_1}{4000} - 5 \times 10^{-3} = 0$$

The constraint equation:

$$i_\Delta = \frac{v_1}{4000}$$

Solving, $v_{\text{oc}} = -80\,\text{V}, \quad v_1 = -60\,\text{V}$



$v_{\text{c}}(0) = 0; \qquad v_{\text{c}}(\infty) = -80\,\text{V}$

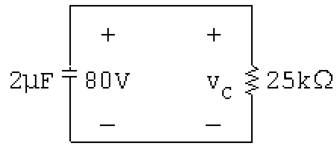
$\tau = RC = (-10{,}000)(1.6 \times 10^{-6}) = -16\,\text{ms}; \qquad \frac{1}{\tau} = -62.5$

$v_{\text{c}} = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau} = -80 + 80e^{62.5t} = 14{,}400$

Solve for the time of the maximum voltage rating:

$e^{62.5t} = 181; \qquad 62.5t = \ln 181; \qquad t = 83.09\,\text{ms}$

P 7.89 [**a**]



$$\tau = (25)(2) \times 10^{-3} = 50\,\text{ms}; \qquad 1/\tau = 20$$

$$v_c(0^+) = 80\,\text{V}; \qquad v_c(\infty) = 0$$

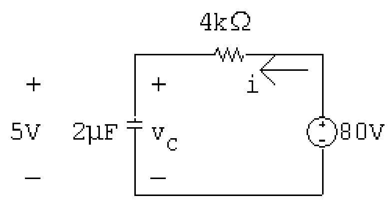
$$v_c = 80e^{-20t}\,\text{V}$$

$$\therefore \quad 80e^{-20t} = 5; \qquad e^{20t} = 16; \qquad t = \frac{\ln 16}{20} = 138.63\,\text{ms}$$

[**b**] $0^+ \leq t \leq 138.63^-$ ms:

$$i = (2 \times 10^{-6})(-1600e^{-20t}) = -3.2e^{-20t}\,\text{mA}$$

$t \geq 138.63^+$ ms:



$$\tau = (2)(4) \times 10^{-3} = 8\,\text{ms}; \qquad 1/\tau = 125$$

$$v_c(138.63^+\,\text{ms}) = 5\,\text{V}; \qquad v_c(\infty) = 80\,\text{V}$$

$$v_c = 80 - 75e^{-125(t-0.13863)}\,\text{V}, \qquad t \geq 138.63\,\text{ms}$$

$$i = 2 \times 10^{-6}(9375)e^{-125(t-0.13863)}$$

$$= 18.75e^{-125(t-0.13863)}\,\text{mA}, \qquad t \geq 138.63^+\,\text{ms}$$

[**c**] $80 - 75e^{-125\Delta t} = 0.85(80) = 68$

$$80 - 68 = 75e^{-125\Delta t} = 12$$

$$e^{125\Delta t} = 6.25; \qquad \Delta t = \frac{\ln 6.25}{125} \cong 14.66\,\text{ms}$$

P 7.90 $v_o = -\dfrac{1}{R(0.5 \times 10^{-6})}\displaystyle\int_0^t 4\,dx + 0 = \dfrac{-4t}{R(0.5 \times 10^{-6})}$

$$\frac{-4(15 \times 10^{-3})}{R(0.5 \times 10^{-6})} = -10$$

$$\therefore \qquad R = \frac{-4(15 \times 10^{-3})}{-10(0.5 \times 10^{-6})} = 12\,\text{k}\Omega$$

P 7.91 $v_o = \dfrac{-4t}{R(0.5\times10^{-6})} + 6 = \dfrac{-4(40\times10^{-3})}{R(0.5\times10^{-6})} + 6 = -10$

$$\therefore \qquad R = \frac{-4(40\times10^{-3})}{-16(0.5\times10^{-6})} = 20\,\text{k}\Omega$$

P 7.92 $RC = (80\times10^3)(250\times10^{-9}) = 20\,\text{ms}; \qquad \dfrac{1}{RC} = 50$

