
The Operational Amplifier

Assessment Problems

AP 5.1 [a] This is an inverting amplifier, so

$$v_o = (-R_f/R_i)v_s = (-80/16)v_s, \quad \text{so} \quad v_o = -5v_s$$

$$v_s(\text{ V}) \quad 0.4 \quad 2.0 \quad 3.5 \quad -0.6 \quad -1.6 \quad -2.4$$

$$v_o(\text{ V}) \quad -2.0 \quad -10.0 \quad -15.0 \quad 3.0 \quad 8.0 \quad 10.0$$

Two of the values, 3.5 V and -2.4 V, cause the op amp to saturate.

[b] Use the negative power supply value to determine the largest input voltage:

$$-15 = -5v_s, \quad v_s = 3 \text{ V}$$

Use the positive power supply value to determine the smallest input voltage:

$$10 = -5v_s, \quad v_s = -2 \text{ V}$$

$$\text{Therefore} \quad -2 \leq v_s \leq 3 \text{ V}$$

AP 5.2 From Assessment Problem 5.1

$$v_o = (-R_f/R_i)v_s = (-R_x/16,000)v_s = (-R_x/16,000)(-0.640)$$

$$= 0.64R_x/16,000 = 4 \times 10^{-5}R_x$$

Use the negative power supply value to determine one limit on the value of R_x :

$$4 \times 10^{-5}R_x = -15 \quad \text{so} \quad R_x = -15/4 \times 10^{-5} = -375 \text{ k}\Omega$$

Since we cannot have negative resistor values, the lower limit for R_x is 0. Now use the positive power supply value to determine the upper limit on the value of R_x :

$$4 \times 10^{-5} R_x = 10 \quad \text{so} \quad R_x = 10/4 \times 10^{-5} = 250 \text{ k}\Omega$$

Therefore,

$$0 \leq R_x \leq 250 \text{ k}\Omega$$

AP 5.3 [a] This is an inverting summing amplifier so

$$v_o = (-R_f/R_a)v_a + (-R_f/R_b)v_b = -(250/5)v_a - (250/25)v_b = -50v_a - 10v_b$$

Substituting the values for v_a and v_b :

$$v_o = -50(0.1) - 10(0.25) = -5 - 2.5 = -7.5 \text{ V}$$

[b] Substitute the value for v_b into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50v_a - 10(0.25) = -50v_a - 2.5 = -10 \text{ V}$$

$$\text{Therefore } 50v_a = 7.5, \quad \text{so } v_a = 0.15 \text{ V}$$

[c] Substitute the value for v_a into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50(0.10) - 10v_b = -5 - 10v_b = -10 \text{ V};$$

$$\text{Therefore } 10v_b = 5, \quad \text{so } v_b = 0.5 \text{ V}$$

[d] The effect of reversing polarity is to change the sign on the v_b term in each equation from negative to positive.

Repeat part (a):

$$v_o = -50v_a + 10v_b = -5 + 2.5 = -2.5 \text{ V}$$

Repeat part (b):

$$v_o = -50v_a + 2.5 = -10 \text{ V}; \quad 50v_a = 12.5, \quad v_a = 0.25 \text{ V}$$

Repeat part (c), using the value of the positive power supply:

$$v_o = -5 + 10v_b = 15 \text{ V}; \quad 10v_b = 20; \quad v_b = 2.0 \text{ V}$$

AP 5.4 [a] Write a node voltage equation at v_n ; remember that for an ideal op amp, the current into the op amp at the inputs is zero:

$$\frac{v_n}{4500} + \frac{v_n - v_o}{63,000} = 0$$

Solve for v_o in terms of v_n by multiplying both sides by 63,000 and collecting terms:

$$14v_n + v_n - v_o = 0 \quad \text{so} \quad v_o = 15v_n$$

Now use voltage division to calculate v_p . We can use voltage division because the op amp is ideal, so no current flows into the non-inverting input terminal and the 400 mV divides between the 15 k Ω resistor and the R_x resistor:

$$v_p = \frac{R_x}{15,000 + R_x}(0.400)$$

Now substitute the value $R_x = 60$ k Ω :

$$v_p = \frac{60,000}{15,000 + 60,000}(0.400) = 0.32 \text{ V}$$

Finally, remember that for an ideal op amp, $v_n = v_p$, so substitute the value of v_p into the equation for v_o

$$v_o = 15v_n = 15v_p = 15(0.32) = 4.8 \text{ V}$$

- [b] Substitute the expression for v_p into the equation for v_o and set the resulting equation equal to the positive power supply value:

$$v_o = 15 \left(\frac{0.4R_x}{15,000 + R_x} \right) = 5$$

$$15(0.4R_x) = 5(15,000 + R_x) \quad \text{so} \quad R_x = 75 \text{ k}\Omega$$

- AP 5.5 [a] Since this is a difference amplifier, we can use the expression for the output voltage in terms of the input voltages and the resistor values given in Eq. 5.22:

$$v_o = \frac{20(60)}{10(24)}v_b - \frac{50}{10}v_a$$

Simplify this expression and substitute in the value for v_b :

$$v_o = 5(v_b - v_a) = 20 - 5v_a$$

Set this expression for v_o to the positive power supply value:

$$20 - 5v_a = 10 \text{ V} \quad \text{so} \quad v_a = 2 \text{ V}$$

Now set the expression for v_o to the negative power supply value:

$$20 - 5v_a = -10 \text{ V} \quad \text{so} \quad v_a = 6 \text{ V}$$

Therefore $2 \leq v_a \leq 6 \text{ V}$

[b] Begin as before by substituting the appropriate values into Eq. 5.22:

$$v_o = \frac{8(60)}{10(12)}v_b - 5v_a = 4v_b - 5v_a$$

Now substitute the value for v_b :

$$v_o = 4(4) - 5v_a = 16 - 5v_a$$

Set this expression for v_o to the positive power supply value:

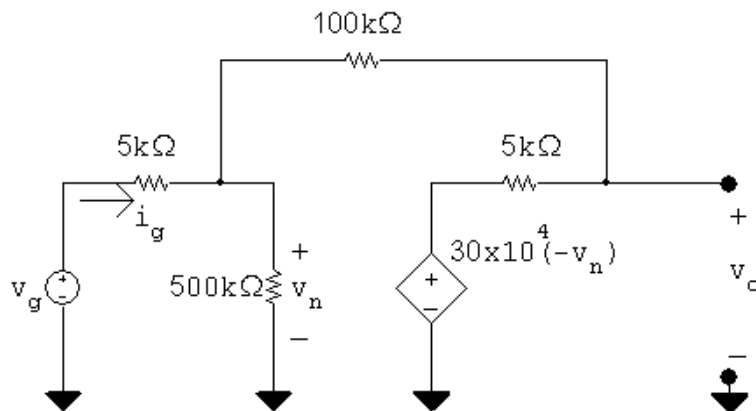
$$16 - 5v_a = 10 \text{ V} \quad \text{so} \quad v_a = 1.2 \text{ V}$$

Now set the expression for v_o to the negative power supply value:

$$16 - 5v_a = -10 \text{ V} \quad \text{so} \quad v_a = 5.2 \text{ V}$$

Therefore $1.2 \leq v_a \leq 5.2 \text{ V}$

AP 5.6 [a] Replace the op amp with the more realistic model of the op amp from Fig. 5.15:



Write the node voltage equation at the left hand node:

$$\frac{v_n}{500,000} + \frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100,000} = 0$$

Multiply both sides by 500,000 and simplify:

$$v_n + 100v_n - 100v_g + 5v_n - 5v_o = 0 \quad \text{so} \quad 21.2v_n - v_o = 20v_g$$

Write the node voltage equation at the right hand node:

$$\frac{v_o - 300,000(-v_n)}{5000} + \frac{v_o - v_n}{100,000} = 0$$

Multiply through by 100,000 and simplify:

$$20v_o + 6 \times 10^6 v_n + v_o - v_n = 0 \quad \text{so} \quad 6 \times 10^6 v_n + 21v_o = 0$$

Use Cramer's method to solve for v_o :

$$\Delta = \begin{vmatrix} 21.2 & -1 \\ 6 \times 10^6 & 21 \end{vmatrix} = 6,000,445.2$$

$$N_o = \begin{vmatrix} 21.2 & 20v_g \\ 6 \times 10^6 & 0 \end{vmatrix} = -120 \times 10^6 v_g$$

$$v_o = \frac{N_o}{\Delta} = -19.9985v_g; \quad \text{so } \frac{v_o}{v_g} = -19.9985$$

[b] Use Cramer's method again to solve for v_n :

$$N_1 = \begin{vmatrix} 20v_g & -1 \\ 0 & 21 \end{vmatrix} = 420v_g$$

$$v_n = \frac{N_1}{\Delta} = 6.9995 \times 10^{-5} v_g$$

$$v_g = 1 \text{ V}, \quad v_n = 69.995 \mu \text{ V}$$

[c] The resistance seen at the input to the op amp is the ratio of the input voltage to the input current, so calculate the input current as a function of the input voltage:

$$i_g = \frac{v_g - v_n}{5000} = \frac{v_g - 6.9995 \times 10^{-5} v_g}{5000}$$

Solve for the ratio of v_g to i_g to get the input resistance:

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 6.9995 \times 10^{-5}} = 5000.35 \Omega$$

[d] This is a simple inverting amplifier configuration, so the voltage gain is the ratio of the feedback resistance to the input resistance:

$$\frac{v_o}{v_g} = -\frac{100,000}{5000} = -20$$

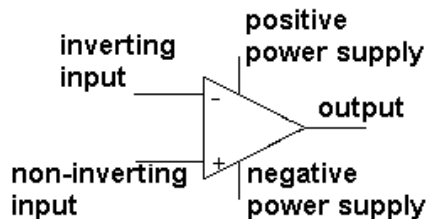
Since this is now an ideal op amp, the voltage difference between the two input terminals is zero; since $v_p = 0$, $v_n = 0$

Since there is no current into the inputs of an ideal op amp, the resistance seen by the input voltage source is the input resistance:

$$R_g = 5000 \Omega$$

Problems

P 5.1 [a] The five terminals of the op amp are identified as follows:



- [b] The input resistance of an ideal op amp is infinite, which constrains the value of the input currents to 0. Thus, $i_n = 0$ A.
- [c] The open-loop voltage gain of an ideal op amp is infinite, which constrains the difference between the voltage at the two input terminals to 0. Thus, $(v_p - v_n) = 0$.
- [d] Write a node voltage equation at v_n :

$$\frac{v_n - 2}{4000} + \frac{v_n - v_o}{12,000} = 0$$

But $v_p = 0$ and $v_n = v_p = 0$. Thus,

$$\frac{-2}{4000} - \frac{v_o}{12,000} = 0 \quad \text{so} \quad v_o = -6 \text{ V}$$

P 5.2 [a] Let the value of the voltage source be v_s :

$$\frac{v_n - v_s}{4000} + \frac{v_n - v_o}{12,000} = 0$$

But $v_n = v_p = 0$. Therefore,

$$v_o = -\frac{12,000}{4000}v_s = -3v_s$$

When $v_s = -6$ V, $v_o = -3(-6) = 18$ V; saturates at $v_o = 15$ V.

