Ch8 Integration By

- التعويض Substitution — Ch1-Ch5, Ch6, Ch7
- تجزيء Parts — 8.1
- Trigonometric Substitution — 8.3
- Trigonometric Power — 8.2
- Partial fraction — 8.4
- 8.7 Improper Integral

8.1 Integration By Parts

$$\int u\,dv = uv - \int v\,du$$

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

Exp

$$\int x\cos x\,dx = \int u\,dv$$

$u = x$ $\quad$ $dv = \cos x\,dx$

$du = dx$ $\quad$ $v = \sin x$

$$= uv - \int v\,du$$

$$= uv - \int v\,du$$

$$= x\sin x - \int \sin x\,dx$$

$$= x\sin x - -\cos x + C$$

$$= x\sin x + \cos x + C$$

$$= x\cos x + \cancel{\sin x} - \cancel{\sin x} + 0$$

الأعم

$$\int x \cos x\,dx$$

$\frac{d}{dx}$	$\int$
x	$\cos x$
1	$\xrightarrow{(+)} \sin x$
0	$\xrightarrow{(-)} -\cos x$

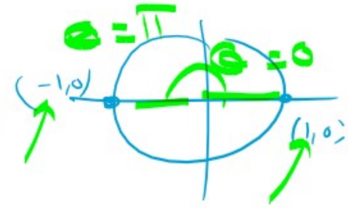
$$= \boxed{x\sin x + \cos x} + C$$

$$\int_0^{\pi} x\cos x\,dx = x\sin x + \cos x \Big|_0^{\pi}$$

$$= (\pi \sin\pi + \cos\pi) - (0\sin 0 + \cos 0)$$

$$= (0 - 1) - (0 + 1) = -2$$

$$= -1 - 1$$



$$\int \frac{1}{x}\,dx = \ln|x| + C$$

Exp $\int \frac{1}{x}\,dx = \ln|x| + c$

Exp $\int \ln x \, dx = \int u\,dv$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}\,dx \qquad v = x$

$= uv - \int v\,du$

$= (\ln x)\,x - \int \cancel{x}\,\frac{1}{\cancel{x}}\,dx$

$= x\ln x - x + C$

Exp $\int x^3 e^x \, dx = \int u\,dv$

$= x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + c$

$\frac{d}{dx}$		$\int$
x^3	$(+)$	e^x
$3x^2$	$(-)$	e^x
$6x$	$(+)$	e^x
6	$(-)$	e^x
0		e^x

Exp $\int x^2 e^x \, dx$

$= x^2 e^x - 2x e^x + 2e^x + c$

x^2		e^x
$2x$	$+$	e^x
2	$-$	e^x

or

$2 \quad e^x$

$+$

$0 \quad e^x$

or

$$\int x^2 e^x\,dx = \int u\,dv$$

$u = x^2 \qquad dv = e^x\,dx$

$du = 2x\,dx \qquad v = e^x$

$$= uv - \int v\,du$$

$$= x^2 e^x - 2\int x e^x\,dx$$

$u = 2x \qquad dv = e^x\,dx$

$du = 2\,dx \qquad v = e^x$

$$= x^2 e^x - \int u\,dv$$

$$= x^2 e^x - \left[uv - \int v\,du\right]$$

$$= x^2 e^x - \left[2x e^x - \int 2 e^x\,dx\right]$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$

Exp

$$\int e^x \sin x\,dx = \int u\,dv$$

$u = e^x \qquad dv = \sin x\,dx$

$du = e^x\,dx \qquad v = -\cos x$

$$= uv - \int v\,du$$

$u = e^x \qquad dv = \cos x\,dx$

$du = e^x\,dx \qquad v = \sin x$

$$= -e^x \cos x - \int -\cos x\, e^x\,dx$$

$$= -e^x \cos x + \int e^x \cos x\,dx$$

$$= -e^x \cos x + \int u\,dv$$

$$\int e^x \quad \ldots \quad e^x \cos x + uv - \int v\,du$$

$$\int e^x \sin x \, dx = -e^x \cos x + uv - \int v\,du$$

$$= -e^x \cos x + e^x \sin x - \boxed{\int \sin x \, e^x \, dx}$$

$$2\int e^x \sin x \, dx = -e^x \cos x + e^x \sin x$$

$$\int e^x \sin x \, dx = \frac{1}{2}\left[-\cos x + \sin x\right] e^x + C$$

Exp $\int e^x \sin x \, dx$

$$= -e^x \cos x + e^x \sin x + \int e^x(-\sin x)\,dx$$

$\frac{d}{dx}$	$\int$
e^x	$\sin x$
e^x (+)	$-\cos x$
e^x (−)	$-\sin x$

stop

0

$$2\int e^x \sin x \, dx = -e^x \cos x + e^x \sin x$$

$$\int e^x \sin x \, dx = \frac{e^x}{2}\left[-\cos x + \sin x\right] + C$$

x^{12}

$\int 12 \; x$

$\int x^{12} e^{x} \, dx$

$\int_{0}^{\pi} \sin x \cos x \, dx =$ نفس الجواب

$\to$ 8.2

حل 1

حل 2

نفس الحل

$y = \boxed{\sqrt{5}} \, \pi$ ...