[a] $t < 0: \qquad v_o = 0$

[b] $0 \le t \le 2\,\text{s}:$

$$v_o = -50\int_0^t 0.075\,dx = -3.75t\,\text{V}$$

[c] $2\,\text{s} \le t \le 4\,\text{s};$

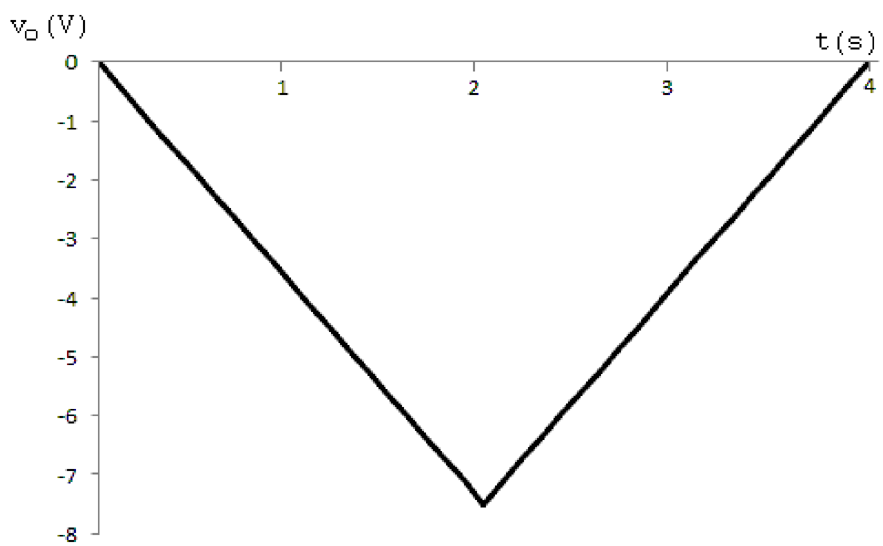
$$v_o(2) = -3.75(2) = -7.5\,\text{V}$$

$$v_o(t) = -50\int_2^t -0.075\,dx - 7.5 = 3.75(t-2) - 7.5 = 3.75t - 15\,\text{V}$$

[d] $t \ge 4\,\text{s}:$

$$v_o(4) = 15 - 15 = 0\,\text{V}$$

$$v_o(t) = 0\,\text{V}$$



P 7.93 Write a KCL equation at the inverting input to the op amp, where the voltage is 0:

$$\frac{0 - v_g}{R_i} + \frac{0 - v_o}{R_f} + C_f\frac{d}{dt}(0 - v_o) = 0$$

$$\therefore \qquad \frac{dv_o}{dt} + \frac{1}{R_fC_f}v_o = -\frac{v_g}{R_iC_f}$$

Note that this first-order differential equation is in the same form as Eq. 7.50 if $I_s = -v_g/R_i$. Therefore, its solution is the same as Eq. 7.51:

$$v_o = \frac{-v_gR_f}{R_i} + \left(V_o - \frac{-v_gR_f}{R_i}\right)e^{-t/R_fC_f}$$

[a] $v_o = 0, \qquad t < 0$

[b] $R_fC_f = (4 \times 10^6)(250 \times 10^{-9}) = 1; \qquad \frac{1}{R_fC_f} = 1$

$$\frac{-v_gR_f}{R_i} = \frac{-(0.075)(4 \times 10^6)}{80{,}000} = -3.75$$

$$V_o = v_o(0) = 0$$

$$\therefore \qquad v_o = -3.75 + (0 + 3.75)e^{-t} = -3.75(1 - e^{-t})\,\text{V}, \qquad 0 \le t \le 2\,\text{s}$$