When $v_s = -3.5$ V, $v_o = -3(-3.5) = 10.5$ V.

When $v_s = -1.25$ V, $v_o = -3(-1.25) = 3.75$ V.

When $v_s = 2.4$ V, $v_o = -3(2.4) = -7.2$ V.

When $v_s = 4.5$ V, $v_o = -3(4.5) = -13.5$ V.

When $v_s = 5.4$ V, $v_o = -3(5.4) = -16.2$ V; saturates at $v_o = -15$ V.

[b] $-3v_s = 15 \quad \text{so} \quad v_s = \frac{15}{-3} = -5 \text{ V}$

$$-3v_s = -15 \quad \text{so} \quad v_s = \frac{-15}{-3} = 5 \text{ V}$$

The range of source voltages that avoids saturation is $-5 \text{ V} \leq v_s \leq 5 \text{ V}$.

$$\text{P 5.3} \quad v_p = \frac{5000}{5000 + 10,000}(6) = 2 \text{ V} = v_n$$

$$\frac{v_n + 5}{3000} + \frac{v_n - v_o}{6000} = 0$$

$$2(2 + 5) + (2 - v_o) = 0$$

$$v_o = 16 \text{ V}$$

$$i_L = \frac{v_o}{8000} = \frac{16}{8000} = 2000 \times 10^{-6}$$

$$i_L = 2 \text{ mA}$$

$$\text{P 5.4} \quad \frac{v_b - v_a}{5000} + \frac{v_b - v_o}{40,000} = 0, \quad \text{therefore } v_o = 9v_b - 8v_a$$

$$[\mathbf{a}] \quad v_a = 1.5 \text{ V}, \quad v_b = 0 \text{ V}, \quad v_o = -12 \text{ V}$$

$$[\mathbf{b}] \quad v_a = -0.5 \text{ V}, \quad v_b = 0 \text{ V}, \quad v_o = 4 \text{ V}$$

$$[\mathbf{c}] \quad v_a = 1 \text{ V}, \quad v_b = 2.5 \text{ V}, \quad v_o = 14.5 \text{ V}$$

$$[\mathbf{d}] \quad v_a = 2.5 \text{ V}, \quad v_b = 1 \text{ V}, \quad v_o = -11 \text{ V}$$

$$[\mathbf{e}] \quad v_a = 2.5 \text{ V}, \quad v_b = 0 \text{ V}, \quad v_o = -20 \text{ V} \quad (\text{saturates at } -16 \text{ V})$$

$$[\mathbf{f}] \quad \text{If } v_b = 2 \text{ V}, \quad v_o = 18 - 8v_a = \pm 16$$

$$\therefore 0.25 \text{ V} \leq v_a \leq 4.25 \text{ V}$$

$$\text{P 5.5} \quad v_o = -(0.5 \times 10^{-3})(10,000) = -5 \text{ V}$$

$$\therefore i_o = \frac{v_o}{5000} = \frac{-5}{5000} = -1 \text{ mA}$$

$$\text{P 5.6} \quad [\mathbf{a}] \quad i_a = \frac{240 \times 10^{-3}}{8000} = 30 \mu\text{A}$$

$$v_1 = -40 \times 10^3 i_2 = -3 \text{ V}$$

$$[\mathbf{b}] \quad \frac{0 - v_a}{60,000} = i_a \quad \text{so} \quad v_a = -60,000 i_a = -1.8 \text{ V}$$

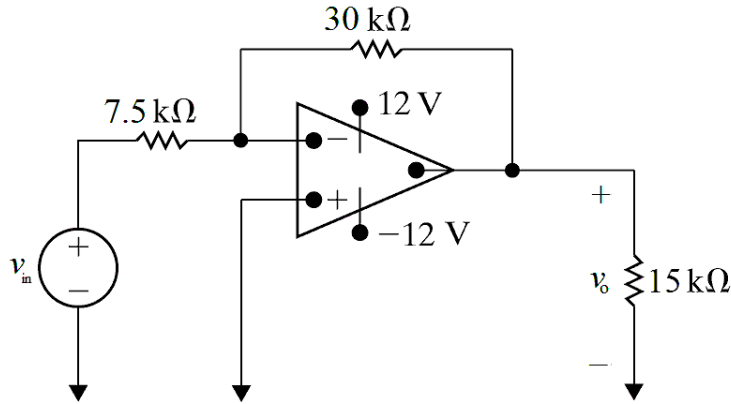
$$[\mathbf{c}] \quad \frac{v_a}{60,000} + \frac{v_a}{40,000} + \frac{v_a - v_o}{30,000} = 0$$

$$\therefore v_o = 2.25v_a = -4.05 \text{ V}$$

$$[\mathbf{d}] \quad i_o = \frac{-v_o}{20,000} + \frac{v_a - v_o}{30,000} = 277.5 \mu\text{A}$$

P 5.7 Since the current into the inverting input terminal of an ideal op-amp is zero, the voltage across the $2.2\text{ M}\Omega$ resistor is $(2.2 \times 10^6)(3.5 \times 10^{-6})$ or 7.7 V . Therefore the voltmeter reads 7.7 V .

P 5.8 [a] $\frac{30,000}{R_{\text{in}}} = 4$ so $R_{\text{in}} = \frac{30,000}{4} = 7500 = 7.5\text{ k}\Omega$



[b] $-4v_{\text{in}} = 12$ so $v_{\text{in}} = \frac{12}{-4} = -3\text{ V}$

$-4v_{\text{in}} = -12$ so $v_{\text{in}} = \frac{12}{4} = 3\text{ V}$

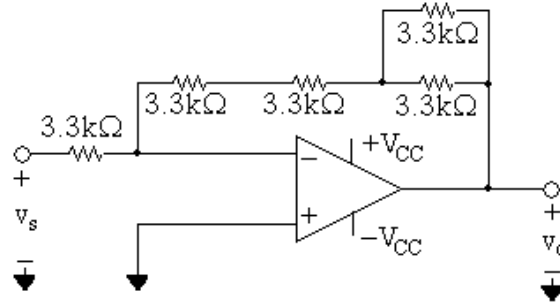
$\therefore -3\text{ V} \leq v_{\text{in}} \leq 3\text{ V}$

[c] $-\frac{R_f}{7500}(2) = -12$ so $R_f = 45\text{ k}\Omega$

$$\left| \frac{v_o}{v_{\text{in}}} \right| = \frac{R_f}{R_{\text{in}}} = \frac{45,000}{7500} = 6$$

The amplifier has a gain of 6.

P 5.9 [a] The gain of an inverting amplifier is the negative of the ratio of the feedback resistor to the input resistor. If the gain of the inverting amplifier is to be 2.5, the feedback resistor must be 2.5 times as large as the input resistor. There are many possible designs that use a resistor value chosen from Appendix H. We present one here that uses $3.3\text{ k}\Omega$ resistors. Use a single $3.3\text{ k}\Omega$ resistor as the input resistor. Then construct a network of $3.3\text{ k}\Omega$ resistors whose equivalent resistance is $2.5(3.3) = 8.25\text{ k}\Omega$ by connecting two resistors in parallel and connecting the parallel resistors in series with two other resistors. The resulting circuit is shown here:



- [b] To amplify signals in the range -2 V to 3 V without saturating the op amp, the power supply voltages must be greater than or equal to the product of the input voltage and the amplifier gain.

$$-2.5(-2) = 5\text{ V} \quad \text{and} \quad -2.5(3) = -7.5\text{ V}$$

Thus, the power supplies should have values of -7.5 V and 5 V .

- P 5.10 [a] Let v_{Δ} be the voltage from the potentiometer contact to ground. Then

$$\frac{0 - v_g}{2000} + \frac{0 - v_{\Delta}}{50,000} = 0$$

$$-25v_g - v_{\Delta} = 0, \quad \therefore v_{\Delta} = -25(40 \times 10^{-3}) = -1\text{ V}$$

$$\frac{v_{\Delta}}{\alpha R_{\Delta}} + \frac{v_{\Delta} - 0}{50,000} + \frac{v_{\Delta} - v_o}{(1 - \alpha)R_{\Delta}} = 0$$

$$\frac{v_{\Delta}}{\alpha} + 2v_{\Delta} + \frac{v_{\Delta} - v_o}{1 - \alpha} = 0$$

$$v_{\Delta} \left(\frac{1}{\alpha} + 2 + \frac{1}{1 - \alpha} \right) = \frac{v_o}{1 - \alpha}$$

$$\therefore v_o = -1 \left[1 + 2(1 - \alpha) + \frac{(1 - \alpha)}{\alpha} \right]$$

$$\text{When } \alpha = 0.2, \quad v_o = -1(1 + 1.6 + 4) = -6.6\text{ V}$$

$$\text{When } \alpha = 1, \quad v_o = -1(1 + 0 + 0) = -1\text{ V}$$

$$\therefore -6.6\text{ V} \leq v_o \leq -1\text{ V}$$

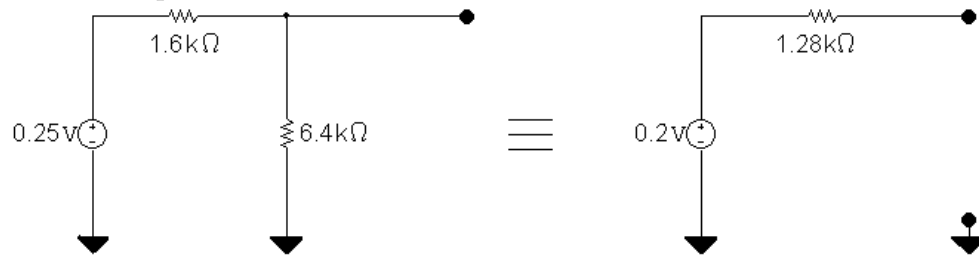
$$\text{[b]} -1 \left[1 + 2(1 - \alpha) + \frac{(1 - \alpha)}{\alpha} \right] = -7$$

$$\alpha + 2\alpha(1 - \alpha) + (1 - \alpha) = 7\alpha$$

$$\alpha + 2\alpha - 2\alpha^2 + 1 - \alpha = 7\alpha$$

$$\therefore 2\alpha^2 + 5\alpha - 1 = 0 \quad \text{so} \quad \alpha \cong 0.186$$

- P 5.11 [a] Replace the combination of v_g , $1.6\text{ k}\Omega$, and the $6.4\text{ k}\Omega$ resistors with its Thévenin equivalent.



$$\text{Then } v_o = \frac{-[12 + \sigma 50]}{1.28}(0.20)$$

At saturation $v_o = -5\text{ V}$; therefore

$$-\left(\frac{12 + \sigma 50}{1.28}\right)(0.2) = -5, \quad \text{or } \sigma = 0.4$$

Thus for $0 \leq \sigma \leq 0.40$ the operational amplifier will not saturate.

