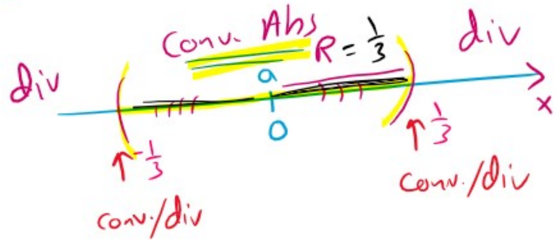
$(-\frac{1}{3}, \frac{1}{3})$ $\frac{1}{3}$

(41) Find IC, R, Conv. Abs., Conv. ~~Conditionally~~

$$\sum_{n=0}^{\infty} 3^n x^n \qquad 3^n \to a_n, \quad a=0$$

$$\sum_{n=0}^{\infty} a_n (x-a)^n \qquad \boxed{a=0} \text{ center}$$



Apply RT $\Rightarrow$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{3^{n+1} x^{n+1}}{\cancel{3^n}\, \cancel{x^n}} \right|$$

$$= |x| \left(\lim_{n\to\infty} 3 \right) = 3|x| \boxed{<} 1$$

$$|x| < \frac{1}{3}$$

$$-\frac{1}{3} < x < \frac{1}{3}$$

$IC = (-\frac{1}{3}, \frac{1}{3})$ $\to$ conv. Abs.

$x = -\frac{1}{3} \Rightarrow \sum 3^n \left(-\frac{1}{3}\right)^n = \sum \cancel{3^n} \cancel{\left(\frac{1}{3}\right)^n} (-1)^n = \sum (-1)^n$ div by n^{th} term test

$x = \frac{1}{3} \Rightarrow \sum \cancel{3^n} \cancel{\left(\frac{1}{3}\right)^n} = \sum_{n=1}^{\infty} 1$ div by n^{th} term test

Hence, $\sum 3^n x^n$ conv. Abs. $\forall\, x \in (-\frac{1}{3}, \frac{1}{3})$

$$\sum_{n=0}^{\infty} 3^n x^n = \sum_{n=0}^{\infty} (3x)^n = \boxed{1} + 3x + (3x)^2 + (3x)^3 + \cdots$$

Conv. geometric Series if $\Downarrow$ $|x| < \frac{1}{3}$

$$= \frac{\boxed{1}}{1 - r} \qquad |r| < 1$$

$|3x| < 1$

$$= \frac{1}{1 - 3x} \quad r = 3x \rightarrow |r| < 1$$

$$|3x| < 1$$

$$-1 < 3x < 1$$

$$-\tfrac{1}{3} < x < \tfrac{1}{3}$$

$$|x| < \tfrac{1}{3}$$

when $x = 0 \Rightarrow 1$

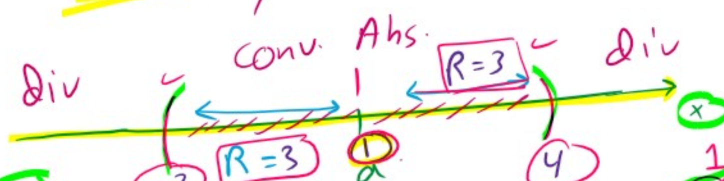
$x = \frac{1}{6} \Rightarrow \sum (3x)^n = \frac{1}{1 - 3\left(\frac{1}{6}\right)}$

$$= \frac{1}{1 - \frac{1}{2}} = \frac{1}{\frac{1}{2}} = \underline{\underline{2}}$$

$$\sum_{n=0}^{\infty} 1 = 1 + 1 + 1 + 1 + 1 + \cdots = \infty$$

$\sum_{n=0}^{\infty} \underbrace{1}_{a_n}$ div by n^{th} term test since $\lim_{n\to\infty} a_n = \lim_{n\to\infty} 1 = 1 \neq 0$

Exp $\displaystyle\sum_{n=1}^{\infty} \underbrace{\frac{(x-1)^n}{n^3\, 3^n}}_{a_n}$ $\qquad \sum a_n (x-a)^n \Rightarrow \underline{\underline{a = 1}}$



Apply RT $\Rightarrow$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(x-1)^{n+1}}{(n+1)^3\, 3^{n+1}} \cdot \frac{n^3\, 3^n}{(x-1)^n} \right| = \frac{1}{3} |x-1| \lim_{n\to\infty} \frac{n^3}{(n+1)^3}$$

$$= \boxed{\frac{1}{3}|x-1|} \boxed{<} \underline{1}$$

$$|x-1| < 3$$

$$-3 < x - 1 < 3$$

$$\boxed{-2 < x < 4}$$

conv. Abs.

after we check

The power series conv. Abs $\forall\ x \in (-2, 4) \Rightarrow [-2, 4]$

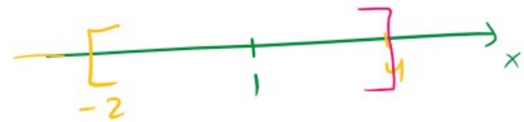
$n=3$

conv. ...

The power series Conv. Abs $\forall\ x \in (-2, 4) \nRightarrow [-2, 4]$

$\boxed{R = 3}$

$\boxed{x = -2} \Rightarrow \sum \frac{(-3)^n}{n^3\, 3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ → Alternating p-series with $p = 3$ $\Rightarrow$ converges " by AST "



$IC = [-2, 4]$

$x = 4 \Rightarrow \sum \frac{\cancel{3^n}}{n^3\, \cancel{3^n}} = \sum \frac{1}{n^3}$ → p-series with $p = 3$ $\Rightarrow$ Conv.