[c] $\frac{1}{R_fC_f} = 1$

$$\frac{-v_gR_f}{R_i} = \frac{-(-0.075)(4 \times 10^6)}{80{,}000} = 3.75$$

$$V_o = v_o(2) = -3.75(1 - e^{-2}) = -3.24\,\text{V}$$

$$\therefore \qquad v_o = 3.75 + [-3.24 - 3.75]e^{-(t-2)}$$

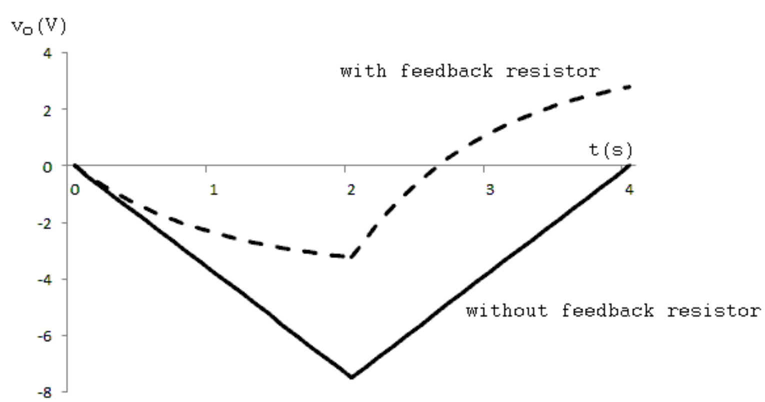
$$= 3.75 - 6.99e^{-(t-2)}\,\text{V}, \quad 2\,\text{s} \le t \le 4\,\text{s}$$

[d] $\frac{1}{R_fC_f} = 1$

$$\frac{-v_gR_f}{R_i} = 0$$

$$V_o = v_o(4) = 3.75 - 6.99e^{-2} = 2.8\,\text{V}$$

$$v_o = 0 + (2.8 - 0)e^{-(t-4)} = 2.8e^{-(t-4)}\,\text{V}, \qquad 4\,\text{s} \le t$$



P 7.94 **[a]** $\dfrac{C dv_p}{dt} + \dfrac{v_p - v_b}{R} = 0;$ therefore $\dfrac{dv_p}{dt} + \dfrac{1}{RC}v_p = \dfrac{v_b}{RC}$

$$\frac{v_n - v_a}{R} + C\frac{d(v_n - v_o)}{dt} = 0;$$

$$\text{therefore} \quad \frac{dv_o}{dt} = \frac{dv_n}{dt} + \frac{v_n}{RC} - \frac{v_a}{RC}$$

But $v_n = v_p$

Therefore $\dfrac{dv_n}{dt} + \dfrac{v_n}{RC} = \dfrac{dv_p}{dt} + \dfrac{v_p}{RC} = \dfrac{v_b}{RC}$

Therefore $\dfrac{dv_o}{dt} = \dfrac{1}{RC}(v_b - v_a);$ $\quad v_o = \dfrac{1}{RC}\displaystyle\int_0^t (v_b - v_a)\, dy$

[b] The output is the integral of the difference between v_b and v_a and then scaled by a factor of $1/RC$.

[c] $v_o = \dfrac{1}{RC}\displaystyle\int_0^t (v_b - v_a)\, dx$

$$RC = (50 \times 10^3)(10 \times 10^{-9}) = 0.5\,\text{ms}$$

$$v_b - v_a = -25\,\text{mV}$$

$$v_o = \frac{1}{0.0005}\int_0^t -25 \times 10^{-3} dx = -50t$$

$$-50t_{\text{sat}} = -6; \qquad t_{\text{sat}} = 120\,\text{ms}$$

P 7.95 Use voltage division to find the voltage at the non-inverting terminal:

$$v_p = \frac{80}{100}(-45) = -36\,\text{V} = v_n$$

Write a KCL equation at the inverting terminal:

$$\frac{-36 - 14}{80{,}000} + 2.5 \times 10^{-6}\frac{d}{dt}(-36 - v_o) = 0$$

$$\therefore \qquad 2.5 \times 10^{-6}\frac{dv_o}{dt} = \frac{-50}{80{,}000}$$

Separate the variables and integrate:

$$\frac{dv_o}{dt} = -250 \quad \therefore \quad dv_o = -250dt$$

$$\int_{v_o(0)}^{v_o(t)} dx = -250\int_0^t dy \quad \therefore \quad v_o(t) - v_o(0) = -250t$$