عدد ثابت

$y' = 0$

Exp جزئية

$\int_{1}^{2} x \ln x \, dx$

$\int \ln x \, dx = x \ln x - x + c$

① $\begin{cases} u = x & dv = \ln x \, dx \\ du = dx & v = x \ln x - x \end{cases}$

② $\begin{cases} u = \ln x & dv = x \, dx \\ du = \frac{1}{x} dx & v = \frac{x^2}{2} \end{cases}$

$= \int_{1}^{2} u \, dv$

$= uv \Big|_{1}^{2} - \int_{1}^{2} v \, du$

② $= \ln x \left(\frac{x^2}{2} \right) \Big|^{2} - \int^{2} \frac{x^2}{2} \cdot \frac{1}{x} \, dx$

② $= \ln x\left(\frac{x^2}{2}\right)\Big|_1^2 - \int_1^2 \frac{x^{\cancel{2}}}{2}\,\frac{1}{\cancel{x}}\,dx$

$= \left[(\ln 2)(2) - (\cancel{\ln 1})(\tfrac{1}{2})\right] - \frac{1}{2}\,\frac{x^2}{2}\Big|_1^2$

$= 2\ln 2 - \frac{1}{4}\left(2^2 - 1^2\right)$

$= 2\ln 2 - \frac{1}{4}(3) \quad = \ln 4 - \frac{3}{4}$

$\int e^x \ln x \, dx$

$\frac{d}{dx}$	$\int$
$\ln x$	e^x
$\frac{1}{x}$	e^x
$-\frac{1}{x^2}$	e^x

مش دائماً بتزبط

$\int_1^2 x \ln x \, dx$

$\frac{d}{dx}$	$\int$
x	$\ln x$
1	$x\ln x - x$
0	$\boxed{\int x\ln x\,dx - \frac{x^2}{2}}$

$+$, $-$

$\int_1^2 x\ln x\, dx = x(x\ln x - \cancel{x})\Big|_1^2 - \left[\int_1^2 x\ln x\,dx - \frac{x^2}{2}\right]$

$$\int_1^2 x\ln x\,dx = x(x\ln x - x)\Big|_1^2 - \left[\int_1^2 x\ln x\,dx - \frac{x^2}{2}\right] \quad 0$$

$$\boxed{\int x\ln x\,dx - \frac{x^2}{2}}$$

$$2\int_1^2 x\ln x\,dx = x^2(\ln x - 1)\Big|_1^2 + \frac{x^2}{2}\Big|_1^2$$

$$= x^2\ln x - x^2 + \frac{x^2}{2}\Big|_1^2$$

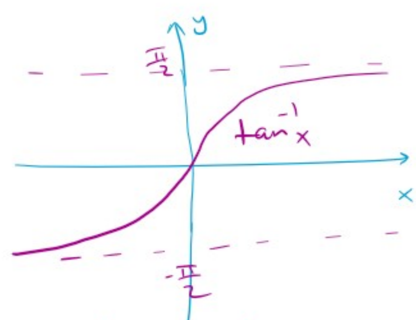
$$= \frac{x^2}{2}\ln x - \frac{x^2}{4}\Big|_1^2$$

$$= \left(\frac{4}{2}\ln 2 - 1\right) - \left(\frac{1}{2}\cancel{\ln 1} - \frac{1}{4}\right)$$

$$= 2\ln 2 - 1 + \frac{1}{4}$$

$$= 2\ln 2 - \frac{3}{4}$$

(11) $\int \tan^{-1} y \; dy$



$\int u\,dv$

$u = \tan^{-1} y \qquad dv = dy$

$du = \frac{dy}{1+y^2} \qquad v = y$

du … $1+y^2$

$$= uv - \int v\,du$$

$$= y\tan^{-1}y - \int y\,\frac{dy}{1+y^2}$$

$$= y\tan^{-1}y - \frac{1}{2}\int \frac{2y}{1+y^2}\,dy$$

$$= y\tan^{-1}y - \frac{1}{2}\ln\left|1+y^2\right| + C$$

$$= y\tan^{-1}y - \frac{1}{2}\ln\left(1+y^2\right) + C$$

$$\tan^{-1}y \neq (\tan y)^{-1} = \frac{1}{\tan y} = \cot y$$

25) $$\int e^{\sqrt{3s+9}}\,ds = \int e^{\sqrt{3x+9}}\,dx$$

$u = e^{\sqrt{3x+9}}$ $\qquad$ $dv = dx$

$du = \frac{3}{2\sqrt{3x+9}}\,e^{\sqrt{3x+9}}$ $\qquad$ $v = x$

$du = \frac{3}{2\sqrt{3x+9}} \quad e$

$$\int e^{\sqrt{3x+9}}\,dx = \int u\,dv = uv - \int v\,du$$

$$= x\,e^{\sqrt{3x+9}} - \int x \left(\frac{3}{2\sqrt{3x+9}}\right) e^{\sqrt{3x+9}}$$