The power series conv. $\forall\ x \in [-2, 4]$ ✓

at $x = -2$, $x = +4$ $\Rightarrow$ Does the series Conv. Abs?

if No ✗ $\Downarrow$ $x = -2$, $x = 4$ Conv. Conditionally

if Yes ✓ $\Downarrow$ $x = -2$, $x = 4$ Conv. Abs.

$\exists$ no points (x) where the power series conv. conditionally

Hence, the power series conv. Abs $\forall\ x \in [-2, 4]$

Exp $\sum_{n=0}^{\infty} n!\, x^n$

$\sum a_n (x - a)^n \Rightarrow a = 0$

$\boxed{R = 0}$

Exp $\sum_{n=0} n!\, x$

Apply RT $\Rightarrow$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\cancel{(n+1)!}^{\,n+1}\ \cancel{x^{n+1}}^{\,x}}{\cancel{n!}\ \cancel{x^n}}\right| = |x|\ \boxed{\lim_{n\to\infty}(n+1)} \to \infty$$

$\boxed{>}\ 1$

$\boxed{R=0}$

Conv. div div

0 x

This infinit series diverges for every x except $x=0$

since when $\underline{x=0} \Rightarrow \sum n!\, x^n = \sum_{n=0}^{\infty} 0 = 0+0+0+\cdots = \underline{0}$ conv.

Th Assume $\sum_{n=0}^{\infty} a_n x^n = \underline{A(x)}$ and $\sum_{n=0}^{\infty} b_n x^n = \underline{B(x)}$

$a=0$

converge abs. on $\underline{|x|<R}$

Then

$-R$ 0 R

$$\left(\sum_{n=0}^{\infty} a_n x^n\right)\left(\sum_{n=0}^{\infty} b_n x^n\right)$$ converges Abs. to $A(x)\,B(x)$ on $\underline{|x|<R}$

Th If $\sum_{n=0}^{\infty} a_n x^n$ conv. Abs. on $|x|<R$ then

$a=0$

$\sum \quad a_n \quad ^n$ conv. Abs. on $|f(x)|<R$

$$\sum a_n \left(f(x)\right)^n \text{ conv. Abs. on } |f(x)| < R$$

for any cont. function f

Exp $\displaystyle\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$ if $|x| < 1$

This means $\displaystyle\sum_{n=0}^{\infty} x^n$ conv. Abs. to $\dfrac{1}{1-x}$ on $|x|<1$
$-1 < x < 1$

$$\sum_{n=0}^{\infty} (4x^2)^n = 1 + 4x^2 + (4x^2)^2 + (4x^2)^3 + \cdots = \frac{1}{1-4x^2} \text{ if } |4x^2| < 1$$

$4|x^2| < 1$

$4x^2 < 1$

$\sqrt{x^2} < \sqrt{\frac{1}{4}}$

$|x| < \frac{1}{2}$

$\boxed{-\frac{1}{2} < x < \frac{1}{2}}$

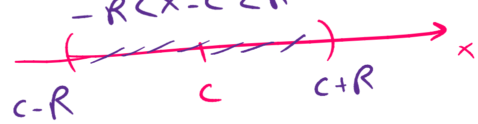
$f = 4x^2$ cont.

Th (Term by Term Differentiation)

if $\displaystyle f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 + \cdots$

conv. Abs on $|x-c| < R$

$-R < x-c < R$



$c-R < x < c+R$

if f has all derivatives on $|x-c|<R$

then

$$f'(x) = \sum_{n=1}^{\infty} n a_n (x-c)^{n-1} = 0 + a_1 + 2a_2(x-c) + 3a_3(x-c)^2 + \cdots$$

on $|x-c| < R$

$\sum_{n=1}$ on $|x-c|<R$

$$f''(x) = \sum_{n=2}^{\infty} n(n-1)a_n(x-c)^{n-2} = 0 + 0 + 2a_2 + 6a_3(x-c) + \cdots$$

Th (Term by Term Integration Th)

Assume $f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$ conv. Abs on $|x-c|<R$

$$= a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 + \cdots$$

Then $\int f(x)dx = \sum_{n=0}^{\infty} a_n \frac{(x-c)^{n+1}}{n+1} + C$ on $|x-c|<R$

Exp Identify this function $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$, $|x| \le 1$

(a) $\sin^{-1} x$ (b) $\cos^{-1} x$ (c) $\tan^{-1} x$ (d) $\sec^{-1} x$ ---

Compare with $\sum_{n=0}^{\infty} a_n (x-a)^n \Rightarrow \boxed{a=0}$

x^3 x^5 x^7 conv. Abs.

$n=0$

$$f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \quad \text{on } |x| \le 1 \Rightarrow f(0) = 0$$

conv. Abs.

$-1 \quad 0 \quad 1$

$$f'(x) = \boxed{1} - x^2 + x^4 - x^6 + \cdots \quad \text{on } |x| < 1$$

conv.

geometric series

$r = |x^2| = x^2 < 1$

$|x| < 1$

$$= \frac{\boxed{1}}{1 - \boxed{-x^2}}$$

$$\frac{-x^2}{1} = \frac{x^4}{-x^2} = -x^2$$

$$f'(x) = \frac{1}{1+x^2}$$

$$\int f'(x)\,dx = \int \frac{1}{1+x^2}\,dx$$

$$\boxed{f(x) = \tan^{-1} x + C}$$

we need to find C

But $f(0) = 0$

$$0 = \tan^{-1} 0 + C \quad \Rightarrow \quad 0 = 0 + C$$

$$\boxed{C = 0}$$

$$\boxed{f(x) = \tan^{-1} x}$$