$v_o(0) = -36 + 56 = 20\,\text{V}$

$v_o(t) = -250t + 20$

Find the time when the voltage reaches 0:

$0 = -250t + 20 \quad \therefore \quad t = \dfrac{20}{250} = 80\,\text{ms}$

P 7.96 **[a]** $RC = (1000)(800 \times 10^{-12}) = 800 \times 10^{-9}; \qquad \dfrac{1}{RC} = 1{,}250{,}000$

$0 \le t \le 1\,\mu\text{s}$:

$v_g = 2 \times 10^6 t$

$$v_o = -1.25 \times 10^6 \int_0^t 2 \times 10^6 x\,dx + 0$$

$$= -2.5 \times 10^{12} \frac{x^2}{2} \Big|_0^t = -125 \times 10^{10} t^2\,\text{V}, \quad 0 \le t \le 1\,\mu\text{s}$$

$v_o(1\,\mu\text{s}) = -125 \times 10^{10}(1 \times 10^{-6})^2 = -1.25\,\text{V}$

$1\,\mu\text{s} \le t \le 3\,\mu\text{s}$:

$v_g = 4 - 2 \times 10^6 t$

$$v_o = -125 \times 10^4 \int_{1\times10^{-6}}^t (4 - 2 \times 10^6 x)\,dx - 1.25$$

$$= -125 \times 10^4 \left[4x \Big|_{1\times10^{-6}}^t - 2 \times 10^6 \frac{x^2}{2} \Big|_{1\times10^{-6}}^t \right] - 1.25$$

$$= -5 \times 10^6 t + 5 + 125 \times 10^{10} t^2 - 1.25 - 1.25$$

$$= 125 \times 10^{10} t^2 - 5 \times 10^6 t + 2.5\,\text{V}, \quad 1\,\mu\text{s} \le t \le 3\,\mu\text{s}$$

$v_o(3\,\mu\text{s}) = 125 \times 10^{10}(3 \times 10^{-6})^2 - 5 \times 10^6(3 \times 10^{-6}) + 2.5$

$= -1.25$

$3\,\mu\text{s} \le t \le 4\,\mu\text{s}$:

$v_g = -8 + 2 \times 10^6 t$

$$v_o = -125 \times 10^4 \int_{3\times10^{-6}}^t (-8 + 2 \times 10^6 x)\,dx - 1.25$$

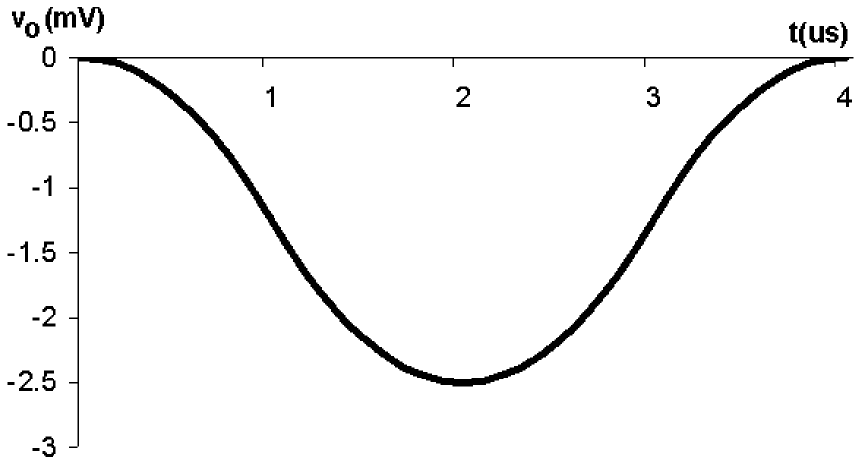
$$= -125 \times 10^4 \left[-8x \Big|_{3\times10^{-6}}^t + 2 \times 10^6 \frac{x^2}{2} \Big|_{3\times10^{-6}}^t \right] - 1.25$$

$$= 10^7 t - 30 - 125 \times 10^{10} t^2 + 11.25 - 1.25$$

$$= -125 \times 10^{10} t^2 + 10^7 t - 20\,\text{V}, \quad 3\,\mu\text{s} \le t \le 4\,\mu\text{s}$$

