8

Natural and Step Responses of RLC Circuits

Assessment Problems

AP 8.1 **[a]** $\dfrac{1}{(2RC)^2} = \dfrac{1}{LC}$, therefore $C = 500\,\text{nF}$

[b] $\alpha = 5000 = \dfrac{1}{2RC}$, therefore $C = 1\,\mu\text{F}$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - \frac{(10^3)(10^6)}{20}} = (-5000 \pm j5000)\ \text{rad/s}$$

[c] $\dfrac{1}{\sqrt{LC}} = 20{,}000$, therefore $C = 125\,\text{nF}$

$$s_{1,2} = \left[-40 \pm \sqrt{(40)^2 - 20^2}\right] 10^3,$$

$$s_1 = -5.36\,\text{krad/s}, \qquad s_2 = -74.64\,\text{krad/s}$$

AP 8.2

$$\begin{aligned}
i_L &= \frac{1}{50 \times 10^{-3}} \int_0^t [-14e^{-5000x} + 26e^{-20{,}000x}]\,dx + 30 \times 10^{-3} \\
&= 20\left\{ \frac{-14e^{-5000x}}{-5000}\Bigg|_0^t + \frac{26e^{-20{,}000t}}{-20{,}000}\Bigg|_0^t \right\} + 30 \times 10^{-3} \\
&= 56 \times 10^{-3}(e^{-5000t} - 1) - 26 \times 10^{-3}(e^{-20{,}000t} - 1) + 30 \times 10^{-3} \\
&= [56e^{-5000t} - 56 - 26e^{-20{,}000t} + 26 + 30]\,\text{mA} \\
&= 56e^{-5000t} - 26e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0
\end{aligned}$$

AP 8.3 From the given values of R, L, and C, $s_1 = -10\,\text{krad/s}$ and $s_2 = -40\ \text{krad/s}$.

[a] $v(0^-) = v(0^+) = 0$, therefore $i_R(0^+) = 0$

[b] $i_C(0^+) = -(i_L(0^+) + i_R(0^+)) = -(-4+0) = 4\,\text{A}$

[c] $C\dfrac{dv_c(0^+)}{dt} = i_c(0^+) = 4, \qquad \text{therefore} \quad \dfrac{dv_c(0^+)}{dt} = \dfrac{4}{C} = 4\times 10^8\,\text{V/s}$

[d] $v = [A_1e^{-10{,}000t} + A_2e^{-40{,}000t}]\,\text{V}, \qquad t \geq 0^+$

$$v(0^+) = A_1 + A_2, \qquad \frac{dv(0^+)}{dt} = -10{,}000A_1 - 40{,}000A_2$$

Therefore $A_1 + A_2 = 0, \qquad -A_1 - 4A_2 = 40{,}000; \qquad A_1 = 40{,}000/3\,\text{V}$

[e] $A_2 = -40{,}000/3\,\text{V}$

[f] $v = [40{,}000/3][e^{-10{,}000t} - e^{-40{,}000t}]\,\text{V}, \qquad t \geq 0$

AP 8.4 [a] $\dfrac{1}{2RC} = 8000, \qquad \text{therefore} \quad R = 62.5\,\Omega$

[b] $i_R(0^+) = \dfrac{10\,\text{V}}{62.5\,\Omega} = 160\,\text{mA}$

$$i_C(0^+) = -(i_L(0^+) + i_R(0^+)) = -80 - 160 = -240\,\text{mA} = C\frac{dv(0^+)}{dt}$$

Therefore $\dfrac{dv(0^+)}{dt} = \dfrac{-240\,\text{m}}{C} = -240\,\text{kV/s}$

[c] $B_1 = v(0^+) = 10\,\text{V}, \qquad \dfrac{dv_c(0^+)}{dt} = \omega_d B_2 - \alpha B_1$

Therefore $6000B_2 - 8000B_1 = -240{,}000, \qquad B_2 = (-80/3)\,\text{V}$

[d] $i_L = -(i_R + i_C); \qquad i_R = v/R; \qquad i_C = C\dfrac{dv}{dt}$

$$v = e^{-8000t}[10\cos 6000t - \frac{80}{3}\sin 6000t]\,\text{V}$$

Therefore $i_R = e^{-8000t}[160\cos 6000t - \dfrac{1280}{3}\sin 6000t]\,\text{mA}$

$$i_C = e^{-8000t}[-240\cos 6000t + \frac{460}{3}\sin 6000t]\,\text{mA}$$

$$i_L = 10e^{-8000t}[8\cos 6000t + \frac{82}{3}\sin 6000t]\,\text{mA}, \qquad t \geq 0$$

AP 8.5 [a] $\left(\dfrac{1}{2RC}\right)^2 = \dfrac{1}{LC} = \dfrac{10^6}{4}, \qquad \text{therefore} \quad \dfrac{1}{2RC} = 500, \quad R = 100\,\Omega$

[b] $0.5CV_0^2 = 12.5\times 10^{-3}, \qquad \text{therefore} \quad V_0 = 50\,\text{V}$

[c] $0.5LI_0^2 = 12.5\times 10^{-3}, \qquad I_0 = 250\,\text{mA}$

[d] $D_2 = v(0^+) = 50, \qquad \dfrac{dv(0^+)}{dt} = D_1 - \alpha D_2$

$i_R(0^+) = \dfrac{50}{100} = 500\,\text{mA}$

Therefore $\quad i_C(0^+) = -(500 + 250) = -750\,\text{mA}$

Therefore $\quad \dfrac{dv(0^+)}{dt} = -750 \times \dfrac{10^{-3}}{C} = -75{,}000\,\text{V/s}$

Therefore $\quad D_1 - \alpha D_2 = -75{,}000; \qquad \alpha = \dfrac{1}{2RC} = 500, \quad D_1 = -50{,}000\ \text{V/s}$

[e] $v = [50e^{-500t} - 50{,}000te^{-500t}]\,\text{V}$

$i_R = \dfrac{v}{R} = [0.5e^{-500t} - 500te^{-500t}]\,\text{A}, \qquad t \geq 0^+$

AP 8.6 **[a]** $i_R(0^+) = \dfrac{V_0}{R} = \dfrac{40}{500} = 0.08\,\text{A}$

[b] $i_C(0^+) = I - i_R(0^+) - i_L(0^+) = -1 - 0.08 - 0.5 = -1.58\,\text{A}$

[c] $\dfrac{di_L(0^+)}{dt} = \dfrac{V_o}{L} = \dfrac{40}{0.64} = 62.5\,\text{A/s}$

[d] $\alpha = \dfrac{1}{2RC} = 1000; \qquad \dfrac{1}{LC} = 1{,}562{,}500; \qquad s_{1,2} = -1000 \pm j750\ \text{rad/s}$

[e] $i_L = i_f + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t, \qquad i_f = I = -1\,\text{A}$

$i_L(0^+) = 0.5 = i_f + B_1', \qquad \text{therefore} \quad B_1' = 1.5\,\text{A}$

$\dfrac{di_L(0^+)}{dt} = 62.5 = -\alpha B_1' + \omega_d B_2', \qquad \text{therefore} \quad B_2' = (25/12)\,\text{A}$

Therefore $\quad i_L(t) = -1 + e^{-1000t}[1.5\cos 750t + (25/12)\sin 750t]\,\text{A}, \qquad t \geq 0$

[f] $v(t) = \dfrac{L di_L}{dt} = 40e^{-1000t}[\cos 750t - (154/3)\sin 750t]V \qquad t \geq 0$

AP 8.7 **[a]** $i(0^+) = 0$, since there is no source connected to L for $t < 0$.

[b] $v_c(0^+) = v_C(0^-) = \left(\dfrac{15\,\text{k}}{15\,\text{k} + 9\,\text{k}}\right)(80) = 50\,\text{V}$

[c] $50 + 80i(0^+) + L\dfrac{di(0^+)}{dt} = 100, \qquad \dfrac{di(0^+)}{dt} = 10{,}000\,\text{A/s}$

[d] $\alpha = 8000; \qquad \dfrac{1}{LC} = 100 \times 10^6; \qquad s_{1,2} = -8000 \pm j6000\ \text{rad/s}$

[e] $i = i_f + e^{-\alpha t}[B_1'\cos\omega_d t + B_2'\sin\omega_d t]; \qquad i_f = 0, \quad i(0^+) = 0$

Therefore $\quad B_1' = 0; \qquad \dfrac{di(0^+)}{dt} = 10{,}000 = -\alpha B_1' + \omega_d B_2'$

Therefore $\quad B_2' = 1.67\,\text{A}; \qquad i = 1.67e^{-8000t}\sin 6000t\,\text{A}, \qquad t \geq 0$

AP 8.8 $v_c(t) = v_f + e^{-\alpha t}[B_1' \cos \omega_d t + B_2' \sin \omega_d t], \qquad v_f = 100\,\text{V}$

$$v_c(0^+) = 50\,\text{V}; \qquad \frac{dv_c(0^+)}{dt} = 0; \qquad \text{therefore} \quad 50 = 100 + B_1'$$

$$B_1' = -50\,\text{V}; \qquad 0 = -\alpha B_1' + \omega_d B_2'$$

$$\text{Therefore} \quad B_2' = \frac{\alpha}{\omega_d} B_1' = \left(\frac{8000}{6000}\right)(-50) = -66.67\,\text{V}$$

$$\text{Therefore} \quad v_c(t) = 100 - e^{-8000t}[50 \cos 6000t + 66.67 \sin 6000t]\,\text{V}, \qquad t \geq 0$$

Problems

P 8.1 **[a]** $i_R(0) = \frac{25}{125} = 200\,\text{mA}$

$$i_L(0) = -300\,\text{mA}$$

$$i_C(0) = -i_L(0) - i_R(0) = 300 - 200 = 100\,\text{mA}$$

[b] $\alpha = \frac{1}{2RC} = \frac{1}{2(125)(5 \times 10^{-6})} = 800$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(200 \times 10^{-3})(8 \times 10^{-6})}} = 1000$$

$\alpha^2 < \omega_0^2$ The response is underdamped

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{1000^2 - 800^2} = 600$$

$$v = B_1 e^{-800t} \cos 600t + B_2 e^{-800t} \sin 600t$$

$$v(0) = B_1 = 25$$

$$\frac{dv}{dt}(0) = -\alpha B_1 + \omega_d B_2 = \frac{1}{C} i_C(0)$$

$$\text{So,} \quad -800(25) + 600 B_2 = \frac{1}{5 \times 10^{-6}}(0.1) = 20{,}000$$

$$\therefore \quad B_2 = \frac{20{,}000 + 800(25)}{600} = 66.67$$

$$v = 25e^{-800t} \cos 600t + 66.67e^{-800t} \sin 600t\,\text{V}, \qquad t \geq 0$$

[c] $$i_C = C\frac{dv}{dt}$$

$$= 5\times10^{-6}[20{,}000e^{-800t}\cos 600t - 68{,}333.33e^{-800t}\sin 600t]$$

$$= 100e^{-800t}\cos 600t - 341.67e^{-800t}\sin 600t \text{ mA}$$

$$i_R = \frac{v}{R} = 200e^{-800t}\cos 600t + 533.36e^{-800t}\sin 600t \text{ mA}$$

$$i_L = -i_C - i_R = -300e^{-800t}\cos 600t - 191.7e^{-800t}\sin 600t \text{ mA}, \quad t \geq 0$$

P 8.2 $$\frac{1}{2RC} = \frac{1}{2(100)(5\times10^{-6})} = 1000$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(200\times10^{-3})(5\times10^{-6})}} = 1000$$

$\alpha^2 = \omega_0^2$ So the response is critically damped

$$v(t) = D_1te^{-1000t} + D_2e^{-1000t}$$

$$v(0^+) = 25\text{ V} = D_2$$

$$\frac{dv(0^+)}{dt} = D_1 - \alpha D_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right)$$

So, $$D_1 - 1000(25) = \frac{1}{5\times10^{-6}}\left(0.3 - \frac{25}{100}\right)$$

$\therefore$ $$D_1 = 35{,}000$$

$$v(t) = 35{,}000e^{-1000t} + 25e^{-1000t}\text{ V}, \qquad t \geq 0$$

P 8.3 $$\alpha = \frac{1}{2RC} = \frac{1}{2(80)(5\times10^{-6})} = 1250$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(200\times10^{-3})(5\times10^{-6})}} = 1000$$

$\alpha^2 > \omega_0^2$ So the response is overdamped

$$s_{1,2} = -1250 \pm \sqrt{1250^2 - 1000^2} = -500, -2000$$

$$v(t) = A_1e^{-500t} + A_2e^{-2000t}$$

$v(0) = A_1 + A_2 = 25$

$\frac{dv}{dt}(0) = -500A_1 - 2000A_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right) = \frac{1}{5 \times 10^{-6}}\left(0.3 - \frac{25}{80}\right) = -2500$

Solving, $A_1 = 31.67,$ $A_2 = -6.67$

$v(t) = 31.67e^{-500t} - 6.67e^{-2000t}\,\text{V}, \qquad t \geq 0$

P 8.4 **[a]** $\alpha = \frac{1}{2RC} = \frac{10^{12}}{(4000)(10)} = 25{,}000$

$\omega_o^2 = \frac{1}{LC} = \frac{10^{12}}{(250)(10)} = 4 \times 10^8$

$s_{1,2} = -25{,}000 \pm \sqrt{625 \times 10^6 - 400 \times 10^6} = -25{,}000 \pm 15{,}000$

$s_1 = -10{,}000$ rad/s

$s_2 = -40{,}000$ rad/s

[b] overdamped

[c] $\omega_d = \sqrt{\omega_o^2 - \alpha^2}$

$\therefore \quad \alpha^2 = \omega_o^2 - \omega_d^2 = 4 \times 10^8 - 144 \times 10^6 = 256 \times 10^6$

$\alpha = 16 \times 10^3 = 16{,}000$

$\frac{1}{2RC} = 16{,}000; \qquad \therefore \quad R = \frac{10^9}{(32{,}000)(10)} = 3125\,\Omega$

[d] $s_1 = -16{,}000 + j12{,}000$ rad/s; $\qquad s_2 = -16{,}000 - j12{,}000$ rad/s

[e] $\alpha = 4 \times 10^4 = \frac{1}{2RC}; \qquad \therefore \quad R = \frac{1}{2C(4 \times 10^4)} = 2500\,\Omega$

P 8.5 **[a]** $-\alpha + \sqrt{\alpha^2 - \omega_o^2} = -1000$

$-\alpha - \sqrt{\alpha^2 - \omega_o^2} = -4000$

Adding the above equations, $\quad -2\alpha = -5000$

$\alpha = 2500\,\text{rad/s}$

$-2500 \pm \sqrt{2500^2 - \omega_0^2} = -1000 \quad$ so $\quad \sqrt{2500^2 - \omega_0^2} = 1500$