[b] When $\sigma = 0.272$, $v_o = \frac{-(12 + 13.6)}{1.28}(0.20) = -4\text{ V}$

$$\text{Also } \frac{v_o}{10} + \frac{v_o}{25.6} + i_o = 0$$

$$\therefore i_o = -\frac{v_o}{10} - \frac{v_o}{25.6} = \frac{4}{10} + \frac{4}{25.6}\text{ mA} = 556.25\text{ }\mu\text{A}$$

- P 5.12 [a] This circuit is an example of an inverting summing amplifier.

[b] $v_o = -\frac{220}{44}v_a - \frac{220}{27.5}v_b - \frac{220}{80}v_c = -5 - 12 + 11 = -6\text{ V}$

[c] $v_o = -6 - 8v_b = \pm 10$

$$\therefore v_b = -0.5\text{ V} \quad \text{when } v_o = 10\text{ V};$$

$$v_b = 2\text{ V} \quad \text{when } v_o = -10\text{ V}$$

$$\therefore -0.5\text{ V} \leq v_b \leq 2\text{ V}$$

P 5.13 $v_o = -\left[\frac{R_f}{4000}(0.2) + \frac{R_f}{5000}(0.15) + \frac{R_f}{20,000}(0.4)\right]$

$$-6 = -0.1 \times 10^{-3} R_f; \quad R_f = 60\text{ k}\Omega; \quad \therefore 0 \leq R_f \leq 60\text{ k}\Omega$$

- P 5.14 [a] Write a KCL equation at the inverting input to the op amp:

$$\frac{v_d - v_a}{60,000} + \frac{v_d - v_b}{20,000} + \frac{v_d - v_c}{36,000} + \frac{v_d}{270,000} + \frac{v_d - v_o}{180,000} = 0$$

Multiply through by 180,000, plug in the values of input voltages, and rearrange to solve for v_o :

$$\begin{aligned} v_o &= 180,000 \left(\frac{3}{60,000} + \frac{-3}{20,000} + \frac{1}{36,000} + \frac{6}{270,000} + \frac{6}{180,000} \right) \\ &= -3 \text{ V} \end{aligned}$$

[b] Write a KCL equation at the inverting input to the op amp. Use the given values of input voltages in the equation:

$$\frac{6-3}{60,000} + \frac{6-9}{20,000} + \frac{6-v_c}{36,000} + \frac{6}{270,000} + \frac{6-v_o}{180,000} = 0$$

Simplify and solve for v_o :

$$9 - 27 + 30 - 5v_c + 4 + 6 - v_o = 0 \quad \text{so} \quad v_o = 22 - 5v_c$$

Set v_o to the positive power supply voltage and solve for v_c :

$$22 - 5v_c = 10 \quad \therefore \quad v_c = 2.4 \text{ V}$$

Set v_o to the negative power supply voltage and solve for v_c :

$$22 - 5v_c = -10 \quad \therefore \quad v_c = 6.4 \text{ V}$$

Therefore,

$$2.4 \text{ V} \leq v_c \leq 6.4 \text{ V}$$

P 5.15 [a]
$$\frac{6-3}{60,000} + \frac{6-9}{20,000} + \frac{6-5}{36,000} + \frac{6}{270,000} + \frac{6-v_o}{R_f} = 0$$

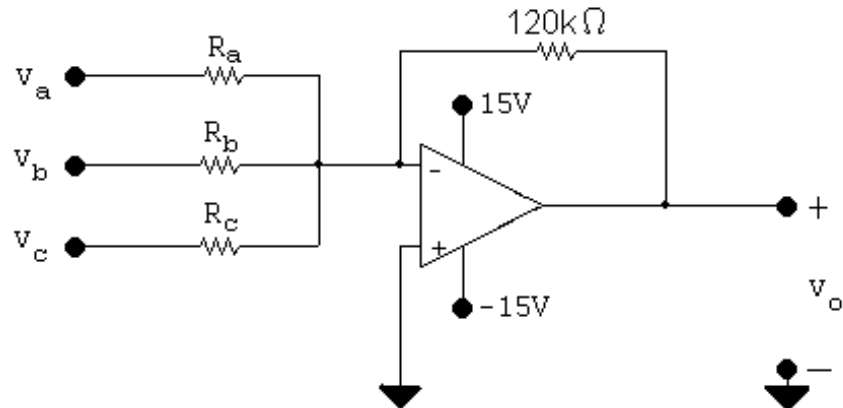
$$\frac{6-v_o}{R_f} = 5 \times 10^{-5} \quad \text{so} \quad R_f = \frac{6-v_o}{5 \times 10^{-5}}$$

For $v_o = -10 \text{ V}$, $R_f = 320 \text{ k}\Omega$

For $v_o = 10 \text{ V}$, $R_f < 0$ so this solution is not possible.

[b]
$$i_o = -(i_f + i_{16k}) = - \left[\frac{-10-6}{320,000} + \frac{-10}{16,000} \right] = 675 \mu\text{A}$$

P 5.16 [a]



$$\frac{120,000}{R_a} = 8 \quad \text{so} \quad R_a = \frac{120,000}{8} = 15 \text{ k}\Omega$$

$$\frac{120,000}{R_b} = 5 \quad \text{so} \quad R_b = \frac{120,000}{5} = 24 \text{ k}\Omega$$

$$\frac{120,000}{R_c} = 12 \quad \text{so} \quad R_c = \frac{120,000}{12} = 10 \text{ k}\Omega$$

$$\text{[b]} \quad -[8(2) + 5v_b + 12(-1)] = -4 - 5v_b$$

$$-4 - 5v_b = -15 \quad \text{so} \quad 5v_b = 11 \quad \text{thus} \quad v_b = \frac{11}{5} = 2.2 \text{ V}$$

$$-4 - 5v_b = 15 \quad \text{so} \quad -5v_b = 19 \quad \text{thus} \quad v_b = \frac{-19}{5} = -3.8 \text{ V}$$

Thus,

$$-3.8 \text{ V} \leq v_b \leq 2.2 \text{ V}$$

P 5.17 We want the following expression for the output voltage:

$$v_o = -(8v_a + 4v_b + 10v_c + 6v_d)$$

This is an inverting summing amplifier, so each input voltage is amplified by a gain that is the ratio of the feedback resistance to the resistance in the forward path for the input voltage. Pick a feedback resistor with divisors of 8, 4, 10, and 6 – say 120 kΩ:

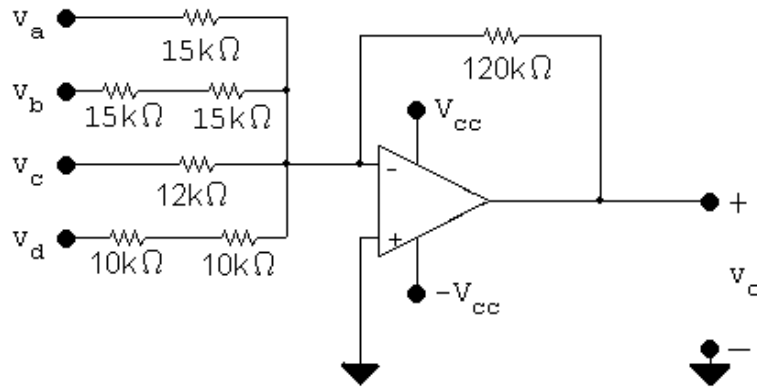
$$v_o = - \left[\frac{120\text{k}}{R_a} v_a + \frac{120\text{k}}{R_b} v_b + \frac{120\text{k}}{R_c} v_c + \frac{120\text{k}}{R_d} v_d \right]$$

Solve for each input resistance value to yield the desired gain:

$$\therefore \quad R_a = 120,000/8 = 15 \text{ k}\Omega \quad R_c = 120,000/10 = 12 \text{ k}\Omega$$

$$R_b = 120,000/4 = 30 \text{ k}\Omega \quad R_d = 120,000/6 = 20 \text{ k}\Omega$$

Now create the 5 resistor values needed from the realistic resistor values in Appendix H. Note that $R_f = 120\text{ k}\Omega$, $R_a = 15\text{ k}\Omega$, and $R_c = 12\text{ k}\Omega$ are already values from Appendix H. Create $R_b = 30\text{ k}\Omega$ by combining two $15\text{ k}\Omega$ resistors in series. Create $R_d = 20\text{ k}\Omega$ by combining two $10\text{ k}\Omega$ resistors in series. Of course there are many other acceptable possibilities. The final circuit is shown here:



- P 5.18 [a] The circuit shown is a non-inverting amplifier.
 [b] We assume the op amp to be ideal, so $v_n = v_p = 2\text{ V}$. Write a KCL equation at v_n :

$$\frac{2}{25,000} + \frac{2 - v_o}{150,000} = 0$$

Solving,

$$v_o = 14\text{ V}.$$

- P 5.19 [a] This circuit is an example of the non-inverting amplifier.
 [b] Use voltage division to calculate v_p :

$$v_p = \frac{8000}{8000 + 32,000} v_s = \frac{v_s}{5}$$

Write a KCL equation at $v_n = v_p = v_s/5$:

$$\frac{v_s/5}{7000} + \frac{v_s/5 - v_o}{56,000} = 0$$

Solving,

$$v_o = 8v_s/5 + v_s/5 = 1.8v_s$$

- [c] $1.8v_s = 12$ so $v_s = 6.67\text{ V}$

$$1.8v_s = -15 \quad \text{so} \quad v_s = -8.33\text{ V}$$

Thus, $-8.33\text{ V} \leq v_s \leq 6.67\text{ V}$.

P 5.20 [a] $v_p = v_n = \frac{68}{80}v_g = 0.85v_g$

$$\therefore \frac{0.85v_g}{30,000} + \frac{0.85v_g - v_o}{63,000} = 0;$$

$$\therefore v_o = 2.635v_g = 2.635(4), \quad v_o = 10.54 \text{ V}$$

[b] $v_o = 2.635v_g = \pm 12$

$$v_g = \pm 4.55 \text{ V}, \quad -4.55 \leq v_g \leq 4.55 \text{ V}$$

[c] $\frac{0.85v_g}{30,000} + \frac{0.85v_g - v_o}{R_f} = 0$

$$\left(\frac{0.85R_f}{30,000} + 0.85 \right) v_g = v_o = \pm 12$$

$$\therefore 1.7R_f + 51 = \pm 360; \quad 1.7R_f = 360 - 51; \quad R_f = 181.76 \text{ k}\Omega$$

P 5.21 [a] From Eq. 5.18,

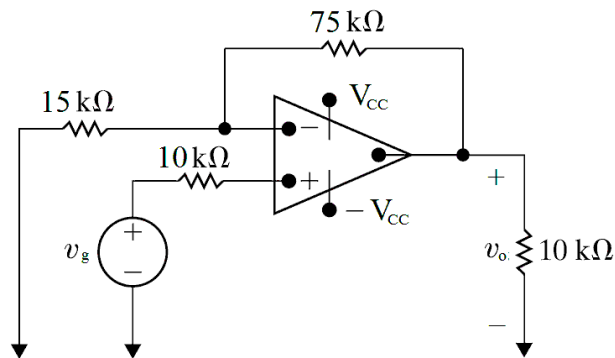
$$v_o = \frac{R_s + R_f}{R_s}v_g \quad \text{so} \quad \frac{v_o}{v_g} = 1 + \frac{R_f}{R_s} = 6$$

So,

$$\frac{R_f}{R_s} = 5$$

Thus,

$$R_s = \frac{R_f}{5} = \frac{75,000}{5} = 15 \text{ k}\Omega$$



[b] $v_o = 6v_g$

When $v_g = -2.5 \text{ V}$, $v_o = 6(-2.5) = -15 \text{ V}$.