$v_o(4\,\mu\text{s}) = -125 \times 10^{10}(4 \times 10^{-6})^2 + 10^7(4 \times 10^{-6}) - 20 = 0$

[b]



[c] The output voltage will also repeat. This follows from the observation that at $t = 4\,\mu$s the output voltage is zero, hence there is no energy stored in the capacitor. This means the circuit is in the same state at $t = 4\,\mu$s as it was at $t = 0$, thus as v_g repeats itself, so will v_o.

P 7.97 [a] T_2 is normally ON since its base current i_{b2} is greater than zero, i.e., $i_{b2} = V_{CC}/R$ when T_2 is ON. When T_2 is ON, $v_{ce2} = 0$, therefore $i_{b1} = 0$. When $i_{b1} = 0$, T_1 is OFF. When T_1 is OFF and T_2 is ON, the capacitor C is charged to V_{CC}, positive at the left terminal. This is a stable state; there is nothing to disturb this condition if the circuit is left to itself.

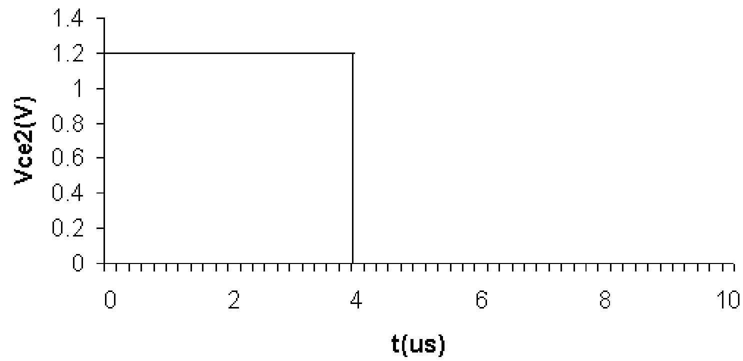
[b] When S is closed momentarily, v_{be2} is changed to $-V_{CC}$ and T_2 snaps OFF. The instant T_2 turns OFF, v_{ce2} jumps to $V_{CC}R_1/(R_1 + R_L)$ and i_{b1} jumps to $V_{CC}/(R_1 + R_L)$, which turns T_1 ON.

[c] As soon as T_1 turns ON, the charge on C starts to reverse polarity. Since v_{be2} is the same as the voltage across C, it starts to increase from $-V_{CC}$ toward $+V_{CC}$. However, T_2 turns ON as soon as $v_{be2} = 0$. The equation for v_{be2} is $v_{be2} = V_{CC} - 2V_{CC}e^{-t/RC}$. $v_{be2} = 0$ when $t = RC\ \ln 2$, therefore T_2 stays OFF for $RC\ \ln 2$ seconds.

P 7.98 [a] For $t < 0$, $v_{ce2} = 0$. When the switch is momentarily closed, v_{ce2} jumps to

$$v_{ce2} = \left(\frac{V_{CC}}{R_1 + R_L}\right) R_1 = \frac{6(5)}{25} = 1.2\,\text{V}$$

T_2 remains open for $(23{,}083)(250) \times 10^{-12}\ \ln 2 \cong 4\,\mu$s.