$\therefore \quad -\omega_0^2 = 1500^2 - 2500^2 \quad$ thus $\quad \omega_0 = 2000$

$\omega_0^2 = \frac{1}{LC} = \frac{1}{(0.01)C} = 2000^2 \quad$ so $\quad C = \frac{1}{(0.01)2000^2} = 25\,\mu\text{F}$

$\alpha = \frac{1}{2RC} = \frac{1}{2R(25 \times 10^{-6})} = 2500 \quad$ so $\quad R = \frac{1}{2(25 \times 10^{-6})(2500)} = 8\,\Omega$

[**b**] $i_\mathrm{R} = \dfrac{v(t)}{R} = 5e^{-1000t} - 11.25e^{-4000t}\,\mathrm{A}, \qquad t \ge 0^+$

$$i_\mathrm{C} = C\frac{dv(t)}{dt} = 9e^{-4000t} - e^{-1000t}\,\mathrm{A}, \qquad t \ge 0^+$$

$$i_\mathrm{L} = -(i_\mathrm{R} + i_\mathrm{C}) = 2.25e^{-4000t} - 4e^{-1000t}\,\mathrm{A}, \qquad t \ge 0$$

P 8.6 [**a**] $\alpha = 400; \qquad \omega_d = 300$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

$$\therefore \quad \omega_o^2 = \omega_d^2 + \alpha^2 = 9\times10^4 + 16\times10^4 = 25\times10^4$$

$$\frac{1}{LC} = 25\times10^4$$

$$L = \frac{1}{(25\times10^4)(250\times10^{-6})} = 16\,\mathrm{mH}$$

[**b**] $\alpha = \dfrac{1}{2RC}$

$$\therefore \quad R = \frac{1}{2\alpha C} = \frac{1}{(800)(250\times10^{-6})} = 5\,\Omega$$

[**c**] $V_o = v(0) = 120\,\mathrm{V}$

[**d**] $I_o = i_\mathrm{L}(0) = -i_\mathrm{R}(0) - i_\mathrm{C}(0)$

$$i_\mathrm{R}(0) = \frac{120}{5} = 24\,\mathrm{A}$$

$$i_\mathrm{C}(0) = C\frac{dv}{dt}(0) = 250\times10^{-6}[-400(120) + 300(80)] = -6\,\mathrm{A}$$

$$\therefore \quad I_o = -24 + 6 = -18\,\mathrm{A}$$

[**e**] $i_\mathrm{C}(t) = 250\times10^{-6}\dfrac{dv(t)}{dt} = e^{-400t}(-17\sin 300t - 6\cos 300t)\,\mathrm{A}$

$$i_\mathrm{R}(t) = \frac{v(t)}{5} = e^{-400t}(24\cos 300t + 16\sin 300t)\,\mathrm{A}$$

$$i_\mathrm{L}(t) = -i_\mathrm{R}(t) - i_\mathrm{C}(t)$$

$$= e^{-400t}(-18\cos 300t + \sin 300t)\,\mathrm{A}, \quad t \ge 0$$

Check:

$$L\frac{di_\mathrm{L}}{dt} = 16\times10^{-3}e^{-400t}[7500\cos 300t + 5000\sin 300t]$$

$$v(t) = e^{-400t}[120\cos 300t + 80\sin 300t]\,\mathrm{V}$$

P 8.7 **[a]** $\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} = (80)^2$

$$\therefore \quad C = \frac{1}{2(50)(80)} = 125\,\mu\text{F}$$

$$\frac{1}{LC} = 80^2$$

$$\therefore \quad L = \frac{1}{80^2(125 \times 10^{-6})} = 1.25\,\text{H}$$

$$v(0) = D_2 = 5\,\text{V}$$

$$\frac{dv}{dt}(0) = D_1 - \alpha D_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right)$$

$$\therefore \qquad D_1 - 80(5) = \frac{1}{125 \times 10^{-6}}\left(0.025 - \frac{5}{50}\right) \qquad \text{so} \qquad D_1 = -200$$

[b] $v = -200te^{-80t} + 5e^{-80t}\,\text{V}, \quad t \geq 0$

$$\frac{dv}{dt} = [16{,}000t - 600]e^{-80t}$$

$$i_{\text{C}} = C\frac{dv}{dt} = (2000t - 75)e^{-80t}\,\text{mA}, \quad t \geq 0^+$$

P 8.8 **[a]** $\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(20 \times 10^{-3})(500 \times 10^{-9})}} = 10{,}000$

$$\alpha = \frac{1}{2RC} = 10{,}000; \qquad R = \frac{1}{2(10{,}000)(500 \times 10^{-9})} = 100\,\Omega$$

[b] $v(t) = D_1te^{-10{,}000t} + D_2e^{-10{,}000t}$

$$v(0) = 40\,\text{V} = D_2$$

$$\frac{dv}{dt}(0) = D_1 - \alpha D_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right)$$

$$\therefore \qquad D_1 - 10{,}000D_2 = \frac{1}{500 \times 10^{-9}}\left(-0.12 - \frac{40}{100}\right) \qquad \text{so} \qquad D_1 = 640{,}000$$

$$\therefore \quad v(t) = (40 - 640{,}000t)e^{-10{,}000t}\,\text{V}, \qquad t \geq 0$$

[c] $i_{\text{C}}(t) = 0$ when $\frac{dv}{dt}(t) = 0$

$$\frac{dv}{dt} = (64 \times 10^8t - 1040 \times 10^3)e^{-10{,}000t}$$

$$\frac{dv}{dt} = 0 \text{ when } 640 \times 10^8t_1 = 1040 \times 10^3; \qquad \therefore \quad t_1 = 162.5\,\mu\text{s}$$

$$v(162.5\mu\text{s}) = (-64)e^{-1.625} = -12.6\,\text{V}$$

[d] $w(0) = \frac{1}{2}(500 \times 10^{-9})(40)^2 + \frac{1}{2}(0.02)(0.012)^2 = 544\,\mu\text{J}$

$$w(162.5\,\mu\text{s}) = \frac{1}{2}(500 \times 10^{-9})(12.6)^2 + \frac{1}{2}(0.02)\left(\frac{12.6}{100}\right)^2 = 198.5\,\mu\text{J}$$

$$\%\text{ remaining } = \frac{198.5}{544}(100) = 36.5\%$$

P 8.9 $\alpha = 500/2 = 250$

$$R = \frac{1}{2\alpha C} = \frac{10^6}{(500)(18)} = 1000\,\Omega$$

$$v(0^+) = -11 + 20 = 9\,\text{V}$$

$$i_{\text{R}}(0^+) = \frac{9}{1000} = 9\,\text{mA}$$

$$\frac{dv}{dt} = 1100e^{-100t} - 8000e^{-400t}$$

$$\frac{dv(0^+)}{dt} = 1100 - 8000 = -6900\,\text{V/s}$$

$$i_{\text{C}}(0^+) = 2 \times 10^{-6}(-6900) = -13.8\,\text{mA}$$

$$i_{\text{L}}(0^+) = -[i_{\text{R}}(0^+) + i_{\text{C}}(0^+)] = -[9 - 13.8] = 4.8\,\text{mA}$$

P 8.10 **[a]** $\alpha = \frac{1}{2RC} = 1250, \qquad \omega_o = 10^3, \qquad \text{therefore overdamped}$

$$s_1 = -500, \qquad s_2 = -2000$$

therefore $v = A_1e^{-500t} + A_2e^{-2000t}$

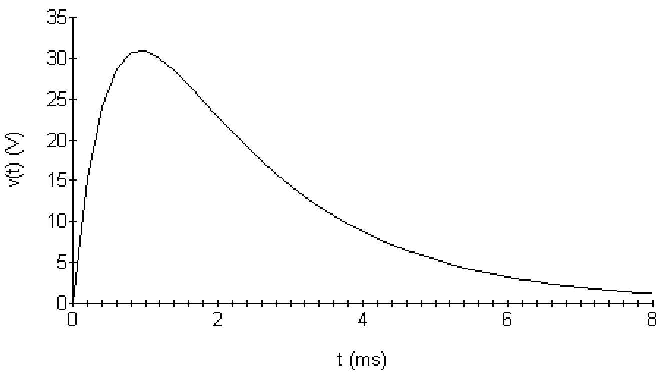
$$v(0^+) = 0 = A_1 + A_2; \qquad \left[\frac{dv(0^+)}{dt}\right] = \frac{i_{\text{C}}(0^+)}{C} = 98{,}000\,\text{V/s}$$

Therefore $-500A_1 - 2000A_2 = 98{,}000$

$$A_1 = \frac{+980}{15}, \quad A_2 = \frac{-980}{15}$$

$$v(t) = \left[\frac{980}{15}\right][e^{-500t} - e^{-2000t}]\,\text{V}, \qquad t \geq 0$$

[b]

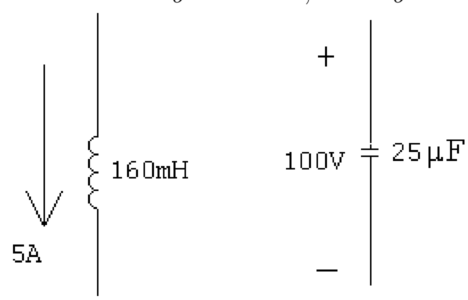


Example 8.4: $v_{\max} \cong 74.1\,\text{V}$ at 1.4 ms

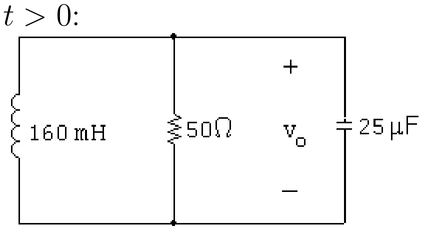
Example 8.5: $v_{\max} \cong 36.1\,\text{V}$ at 1.0 ms

Problem 8.17: $v_{\max} \cong 30.9$ at 0.92 ms

P 8.11 $t < 0:$ $V_o = 100\,\text{V},$ $I_o = 5\,\text{A}$



$t > 0$:



$$\alpha = \frac{1}{2RC} = \frac{1}{2(50)(25 \times 10^{-6})} = 400 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(160 \times 10^{-3})(25 \times 10^{-6})}} = 500$$

$\alpha^2 < \omega_0^2$ Response is underdamped

$$\omega_d = \sqrt{500^2 - 400^2} = 300$$

$$\therefore \quad v_o = B_1 e^{-400t}\cos 300t + B_2 e^{-400t}\sin 300t$$

$$v_o(0) = B_1 = 100$$

$$\frac{dv_o}{dt}(0) = -\alpha B_1 + \omega_d B_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right)$$

$$\therefore \qquad -(400)(100) + 300B_2 = \frac{1}{25\times 10^{-6}}\left(-5 - \frac{100}{50}\right) \qquad \text{so} \qquad B_2 = -800$$

$$\therefore \quad v_o = 100e^{-400t}\cos 300t - 800e^{-400t}\sin 300t\,\text{V}, \qquad t \geq 0$$

P 8.12 $\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(160\times 10^{-3})(25\times 10^{-6})}} = 500$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(40)(25\times 10^{-6})} = 500$$

$\alpha^2 = \omega_0^2$ Response is critically damped

$$v_o(t) = D_1 t e^{-500t} = D_2 e^{-500t}$$

$$v_o(0) = D_2 = 100\,\text{V}$$

$$\frac{dv_o}{dt}(0) = D_1 - \alpha D_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right)$$

$$\therefore \qquad D_1 - 500(100) = \frac{1}{25\times 10^{-6}}\left(-5 - \frac{100}{40}\right) \qquad \text{so} \qquad D_1 = -250{,}000$$

$$v_o(t) = -250{,}000te^{-500t} + 100e^{-500t}\,\text{V}, \qquad t \geq 0$$

P 8.13 $\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(160\times 10^{-3})(25\times 10^{-6})}} = 500$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(32)(25\times 10^{-6})} = 625$$

$\therefore \quad \alpha^2 > \omega_o^2$ (overdamped)

$$s_{1,2} = -625 \pm \sqrt{625^2 - 500^2} = -250, -1000$$

$$v_o(t) = A_1 e^{-250t} + A_2 e^{-1000t}$$

$$v_o(0) = A_1 + A_2 = 100\,\text{V}$$

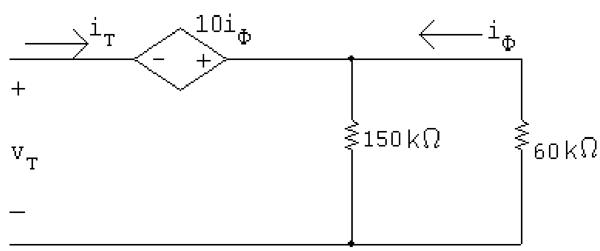
$$\frac{dv_o}{dt}(0) = -250A_1 - 1000A_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right)$$

$$\therefore \qquad -250A_1 - 1000A_2 = \frac{1}{25 \times 10^{-6}}\left(-5 - \frac{100}{32}\right) = -325{,}000$$

Solving, $\quad A_1 = -300, \qquad A_2 = 400$

$$v_o(t) = -300e^{-250t} + 400e^{-1000t}\,\text{V}, \qquad t \geq 0$$

P 8.14



$$v_T = -10i_\phi + i_T\left(\frac{(150)(60)}{210}\right) = -10\frac{-i_T(150)}{210} + i_t\frac{9000}{210}$$

$$\frac{v_T}{i_T} = \frac{1500 + 9000}{210} = 50\,\Omega$$

$$V_o = \frac{4000}{10{,}000}(50) = 20\,\text{V}; \qquad I_o = 0$$

$$i_\text{C}(0) = -i_R(0) - i_\text{L}(0) = -\frac{20}{50} = -0.4\,\text{A}$$

$$\frac{i_\text{C}(0)}{C} = \frac{-0.4}{8 \times 10^{-6}} = -50{,}000$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(51.2 \times 10^{-3})(8 \times 10^{-6})}} = 1562.5 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{(2)(50)(8 \times 10^{-6})} = 1250 \text{ rad/s}$$

$\alpha^2 < \omega_0^2 \qquad$ so the response is underdamped

$$\omega_d = \sqrt{1562.5^2 - 1250^2} = 937.5$$

$$v_o = B_1 e^{-1250t}\cos 937.5t + B_2 e^{-1250t}\sin 937.5t$$

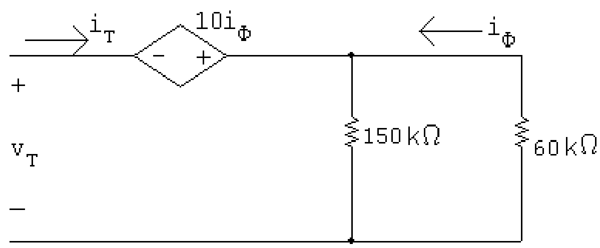
$$v_o(0) = B_1 20\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -\alpha B_1 + \omega_d B_2 = \frac{i_C(0)}{C}$$

$$\therefore \qquad -1250(20) + 937.5B_2 = -50{,}000 \qquad \text{so} \qquad B_2 = -26.67$$

$$v_o = 20e^{-1250t}\cos 937.5t - 26.67e^{-1250t}\sin 937.5t\,\text{V}, \qquad t \geq 0$$