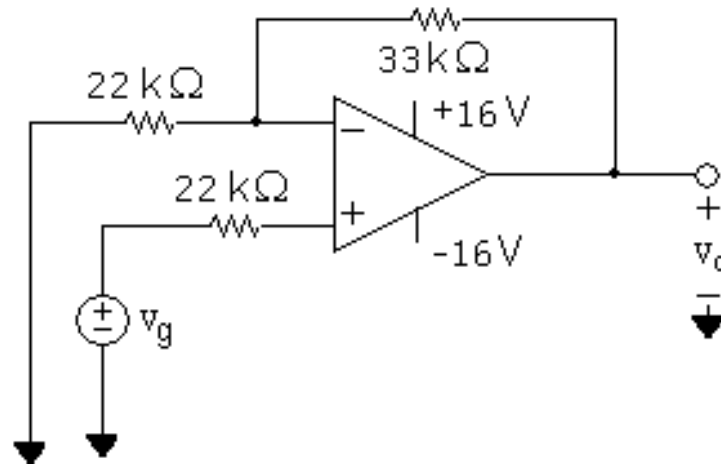
When $v_g = 1.5 \text{ V}$, $v_o = 6(1.5) = 9 \text{ V}$.

The power supplies can be set at 9 V and -15 V.

P 5.22 [a] From the equation for the non-inverting amplifier,

$$\frac{R_s + R_f}{R_s} = 2.5 \quad \text{so} \quad R_s + R_f = 2.5R_s \quad \text{and therefore} \quad R_f = 1.5R_s$$

Choose $R_s = 22 \text{ k}\Omega$, which is a component in Appendix H. Then $R_f = (1.5)(22) = 33 \text{ k}\Omega$, which is also a resistor value in Appendix H. The resulting non-inverting amplifier circuit is shown here:



[b] $v_o = 2.5v_g = 16$ so $v_g = 6.4 \text{ V}$

$$v_o = 2.5v_g = -16 \quad \text{so} \quad v_g = -6.4 \text{ V}$$

Therefore,

$$-6.4 \text{ V} \leq v_g \leq 6.4 \text{ V}$$

P 5.23 [a] This circuit is an example of a non-inverting summing amplifier.

[b] Write a KCL equation at v_p and solve for v_p in terms of v_s :

$$\frac{v_p - 5}{16,000} + \frac{v_p - v_s}{24,000} = 0$$

$$3v_p - 15 + 2v_p - 2v_s = 0 \quad \text{so} \quad v_p = 2v_s/5 + 3$$

Now write a KCL equation at v_n and solve for v_o :

$$\frac{v_n}{24,000} + \frac{v_n - v_o}{96,000} = 0 \quad \text{so} \quad v_o = 5v_n$$

Since we assume the op amp is ideal, $v_n = v_p$. Thus,

$$v_o = 5(2v_s/5 + 3) = 2v_s + 15$$

[c] $2v_s + 15 = 10$ so $v_s = -2.5 \text{ V}$

$$2v_s + 15 = -10 \quad \text{so} \quad v_s = -12.5 \text{ V}$$

Thus, $-12.5 \text{ V} \leq v_s \leq -2.5 \text{ V}$.

$$\text{P 5.24 [a]} \quad \frac{v_p - v_a}{R_a} + \frac{v_p - v_b}{R_b} + \frac{v_p - v_c}{R_c} = 0$$

$$\therefore v_p = \frac{R_b R_c}{D} v_a + \frac{R_a R_c}{D} v_b + \frac{R_a R_b}{D} v_c$$

$$\text{where } D = R_b R_c + R_a R_c + R_a R_b$$

$$\frac{v_n}{20,000} + \frac{v_n - v_o}{100,000} = 0$$

$$\left(\frac{100,000}{20,000} + 1 \right) v_n = 6v_n = v_o$$

$$\therefore v_o = \frac{6R_b R_c}{D} v_a + \frac{6R_a R_c}{D} v_b + \frac{6R_a R_b}{D} v_c$$

By hypothesis,

$$\frac{6R_b R_c}{D} = 1; \quad \frac{6R_a R_c}{D} = 2; \quad \frac{6R_a R_b}{D} = 3$$

Then

$$\frac{6R_a R_b / D}{6R_a R_c / D} = \frac{3}{2} \quad \text{so} \quad R_b = 1.5R_c$$

But from the circuit

$$R_b = 15 \text{ k}\Omega \quad \text{so} \quad R_c = 10 \text{ k}\Omega$$

Similarly,

$$\frac{6R_b R_c / D}{6R_a R_b / D} = \frac{1}{3} \quad \text{so} \quad 3R_c = R_a$$

Thus,

$$R_a = 30 \text{ k}\Omega$$

$$\text{[b]} \quad v_o = 1(0.7) + 2(0.4) + 3(1.1) = 4.8 \text{ V}$$

$$v_n = v_o / 6 = 0.8 \text{ V} = v_p$$

$$i_a = \frac{v_a - v_p}{30,000} = \frac{0.7 - 0.8}{30,000} = -3.33 \mu\text{A}$$

$$i_b = \frac{v_b - v_p}{15,000} = \frac{0.4 - 0.8}{15,000} = -26.67 \mu\text{A}$$

$$i_c = \frac{v_c - v_p}{10,000} = \frac{1.1 - 0.8}{10,000} = 30 \mu\text{A}$$

Check:

$$i_a + i_b + i_c = 0? \quad -3.33 - 26.67 + 30 = 0 \text{ (checks)}$$

- P 5.25 [a] Assume v_a is acting alone. Replacing v_b with a short circuit yields $v_p = 0$, therefore $v_n = 0$ and we have

$$\frac{0 - v_a}{R_a} + \frac{0 - v'_o}{R_b} + i_n = 0, \quad i_n = 0$$

Therefore

$$\frac{v'_o}{R_b} = -\frac{v_a}{R_a}, \quad v'_o = -\frac{R_b}{R_a}v_a$$

Assume v_b is acting alone. Replace v_a with a short circuit. Now

$$v_p = v_n = \frac{v_b R_d}{R_c + R_d}$$

$$\frac{v_n}{R_a} + \frac{v_n - v''_o}{R_b} + i_n = 0, \quad i_n = 0$$

$$\left(\frac{1}{R_a} + \frac{1}{R_b}\right) \left(\frac{R_d}{R_c + R_d}\right) v_b - \frac{v''_o}{R_b} = 0$$

$$v''_o = \left(\frac{R_b}{R_a} + 1\right) \left(\frac{R_d}{R_c + R_d}\right) v_b = \frac{R_d}{R_a} \left(\frac{R_a + R_b}{R_c + R_d}\right) v_b$$

$$v_o = v'_o + v''_o = \frac{R_d}{R_a} \left(\frac{R_a + R_b}{R_c + R_d}\right) v_b - \frac{R_b}{R_a} v_a$$

$$[b] \frac{R_d}{R_a} \left(\frac{R_a + R_b}{R_c + R_d}\right) = \frac{R_b}{R_a}, \quad \text{therefore } R_d(R_a + R_b) = R_b(R_c + R_d)$$

$$R_d R_a = R_b R_c, \quad \text{therefore } \frac{R_a}{R_b} = \frac{R_c}{R_d}$$

$$\text{When } \frac{R_d}{R_a} \left(\frac{R_a + R_b}{R_c + R_d}\right) = \frac{R_b}{R_a}$$

$$\text{Eq. (5.22) reduces to } v_o = \frac{R_b}{R_a} v_b - \frac{R_b}{R_a} v_a = \frac{R_b}{R_a} (v_b - v_a).$$

- P 5.26 [a] This is a difference amplifier circuit.

- [b] Use Eq. 5.22 with $R_a = 5 \text{ k}\Omega$, $R_b = 20 \text{ k}\Omega$, $R_c = 8 \text{ k}\Omega$, $R_d = 2 \text{ k}\Omega$, and $v_b = 5 \text{ V}$:

$$v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)} v_b - \frac{R_b}{R_a} v_a = \frac{2(5 + 20)}{5(8 + 2)} (5) - \frac{20}{5} v_a = 5 - 4v_a$$

$$[c] \frac{2000(5000 + R_f)}{5000(8000 + 2000)} (5) - \frac{R_f}{5000} (2) = \frac{5000 + R_f}{5000} - \frac{2R_f}{5000} = 1 - \frac{R_f}{5000}$$

$$1 - \frac{R_f}{5000} = 10 \quad \text{so} \quad R_f < 0 \text{ which is not a possible solution.}$$

$$1 - \frac{R_f}{5000} = -10 \quad \text{so} \quad R_f = 5000(11) = 55 \text{ k}\Omega$$

$$\text{P 5.27 [a]} \quad v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \frac{R_b}{R_a}v_a = \frac{120(24 + 75)}{24(130 + 120)}(5) - \frac{75}{24}(8)$$

$$v_o = 9.9 - 25 = -15.1 \text{ V}$$

$$\text{[b]} \quad \frac{v_1 - 8}{24,000} + \frac{v_1 + 15.1}{75,000} = 0 \quad \text{so} \quad v_1 = 2.4 \text{ V}$$

$$i_a = \frac{8 - 2.4}{24,000} = 233 \mu \text{ A}$$

$$R_{\text{ina}} = \frac{v_a}{i_a} = \frac{8}{233 \times 10^{-6}} = 34.3 \text{ k}\Omega$$

$$\text{[c]} \quad R_{\text{inb}} = R_c + R_d = 250 \text{ k}\Omega$$

$$\text{P 5.28} \quad v_p = \frac{1500}{9000}(-18) = -3 \text{ V} = v_n$$

$$\frac{-3 + 18}{1600} + \frac{-3 - v_o}{R_f} = 0$$

$$\therefore v_o = 0.009375R_f - 3$$

$$v_o = 9 \text{ V}; \quad R_f = 1280 \Omega$$

$$v_o = -9 \text{ V}; \quad R_f = -640 \Omega$$

$$\text{But } R_f \geq 0, \quad \therefore R_f = 1.28 \text{ k}\Omega$$

$$\text{P 5.29} \quad v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \frac{R_b}{R_a}v_a$$

$$\text{By hypothesis: } R_b/R_a = 4; \quad R_c + R_d = 470 \text{ k}\Omega; \quad \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)} = 3$$

$$\therefore \frac{R_d(R_a + 4R_a)}{R_a \cdot 470,000} = 3 \quad \text{so} \quad R_d = 282 \text{ k}\Omega; \quad R_c = 188 \text{ k}\Omega$$