خطأ

$$\int e^{\sqrt{3x+9}}\,dx$$

$y = \sqrt{3x+9}$

$dy = \frac{3}{2\sqrt{3x+9}}\,dx$

$= \frac{3}{2y}\,dx$

$$\int e^{y}\,\frac{2y\,dy}{3}$$

$$\int \frac{2}{3} y\, e^{y}\,dy$$

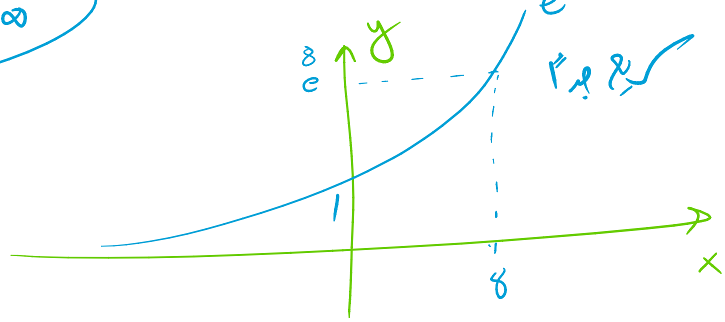
$\frac{2}{3}y$		e^y
	$+$	e^y
$\frac{2}{3}$		
	$-$	e^y
0		

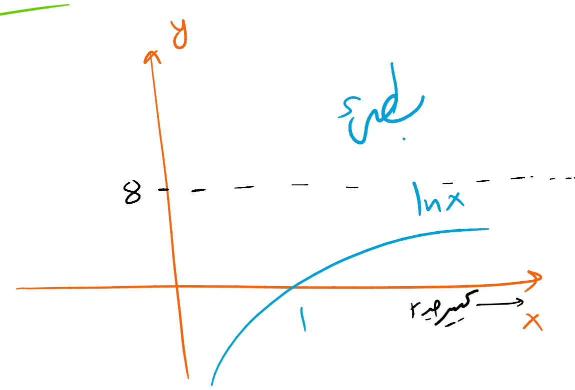
$$= \frac{2}{3} y\, e^{y} - \frac{2}{3} e^{y} + c$$

$$= \frac{2}{3} e^{y}\left[y - 1\right] + c$$

$$= \frac{2}{3} e^{\sqrt{3x+9}}\left[\sqrt{3x+9} - 1\right] + c$$

$x \to \infty$





$$\lim_{x\to\infty} e^x = \infty$$

$\ln 8 \approx 2.08$

$$\lim_{x\to\infty} \ln x = \infty$$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \begin{cases} 0 \Rightarrow g \text{ goes faster than } f \text{ as } x\to\infty \\ \infty \Rightarrow f \text{ is faster than } g \text{ as } x\to\infty \\ \text{رقم موجب} \Rightarrow f \text{ and } g \text{ grow at same rate as } x\to\infty \end{cases}$$

Exp $\log_3 x$, $\ln x$

$$\lim_{x\to\infty} \frac{\log_3 x}{\ln x} = \lim_{x\to\infty} \frac{\frac{\cancel{\ln x}}{\ln 3}}{\cancel{\ln x}} = \frac{1}{\ln 3}$$

رقم

$\log_3 x$ and $\ln x$ grow at same rate.

$\log_3 x$ and $\ln x$ grow at same ...

(2) x^2 , $(\ln 2)^x$

$\lim\limits_{x\to\infty} \left(\frac{1}{2}\right)^x = 0$

$\lim\limits_{x\to\infty} (2)^x = \infty$

$\lim\limits_{x\to\infty} \frac{(\ln 2)^x}{x^2} = 0$

$\lim\limits_{x\to\infty} \frac{1}{x^2} \to 0$ $\lim\limits_{x\to\infty} (\ln 2)^x \to 0$

$\lim\limits_{x\to\infty} a^x = \begin{cases} 0 & 0<a<1 \\ \infty & a>1 \end{cases}$

$1 < 2 < e \approx 2.718$

$\ln 1 < \ln 2 < \ln e$

$\boxed{0 < \ln 2 < 1}$

0

x^2 is faster than $(\ln 2)^x$ as $x \to \infty$

Exp $(\ln 2)^x$, e^x , x^2

$\frac{\infty}{\infty}$

e^x faster than x^2

x^2 faster $(\ln 2)^x$

$\Rightarrow \lim\limits_{x\to\infty} \frac{e^x}{x^2} = \infty$

$\lim \frac{e^x}{2x}$

$\lim \frac{e^x}{2}$

$(\ln 2)^x < x^2 < e^x$

$(\ln x)^x < x^2 < e^x$

$\lim \frac{e^x}{2}$