P 8.15



$$v_T = -10i_\phi + i_T\left(\frac{(150)(60)}{210}\right) = -10\frac{-i_T(150)}{210} + i_t\frac{9000}{210}$$

$$\frac{v_T}{i_T} = \frac{1500 + 9000}{210} = 50\,\Omega$$

$$V_o = \frac{4000}{10{,}000}(50) = 20\,\text{V}; \qquad I_o = 0$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(80\times 10^{-3})(8\times 10^{-6})}} = 1250 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(50)(8\times 10^{-6})} = 1250 \text{ rad/s}$$

$$\alpha^2 = \omega_0^2 \qquad \text{so the response is critically damped}$$

$$v_o = D_1 t e^{-1250t} + D_2 e^{-1250t}$$

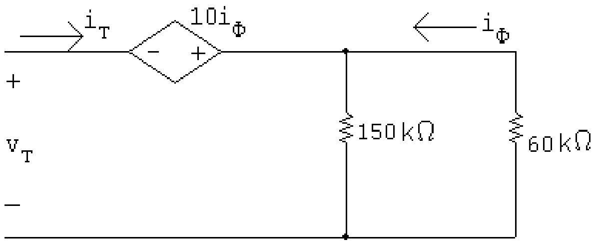
$$v_o(0) = D_2 = 20\,\text{V}$$

$$\frac{dv_o}{dt}(0) = D_1 - \alpha D_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right)$$

$$\therefore \qquad D_1 - 1250(20) = \frac{1}{8\times 10^{-6}}\left(0 - \frac{20}{50}\right) \qquad \text{so} \qquad D_1 = -25{,}000$$

$$v_o = -25{,}000te^{-1250t} + 20e^{-1250t}\,\text{V}, \qquad t \geq 0$$

P 8.16



$$v_T = -10i_\phi + i_T\left(\frac{(150)(60)}{210}\right) = -10\frac{-i_T(150)}{210} + i_t\frac{9000}{210}$$

$$\frac{v_T}{i_T} = \frac{1500 + 9000}{210} = 50\,\Omega$$

$$V_o = \frac{4000}{10{,}000}(50) = 20\,\text{V}; \qquad I_o = 0$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(125\times10^{-3})(8\times10^{-6})}} = 1000 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(50)(8\times10^{-6})} = 1250 \text{ rad/s}$$

$\alpha^2 > \omega_0^2$ so the response is overdamped

$$s_{1,2} = -1250 \pm \sqrt{1250^2 - 1000^2} = -500, -2000$$

$$v_o = A_1 e^{-500t} + A_2 e^{-1000t}$$

$$v_o(0) = A_1 + A_2 = 20\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -500A_1 - 2000A_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right)$$

$$\therefore \qquad -500A_1 - 2000A_2 = \frac{1}{8\times10^{-6}}\left(0 - \frac{20}{50}\right) = -50{,}000$$

Solving, $A_1 = -6.67$, $A_2 = 26.67$

$$v_o = -6.67e^{-500t} + 26.67e^{-2000t}\,\text{V}, \qquad t \geq 0$$

P 8.17 **[a]** $\frac{1}{LC} = 5000^2$

There are many possible solutions. This one begins by choosing $L = 10\,\text{mH}$. Then,

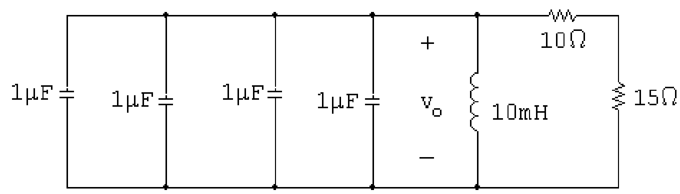
$$C = \frac{1}{(10 \times 10^{-3})(5000)^2} = 4\,\mu\text{F}$$

We can achieve this capacitor value using components from Appendix H by combining four $1\,\mu$F capacitors in parallel.

Critically damped: $\alpha = \omega_0 = 5000$ so $\frac{1}{2RC} = 5000$

$$\therefore \quad R = \frac{1}{2(4 \times 10^{-6})(5000)} = 25\,\Omega$$

We can create this resistor value using components from Appendix H by combining a $10\,\Omega$ resistor and a $15\,\Omega$ resistor in series. The final circuit:



[b] $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -5000 \pm 0$

Therefore there are two repeated real roots at -5000 rad/s.

P 8.18 **[a]** Underdamped response:

$\alpha < \omega_0$ so $\alpha < 5000$

Therefore we choose a larger resistor value than the one used in Problem 8.17. Choose $R = 100\,\Omega$:

$$\alpha = \frac{1}{2(100)(4 \times 10^{-6})} = 1250$$

$$s_{1,2} = -1250 \pm \sqrt{1250^2 - 5000^2} = -1250 \pm j4841.23 \text{ rad/s}$$

[b] Overdamped response:

$\alpha > \omega_0$ so $\alpha > 5000$

Therefore we choose a smaller resistor value than the one used in Problem 8.17. Choose $R = 20\,\Omega$:

$$\alpha = \frac{1}{2(20)(4 \times 10^{-6})} = 6250$$

$$s_{1,2} = -1250 \pm \sqrt{6250^2 - 5000^2} = -1250 \pm 3750$$

$$= -2500 \text{ rad/s}; \quad \text{and} \quad -10{,}000 \text{ rad/s}$$

P 8.19 **[a]** $\alpha = \dfrac{1}{2RC} = 800$ rad/s

$$\omega_o^2 = \frac{1}{LC} = 10^6$$

$$\omega_d = \sqrt{10^6 - 800^2} = 600 \text{ rad/s}$$

$$\therefore \quad v = B_1 e^{-800t}\cos 600t + B_2 e^{-800t}\sin 600t$$

$$v(0) = B_1 = 30$$

$$i_R(0^+) = \frac{30}{5000} = 6\,\text{mA}; \qquad i_C(0^+) = -12\,\text{mA}$$

$$\therefore \quad \frac{dv}{dt}(0^+) = \frac{-0.012}{125 \times 10^{-9}} = -96{,}000 \text{ V/s}$$

$$-96{,}000 = -\alpha B_1 + \omega_d B_2 = -(800)(30) + 600B_2$$

$$\therefore \quad B_2 = -120$$

$$\therefore \quad v = 30e^{-800t}\cos 600t - 120e^{-800t}\sin 600t\,\text{V}, \qquad t \geq 0$$

[b] $\dfrac{dv}{dt} = 6000e^{-800t}(13\sin 600t - 16\cos 600t)$

$$\frac{dv}{dt} = 0 \quad \text{when} \quad 16\cos 600t = 13\sin 600t \quad \text{or} \quad \tan 600t = \frac{16}{13}$$

$$\therefore \quad 600t_1 = 0.8885, \qquad t_1 = 1.48\,\text{ms}$$

$$600t_2 = 0.8885 + \pi, \qquad t_2 = 6.72\,\text{ms}$$

$$600t_3 = 0.8885 + 2\pi, \qquad t_3 = 11.95\,\text{ms}$$

[c] $t_3 - t_1 = 10.47\,\text{ms}; \qquad T_d = \dfrac{2\pi}{\omega_d} = \dfrac{2\pi}{600} = 10.47\,\text{ms}$

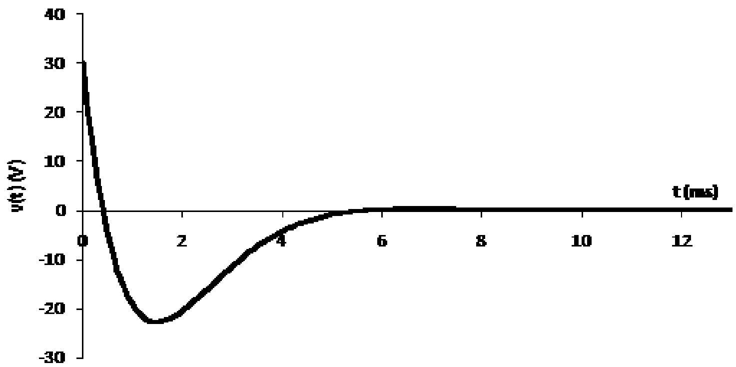
[d] $t_2 - t_1 = 5.24\,\text{ms}; \qquad \dfrac{T_d}{2} = \dfrac{10.48}{2} = 5.24\,\text{ms}$

[e] $v(t_1) = 30e^{-(1.184)}(\cos 0.8885 - 4\sin 0.8885) = -22.7\,\text{V}$

$$v(t_2) = 30e^{-(5.376)}(\cos 4.032 - 4\sin 4.032) = 0.334\,\text{V}$$

$$v(t_3) = 30e^{-(9.56)}(\cos 7.17 - 4\sin 7.17) = -5.22\,\text{mV}$$

[f]



P 8.20 [a] $\alpha = 0; \qquad \omega_d = \omega_o = \sqrt{10^6} = 1000\,\text{rad/s}$

$$v = B_1 \cos \omega_o t + B_2 \sin \omega_o t; \qquad v(0) = B_1 = 30$$

$$C\frac{dv}{dt}(0) = -i_{\text{L}}(0) = -0.006$$

$$-48{,}000 = -\alpha B_1 + \omega_d B_2 = -0 + 1000B_2$$

$$\therefore \quad B_2 = \frac{-48{,}000}{1000} = -48\,\text{V}$$

$$v = 30\cos 1000t - 48\sin 1000t\,\text{V}, \qquad t \geq 0$$

[b] $2\pi f = 1000; \qquad f = \dfrac{1000}{2\pi} \cong 159.15\,\text{Hz}$

[c] $\sqrt{30^2 + 48^2} = 56.6\,\text{V}$

P 8.21 From the form of the solution we have

$$v(0) = A_1 + A_2$$

$$\frac{dv(0^+)}{dt} = -\alpha(A_1 + A_2) + j\omega_d(A_1 - A_2)$$

We know both $v(0)$ and $dv(0^+)/dt$ will be real numbers. To facilitate the algebra we let these numbers be K_1 and K_2, respectively. Then our two simultaneous equations are

$$K_1 = A_1 + A_2$$

$$K_2 = (-\alpha + j\omega_d)A_1 + (-\alpha - j\omega_d)A_2$$

The characteristic determinant is

$$\Delta = \begin{vmatrix} 1 & 1 \\ (-\alpha + j\omega_d) & (-\alpha - j\omega_d) \end{vmatrix} = -j2\omega_d$$

The numerator determinants are

$$N_1 = \begin{vmatrix} K_1 & 1 \\ K_2 & (-\alpha - j\omega_d) \end{vmatrix} = -(\alpha + j\omega_d)K_1 - K_2$$

and $$N_2 = \begin{vmatrix} 1 & K_1 \\ (-\alpha + j\omega_d) & K_2 \end{vmatrix} = K_2 + (\alpha - j\omega_d)K_1$$

It follows that $$A_1 = \frac{N_1}{\Delta} = \frac{\omega_d K_1 - j(\alpha K_1 + K_2)}{2\omega_d}$$

and $$A_2 = \frac{N_2}{\Delta} = \frac{\omega_d K_1 + j(\alpha K_1 + K_2)}{2\omega_d}$$

We see from these expressions that $A_1 = A_2^*$.

P 8.22 By definition, $B_1 = A_1 + A_2$. From the solution to Problem 8.21 we have

$$A_1 + A_2 = \frac{2\omega_d K_1}{2\omega_d} = K_1$$

But K_1 is $v(0)$, therefore, $B_1 = v(0)$, which is identical to Eq. (8.30).
By definition, $B_2 = j(A_1 - A_2)$. From Problem 8.12 we have

$$B_2 = j(A_1 - A_2) = \frac{j[-2j(\alpha K_1 + K_2)]}{2\omega_d} = \frac{\alpha K_1 + K_2}{\omega_d}$$

It follows that

$$K_2 = -\alpha K_1 + \omega_d B_2, \quad \text{but} \quad K_2 = \frac{dv(0^+)}{dt} \quad \text{and} \quad K_1 = B_1.$$

Thus we have

$$\frac{dv}{dt}(0^+) = -\alpha B_1 + \omega_d B_2,$$

which is identical to Eq. (8.31).