Create $R_d = 282 \text{ k}\Omega$ by combining a $270 \text{ k}\Omega$ resistor and a $12 \text{ k}\Omega$ resistor in series. Create $R_c = 188 \text{ k}\Omega$ by combining a $120 \text{ k}\Omega$ resistor and a $68 \text{ k}\Omega$ resistor in series. Also, when $v_o = 0$ we have

$$\frac{v_n - v_a}{R_a} + \frac{v_n}{R_b} = 0$$

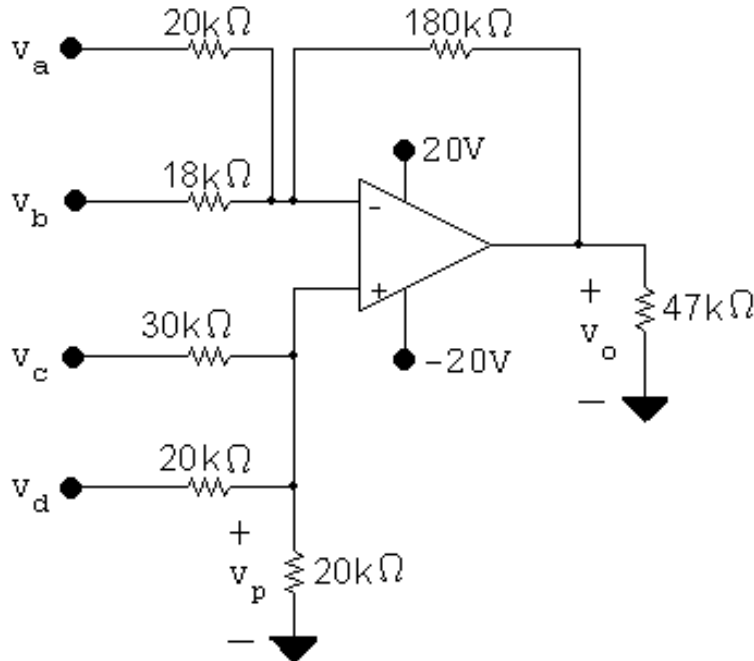
$$\therefore v_n \left(1 + \frac{R_a}{R_b} \right) = v_a; \quad v_n = 0.8v_a$$

$$i_a = \frac{v_a - 0.8v_a}{R_a} = 0.2 \frac{v_a}{R_a}; \quad R_{in} = \frac{v_a}{i_a} = 5R_a = 22 \text{ k}\Omega$$

$$\therefore R_a = 4.4 \text{ k}\Omega; \quad R_b = 17.6 \text{ k}\Omega$$

Create $R_a = 4.4 \text{ k}\Omega$ by combining two $2.2 \text{ k}\Omega$ resistors in series. Create $R_b = 17.6 \text{ k}\Omega$ by combining a $12 \text{ k}\Omega$ resistor and a $5.6 \text{ k}\Omega$ resistor in series.

P 5.30 [a]



$$\frac{v_p}{20,000} + \frac{v_p - v_c}{30,000} + \frac{v_p - v_d}{20,000} = 0$$

$$\therefore 8v_p = 2v_c + 3v_d = 8v_n$$

$$\frac{v_n - v_a}{20,000} + \frac{v_n - v_b}{18,000} + \frac{v_n - v_o}{180,000} = 0$$

$$\begin{aligned} \therefore v_o &= 20v_n - 9v_a - 10v_b \\ &= 20[(1/4)v_c + (3/8)v_d] - 9v_a - 10v_b \\ &= 20(0.75 + 1.5) - 9(1) - 10(2) = 16 \text{ V} \end{aligned}$$

[b] $v_o = 5v_c + 30 - 9 - 20 = 5v_c + 1$

$$\pm 20 = 5v_c + 1$$

$$\therefore v_b = -4.2 \text{ V} \quad \text{and} \quad v_b = 3.8 \text{ V}$$

$$\therefore -4.2 \text{ V} \leq v_b \leq 3.8 \text{ V}$$

P 5.31 $v_p = R_b i_b = v_n$

$$\frac{R_b i_b}{2000} + \frac{R_b i_b - v_o}{R_f} - i_a = 0$$

$$\therefore R_b i_b \left(\frac{1}{2000} + \frac{1}{R_f} \right) - i_a = \frac{v_o}{R_f}$$

$$\therefore R_b i_b \left(1 + \frac{R_f}{2000} \right) - R_f i_a = v_o$$

By hypothesis, $v_o = 8000(i_b - i_a)$. Therefore,

$$R_f = 8 \text{ k}\Omega \quad (\text{use } 3.3 \text{ k}\Omega \text{ and } 4.7 \text{ k}\Omega \text{ resistors in series})$$

$$R_b \left(1 + \frac{8000}{2000} \right) = 8000 \quad \text{so} \quad R_b = 1.6 \text{ k}\Omega$$

To construct the 1.6 kΩ resistor, combine 270 Ω, 330 Ω, and 1 kΩ resistors in series.

P 5.32 [a]
$$v_p = \frac{\alpha R_g}{\alpha R_g + (R_g - \alpha R_g)} v_g \quad v_o = \left(1 + \frac{R_f}{R_1} \right) \alpha v_g - \frac{R_f}{R_1} v_g$$

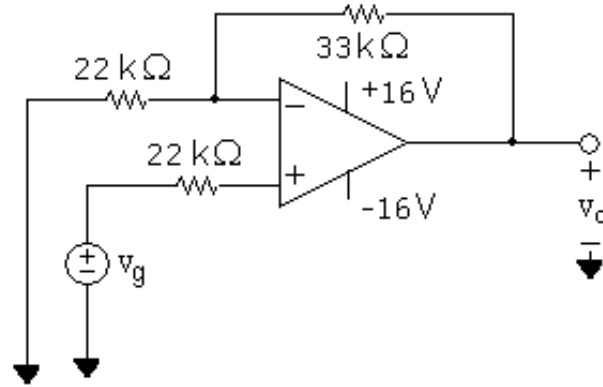
$$v_n = v_p = \alpha v_g \quad = (\alpha v_g - v_g) 4 + \alpha v_g$$

$$\frac{v_n - v_g}{R_1} + \frac{v_n - v_o}{R_f} = 0 \quad = [(\alpha - 1) 4 + \alpha] v_g$$

$$(v_n - v_g) \frac{R_f}{R_1} + v_n - v_o = 0 \quad = (5\alpha - 4) v_g$$

$$= (5\alpha - 4)(2) = 10\alpha - 8$$

α	v_o	α	v_o	α	v_o
0.0	-8 V	0.4	-4 V	0.8	0 V
0.1	-7 V	0.5	-3 V	0.9	1 V
0.2	-6 V	0.6	-2 V	1.0	2 V
0.3	-5 V	0.7	-1 V		



[b] Rearranging the equation for v_o from (a) gives

$$v_o = \left(\frac{R_f}{R_1} + 1\right) v_g \alpha + -\left(\frac{R_f}{R_1}\right) v_g$$

Therefore,

$$\text{slope} = \left(\frac{R_f}{R_1} + 1\right) v_g; \quad \text{intercept} = -\left(\frac{R_f}{R_1}\right) v_g$$

[c] Using the equations from (b),

$$-6 = \left(\frac{R_f}{R_1} + 1\right) v_g; \quad 4 = -\left(\frac{R_f}{R_1}\right) v_g$$

Solving,

$$v_g = -2 \text{ V}; \quad \frac{R_f}{R_1} = 2$$

P 5.33 [a] $A_{\text{dm}} = \frac{(24)(26) + (25)(25)}{(2)(1)(25)} = 24.98$

[b] $A_{\text{cm}} = \frac{(1)(24) - 25(1)}{1(25)} = -0.04$

[c] $\text{CMRR} = \left| \frac{24.98}{0.04} \right| = 624.50$

P 5.34 $A_{\text{cm}} = \frac{(3000)(6000) - (6000)R_x}{3000(6000 + R_x)}$

$$A_{\text{dm}} = \frac{6000(3000 + 6000) + 6000(6000 + R_x)}{2(3000)(6000 + R_x)}$$

$$\frac{A_{\text{dm}}}{A_{\text{cm}}} = \frac{R_x + 15,000}{2(3000 - R_x)}$$

$$\therefore \frac{R_x + 15,000}{2(3000 - R_x)} = \pm 1500 \quad \text{for the limits on the value of } R_x$$

If we use +1500 $R_x = 2994 \Omega$

If we use -1500 $R_x = 3006 \Omega$

$$2994 \Omega \leq R_x \leq 3006 \Omega$$

P 5.35 It follows directly from the circuit that $v_o = -(75/15)v_g = -5v_g$
From the plot of v_g we have $v_g = 0, \quad t < 0$

$$v_g = t \quad 0 \leq t \leq 2$$

$$v_g = 4 - t \quad 2 \leq t \leq 6$$

$$v_g = t - 8 \quad 6 \leq t \leq 10$$

$$v_g = 12 - t \quad 10 \leq t \leq 14$$

$$v_g = t - 16 \quad 14 \leq t \leq 18, \quad \text{etc.}$$

Therefore

$$v_o = -5t \quad 0 \leq t \leq 2$$

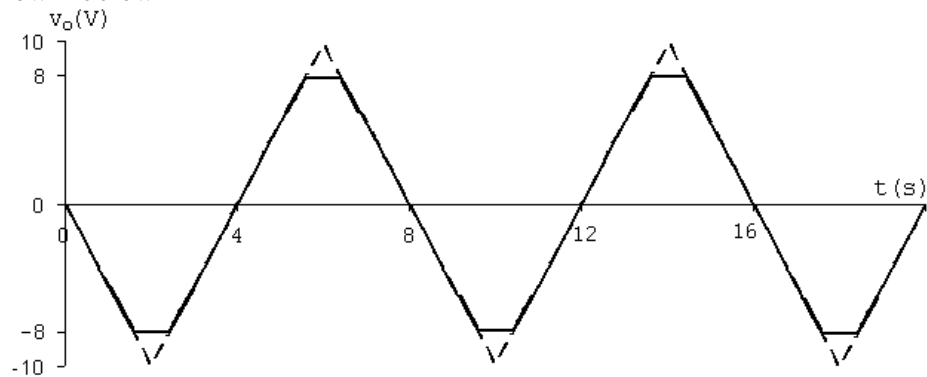
$$v_o = 5t - 20 \quad 2 \leq t \leq 6$$

$$v_o = 40 - 5t \quad 6 \leq t \leq 10$$

$$v_o = 5t - 60 \quad 10 \leq t \leq 14$$

$$v_o = 80 - 5t \quad 14 \leq t \leq 18, \quad \text{etc.}$$

These expressions for v_o are valid as long as the op amp is not saturated. Since the peak values of v_o are ± 9 , the output is clipped at ± 9 . The plot is shown below.