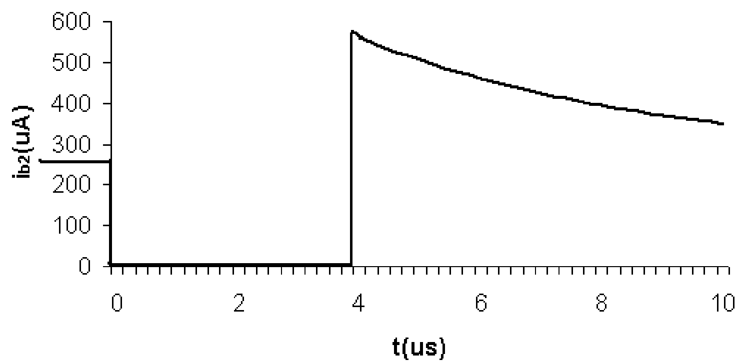


[b] $i_{\rm b2} = \dfrac{V_{CC}}{R} = 259.93\,\mu\text{A}, \qquad -5 \le t \le 0\,\mu\text{s}$

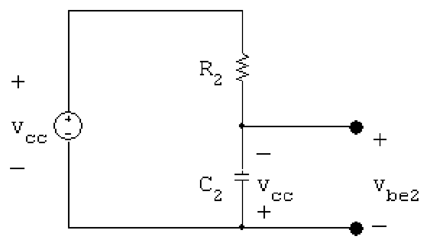
$i_{\rm b2} = 0, \qquad 0 < t < RC\ln 2$

$$i_{\rm b2} = \frac{V_{CC}}{R} + \frac{V_{CC}}{R_{\rm L}} e^{-(t-RC\ln 2)/R_{\rm L}C}$$

$$= 259.93 + 300e^{-0.2\times 10^6(t-4\times 10^{-6})}\,\mu\text{A}, \qquad RC\ln 2 < t$$

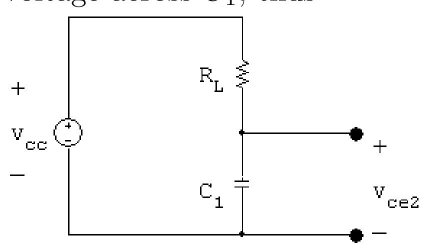


P 7.99 **[a]** While T_2 has been ON, C_2 is charged to V_{CC}, positive on the left terminal. At the instant T_1 turns ON the capacitor C_2 is connected across $b_2 - e_2$, thus $v_{\rm be2} = -V_{CC}$. This negative voltage snaps T_2 OFF. Now the polarity of the voltage on C_2 starts to reverse, that is, the right-hand terminal of C_2 starts to charge toward $+V_{CC}$. At the same time, C_1 is charging toward V_{CC}, positive on the right. At the instant the charge on C_2 reaches zero, $v_{\rm be2}$ is zero, T_2 turns ON. This makes $v_{\rm be1} = -V_{CC}$ and T_1 snaps OFF. Now the capacitors C_1 and C_2 start to charge with the polarities to turn T_1 ON and T_2 OFF. This switching action repeats itself over and over as long as the circuit is energized. At the instant T_1 turns ON, the voltage controlling the state of T_2 is governed by the following circuit:



It follows that $v_{\mathrm{be2}} = V_{CC} - 2V_{CC}e^{-t/R_2C_2}$.

[b] While T_2 is OFF and T_1 is ON, the output voltage v_{ce2} is the same as the voltage across C_1, thus



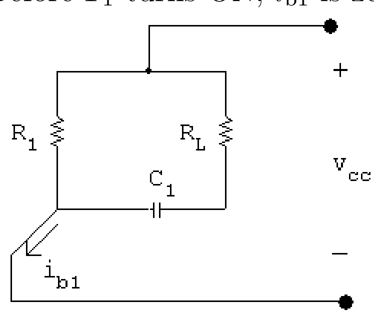
It follows that $v_{\mathrm{ce2}} = V_{CC} - V_{CC}e^{-t/R_\mathrm{L}C_1}$.