P 8.23 **[a]** $2\alpha = 1000; \qquad \alpha = 500\,\text{rad/s}$

$$2\sqrt{\alpha^2 - \omega_o^2} = 600; \qquad \omega_o = 400\,\text{rad/s}$$

$$C = \frac{1}{2\alpha R} = \frac{1}{2(500)(250)} = 4\,\mu F$$

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{(400)^2(4\times 10^{-6})} = 1.5625\,\mathrm{H}$$

$$i_C(0^+) = A_1 + A_2 = 45\,\mathrm{mA}$$

$$\frac{di_C}{dt} + \frac{di_L}{dt} + \frac{di_R}{dt} = 0$$

$$\frac{di_C(0)}{dt} = -\frac{di_L(0)}{dt} - \frac{di_R(0)}{dt}$$

$$\frac{di_L(0)}{dt} = \frac{0}{1.5625} = 0\,\mathrm{A/s}$$

$$\frac{di_R(0)}{dt} = \frac{1}{R}\frac{dv(0)}{dt} = \frac{1}{R}\frac{i_C(0)}{C} = \frac{45\times 10^{-3}}{(250)(4\times 10^{-6})} = 45\,\mathrm{A/s}$$

$$\therefore \quad \frac{di_C(0)}{dt} = 0 - 45 = -45\,\mathrm{A/s}$$

$$\therefore \quad 200A_1 + 800A_2 = 45; \qquad A_1 + A_2 = 0.045$$

Solving, $A_1 = -15\,\mathrm{mA}; \qquad A_2 = 60\,\mathrm{mA}$

$$\therefore \quad i_C = -15e^{-200t} + 60e^{-800t}\,\mathrm{mA}, \qquad t \geq 0^+$$

[b] By hypothesis

$$v = A_3e^{-200t} + A_4e^{-800t}, \qquad t \geq 0$$

$$v(0) = A_3 + A_4 = 0$$

$$\frac{dv(0)}{dt} = \frac{45\times 10^{-3}}{4\times 10^{-6}} = 11{,}250\,\mathrm{V/s}$$

$$-200A_3 - 800A_4 = 11{,}250; \qquad \therefore \quad A_3 = 18.75\,\mathrm{V}; \qquad A_4 = -18.75\,\mathrm{V}$$

$$v = 18.75e^{-200t} - 18.75e^{-800t}\,\mathrm{V}, \qquad t \geq 0$$

[c] $i_R(t) = \dfrac{v}{250} = 75e^{-200t} - 75e^{-800t}\,\mathrm{mA}, \qquad t \geq 0^+$

[d] $i_L = -i_R - i_C$

$$i_L = -60e^{-200t} + 15e^{-800t}\,\mathrm{mA}, \qquad t \geq 0$$

P 8.24 **[a]** $v = L\left(\dfrac{di_L}{dt}\right) = 16[e^{-20{,}000t} - e^{-80{,}000t}]\,\mathrm{V}, \qquad t \geq 0$

[b] $i_R = \dfrac{v}{R} = 40[e^{-20{,}000t} - e^{-80{,}000t}]\,\mathrm{mA}, \qquad t \geq 0^+$

[c] $i_C = I - i_L - i_R = [-8e^{-20{,}000t} + 32e^{-80{,}000t}]\,\mathrm{mA}, \qquad t \geq 0^+$

P 8.25 **[a]** $v = L\left(\frac{di_{\rm L}}{dt}\right) = 40e^{-32{,}000t}\sin 24{,}000t\,{\rm V}, \qquad t \geq 0$

[b] $i_{\rm C}(t) = I - i_{\rm R} - i_{\rm L} = 24 \times 10^{-3} - \frac{v}{625} - i_{\rm L}$

$$= [24e^{-32{,}000t}\cos 24{,}000t - 32e^{-32{,}000t}\sin 24{,}000t]\,{\rm mA}, \qquad t \geq 0^+$$

P 8.26 $v = L\left(\frac{di_{\rm L}}{dt}\right) = 960{,}000te^{-40{,}000t}\,{\rm V}, \qquad t \geq 0$

P 8.27 $\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(25 \times 10^{-3})(62.5 \times 10^{-6})}} = 800$ rad/s

$$\alpha = \frac{1}{2RC} = \frac{1}{2(12.5)(62.5 \times 10^{-6})} = 640 \text{ rad/s} \qquad \therefore \quad \text{underdamped}$$

$$\omega_d = \sqrt{800^2 - 640^2} = 480$$

$$I_f = 2\,{\rm A}$$

$$i_L = 2 + B_1'e^{-640t}\cos 480t + B_2'e^{-640t}\sin 480t$$

$$i_L(0) = 2 + B_1' = 1 \qquad \text{so} \qquad B_1' = -1$$

$$\frac{di_L}{dt}(0) = -\alpha B_1' + \omega_d B_2' = \frac{V_0}{L}$$

$$\therefore \qquad -640(-1) + 480B_2' = \frac{50}{25 \times 10^{-3}} \qquad \text{so} \qquad B_2' = 2.83$$

$$i_L(t) = 2 - e^{-640t}\cos 480t + 2.83e^{-640t}\sin 480t\,{\rm A}, \quad t \geq 0$$

P 8.28 $\alpha = \frac{1}{2RC} = \frac{1}{2(8)(62.5 \times 10^{-6})} = 1000$ rad/s

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(25 \times 10^{-3})(62.5 \times 10^{-6})}} = 800 \text{ rad/s}$$

Overdamped: $s_{1,2} = -1000 \pm \sqrt{1000^2 - 800^2} = -400, -1600$ rad/s

$$I_f = 2\,{\rm A}$$

$$i_L = 2 + A_1'e^{-400t} + A_2'e^{-1600t}$$

$$i_L(0) = 2 + A_1' + A_2' = 1 \qquad \text{so} \qquad A_1' + A_2' = -1$$

$$\frac{di_L}{dt}(0) = -400A_1' - 1600A_2' = \frac{V_0}{L} = \frac{50}{25 \times 10^{-3}} = 2000$$

$$\text{Solving,} \qquad A_1' = \frac{1}{3}, \qquad A_2' = -\frac{4}{3}$$

$$i_L(t) = 2 + \frac{1}{3}e^{-400t} - \frac{4}{3}e^{-1600t}\,\text{A}, \quad t \geq 0$$

P 8.29 $\alpha = \frac{1}{2RC} = \frac{1}{2(10)(62.5 \times 10^{-6})} = 800$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(25 \times 10^{-3})(62.5 \times 10^{-6})}} = 800 \text{ rad/s}$$

$$\alpha^2 = \omega_0^2 \qquad \text{Critically damped}$$

$$I_f = 2\,\text{A}$$

$$i_L = 2 + D_1' t e^{-800t} + D_2' e^{-800t}$$

$$i_L(0) = 2 + D_2' = 1; \qquad \therefore \quad D_2' = -1\,\text{A}$$

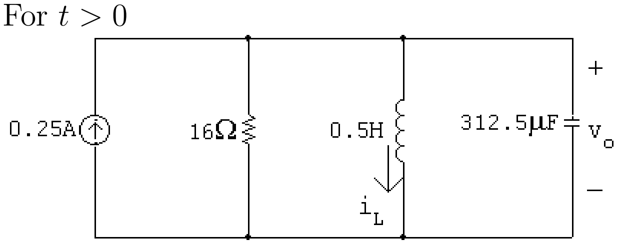
$$\frac{di_L}{dt}(0) = D_1' - \alpha D_2' = \frac{V_0}{L}$$

$$\therefore \qquad D_1' - 800(-1) = \frac{50}{25 \times 10^{-3}} \qquad \text{so} \qquad D_1' = 1200$$

$$i_L = 2 + 1200te^{-800t} - e^{-800t}\,\text{A}, \quad t \geq 0$$

P 8.30 $i_L(0^-) = i_L(0^+) = \frac{4}{16} = 0.25\,\text{A}$

For $t > 0$



$$\alpha = \frac{1}{2RC} = 100\,\text{rad/s}; \qquad \omega_o^2 = \frac{1}{LC} = 80^2 \quad \text{so} \quad \omega_o = 80 \text{ rad/s}$$

$$s_{1,2} = -100 \pm \sqrt{100^2 - 80^2} = -40, -160 \text{ rad/s}$$

$$v_o(\infty) = 0 = V_f; \qquad A_1' + A_2' = v(0) = 0$$

$$v_o = A_1' e^{-40t} + A_2' e^{-160t}$$

$$i_C(0^+) = -0.25 + 0.25 + 0 = 0$$

$$\therefore \quad \frac{dv_o}{dt} = 0 = -40A_1' - 160A_2'$$

Solving, $\quad A_1' = 0; \qquad A_2' = 0$

$\therefore \quad v_o = 0$ for $t \geq 0$

Note: $\quad v_o(0) = 0; \qquad v_o(\infty) = 0; \qquad \dfrac{dv_o(0)}{dt} = 0$

Hence, the 0.25 A current circulates between the current source and the ideal inductor in the equivalent circuit. In the original circuit, the 4 V source sustains a current of 0.25 A in the inductor. This is an example of a circuit going directly into steady state when the switch is closed. There is no transient period, or interval.

P 8.31 $\quad \alpha = \dfrac{1}{2RC} = \dfrac{1}{2(20)(31.25 \times 10^{-6})} = 800 \text{ rad/s}$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(50 \times 10^{-3})(31.25 \times 10^{-6})}} = 800 \text{ rad/s}$$

$\alpha^2 = \omega_0^2 \qquad$ Critically damped

$$V_0 = v_C(0) = 60\,\text{V}; \qquad I_0 = i_o(0) = 0; \qquad I_f = i_o(\infty) = \frac{60}{20} = 3\,\text{A}$$

$$i_o = 3 + D_1' t e^{-800t} + D_2' e^{-800t}$$

$i_o(0) = 3 + D_2' = 0 \qquad$ so $\qquad D_2' = -3$

$\dfrac{di_o}{dt}(0) = D_1' - \alpha D_2' = \dfrac{V_0}{L} \qquad$ so $\qquad D_1' - 800(-3) = \dfrac{60}{50 \times 10^{-3}}$

Solving, $\quad D_1' = -1200$

$$i_o(t) = 3 - 1200te^{-800t} - 3e^{-800t}\,\text{A}, \qquad t \geq 0$$

P 8.32 **[a]** $\alpha = \frac{1}{2RC} = \frac{1}{2(20)(31.25 \times 10^{-6})} = 800 \text{ rad/s}$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(50 \times 10^{-3})(31.25 \times 10^{-6})}} = 800 \text{ rad/s}$$

$\alpha^2 = \omega_0^2$ Critically damped

$V_f = v_C(\infty) = 0$

$v_o = D_1' te^{-800t} + D_2' e^{-800t}$

$v_o(0) = D_2' = 60$

$\frac{v_o}{dt}(0) = D_1' - \alpha D_2' = 0$ so $D_1' = (800)(60) = 48{,}000$

$\therefore \quad v_o(t) = 48{,}000te^{-800t} + 60e^{-800t}\,\text{V}, \quad t \geq 0$

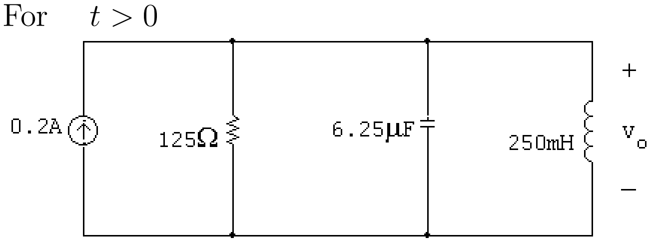
[b] $v_o = L\frac{di_o}{dt} = (50 \times 10^{-3})\frac{d}{dt}(3 - 1200te^{-800t} - 3e^{-800t})$

$v_o(t) = (50 \times 10^{-3})(960{,}000te^{-800t} + 1200e^{-800t})$

$= 48{,}000te^{-800t} + 60e^{-800t}\,\text{V}, \quad t \geq 0$

Thus the solutions for $i_o(t)$ and $v_o(t)$ agree.

P 8.33 For $t > 0$



$\alpha = \frac{1}{2RC} = 640; \qquad \frac{1}{LC} = 64 \times 10^4$

$\omega_d = \sqrt{800^2 - 640^2} = 480$

$i_o = I_f + B_1' e^{-640t}\cos 480t + B_2' e^{-640t}\sin 480t$

$I_f = \frac{25}{125} = 0.2\,\text{A}$

$i_o(0) = 0.2 + B_1' = 0$ so $B_1' = -0.2$

$\frac{di_o}{dt}(0) = -\alpha B_1' + \omega_d B_2' = \frac{V_0}{L}$ so $-640(-0.2) + 480B_2' = 0$

Solving, $B_2' = -0.267$

$i_o(t) = 0.2 - 0.2e^{-640t}\cos 480t - 0.267e^{-640t}\sin 480t\,\text{A}, \quad t \geq 0$

P 8.34 **[a]** $\alpha = \frac{1}{2RC} = 640; \qquad \frac{1}{LC} = 64 \times 10^4$

$$\omega_d = \sqrt{800^2 - 640^2} = 480$$

$$v_o = V_f + B_1' e^{-640t}\cos 480t + B_2' e^{-640t}\sin 480t$$

$$V_f = 0; \qquad V_o(0^+) = 0; \qquad i_C(0^+) = 0.2\,\text{A}$$

$$v_o(0) = 0 + B_1' = 0 \qquad \text{so} \qquad B_1' = 0$$

$$\frac{dv_o}{dt}(0) = -640(0) + 480B_2' = \frac{i_C(0^+)}{6.25 \times 10^{-6}} = 32{,}000 \qquad \text{so} \qquad B_2' = 66.67$$

$$\therefore \qquad v_o(t) = 66.67e^{-640t}\sin 480t\,\text{V}, \quad t \geq 0$$

[b] From the solution to Problem 8.33,

$$i_o(t) = 0.2 - 0.2e^{-640t}\cos 480t - 0.267e^{-640t}\sin 480t\,\text{A}$$

$$v_o = L\frac{di_o}{dt} = (0.25)(266.67e^{-640t}\sin 480t) = 66.67e^{-640t}\sin 480t\,\text{V}, \quad t \geq 0$$

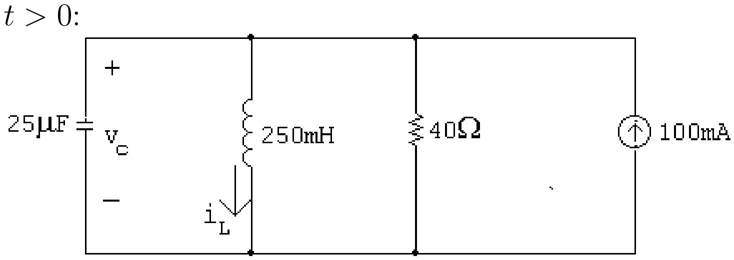
Thus the solutions to Problems 8.33 and 8.34 are consistent.

P 8.35 $t < 0$:

$$V_0 = v_o(0^-) = v_o(0^+) = \frac{3000}{4000}(100) = 75\,\text{V}$$

$$I_0 = i_{\text{L}}(0^-) = i_{\text{L}}(0^+) = 100\,\text{mA}$$

$t > 0$:



$$\alpha = \frac{1}{2RC} = \frac{1}{2(40)(25 \times 10^{-6})} = 500\,\text{rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(250 \times 10^{-3})(25 \times 10^{-6})}} = 400$$

$$\therefore \quad \alpha^2 > \omega_o^2 \quad \text{overdamped}$$

$$s_{1,2} = -500 \pm \sqrt{500^2 - 400^2} = -200, -800$$

[a] $i_L = I_f + A_1 e^{-200t} + A_2 e^{-800t}$

$I_f = 100\,\text{mA}$

$i_L(0) = 0.1 + A_1 + A_2 = 0.1 \qquad \text{so} \qquad A_1 + A_2 = 0$

$$\frac{di_L}{dt}(0) = -200A_1 - 800A_2 = \frac{V_0}{L} = \frac{75}{0.25} = 300$$

Solving, $\quad A_1 = 0.5, \qquad A_2 = -0.5$

$\therefore \qquad i_L(t) = 0.1 + 0.5e^{-200t} - 0.5e^{-800t}\,\text{A}$

[b] $v_C(t) = v_L(t) = L\dfrac{di_L}{dt} = (0.25)(-100e^{-200t} + 400e^{-800t})$

$$= -25e^{-200t} + 100e^{-800t}\,\text{V}, \qquad t \geq 0$$

P 8.36 **[a]** $w_\text{L} = \displaystyle\int_0^\infty p\,dt = \int_0^\infty v_o i_\text{L}\,dt$

$v_o = -25e^{-200t} + 100e^{-800t}\,\text{V}$

$i_\text{L} = 0.1 + 0.5e^{-200t} - 0.5e^{-800t}\,\text{A}$

$p = -2.5e^{-200t} - 12.5e^{-400t} + 10e^{-800t} + 62.5e^{-1000t} - 50e^{-1600t}\,\text{W}$

$$w_\text{L} = -2.5\int_0^\infty e^{-200t}\,dt - 12.5\int_0^\infty e^{-400t}\,dt + 10\int_0^\infty e^{-800t}\,dt$$

$$+62.5\int_0^\infty e^{-1000t}\,dt - 50\int_0^\infty e^{-1600t}\,dt$$

$$= -2.5\frac{e^{-200t}}{-200}\Big|_0^\infty + -12.5\frac{e^{-400t}}{-400}\Big|_0^\infty$$

$$= 10\frac{e^{-800t}}{-800}\Big|_0^\infty + 62.5\frac{e^{-1000t}}{-1000}\Big|_0^\infty$$

$$-\ 50\frac{e^{-1600t}}{-1600}\Big|_0^\infty$$

All the upper limits evaluate to zero hence

$$w_\text{L} = \frac{-2.5}{200} - \frac{-12.5}{400} + \frac{10}{800} + \frac{62.5}{1000} - \frac{50}{1600} = 0\,\text{J}$$

Since the initial and final values of the current in the inductor are the same, the initial and final values of the energy in the inductor are the same. Thus, there is no net energy delivered to the inductor.