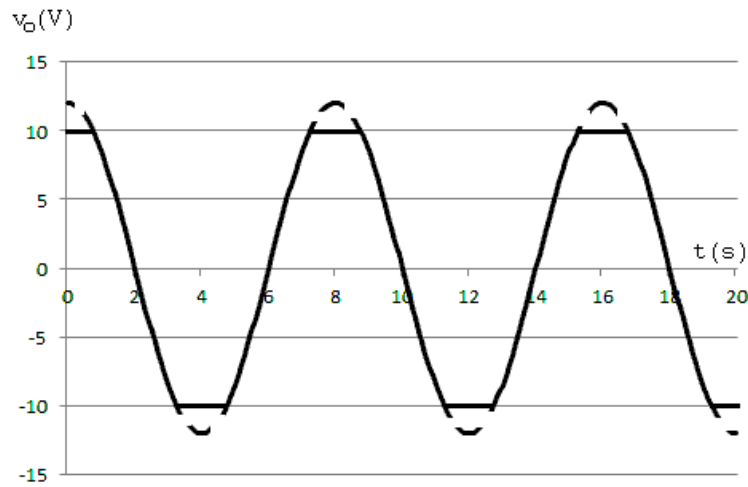
P 5.36 $v_p = \frac{5.4}{7.2}v_g = 0.75v_g = 3 \cos(\pi/4)t \text{ V}$

$$\frac{v_n}{20,000} + \frac{v_n - v_o}{60,000} = 0$$

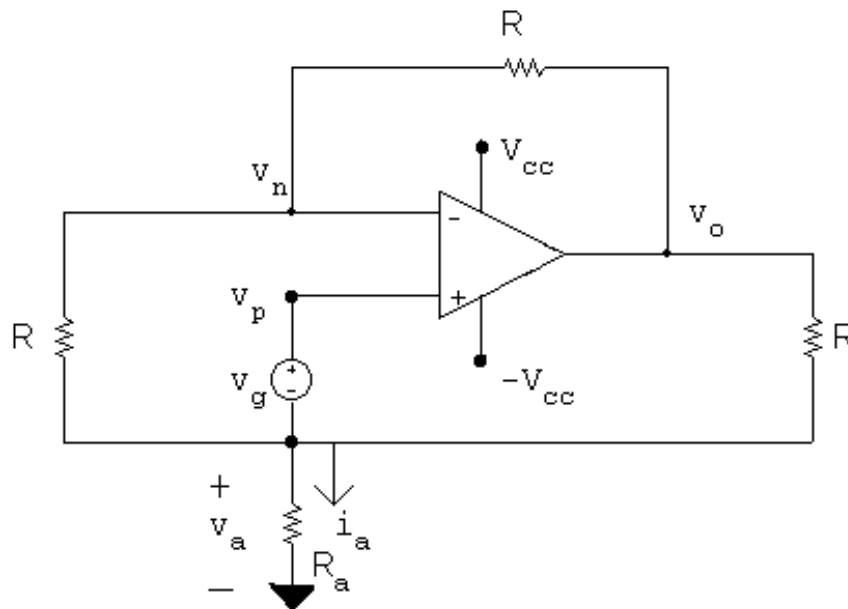
$$4v_n = v_o; \quad v_n = v_p$$

$$\therefore v_o = 12 \cos(\pi/4)t \text{ V} \quad 0 \leq t \leq \infty$$

but saturation occurs at $v_o = \pm 10 \text{ V}$



P 5.37 [a]



$$\frac{v_n - v_a}{R} + \frac{v_n - v_o}{R} = 0$$

$$2v_n - v_a = v_o$$

$$\frac{v_a}{R_a} + \frac{v_a - v_n}{R} + \frac{v_a - v_o}{R} = 0$$

$$v_a \left[\frac{1}{R_a} + \frac{2}{R} \right] - \frac{v_n}{R} = \frac{v_o}{R}$$

$$v_a \left(2 + \frac{R}{R_a} \right) - v_n = v_o$$

$$v_n = v_p = v_a + v_g$$

$$\therefore 2v_n - v_a = 2v_a + 2v_g - v_a = v_a + 2v_g$$

$$\therefore v_a - v_o = -2v_g \quad (1)$$

$$2v_a + v_a \left(\frac{R}{R_a} \right) - v_a - v_g = v_o$$

$$\therefore v_a \left(1 + \frac{R}{R_a} \right) - v_o = v_g \quad (2)$$

Now combining equations (1) and (2) yields

$$-v_a \frac{R}{R_a} = -3v_g$$

$$\text{or } v_a = 3v_g \frac{R_a}{R}$$

$$\text{Hence } i_a = \frac{v_a}{R_a} = \frac{3v_g}{R} \quad \text{Q.E.D.}$$

[b] At saturation $v_o = \pm V_{cc}$

$$\therefore v_a = \pm V_{cc} - 2v_g \quad (3)$$

and

$$\therefore v_a \left(1 + \frac{R}{R_a} \right) = \pm V_{cc} + v_g \quad (4)$$

Dividing Eq (4) by Eq (3) gives

$$1 + \frac{R}{R_a} = \frac{\pm V_{cc} + v_g}{\pm V_{cc} - 2v_g}$$

$$\therefore \frac{R}{R_a} = \frac{\pm V_{cc} + v_g}{\pm V_{cc} - 2v_g} - 1 = \frac{3v_g}{\pm V_{cc} - 2v_g}$$

$$\text{or } R_a = \frac{(\pm V_{cc} - 2v_g)}{3v_g} R \quad \text{Q.E.D.}$$

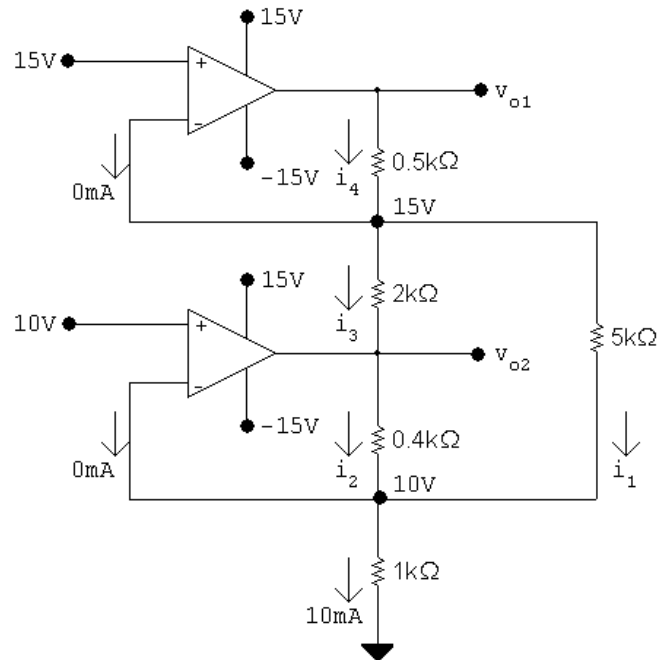
P 5.38 [a] $v_p = v_s, \quad v_n = \frac{R_1 v_o}{R_1 + R_2}, \quad v_n = v_p$

$$\text{Therefore } v_o = \left(\frac{R_1 + R_2}{R_1} \right) v_s = \left(1 + \frac{R_2}{R_1} \right) v_s$$

[b] $v_o = v_s$

[c] Because $v_o = v_s$, thus the output voltage follows the signal voltage.

P 5.39



$$i_1 = \frac{15 - 10}{5000} = 1 \text{ mA}$$

$$i_2 + i_1 + 0 = 10 \text{ mA}; \quad i_2 = 9 \text{ mA}$$

$$v_{o2} = 10 + (400)(9) \times 10^{-3} = 13.6 \text{ V}$$

$$i_3 = \frac{15 - 13.6}{2000} = 0.7 \text{ mA}$$

$$i_4 = i_3 + i_1 = 1.7 \text{ mA}$$

$$v_{o1} = 15 + 1.7(0.5) = 15.85 \text{ V}$$

P 5.40 [a] $p_{16\text{k}\Omega} = \frac{(320 \times 10^{-3})^2}{(16 \times 10^3)} = 6.4 \mu\text{W}$

[b] $v_{16\text{k}\Omega} = \left(\frac{16}{64}\right)(320) = 80 \text{ mV}$

$$p_{16\text{k}\Omega} = \frac{(80 \times 10^{-3})^2}{(16 \times 10^3)} = 0.4 \mu\text{W}$$

[c] $\frac{p_a}{p_b} = \frac{6.4}{0.4} = 16$

[d] Yes, the operational amplifier serves several useful purposes:

- First, it enables the source to control 16 times as much power delivered to the load resistor. When a small amount of power controls a larger amount of power, we refer to it as *power amplification*.
- Second, it allows the full source voltage to appear across the load resistor, no matter what the source resistance. This is the *voltage follower* function of the operational amplifier.
- Third, it allows the load resistor voltage (and thus its current) to be set without drawing any current from the input voltage source. This is the *current amplification* function of the circuit.

P 5.41 [a] Let v_{o1} = output voltage of the amplifier on the left. Let v_{o2} = output voltage of the amplifier on the right. Then

$$v_{o1} = \frac{-47}{10}(1) = -4.7 \text{ V}; \quad v_{o2} = \frac{-220}{33}(-0.15) = 1.0 \text{ V}$$

$$i_a = \frac{v_{o2} - v_{o1}}{1000} = 5.7 \text{ mA}$$

[b] $i_a = 0$ when $v_{o1} = v_{o2}$ so from (a) $v_{o2} = 1 \text{ V}$

Thus

$$\frac{-47}{10}(v_L) = 1$$

$$v_L = -\frac{10}{47} = -212.77 \text{ mV}$$

P 5.42 [a] Assume the op-amp is operating within its linear range, then

$$i_L = \frac{8}{4000} = 2 \text{ mA}$$

$$\text{For } R_L = 4 \text{ k}\Omega \quad v_o = (4 + 4)(2) = 16 \text{ V}$$

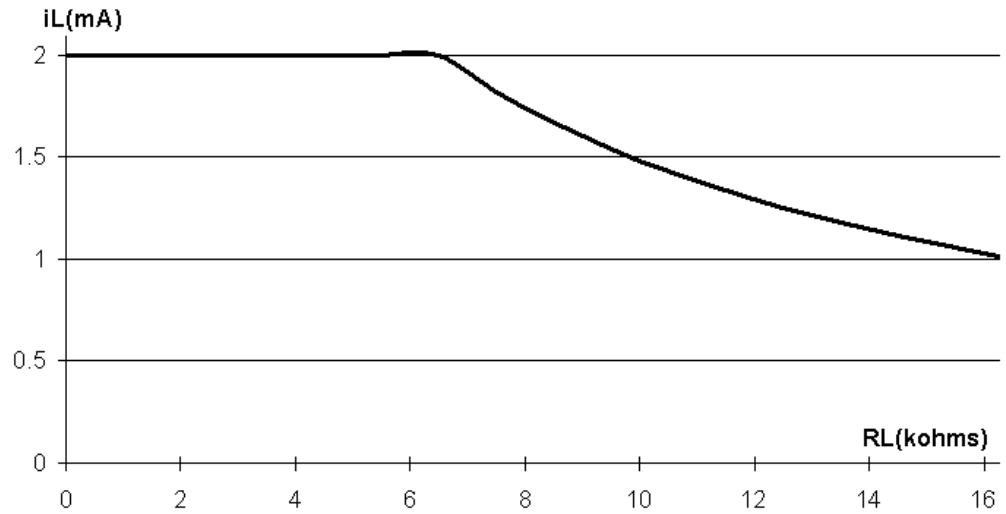
Now since $v_o < 20 \text{ V}$ our assumption of linear operation is correct, therefore

$$i_L = 2 \text{ mA}$$

[b] $20 = 2(4 + R_L)$; $R_L = 6 \text{ k}\Omega$

[c] As long as the op-amp is operating in its linear region i_L is independent of R_L . From (b) we found the op-amp is operating in its linear region as long as $R_L \leq 6 \text{ k}\Omega$. Therefore when $R_L = 6 \text{ k}\Omega$ the op-amp is saturated. We can estimate the value of i_L by assuming $i_p = i_n \ll i_L$. Then $i_L = 20/(4000 + 16,000) = 1 \text{ mA}$. To justify neglecting the current into the op-amp assume the drop across the $50 \text{ k}\Omega$ resistor is negligible, since the input resistance to the op-amp is at least $500 \text{ k}\Omega$. Then $i_p = i_n = (8 - 4)/(500 \times 10^3) = 8 \mu\text{A}$. But $8 \mu\text{A} \ll 1 \text{ mA}$, hence our assumption is reasonable.