[c] T_2 will be OFF until v_{be2} reaches zero. As soon as v_{be2} is zero, i_{b2} will become positive and turn T_2 ON. $v_{\mathrm{be2}} = 0$ when $V_{CC} - 2V_{CC}e^{-t/R_2C_2} = 0$, or when $t = R_2C_2\,\ln 2$.

[d] When $t = R_2C_2\,\ln 2$, we have

$$v_{\mathrm{ce2}} = V_{CC} - V_{CC}e^{-[(R_2C_2\,\ln 2)/(R_\mathrm{L}C_1)]} = V_{CC} - V_{CC}e^{-10\,\ln 2} \cong V_{CC}$$

[e] Before T_1 turns ON, i_{b1} is zero. At the instant T_1 turns ON, we have

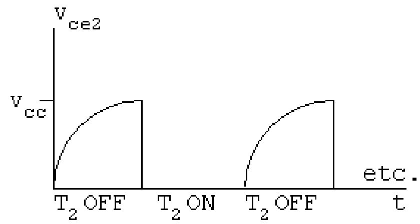


$$i_{b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_\mathrm{L}}e^{-t/R_\mathrm{L}C_1}$$

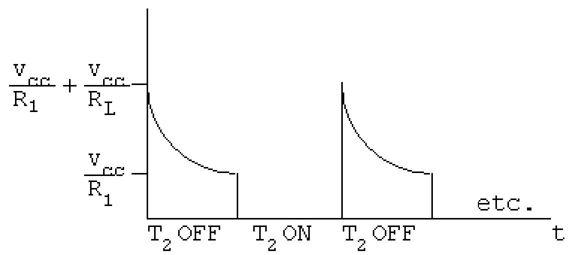
[f] At the instant T_2 turns back ON, $t = R_2C_2\,\ln 2$; therefore

$$i_{\mathrm{b1}} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_\mathrm{L}}e^{-10\,\ln 2} \cong \frac{V_{CC}}{R_1}$$

[g]



[h]



P 7.100 **[a]** $t_{\rm OFF2} = R_2C_2 \ln 2 = 18 \times 10^3(2 \times 10^{-9}) \ln 2 \cong 25\,\mu$s

[b] $t_{\rm ON2} = R_1C_1 \ln 2 \cong 25\,\mu$s

[c] $t_{\rm OFF1} = R_1C_1 \ln 2 \cong 25\,\mu$s

[d] $t_{\rm ON1} = R_2C_2 \ln 2 \cong 25\,\mu$s

[e] $i_{\rm b1} = \dfrac{9}{3} + \dfrac{9}{18} = 3.5\,$mA

[f] $i_{\rm b1} = \dfrac{9}{18} + \dfrac{9}{3}e^{-6\ln 2} \cong 0.5469\,$mA

[g] $v_{\rm ce2} = 9 - 9e^{-6\ln 2} \cong 8.86\,$V

P 7.101 **[a]** $t_{\rm OFF2} = R_2C_2 \ln 2 = (18 \times 10^3)(2.8 \times 10^{-9}) \ln 2 \cong 35\,\mu$s

[b] $t_{\rm ON2} = R_1C_1 \ln 2 \cong 37.4\,\mu$s

[c] $t_{\rm OFF1} = R_1C_1 \ln 2 \cong 37.4\,\mu$s

[d] $t_{\rm ON1} = R_2C_2 \ln 2 = 35\,\mu$s

[e] $i_{\rm b1} = 3.5\,$mA

[f] $i_{\rm b1} = \dfrac{9}{18} + 3e^{-5.6\ln 2} \cong 0.562\,$mA

[g] $v_{\rm ce2} = 9 - 9e^{-5.6\ln 2} \cong 8.81\,$V

Note in this circuit T_2 is OFF 35 μs and ON 37.4 μs of every cycle, whereas T_1 is ON 35 μs and OFF 37.4 μs every cycle.