[b] $v_o = -25e^{-200t} + 100e^{-800t}\,\text{V}$

$$i_\text{R} = \frac{v_o}{40} = -0.625e^{-200t} + 2.5e^{-800t}\,\text{A}$$

$$p_\text{R} = v_o i_\text{R} = 15.625e^{-400t} - 125e^{-1000t} + 250e^{-1600t}\,\text{W}$$

$$w_\text{R} = \int_0^\infty p_\text{R}\,dt$$

$$= 15.625\int_0^\infty e^{-400t}\,dt - 125\int_0^\infty e^{-1000t}\,dt + 250\int_0^\infty e^{-1600t}\,dt$$

$$= 15.625\frac{e^{-400t}}{-400}\Big|_0^\infty - 125\frac{e^{-1000t}}{-1000}\Big|_0^\infty + 250\frac{e^{-1600t}}{-1600}\Big|_0^\infty$$

Since all the upper limits evaluate to zero we have

$$w_\text{R} = \frac{15.625}{400} - \frac{125}{400} + \frac{250}{1600} = 70.3125\,\text{mJ}$$

[c] $0.1 = i_\text{R} + i_\text{C} + i_\text{L} \quad (\text{mA})$

$$i_\text{C} = 0.1 - (-0.625e^{-200t} + 2.5e^{-800t}) - (0.1 + 0.5e^{-200t} - 0.5e^{-800t})$$

$$= 0.125e^{-200t} - 2e^{-800t}\,\text{A}$$

$$p_\text{C} = v_o i_\text{C} = [-25e^{-200t} + 100e^{-800t}][0.125e^{-200t} - 2e^{-800t}]$$

$$= -3.125e^{-400t} + 62.5e^{-1000t} - 200e^{-1600t}\,\text{W}$$

$$w_\text{C} = -3.125\int_0^\infty e^{-400t}\,dt + 62.5\int_0^\infty e^{-1000t}\,dt - 200\int_0^\infty e^{-1600t}\,dt$$

$$= -3.125\frac{e^{-400t}}{-400}\Big|_0^\infty + 62.5\frac{e^{-1000t}}{-1000}\Big|_0^\infty - 200\frac{e^{-1600t}}{-1600}\Big|_0^\infty$$

Since all upper limits evaluate to zero we have

$$w_\text{C} = \frac{-3.125}{400} + \frac{62.5}{1000} - \frac{200}{1600} = -70.3125\,\text{mJ}$$

[d] $i_s = 100\,\text{mA}$

$$p_s(\text{del}) = 0.1v = -2.5e^{-200t} + 10e^{-800t}\,\text{W}$$

$$w_s = -2.5\int_0^\infty e^{-200t}\,dt + 10\int_0^\infty e^{-800t}\,dt$$

$$= -2.5\frac{e^{-200t}}{-200}\Big|_0^\infty + 10\frac{e^{-800t}}{-800}\Big|_0^\infty = \frac{-2.5}{200} + \frac{10}{800} = 0$$

[e] $w_{\rm L} = 0\,{\rm J}$

$w_{\rm R} = 70.3125\,{\rm mJ}$ (absorbed)

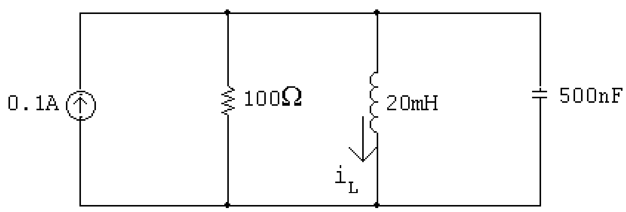
$w_{\rm C} = 70.3125\,{\rm mJ}$ (delivered)

$w_S = 0\,{\rm mJ}$

$\sum w_{\rm del} = w_{\rm abs} = 70.3125\,{\rm mJ}.$

P 8.37 $t < 0:$ $\quad i_{\rm L}(0^-) = \dfrac{36}{300} = 0.12\,{\rm A};$ $\qquad v_{\rm C}(0^-) = 0\,{\rm V}$

The circuit reduces to:



$i_{\rm L}(\infty) = 0.1\,{\rm A}$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(20\times10^{-3})(500\times10^{-9})}} = 10{,}000\ {\rm rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{(100)(500\times10^{-9})} = 10{,}000\ {\rm rad/s}$$

Critically damped:

$$i_L = 0.1 + D_1' te^{-10{,}000t} + D_2' e^{-10{,}000}$$

$$i_L(0) = 0.1 + D_2' = 0.12 \qquad \text{so} \qquad D_2' = 0.02$$

$$\frac{di_L}{dt}(0) = D_1' - \alpha D_2' = \frac{V_0}{L} \qquad \text{so} \qquad D_1' - (10{,}000)(0.02) = 0$$

Solving, $\quad D_1' = 200$

$$i_L(t) = 0.1 + 200te^{-10{,}000t} + 0.02e^{-10{,}000t}\,{\rm A}, \qquad t \ge 0$$

P 8.38 $v_C(0^+) = \frac{1}{2}(80) = 40\,\text{V}$

$$i_L(0^+) = 10\,\text{A}; \qquad i_L(\infty) = \frac{80}{10} = 8\,\text{A}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(5)(250 \times 10^{-6})} = 400 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(16 \times 10^{-3})(250 \times 10^{-6})}} = 500 \text{ rad/s}$$

$$\alpha^2 < \omega_o^2; \qquad \therefore \quad \text{underdamped}$$

$$\omega_d = \sqrt{500^2 - 400^2} = 300 \text{ rad/s}$$

$$i_L = 8 + B_1' e^{-400t} \cos 300t + B_2' e^{-400t} \sin 300t$$

$$i_L(0) = 8 + B_1' = 10; \qquad B_1' = 10 - 8 = 2\,\text{A}$$

$$\frac{di_L}{dt}(0) = -\alpha B_1' + \omega_d B_2' = \frac{V_0}{L}$$

$$\therefore \quad -400(2) + 300B_2' = \frac{40}{0.016} \qquad \text{so} \qquad B_2' = 11\,\text{A}$$

$$\therefore \quad i_L = 8 + 2e^{-400t} \cos 300t + 11e^{-400t} \sin 300t\,\text{A}, \qquad t \geq 0$$

P 8.39 $\alpha = 2000\,\text{rad/s}; \qquad \omega_d = 1500\,\text{rad/s}$

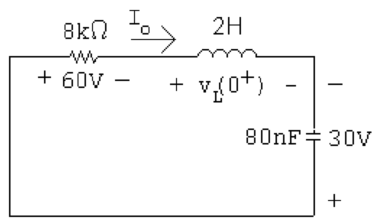
$$\omega_o^2 - \alpha^2 = 225 \times 10^4; \qquad \omega_o^2 = 625 \times 10^4; \quad w_o = 25{,}000\,\text{rad/s}$$

$$\alpha = \frac{R}{2L} = 2000; \qquad R = 4000L$$

$$\frac{1}{LC} = 625 \times 10^4; \qquad L = \frac{1}{(625 \times 10^4)(80 \times 10^{-9})} = 2\,\text{H}$$

$$\therefore \quad R = 8\,\text{k}\Omega$$

$$i(0^+) = B_1 = 7.5\,\text{mA}; \qquad \text{at } t = 0^+$$



$$60 + v_L(0^+) - 30 = 0; \qquad \therefore \qquad v_L(0^+) = -30\,\text{V}$$

$$\frac{di(0^+)}{dt} = \frac{-30}{2} = -15\,\text{A/s}$$

$$\therefore \quad \frac{di(0^+)}{dt} = 1500B_2 - 2000B_1 = -15$$

$$\therefore \quad 1500B_2 = 2000(7.5 \times 10^{-3}) - 15; \qquad \therefore \quad B_2 = 0\,\text{A}$$

$$\therefore \quad i = 7.5e^{-2000t}\cos 1500t\,\text{mA}, \quad t \geq 0$$

P 8.40 From Prob. 8.39 we know v_c will be of the form

$$v_c = B_3e^{-2000t}\cos 1500t + B_4e^{-2000t}\sin 1500t$$

From Prob. 8.39 we have

$$v_c(0) = -30\,\text{V} = B_3$$

and

$$\frac{dv_c(0)}{dt} = \frac{i_C(0)}{C} = \frac{7.5 \times 10^{-3}}{80 \times 10^{-9}} = 93.75 \times 10^3$$

$$\frac{dv_c(0)}{dt} = 1500B_4 - 2000B_3 = 93{,}750$$

$$\therefore \quad 1500B_4 = 2000(-30) + 93{,}750; \qquad B_4 = 22.5\,\text{V}$$

$$v_c(t) = -30e^{-2000t}\cos 1500t + 22.5e^{-2000t}\sin 1500t\,\text{V} \qquad t \geq 0$$

P 8.41 **[a]** $-\alpha + \sqrt{\alpha^2 - \omega_0^2} = -4000; \qquad -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -16{,}000$

$$\therefore \quad \alpha = 10{,}000 \text{ rad/s}, \qquad \omega_0^2 = 64 \times 10^6$$

$$\alpha = \frac{R}{2L} = 10{,}000; \qquad R = 20{,}000L$$

$$\omega_o^2 = \frac{1}{LC} = 64 \times 10^6; \qquad L = \frac{10^9}{64 \times 10^6(31.25)} = 0.5\,\text{H}$$

$$R = 10{,}000\ \Omega$$

[b] $i(0) = 0$

$$L\frac{di(0)}{dt} = v_c(0); \qquad \frac{1}{2}(31.25)\times 10^{-9}v_c^2(0) = 9\times 10^{-6}$$

$$\therefore \quad v_c^2(0) = 576; \qquad v_c(0) = 24\,\text{V}$$

$$\frac{di(0)}{dt} = \frac{24}{0.5} = 48\,\text{A/s}$$

[c] $i(t) = A_1e^{-4000t} + A_2e^{-16{,}000t}$

$$i(0) = A_1 + A_2 = 0$$

$$\frac{di(0)}{dt} = -4000A_1 - 16{,}000A_2 = 48$$

Solving,

$$\therefore \quad A_1 = 4\,\text{mA}; \qquad A_2 = -4\,\text{mA}$$

$$i(t) = 4e^{-4000t} - 4e^{-16{,}000t}\,\text{mA}, \qquad t \geq 0$$

[d] $\dfrac{di(t)}{dt} = -16e^{-4000t} + 64e^{-16{,}000t}$

$$\frac{di}{dt} = 0 \text{ when } 64e^{-16{,}000t} = 16e^{-4000t}$$

$$\text{or } e^{12{,}000t} = 4$$

$$\therefore \quad t = \frac{\ln 4}{12{,}000} = 115.52\,\mu\text{s}$$

[e] $i_{\max} = 4e^{-0.4621} - 4e^{-1.8484} = 1.89\,\text{mA}$

[f] $v_L(t) = 0.5\dfrac{di}{dt} = [-8e^{-1000t} + 32e^{-4000t}]\,\text{V}, \quad t \geq 0^+$

P 8.42 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^9}{(125)(0.32)} = 25\times 10^6$

$$\alpha = \frac{R}{2L} = \omega_o = 5000\,\text{rad/s}$$

$$\therefore \quad R = (5000)(2)L = 1250\,\Omega$$

[b] $i(0) = i_{\text{L}}(0) = 6\,\text{mA}$

$$v_{\text{L}}(0) = 15 - (0.006)(1250) = 7.5\,\text{V}$$

$$\frac{di}{dt}(0) = \frac{7.5}{0.125} = 60\ \text{A/s}$$

[c] $v_C = D_1te^{-5000t} + D_2e^{-5000t}$

$v_C(0) = D_2 = 15\,\text{V}$

$$\frac{dv_C}{dt}(0) = D_1 - 5000D_2 = \frac{i_C(0)}{C} = \frac{-i_L(0)}{C} = -18{,}750$$

$\therefore \quad D_1 = 56{,}250$ V/s

$v_C = 56{,}250te^{-5000t} + 15e^{-5000t}\,\text{V}, \qquad t \geq 0$

P 8.43 **[a]** $\dfrac{1}{LC} = 20{,}000^2$

There are many possible solutions. This one begins by choosing $L = 1\,\text{mH}$. Then,

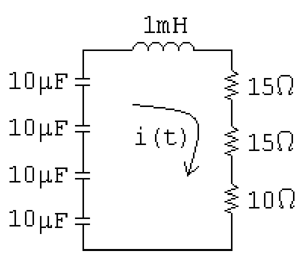
$$C = \frac{1}{(1 \times 10^{-3})(20{,}000)^2} = 2.5\,\mu\text{F}$$

We can achieve this capacitor value using components from Appendix H by combining four $10\,\mu$F capacitors in series.

Critically damped: $\quad \alpha = \omega_0 = 20{,}000 \quad$ so $\quad \dfrac{R}{2L} = 20{,}000$

$\therefore \quad R = 2(10^{-3})(20{,}000) = 40\,\Omega$

We can create this resistor value using components from Appendix H by combining a $10\,\Omega$ resistor and two $15\,\Omega$ resistors in series. The final circuit:



[b] $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -20{,}000 \pm 0$

Therefore there are two repeated real roots at $-20{,}000$ rad/s.