[d]



P 5.43 From Eq. 5.57,

$$\frac{v_{\text{ref}}}{R + \Delta R} = v_n \left(\frac{1}{R + \Delta R} + \frac{1}{R - \Delta R} + \frac{1}{R_f} \right) - \frac{v_o}{R_f}$$

Substituting Eq. 5.59 for $v_p = v_n$:

$$\frac{v_{\text{ref}}}{R + \Delta R} = \frac{v_{\text{ref}} \left(\frac{1}{R + \Delta R} + \frac{1}{R - \Delta R} + \frac{1}{R_f} \right)}{(R - \Delta R) \left(\frac{1}{R + \Delta R} + \frac{1}{R - \Delta R} + \frac{1}{R_f} \right)} - \frac{v_o}{R_f}$$

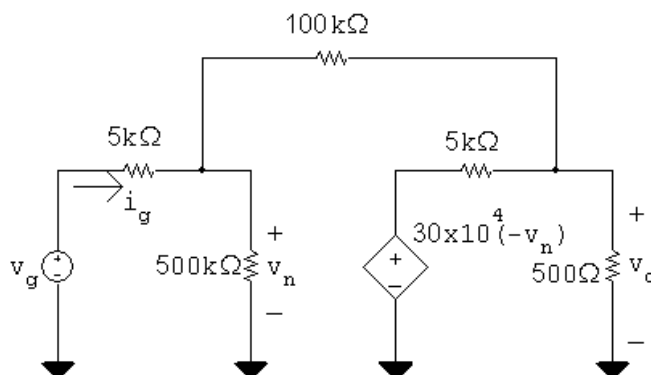
Rearranging,

$$\frac{v_o}{R_f} = v_{\text{ref}} \left(\frac{1}{R - \Delta R} - \frac{1}{R + \Delta R} \right)$$

Thus,

$$v_o = v_{\text{ref}} \left(\frac{2\Delta R}{R^2 - (\Delta R)^2} \right) R_f$$

P 5.44 [a] Replace the op amp with the model from Fig. 5.15:



Write two node voltage equations, one at the left node, the other at the right node:

$$\frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100,000} + \frac{v_n}{500,000} = 0$$

$$\frac{v_o + 3 \times 10^5 v_n}{5000} + \frac{v_o - v_n}{100,000} + \frac{v_o}{500} = 0$$

Simplify and place in standard form:

$$106v_n - 5v_o = 100v_g$$

$$(6 \times 10^6 - 1)v_n + 221v_o = 0$$

Let $v_g = 1$ V and solve the two simultaneous equations:

$$v_o = -19.9844 \text{ V}; \quad v_n = 736.1 \mu\text{V}$$

Thus the voltage gain is $v_o/v_g = -19.9844$.

[b] From the solution in part (a), $v_n = 736.1 \mu\text{V}$.

[c]
$$i_g = \frac{v_g - v_n}{5000} = \frac{v_g - 736.1 \times 10^{-6} v_g}{5000}$$

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 736.1 \times 10^{-6}} = 5003.68 \Omega$$

[d] For an ideal op amp, the voltage gain is the ratio between the feedback resistor and the input resistor:

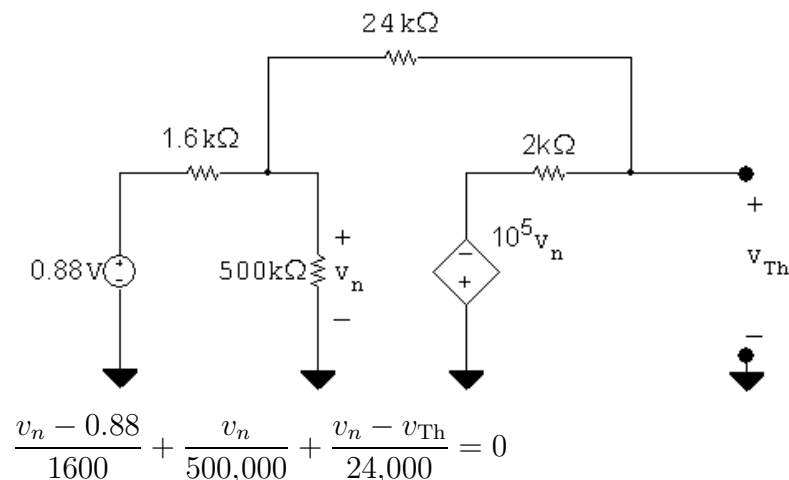
$$\frac{v_o}{v_g} = -\frac{100,000}{5000} = -20$$

For an ideal op amp, the difference between the voltages at the input terminals is zero, and the input resistance of the op amp is infinite.

Therefore,

$$v_n = v_p = 0 \text{ V}; \quad R_g = 5000 \Omega$$

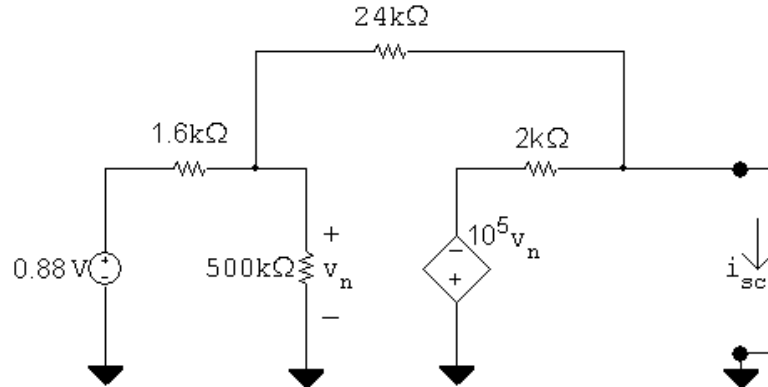
P 5.45 [a]



$$\frac{v_{Th} + 10^5 v_n}{2000} + \frac{v_{Th} - v_n}{24,000} = 0$$

Solving, $v_{Th} = -13.198 \text{ V}$

Short-circuit current calculation:

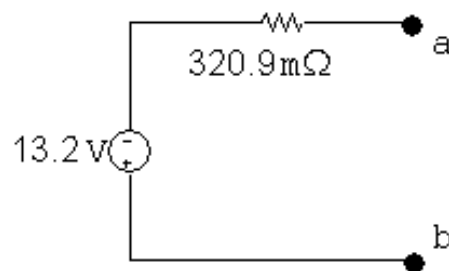


$$\frac{v_n}{500,000} + \frac{v_n - 0.88}{1600} + \frac{v_n - 0}{24,000} = 0$$

$$\therefore v_n = 0.8225 \text{ V}$$

$$i_{sc} = \frac{v_n}{24,000} - \frac{10^5}{2000} v_n = -41.13 \text{ A}$$

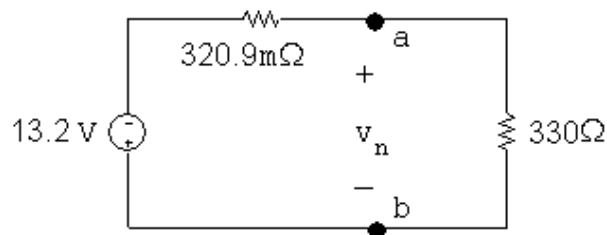
$$R_{Th} = \frac{v_{Th}}{i_{sc}} = 320.9 \text{ m}\Omega$$



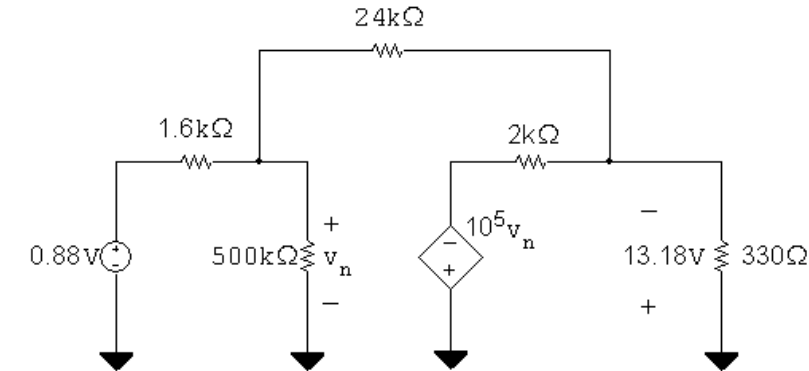
[b] The output resistance of the inverting amplifier is the same as the Thévenin resistance, i.e.,

$$R_o = R_{Th} = 320.9 \text{ m}\Omega$$

[c]



$$v_o = \left(\frac{330}{330.3209} \right) (-13.2) = -13.18 \text{ V}$$



$$\frac{v_n - 0.88}{1600} + \frac{v_n}{500,000} + \frac{v_n + 13.18}{24,000} = 0$$