P 7.102 If $R_1 = R_2 = 50R_{\rm L} = 100\,{\rm k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{100 \times 10^3 \ln 2} = 692.49\,{\rm pF}; \qquad C_2 = \frac{36 \times 10^{-6}}{100 \times 10^3 \ln 2} = 519.37\,{\rm pF}$$

If $R_1 = R_2 = 6R_\mathrm{L} = 12\,\mathrm{k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{12 \times 10^3\ \ln 2} = 5.77\,\mathrm{nF}; \qquad C_2 = \frac{36 \times 10^{-6}}{12 \times 10^3\ \ln 2} = 4.33\,\mathrm{nF}$$

Therefore $692.49\,\mathrm{pF} \leq C_1 \leq 5.77\,\mathrm{nF}$ and $519.37\,\mathrm{pF} \leq C_2 \leq 4.33\,\mathrm{nF}$

P 7.103 **[a]** $0 \leq t \leq 0.5$:

$$i = \frac{21}{60} + \left(\frac{30}{60} - \frac{21}{60}\right) e^{-t/\tau} \qquad \text{where } \tau = L/R.$$

$$i = 0.35 + 0.15e^{-60t/L}$$

$$i(0.5) = 0.35 + 0.15e^{-30/L} = 0.40$$

$$\therefore\ e^{30/L} = 3; \qquad L = \frac{30}{\ln 3} = 27.31\,\mathrm{H}$$

[b] $0 \leq t \leq t_r$, where t_r is the time the relay releases:

$$i = 0 + \left(\frac{30}{60} - 0\right) e^{-60t/L} = 0.5e^{-60t/L}$$

$$\therefore\ 0.4 = 0.5e^{-60t_r/L}; \qquad e^{60t_r/L} = 1.25$$

$$t_r = \frac{27.31 \ln 1.25}{60} \cong 0.10\,\mathrm{s}$$

P 7.104 From the Practical Perspective,

$$v_C(t) = 0.75V_S = V_S(1 - e^{-t/RC})$$

$$0.25 = e^{-t/RC} \qquad \text{so} \qquad t = -RC \ln 0.25$$

In the above equation, t is the number of seconds it takes to charge the capacitor to $0.75V_S$, so it is a period. We want to calculate the heart rate, which is a frequency in beats per minute, so $H = 60/t$. Thus,

$$H = \frac{60}{-RC \ln 0.25}$$

P 7.105 In this problem, $V_{max} = 0.6V_S$, so the equation for heart rate in beats per minute is

$$H = \frac{60}{-RC \ln 0.4}$$

Given $R = 150\,\mathrm{k}\Omega$ and $C = 6\,\mu\mathrm{F}$,

$$H = \frac{60}{-(150{,}000)(6 \times 10^{-6}) \ln 0.4} = 72.76$$

Therefore, the heart rate is about 73 beats per minute.

P 7.106 From the Practical Perspective,

$$v_C(t) = V_{max} = V_S(1 - e^{-t/RC})$$

Solve this equation for the resistance R:

$$\frac{V_{max}}{V_S} = 1 - e^{-t/RC} \qquad \text{so} \qquad e^{-t/RC} = 1 - \frac{V_{max}}{V_S}$$

Then, $$\frac{-t}{RC} = \ln\left(1 - \frac{V_{max}}{V_S}\right)$$

$$\therefore \qquad R = \frac{-t}{C\ln\left(1 - \frac{V_{max}}{V_S}\right)}$$

In the above equation, t is the time it takes to charge the capacitor to a voltage of V_{max}. But t and the heart rate H are related as follows:

$$H = \frac{60}{t}$$

Therefore,

$$R = \frac{-60}{HC\ln\left(1 - \frac{V_{max}}{V_S}\right)}$$

P 7.107 From Problem 7.106,

$$R = \frac{-60}{HC\ln\left(1 - \frac{V_{max}}{V_S}\right)}$$

Note that from the problem statement,

$$\frac{V_{max}}{V_S} = 0.68$$

Therefore,

$$R = \frac{-60}{(70)(2.5 \times 10^{-6})\ln(1 - 0.68)} = 301\,\text{k}\Omega$$