P 8.44 **[a]** Underdamped response:

$\alpha < \omega_0 \qquad$ so $\qquad \alpha < 20{,}000$

Therefore we choose a larger resistor value than the one used in Problem 8.40 to give a smaller value of α. For convenience, pick $\alpha = 16{,}000$ rad/s:

$$\alpha = \frac{R}{2L} = 16{,}000 \qquad \text{so} \qquad R = 2(16{,}000)(10^{-3}) = 32\,\Omega$$

We can create a 32 Ω resistance by combining a 10 Ω resistor and a 22 Ω resistor in series.

$$s_{1,2} = -16{,}000 \pm \sqrt{16{,}000^2 - 20{,}000^2} = -16{,}000 \pm j12{,}000 \text{ rad/s}$$

[b] Overdamped response:

$\alpha > \omega_0$ so $\alpha > 20{,}000$

Therefore we choose a smaller resistor value than the one used in Problem 8.40. Choose $R = 50\,\Omega$, which can be created by combining two 100 Ω resistors in parallel:

$$\alpha = \frac{R}{2L} = 25{,}000$$

$$s_{1,2} = -25{,}000 \pm \sqrt{25{,}000^2 - 20{,}000^2} = -25{,}000 \pm 15{,}000$$

$$= -10{,}000 \text{ rad/s}; \qquad \text{and} \qquad -40{,}000 \text{ rad/s}$$

P 8.45 **[a]** $t < 0$:

$$i_o = \frac{100}{50} = 2\,\text{A}; \qquad v_o = -4(100) = -400\,\text{V}$$

$t > 0$:

$$\alpha = \frac{R}{2L} = \frac{500}{2(0.4)} = 625 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(0.4)(10 \times 10^{-6})}} = 500 \text{ rad/s}$$

$$\alpha^2 < \omega_o^2 \quad \therefore \quad \text{overdamped}$$

$$s_{1,2} = -625 \pm \sqrt{625^2 - 500^2} = -250, -1000 \text{ rad/s}$$

$$i_o = A_1 e^{-250t} + A_2 e^{-1000t}$$

$$i_o(0) = A_1 + A_2 = 2$$

$$\frac{di_o}{dt}(0) = -250A_1 - 1000A_2 = \frac{1}{L}(-V_0 - RI_0) = -1500$$

$$\text{Solving,} \qquad A_1 = \frac{2}{3}; \qquad A_2 = \frac{4}{3}$$

$$\therefore \qquad i_o(t) = \frac{2}{3}e^{-250t} + \frac{4}{3}e^{-1000t}\,\text{A}, \quad t \geq 0$$

[b] $$v_o(t) = \frac{1}{10 \times 10^{-6}} \int_0^t i_o(x)\,dx - 400$$

$$= 10^5 \left(\int_0^t \frac{2}{3} e^{-250x}\,dx + \int_0^t \frac{4}{3} e^{-1000x}\,dx \right) - 400$$

$$= 10^5 \left(\frac{(2/3)e^{-250x}}{-250} \Bigg|_0^t + \frac{(4/3)e^{-1000x}}{-1000} \Bigg|_0^t \right) - 400$$

$$= -266.67e^{-250t} - 133.33e^{-1000t}\,\text{V}, \quad t \geq 0$$

P 8.46 $t < 0:$ $\quad I_0 = -75\,\text{mA}; \quad V_0 = 0$

$t > 0:$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(80 \times 10^{-3})(200 \times 10^{-6})}} = 250$$

$$\alpha = \frac{R}{2L} = \frac{40}{2(80 \times 10^{-3})} = 250$$

Critically damped:

$$i = D_1 te^{-250t} + D_2 e^{-250t}$$

$$i(0) = D_2 = I_0 = -0.075$$

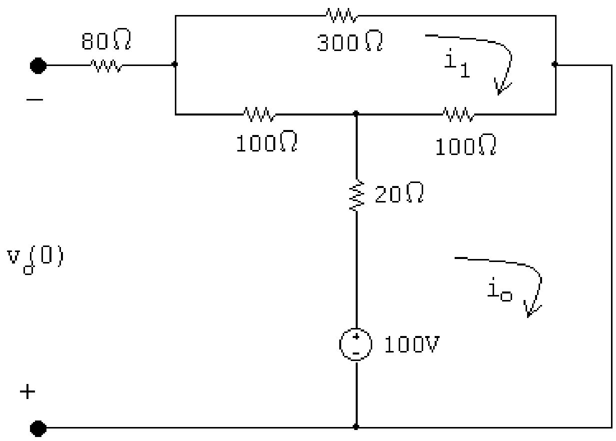
$$\frac{di}{dt}(0) = D_1 - \alpha D_2 = \frac{1}{L}(-V_0 - RI_0)$$

So $\quad D_1 - (250)(-0.075) = \dfrac{1}{80 \times 10^{-3}}(0 - (40)(-0.075))$

Solving, $\quad D_1 = 18.75$

$\therefore \quad i(t) = 18{,}750te^{-250t} - 75e^{-250t}\,\text{mA}, \quad t \geq 0$

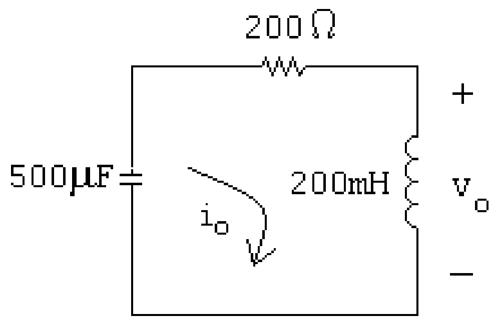
P 8.47 $t < 0$:



$$500i_1 - 100i_o = 0; \qquad -100i_1 + 120i_o = 100$$

Solving, $\quad i_1 = 0.2\,\text{A}; \quad i_o = 1\,\text{A}; \quad V_o = -100 + 20i_o + 100i_1 = -60\,\text{V}$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{200}{2(0.2)} = 500$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(0.2)(36.25 \times 10^{-6})}} = 400$$

$\alpha^2 > \omega_o^2 \quad$ overdamped

$$s_{1,2} = -100 \pm \sqrt{500^2 - 400^2} = -200, -800 \ \text{rad/s}$$

$$i_o = A_1 e^{-200t} + A_2 e^{-800t}$$

$$i_o(0) = A_1 + A_2 = 1$$

$$\frac{di_o}{dt}(0) = -200A_1 - 800A_2 = \frac{1}{L}(-V_0 - RI_0) = -700$$

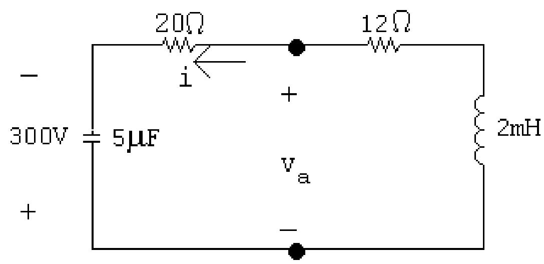
Solving, $A_1 = 166.67\,\text{mA}$; $A_2 = 833.33\,\text{mA}$

$$\therefore \quad i_o(t) = 166.67e^{-200t} + 833.33e^{-800t}\,\text{mA}, \quad t \geq 0$$

$$v_o(t) = L\frac{di_o}{dt} = (0.2)[(-200)0.16667e^{-200t} + (-800)0.83333e^{-800t}]$$

$$= -6.67e^{-200t} - 133.33e^{-800t}\,\text{V}, \quad t \geq 0$$

P 8.48 **[a]** For $t > 0$:



Since $i(0^-) = i(0^+) = 0$

$v_a(0^+) = -300\,\text{V}$

[b] $v_a = 20i + \frac{1}{5 \times 10^{-6}}\int_0^t i\,dx - 300$

$$\frac{dv_a}{dt} = 20\frac{di}{dt} + 200{,}000i$$

$$\frac{dv_a(0^+)}{dt} = 20\frac{di(0^+)}{dt} + 200{,}000i(0^+) = 20\frac{di(0^+)}{dt}$$

$$\frac{di(0^+)}{dt} = \frac{1}{L}(-V_0 - RI_0) = \frac{1}{2 \times 10^{-3}}(300) = 150{,}000$$

$$\therefore \quad \frac{dv_a(0^+)}{dt} = 3 \times 10^6\,\text{V/s}$$

[c] $\alpha = \frac{R}{2L} = \frac{32}{2(2 \times 10^{-3}} = 8000\,\text{rad/s}$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(2 \times 10^{-3})(5 \times 10^{-6})}} = 10{,}000$$

Underdamped:

$$\omega_d = \sqrt{10{,}000^2 - 8000^2} = 6000$$

$$v_a = B_1 e^{-8000t}\cos 6000t + B_2 e^{-8000t}\sin 6000t$$

$$v_a(0) = B_1 = -300\,\text{V}$$

$$\frac{dv_a(0)}{dt} = -8000B_1 + 6000B_2 = 3\times 10^6; \qquad \therefore \quad B_2 = 100\,\text{V}$$

$$v_a(t) = -300e^{-8000t}\cos 6000t + 100e^{-8000t}\sin 6000t\,\text{V}, \quad t \geq 0^+$$

P 8.49 $\alpha = \dfrac{R}{2L} = \dfrac{200}{2(0.025)} = 400\,\text{rad/s}$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(250\times 10^{-3})(16\times 10^{-6})}} = 500\,\text{rad/s}$$

$$\alpha^2 < \omega_0^2: \qquad \text{underdamped}$$

$$\omega_d = \sqrt{500^2 - 400^2} = 300\,\text{rad/s}$$

$$v_o = V_f + B_1' e^{-400t}\cos 300t + B_2' e^{-400t}\sin 300t$$

$$v_o(\infty) = 200(0.08) = 16\,\text{V}$$

$$v_o(0) = 0 = V_f + B_1' = 0 \qquad \text{so} \qquad B_1' = -16\,\text{V}$$

$$\frac{dv_o}{dt}(0) = 0 = -400B_1' + 300B_2' \qquad \text{so} \qquad B_2' = -21.33\,\text{V}$$

$$\therefore \quad v_o(t) = 16 - 16e^{-400t}\cos 300t - 21.33e^{-400t}\sin 300t\,\text{V}, \quad t \geq 0$$

P 8.50 $\alpha = \dfrac{R}{2L} = \dfrac{250}{2(0.025)} = 500\,\text{rad/s}$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(250\times 10^{-3})(16\times 10^{-6})}} = 500\,\text{rad/s}$$

$$\alpha^2 = \omega_0^2: \qquad \text{critically damped}$$

$$v_o = V_f + D_1' te^{-500t} + D_2' e^{-500t}$$

$$v_o(0) = 0 = V_f + D_2'$$

$$v_o(\infty) = (250)(0.08) = 20\,\text{V}; \qquad \therefore \quad D_2' = -20\,\text{V}$$

$$\frac{dv_o}{dt}(0) = 0 = D_1' - \alpha D_2' \qquad \text{so} \qquad D_1' = (500)(-20) = -10{,}000\ \text{V/s}$$

$$\therefore \qquad v_o(t) = 20 - 10{,}000te^{-500t} - 20e^{-500t}\,\text{V}, \quad t \geq 0$$

P 8.51 $\alpha = \dfrac{R}{2L} = \dfrac{312.5}{2(0.025)} = 625\,\text{rad/s}$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(250 \times 10^{-3})(16 \times 10^{-6})}} = 500\,\text{rad/s}$$

$\alpha^2 > \omega_0^2 :$ overdamped

$$s_{1,2} = -625 \pm \sqrt{625^2 - 500^2} = -250, -1000\,\text{rad/s}$$

$$v_o = V_f + A_1' e^{-250t} + A_2' e^{-1000t}$$

$$v_o(0) = 0 = V_f + A_1' + A_2'$$

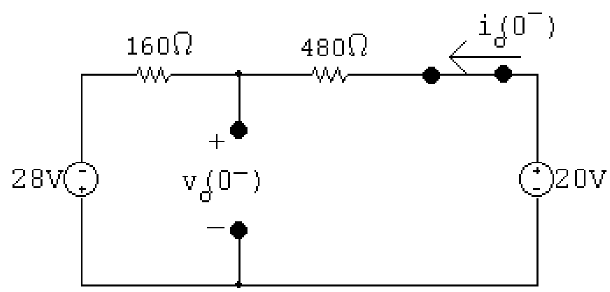
$$v_o(\infty) = (312.5)(08) = 25\,\text{V}; \qquad \therefore \quad A_1' + A_2' = -25\,\text{V}$$

$$\frac{dv_o}{dt}(0) = 0 = -250A_1' - 1000A_2'$$

Solving, $A_1' = -33.33\,\text{V};$ $A_2' = 8.33\,\text{V}$

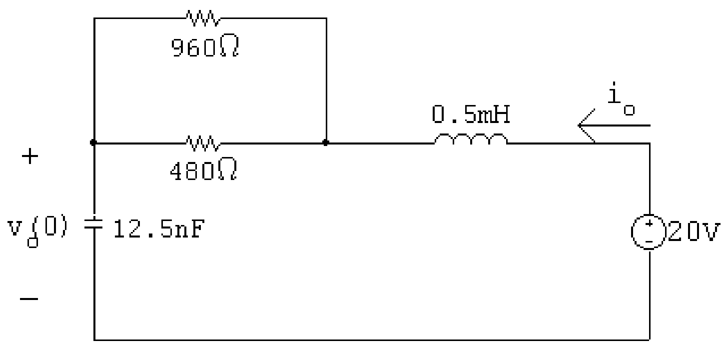
$$v_o(t) = 25 - 33.33e^{-250t} + 8.33e^{-1000t}\,\text{V}, \quad t \geq 0$$

P 8.52 $t < 0$:



$$i_o(0^-) = \frac{20 + 28}{160 + 480} = 75\,\text{mA}$$

$$v_o(0^-) = 20 - 480(0.075) = -16\,\text{V}$$



As $t \to \infty$, $V_f = 20$ V.

$$R_{\text{eq}} = 960 \| 480 = 320\,\Omega$$

$$\alpha = \frac{R_{\text{eq}}}{2L} = \frac{320}{2(0.5 \times 10^{-3})} = 320{,}000 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(0.5 \times 10^{-3})(12.5 \times 10^{-9})}} = 400{,}000\,\text{rad/s}$$

$$\alpha^2 < \omega_0^2: \qquad \text{underdamped}$$

$$\omega_d = \sqrt{400{,}000^2 - 320{,}000^2} = 240{,}000 \text{ rad/s}$$

$$v_o = 20 + B_1' e^{-320{,}000t} \cos 240{,}000t + B_2' e^{-320{,}000t} \sin 240{,}000t$$

$$v_o(0) = 20 + B_1' = -16 \qquad \text{so} \qquad B_1' = -36\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -\alpha B_1' + \omega_d B_2' = \frac{I_0}{C} \qquad \text{so} \qquad -320{,}000(-36) + 240{,}000 B_2' = \frac{75 \times 10^{-3}}{12.5 \times 10^{-9}}$$

$$\text{solving,} \qquad B_2' = -23$$

$$\therefore \qquad v_o(t) = 20 - 36e^{-320{,}000t} \cos 240{,}000t - 23e^{-320{,}000t} \sin 240{,}000t\,\text{V} \quad t \geq 0$$