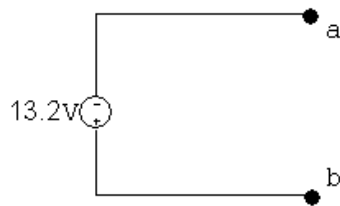
$$\therefore v_n = 942 \mu\text{V}$$

$$i_g = \frac{0.88 - 942 \times 10^{-6}}{1600} = 549.41 \mu\text{A}$$

$$R_g = \frac{0.88}{i_g} = 1601.71 \Omega$$

P 5.46 [a] $v_{\text{Th}} = -\frac{24,000}{1600}(0.88) = -13.2 \text{ V}$

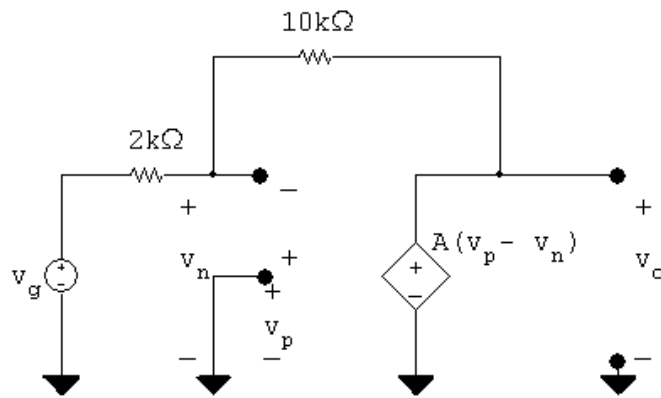
$R_{\text{Th}} = 0$, since op-amp is ideal



[b] $R_o = R_{\text{Th}} = 0 \Omega$

[c] $R_g = 1.6 \text{ k}\Omega$ since $v_n = 0$

P 5.47 [a]



$$\frac{v_n - v_g}{2000} + \frac{v_n - v_o}{10,000} = 0$$

$$\therefore v_o = 6v_n - 5v_g$$

$$\text{Also } v_o = A(v_p - v_n) = -Av_n$$

$$\therefore v_n = \frac{-v_o}{A}$$

$$\therefore v_o \left(1 + \frac{6}{A}\right) = -5v_g$$

$$v_o = \frac{-5A}{(6 + A)}v_g$$

$$[\mathbf{b}] \quad v_o = \frac{-5(150)(1)}{156} = -4.81 \text{ V}$$

$$[\mathbf{c}] \quad v_o = \frac{-5}{1 + (6/A)}(1) = -5 \text{ V}$$

$$[\mathbf{d}] \quad \frac{-5A}{A+6}(1) = -0.99(5) \quad \text{so} \quad -5A = -4.95(A+6)$$

$$\therefore -0.05A = -29.7 \quad \text{so} \quad A = 594$$

$$\text{P 5.48 } [\mathbf{a}] \quad \frac{v_n}{16,000} + \frac{v_n - v_g}{800,000} + \frac{v_n - v_o}{200,000} = 0 \quad \text{or} \quad 55v_n - 4v_o = v_g \quad \text{Eq (1)}$$

$$\frac{v_o}{20,000} + \frac{v_o - v_n}{200,000} + \frac{v_o - 50,000(v_p - v_n)}{8000} = 0$$

$$36v_o - v_n - 125 \times 10^4(v_p - v_n) = 0$$

$$v_p = v_g + \frac{(v_n - v_g)(240)}{800} = (0.7)v_g + (0.3)v_n$$

$$36v_o - v_n - 125 \times 10^4[(0.7)v_g - (0.7)v_n] = 0$$

$$36v_o + 874,999v_n = 875,000v_g \quad \text{Eq (2)}$$

Let $v_g = 1 \text{ V}$ and solve Eqs. (1) and (2) simultaneously:

$$v_n = 999.446 \text{ mV} \quad \text{and} \quad v_o = 13.49 \text{ V}$$

$$\therefore \frac{v_o}{v_g} = 13.49$$

[\mathbf{b}] From part (a), $v_n = 999.446 \text{ mV}$.

$$v_p = (0.7)(1000) + (0.3)(999.446) = 999.834 \text{ mV}$$

[\mathbf{c}] $v_p - v_n = 387.78 \mu\text{V}$

$$[\mathbf{d}] \quad i_g = \frac{(1000 - 999.83)10^{-3}}{24 \times 10^3} = 692.47 \text{ pA}$$

$$[e] \frac{v_g}{16,000} + \frac{v_g - v_o}{200,000} = 0, \quad \text{since } v_n = v_p = v_g$$

$$\therefore v_o = 13.5v_g, \quad \frac{v_o}{v_g} = 13.5$$

$$v_n = v_p = 1 \text{ V}; \quad v_p - v_n = 0 \text{ V}; \quad i_g = 0 \text{ A}$$

P 5.49 [a] Use Eq. 5.61 to solve for R_f ; note that since we are using 1% strain gages, $\Delta = 0.01$:

$$R_f = \frac{v_o R}{2\Delta v_{\text{ref}}} = \frac{(5)(120)}{(2)(0.01)(15)} = 2 \text{ k}\Omega$$

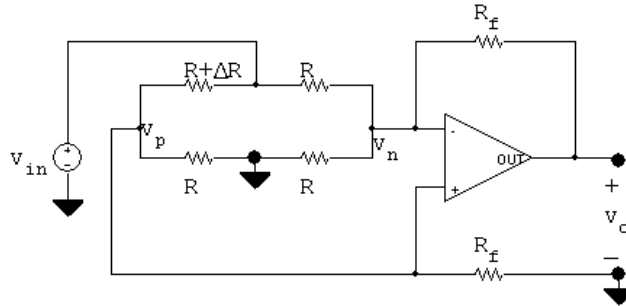
[b] Now solve for Δ given $v_o = 50 \text{ mV}$:

$$\Delta = \frac{v_o R}{2R_f v_{\text{ref}}} = \frac{(0.05)(120)}{2(2000)(15)} = 100 \times 10^{-6}$$

The change in strain gage resistance that corresponds to a 50 mV change in output voltage is thus

$$\Delta R = \Delta R = (100 \times 10^{-6})(120) = 12 \text{ m}\Omega$$

P 5.50 [a]



Let $R_1 = R + \Delta R$

$$\frac{v_p}{R_f} + \frac{v_p}{R} + \frac{v_p - v_{\text{in}}}{R_1} = 0$$

$$\therefore v_p \left[\frac{1}{R_f} + \frac{1}{R} + \frac{1}{R_1} \right] = \frac{v_{\text{in}}}{R_1}$$

$$\therefore v_p = \frac{RR_f v_{\text{in}}}{RR_1 + R_f R_1 + R_f R} = v_n$$

$$\frac{v_n}{R} + \frac{v_n - v_{\text{in}}}{R} + \frac{v_n - v_o}{R_f} = 0$$

$$v_n \left[\frac{1}{R} + \frac{1}{R} + \frac{1}{R_f} \right] - \frac{v_o}{R_f} = \frac{v_{\text{in}}}{R}$$

$$\begin{aligned} \therefore v_n \left[\frac{R + 2R_f}{RR_f} \right] - \frac{v_{in}}{R} &= \frac{v_o}{R_f} \\ \therefore \frac{v_o}{R_f} &= \left[\frac{R + 2R_f}{RR_f} \right] \left[\frac{RR_f v_{in}}{RR_1 + R_f R_1 + R_f R} \right] - \frac{v_{in}}{R} \\ \therefore \frac{v_o}{R_f} &= \left[\frac{R + 2R_f}{RR_1 + R_f R_1 + R_f R} - \frac{1}{R} \right] v_{in} \\ \therefore v_o &= \frac{[R^2 + 2RR_f - R_1(R + R_f) - RR_f]R_f}{R[R_1(R + R_f) + RR_f]} v_{in} \end{aligned}$$

Now substitute $R_1 = R + \Delta R$ and get

$$v_o = \frac{-\Delta R(R + R_f)R_f v_{in}}{R[(R + \Delta R)(R + R_f) + RR_f]}$$

If $\Delta R \ll R$

$$v_o \approx \frac{(R + R_f)R_f(-\Delta R)v_{in}}{R^2(R + 2R_f)}$$

$$[b] \quad v_o \approx \frac{47 \times 10^4(48 \times 10^4)(-95)15}{10^8(95 \times 10^4)} \approx -3.384 \text{ V}$$

$$[c] \quad v_o = \frac{-95(48 \times 10^4)(47 \times 10^4)15}{10^4[(1.0095)10^4(48 \times 10^4) + 47 \times 10^8]} = -3.368 \text{ V}$$

P 5.51 [a] $v_o \approx \frac{(R + R_f)R_f(-\Delta R)v_{in}}{R^2(R + 2R_f)}$

$$v_o = \frac{(R + R_f)(-\Delta R)R_f v_{in}}{R[(R + \Delta R)(R + R_f) + RR_f]}$$

$$\therefore \frac{\text{approx value}}{\text{true value}} = \frac{R[(R + \Delta R)(R + R_f) + RR_f]}{R^2(R + 2R_f)}$$

$$\therefore \text{Error} = \frac{R[(R + \Delta R)(R + R_f) + RR_f] - R^2(R + 2R_f)}{R^2(R + 2R_f)}$$

$$= \frac{\Delta R (R + R_f)}{R (R + 2R_f)}$$

$$\therefore \% \text{ error} = \frac{\Delta R(R + R_f)}{R(R + 2R_f)} \times 100$$

$$[b] \quad \% \text{ error} = \frac{95(48 \times 10^4) \times 100}{10^4(95 \times 10^4)} = 0.48\%$$

$$\text{P 5.52} \quad 1 = \frac{\Delta R(48 \times 10^4)}{10^4(95 \times 10^4)} \times 100$$

$$\therefore \Delta R = \frac{9500}{48} = 197.91667 \Omega$$

$$\therefore \% \text{ change in } R = \frac{197.91667}{10^4} \times 100 \approx 1.98\%$$

P 5.53 [a] It follows directly from the solution to Problem 5.50 that

$$v_o = \frac{[R^2 + 2RR_f - R_1(R + R_f) - RR_f]R_f v_{in}}{R[R_1(R + R_f) + RR_f]}$$

Now $R_1 = R - \Delta R$. Substituting into the expression gives

$$v_o = \frac{(R + R_f)R_f(\Delta R)v_{in}}{R[(R - \Delta R)(R + R_f) + RR_f]}$$

Now let $\Delta R \ll R$ and get

$$v_o \approx \frac{(R + R_f)R_f \Delta R v_{in}}{R^2(R + 2R_f)}$$

[b] It follows directly from the solution to Problem 5.50 that

$$\therefore \frac{\text{approx value}}{\text{true value}} = \frac{R[(R - \Delta R)(R + R_f) + RR_f]}{R^2(R + 2R_f)}$$

$$\therefore \text{Error} = \frac{(R - \Delta R)(R + R_f) + RR_f - R(R + 2R_f)}{R(R + 2R_f)}$$

$$= \frac{-\Delta R(R + R_f)}{R(R + 2R_f)}$$

$$\therefore \% \text{ error} = \frac{-\Delta R(R + R_f)}{R(R + 2R_f)} \times 100$$

[c] $R - \Delta R = 9810 \Omega \quad \therefore \Delta R = 10,000 - 9810 = 190 \Omega$

$$\therefore v_o \approx \frac{(48 \times 10^4)(47 \times 10^4)(190)(15)}{10^8(95 \times 10^4)} \approx 6.768 \text{ V}$$

$$\text{[d] } \% \text{ error} = \frac{-190(48 \times 10^4)(100)}{10^4(95 \times 10^4)} = -0.96\%$$