P 8.53 $i_C(0) = 0; \qquad v_o(0) = 200\,\text{V}$

$$\alpha = \frac{R}{2L} = \frac{25}{2(0.25)} = 50\,\text{rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(250 \times 10^{-3})(2.5 \times 10^{-3})}} = 40 \text{ rad/s}$$

$\therefore \quad \alpha^2 > \omega_o^2; \qquad$ overdamped

$$s_{1,2} = -50 \pm \sqrt{50^2 - 40^2} = -20, -80 \text{ rad/s}$$

$$v_o = V_f + A_1' e^{-20t} + A_2' e^{-80t}$$

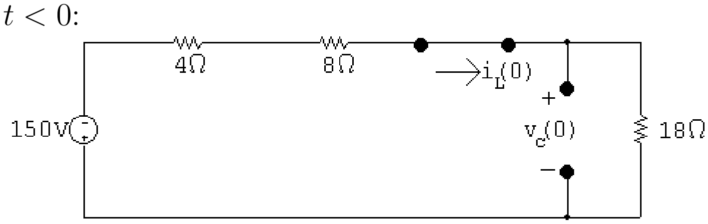
$V_f = 150\,\text{V}; \qquad v_o(0) = 150 + A_1' + A_2' = 200 \qquad$ so $\qquad A_1' + A_2' = 50$

$$\frac{v_o}{dt}(0) = -20A_1' - 80A_2' = \frac{I_0}{C} = 0$$

Solving, $\qquad A_1' = 66.67; \qquad A_2' = -16.67$

$$\therefore \qquad v_o(t) = 150 + 66.67e^{-20t} - 16.67e^{-80t}\,\text{V}, \quad t \geq 0$$

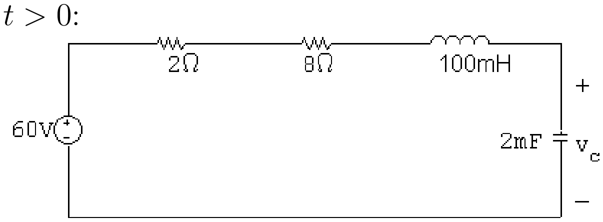
P 8.54 $\quad t < 0$:



$$i_L(0) = \frac{-150}{30} = -5\,\text{A}$$

$$v_C(0) = 18i_L(0) = -90\,\text{V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{10}{2(0.1)} = 50 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(0.1)(2 \times 10^{-3})} = 5000$$

$\omega_o > \alpha^2 \quad \therefore \qquad$ underdamped

$$s_{1,2} = -50 \pm \sqrt{50^2 - 5000} = -50 \pm j50$$

$$v_c = 60 + B_1' e^{-50t}\cos 50t + B_2' e^{-50t}\sin 50t$$

$$v_c(0) = -90 = 60 + B_1' \quad \therefore \quad B_1' = -150$$

$$C\frac{dv_c}{dt}(0) = -5; \qquad \frac{dv_c}{dt}(0) = \frac{-5}{2\times 10^{-3}} = -2500$$

$$\frac{dv_c}{dt}(0) = -50B_1' + 50B_2 = -2500 \quad \therefore \quad B_2' = -200$$

$$v_c = 60 - 150e^{-50t}\cos 50t - 200e^{-50t}\sin 50t\,\text{V}, \quad t \geq 0$$

P 8.55 **[a]** Let i be the current in the direction of the voltage drop $v_o(t)$. Then by hypothesis

$$i = i_f + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$$

$$i_f = i(\infty) = 0, \qquad i(0) = \frac{V_g}{R} = B_1'$$

Therefore $\quad i = B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$

$$L\frac{di(0)}{dt} = 0, \qquad \text{therefore} \quad \frac{di(0)}{dt} = 0$$

$$\frac{di}{dt} = \left[(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t\right]e^{-\alpha t}$$

Therefore $\quad \omega_d B_2' - \alpha B_1' = 0; \qquad B_2' = \dfrac{\alpha}{\omega_d}B_1' = \dfrac{\alpha}{\omega_d}\dfrac{V_g}{R}$

Therefore

$$v_o = L\frac{di}{dt} = -\left\{L\left(\frac{\alpha^2 V_g}{\omega_d R} + \frac{\omega_d V_g}{R}\right)\sin\omega_d t\right\}e^{-\alpha t}$$

$$= -\left\{\frac{LV_g}{R}\left(\frac{\alpha^2}{\omega_d} + \omega_d\right)\sin\omega_d t\right\}e^{-\alpha t}$$

$$= -\frac{V_g L}{R}\left(\frac{\alpha^2 + \omega_d^2}{\omega_d}\right)e^{-\alpha t}\sin\omega_d t$$

$$= -\frac{V_g L}{R}\left(\frac{\omega_o^2}{\omega_d}\right)e^{-\alpha t}\sin\omega_d t$$

$$= -\frac{V_g L}{R\omega_d}\left(\frac{1}{LC}\right)e^{-\alpha t}\sin\omega_d t$$

$$v_o = -\frac{V_g}{RC\omega_d}e^{-\alpha t}\sin\omega_d t\,\text{V}, \quad t \geq 0$$

[b] $\frac{dv_o}{dt} = -\frac{V_g}{\omega_d RC}\{\omega_d \cos\omega_d t - \alpha\sin\omega_d t\}e^{-\alpha t}$

$\frac{dv_o}{dt} = 0 \quad \text{when} \quad \tan\omega_d t = \frac{\omega_d}{\alpha}$

Therefore $\omega_d t = \tan^{-1}(\omega_d/\alpha)$ (smallest t)

$t = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$

P 8.56 **[a]** From Problem 8.55 we have

$$v_o = \frac{-V_g}{RC\omega_d}e^{-\alpha t}\sin\omega_d t$$

$$\alpha = \frac{R}{2L} = \frac{480}{2(8\times10^{-3})} = 30{,}000\,\text{rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(8\times10^{-3})(50\times10^{-9})}} = 50{,}000\,\text{rad/s}$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 40{,}000\,\text{rad/s}$$

$$\frac{-V_g}{RC\omega_d} = \frac{-(-24)}{(480)(50\times10^{-9})(40\times10^3)} = 25$$

$$\therefore \quad v_o = 25e^{-30{,}000t}\sin 40{,}000t\,\text{V}$$

[b] From Problem 8.55

$$t_d = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right) = \frac{1}{40{,}000}\tan^{-1}\left(\frac{40{,}000}{30{,}000}\right)$$

$$t_d = 23.18\,\mu\text{s}$$

[c] $v_{\max} = 25e^{-0.03(23.18)}\sin[(0.04)(23.18)] = 9.98\,\text{V}$

[d] $R = 96\,\Omega; \qquad \alpha = 6000\,\text{rad/s}$

$$\omega_d = 49{,}638.7\,\text{rad/s}$$

$$v_o = 100.73e^{-6000t}\sin 49{,}638.7t\,\text{V}, \quad t \ge 0$$

$$t_d = 29.22\,\mu\text{s}$$

$$v_{\max} = 83.92\,\text{V}$$

P 8.57 **[a]** $v_c = V_f + [B_1' \cos\omega_d t + B_2' \sin\omega_d t]\, e^{-\alpha t}$

$$\frac{dv_c}{dt} = [(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t]e^{-\alpha t}$$

Since the initial stored energy is zero,

$$v_c(0^+) = 0 \quad \text{and} \quad \frac{dv_c(0^+)}{dt} = 0$$

It follows that $B_1' = -V_f$ and $B_2' = \dfrac{\alpha B_1'}{\omega_d}$

When these values are substituted into the expression for $[dv_c/dt]$, we get

$$\frac{dv_c}{dt} = \left(\frac{\alpha^2}{\omega_d} + \omega_d\right) V_f e^{-\alpha t}\sin\omega_d t$$

But $V_f = V$ and $\dfrac{\alpha^2}{\omega_d} + \omega_d = \dfrac{\alpha^2 + \omega_d^2}{\omega_d} = \dfrac{\omega_o^2}{\omega_d}$

Therefore $\dfrac{dv_c}{dt} = \left(\dfrac{\omega_o^2}{\omega_d}\right) V e^{-\alpha t}\sin\omega_d t$

[b] $\dfrac{dv_c}{dt} = 0$ when $\sin\omega_d t = 0$, or $\omega_d t = n\pi$

where $n = 0, 1, 2, 3, \ldots$

Therefore $t = \dfrac{n\pi}{\omega_d}$

[c] When $t_n = \dfrac{n\pi}{\omega_d}$, $\cos\omega_d t_n = \cos n\pi = (-1)^n$

and $\sin\omega_d t_n = \sin n\pi = 0$

Therefore $v_c(t_n) = V[1 - (-1)^n e^{-\alpha n\pi/\omega_d}]$

[d] It follows from [c] that

$$v(t_1) = V + Ve^{-(\alpha\pi/\omega_d)} \quad \text{and} \quad v_c(t_3) = V + Ve^{-(3\alpha\pi/\omega_d)}$$

Therefore $\dfrac{v_c(t_1) - V}{v_c(t_3) - V} = \dfrac{e^{-(\alpha\pi/\omega_d)}}{e^{-(3\alpha\pi/\omega_d)}} = e^{(2\alpha\pi/\omega_d)}$

But $\dfrac{2\pi}{\omega_d} = t_3 - t_1 = T_d$, thus $\alpha = \dfrac{1}{T_d} \ln\dfrac{[v_c(t_1) - V]}{[v_c(t_3) - V]}$

P 8.58 $\dfrac{1}{T_d}\ln\left\{\dfrac{v_c(t_1)-V}{v_c(t_3)-V}\right\}$; $\qquad T_d = t_3 - t_1 = \dfrac{3\pi}{7} - \dfrac{\pi}{7} = \dfrac{2\pi}{7}\,\text{ms}$

$$\alpha = \frac{7000}{2\pi}\ln\left[\frac{63.84}{26.02}\right] = 1000; \qquad \omega_d = \frac{2\pi}{T_d} = 7000\,\text{rad/s}$$

$$\omega_o^2 = \omega_d^2 + \alpha^2 = 49\times 10^6 + 10^6 = 50\times 10^6$$

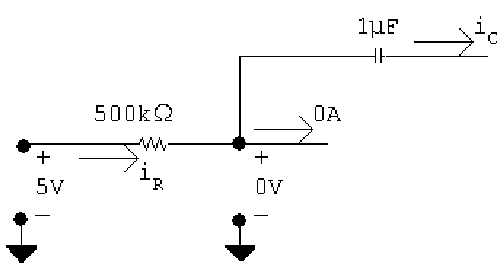
$$L = \frac{1}{(50\times 10^6)(0.1\times 10^{-6})} = 200\,\text{mH}; \qquad R = 2\alpha L = 400\,\Omega$$

P 8.59 At $t = 0$ the voltage across each capacitor is zero. It follows that since the operational amplifiers are ideal, the current in the $500\,\text{k}\Omega$ is zero. Therefore there cannot be an instantaneous change in the current in the $1\,\mu\text{F}$ capacitor. Since the capacitor current equals $C(dv_o/dt)$, the derivative must be zero.

P 8.60 **[a]** From Example 8.13 $\quad \dfrac{d^2v_o}{dt^2} = 2$

therefore $\quad \dfrac{dg(t)}{dt} = 2, \quad g(t) = \dfrac{dv_o}{dt}$

$$g(t) - g(0) = 2t; \qquad g(t) = 2t + g(0); \qquad g(0) = \frac{dv_o(0)}{dt}$$



$$i_\text{R} = \frac{5}{500}\times 10^{-3} = 10\,\mu\text{A} = i_\text{C} = -C\frac{dv_o(0)}{dt}$$

$$\frac{dv_o(0)}{dt} = \frac{-10\times 10^{-6}}{1\times 10^{-6}} = -10 = g(0)$$

$$\frac{dv_o}{dt} = 2t - 10$$

$$dv_o = 2t\,dt - 10\,dt$$

$$v_o - v_o(0) = t^2 - 10t; \qquad v_o(0) = 8\,\text{V}$$

$$v_o = t^2 - 10t + 8, \qquad 0 \le t \le t_\text{sat}$$

[b] $t^2 - 10t + 8 = -9$

$t^2 - 10t + 17 = 0$

$t \cong 2.17\,\text{s}$

P 8.61 Part (1) — Example 8.14, with R_1 and R_2 removed:

[a] $R_a = 100\,\text{k}\Omega$; $C_1 = 0.1\,\mu\text{F}$; $R_b = 25\,\text{k}\Omega$; $C_2 = 1\,\mu\text{F}$

$$\frac{d^2v_o}{dt^2} = \left(\frac{1}{R_aC_1}\right)\left(\frac{1}{R_bC_2}\right)v_g; \qquad \frac{1}{R_aC_1} = 100 \quad \frac{1}{R_bC_2} = 40$$

$$v_g = 250 \times 10^{-3}; \qquad \text{therefore} \quad \frac{d^2v_o}{dt^2} = 1000$$

[b] Since $v_o(0) = 0 = \frac{dv_o(0)}{dt}$, our solution is $v_o = 500t^2$

The second op-amp will saturate when

$v_o = 6\,\text{V}$, or $t_{\text{sat}} = \sqrt{6/500} \cong 0.1095\,\text{s}$

[c] $\frac{dv_{o1}}{dt} = -\frac{1}{R_aC_1}v_g = -25$

[d] Since $v_{o1}(0) = 0$, $v_{o1} = -25t\,\text{V}$

At $t = 0.1095\,\text{s}$, $v_{o1} \cong -2.74\,\text{V}$

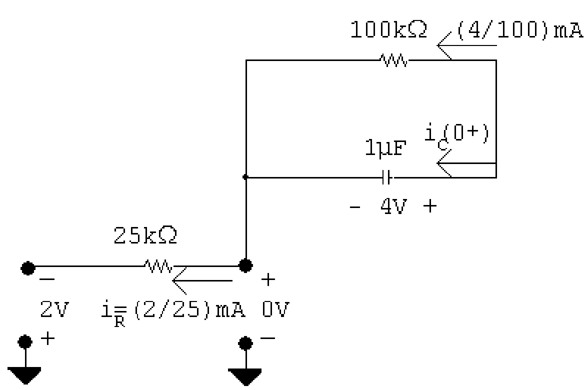
Therefore the second amplifier saturates before the first amplifier saturates. Our expressions are valid for $0 \le t \le 0.1095\,\text{s}$. Once the second op-amp saturates, our linear model is no longer valid.

Part (2) — Example 8.14 with $v_{o1}(0) = -2\,\text{V}$ and $v_o(0) = 4\,\text{V}$:

[a] Initial conditions will not change the differential equation; hence the equation is the same as Example 8.14.

[b] $v_o = 5 + A_1'e^{-10t} + A_2'e^{-20t}$ (from Example 8.14)

$v_o(0) = 4 = 5 + A_1' + A_2'$



$$\frac{4}{100} + i_C(0^+) - \frac{2}{25} = 0$$

$$i_C(0^+) = \frac{4}{100}\,\text{mA} = C\frac{dv_o(0^+)}{dt}$$

$$\frac{dv_o(0^+)}{dt} = \frac{0.04 \times 10^{-3}}{10^{-6}} = 40\,\text{V/s}$$

$$\frac{dv_o}{dt} = -10A'_1e^{-10t} - 20A'_2e^{-20t}$$

$$\frac{dv_o}{dt}(0^+) = -10A'_1 - 20A'_2 = 40$$

Therefore $-A'_1 - 2A'_2 = 4$ and $A'_1 + A'_2 = -1$
Thus, $A'_1 = 2$ and $A'_2 = -3$

$$v_o = 5 + 2e^{-10t} - 3e^{-20t}\,\text{V}$$

[c] Same as Example 8.14:

$$\frac{dv_{o1}}{dt} + 20v_{o1} = -25$$

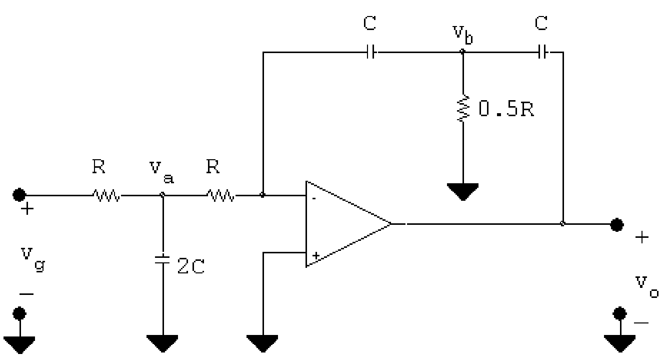
[d] From Example 8.14:

$v_{o1}(\infty) = -1.25\,\text{V}$; $\quad v_1(0) = -2\,\text{V}$ (given)

Therefore

$$v_{o1} = -1.25 + (-2 + 1.25)e^{-20t} = -1.25 - 0.75e^{-20t}\,\text{V}$$

P 8.62 **[a]**



$$2C\frac{dv_a}{dt} + \frac{v_a - v_g}{R} + \frac{v_a}{R} = 0$$

(1) Therefore $\quad \dfrac{dv_a}{dt} + \dfrac{v_a}{RC} = \dfrac{v_g}{2RC}$

$$\frac{0 - v_a}{R} + C\frac{d(0 - v_b)}{dt} = 0$$

(2) Therefore $\quad \dfrac{dv_b}{dt} + \dfrac{v_a}{RC} = 0, \quad v_a = -RC\dfrac{dv_b}{dt}$

$$\frac{2v_b}{R} + C\frac{dv_b}{dt} + C\frac{d(v_b - v_o)}{dt} = 0$$

(3) Therefore $\frac{dv_b}{dt} + \frac{v_b}{RC} = \frac{1}{2}\frac{dv_o}{dt}$

From (2) we have $\frac{dv_a}{dt} = -RC\frac{d^2v_b}{dt^2}$ and $v_a = -RC\frac{dv_b}{dt}$

When these are substituted into (1) we get

(4) $-RC\frac{d^2v_b}{dt^2} - \frac{dv_b}{dt} = \frac{v_g}{2RC}$

Now differentiate (3) to get

(5) $\frac{d^2v_b}{dt^2} + \frac{1}{RC}\frac{dv_b}{dt} = \frac{1}{2}\frac{d^2v_o}{dt^2}$

But from (4) we have

(6) $\frac{d^2v_b}{dt^2} + \frac{1}{RC}\frac{dv_b}{dt} = -\frac{v_g}{2R^2C^2}$

Now substitute (6) into (5)

$$\frac{d^2v_o}{dt^2} = -\frac{v_g}{R^2C^2}$$

[b] When $R_1C_1 = R_2C_2 = RC$: $\frac{d^2v_o}{dt^2} = \frac{v_g}{R^2C^2}$

The two equations are the same except for a reversal in algebraic sign.

[c] Two integrations of the input signal with one operational amplifier.

P 8.63 **[a]** $\frac{d^2v_o}{dt^2} = \frac{1}{R_1C_1R_2C_2}v_g$

$$\frac{1}{R_1C_1R_2C_2} = \frac{10^{-6}}{(100)(400)(0.5)(0.2)\times 10^{-6}\times 10^{-6}} = 250$$

$\therefore \quad \frac{d^2v_o}{dt^2} = 250v_g$

$0 \le t \le 0.5^-$:

$v_g = 80\,\text{mV}$

$$\frac{d^2v_o}{dt^2} = 20$$

Let $g(t) = \frac{dv_o}{dt}$, then $\frac{dg}{dt} = 20$ or $dg = 20\,dt$

$$\int_{g(0)}^{g(t)} dx = 20\int_0^t dy$$

$$g(t) - g(0) = 20t, \quad g(0) = \frac{dv_o}{dt}(0) = 0$$

$$g(t) = \frac{dv_o}{dt} = 20t$$

$$dv_o = 20t\,dt$$

$$\int_{v_o(0)}^{v_o(t)} dx = 20\int_0^t x\,dx; \qquad v_o(t) - v_o(0) = 10t^2, \quad v_o(0) = 0$$

$$v_o(t) = 10t^2\,\text{V}, \quad 0 \le t \le 0.5^-$$

$$\frac{dv_{o1}}{dt} = -\frac{1}{R_1C_1}v_g = -20v_g = -1.6$$

$$dv_{o1} = -1.6\,dt$$

$$\int_{v_{o1}(0)}^{v_{o1}(t)} dx = -1.6\int_0^t dy$$

$$v_{o1}(t) - v_{o1}(0) = -1.6t, \qquad v_{o1}(0) = 0$$

$$v_{o1}(t) = -1.6t\,\text{V}, \qquad 0 \le t \le 0.5^-$$

$0.5^+ \le t \le t_{\text{sat}}$:

$$\frac{d^2v_o}{dt^2} = -10, \qquad \text{let} \quad g(t) = \frac{dv_o}{dt}$$

$$\frac{dg(t)}{dt} = -10; \qquad dg(t) = -10\,dt$$

$$\int_{g(0.5^+)}^{g(t)} dx = -10\int_{0.5}^t dy$$

$$g(t) - g(0.5^+) = -10(t - 0.5) = -10t + 5$$

$$g(0.5^+) = \frac{dv_o(0.5^+)}{dt}$$

$$C\frac{dv_o}{dt}(0.5^+) = \frac{0 - v_{o1}(0.5^+)}{400 \times 10^3}$$

$$v_{o1}(0.5^+) = v_o(0.5^-) = -1.6(0.5) = -0.80\,\text{V}$$

$$\therefore \quad C\frac{dv_{o1}(0.5^+)}{dt} = \frac{0.80}{0.4 \times 10^3} = 2\,\mu\text{A}$$

$$\frac{dv_{o1}}{dt}(0.5^+) = \frac{2 \times 10^{-6}}{0.2 \times 10^{-6}} = 10\,\text{V/s}$$

$$\therefore \quad g(t) = -10t + 5 + 10 = -10t + 15 = \frac{dv_o}{dt}$$

$$\therefore \quad dv_o = -10t\,dt + 15\,dt$$

$$\int_{v_o(0.5^+)}^{v_o(t)} dx = \int_{0.5^+}^{t} -10y\,dy + \int_{0.5^+}^{t} 15\,dy$$

$$v_o(t) - v_o(0.5^+) = -5y^2 \Big|_{0.5}^{t} + 15y \Big|_{0.5}^{t}$$

$$v_o(t) = v_o(0.5^+) - 5t^2 + 1.25 + 15t - 7.5$$

$$v_o(0.5^+) = v_o(0.5^-) = 2.5\,\text{V}$$

$$\therefore \quad v_o(t) = -5t^2 + 15t - 3.75\,\text{V}, \qquad 0.5^+ \le t \le t_{\text{sat}}$$

$$\frac{dv_{o1}}{dt} = -20(-0.04) = 0.8, \qquad 0.5^+ \le t \le t_{\text{sat}}$$

$$dv_{o1} = 0.8\,dt; \qquad \int_{v_{o1}(0.5^+)}^{v_{o1}(t)} dx = 0.8 \int_{0.5^+}^{t} dy$$

$$v_{o1}(t) - v_{o1}(0.5^+) = 0.8t - 0.4; \qquad v_{o1}(0.5^+) = v_{o1}(0.5^-) = -0.8\,\text{V}$$

$$\therefore \quad v_{o1}(t) = 0.8t - 1.2\,\text{V}, \qquad 0.5^+ \le t \le t_{\text{sat}}$$

Summary:

$0 \le t \le 0.5^-\text{s}: \qquad v_{o1} = -1.6t\,\text{V}, \quad v_o = 10t^2\,\text{V}$

$0.5^+\text{s} \le t \le t_{\text{sat}}: \qquad v_{o1} = 0.8t - 1.2\,\text{V}, \quad v_o = -5t^2 + 15t - 3.75\,\text{V}$

[b] $-12.5 = -5t_{\text{sat}}^2 + 15t_{\text{sat}} - 3.75$

$$\therefore \quad 5t_{\text{sat}}^2 - 15t_{\text{sat}} - 8.75 = 0$$

Solving, $\qquad t_{\text{sat}} = 3.5\,\text{sec}$

$$v_{o1}(t_{\text{sat}}) = 0.8(3.5) - 1.2 = 1.6\,\text{V}$$

P 8.64 $\tau_1 = (10^6)(0.5 \times 10^{-6}) = 0.50\,\text{s}$

$$\frac{1}{\tau_1} = 2; \qquad \tau_2 = (5 \times 10^6)(0.2 \times 10^{-6}) = 1\,\text{s}; \qquad \therefore \quad \frac{1}{\tau_2} = 1$$

$$\therefore \quad \frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2v_o = 20$$

$$s^2 + 3s + 2 = 0$$

$(s+1)(s+2) = 0; \qquad s_1 = -1, \quad s_2 = -2$

$v_o = V_f + A_1'e^{-t} + A_2'e^{-2t}; \qquad V_f = \dfrac{20}{2} = 10\,\text{V}$

$v_o = 10 + A_1'e^{-t} + A_2'e^{-2t}$

$v_o(0) = 0 = 10 + A_1' + A_2'; \qquad \dfrac{dv_o}{dt}(0) = 0 = -A_1' - 2A_2'$

$\therefore \quad A_1' = -20, \qquad A_2' = 10\,\text{V}$

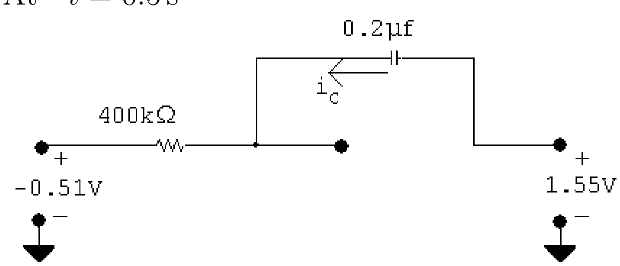
$v_o(t) = 10 - 20e^{-t} + 10e^{-2t}\,\text{V}, \qquad 0 \le t \le 0.5\,\text{s}$

$\dfrac{dv_{o1}}{dt} + 2v_{o1} = -1.6; \qquad \therefore \quad v_{o1} = -0.8 + 0.8e^{-2t}\,\text{V}, \quad 0 \le t \le 0.5\,\text{s}$

$v_o(0.5) = 10 - 20e^{-0.5} + 10e^{-1} = 1.55\,\text{V}$

$v_{o1}(0.5) = -0.8 + 0.8e^{-1} = -0.51\,\text{V}$

At $\quad t = 0.5\,\text{s}$



$i_C = \dfrac{0 + 0.51}{400 \times 10^3} = 1.26\,\mu\text{A}$

$C\dfrac{dv_o}{dt} = 1.26\,\mu\text{A}; \qquad \dfrac{dv_o}{dt} = \dfrac{1.26}{0.2} = 6.32\,\text{V/s}$

$0.5\,\text{s} \le t \le \infty$:

$\dfrac{d^2v_o}{dt^2} + 3\dfrac{dv_o}{dt} + 2 = -10$

$v_o(\infty) = -5$

$\therefore \quad v_o = -5 + A_1'e^{-(t-0.5)} + A_2'e^{-2(t-0.5)}$

$$1.55 = -5 + A_1' + A_2'$$

$$\frac{dv_o}{dt}(0.5) = 6.32 = -A_1' - 2A_2'$$

$$\therefore \quad A_1' + A_2' = 6.55; \qquad -A_1' - 2A_2' = 6.32$$

Solving,

$$A_1' = 19.42; \qquad A_2' = -12.87$$

$$\therefore \quad v_o = -5 + 19.42e^{-(t-0.5)} - 12.87e^{-2(t-0.5)}\,\text{V}, \qquad 0.5 \le t \le \infty$$

$$\frac{dv_{o1}}{dt} + 2v_{o1} = 0.8$$

$$\therefore \quad v_{o1} = 0.4 + (-0.51 - 0.4)e^{-2(t-0.5)} = 0.4 - 0.91e^{-2(t-0.5)}\,\text{V}, \qquad 0.5 \le t \le \infty$$

P 8.65 **[a]**

$$f(t) = \text{inertial force} + \text{frictional force} + \text{spring force}$$
$$= M[d^2x/dt^2] + D[dx/dt] + Kx$$

[b]

$$\frac{d^2x}{dt^2} = \frac{f}{M} - \left(\frac{D}{M}\right)\left(\frac{dx}{dt}\right) - \left(\frac{K}{M}\right)x$$

Given $v_A = \dfrac{d^2x}{dt^2}$, then

$$v_B = -\frac{1}{R_1C_1}\int_0^t \left(\frac{d^2x}{dy^2}\right) dy = -\frac{1}{R_1C_1}\frac{dx}{dt}$$

$$v_{\text{C}} = -\frac{1}{R_2C_2}\int_0^t v_B\,dy = \frac{1}{R_1R_2C_1C_2}x$$

$$v_D = -\frac{R_3}{R_4}\cdot v_B = \frac{R_3}{R_4R_1C_1}\frac{dx}{dt}$$

$$v_E = \left[\frac{R_5 + R_6}{R_6}\right]v_{\text{C}} = \left[\frac{R_5 + R_6}{R_6}\right]\cdot\frac{1}{R_1R_2C_1C_2}\cdot x$$

$$v_F = \left[\frac{-R_8}{R_7}\right]f(t), \qquad v_A = -(v_D + v_E + v_F)$$

Therefore $$\frac{d^2x}{dt^2} = \left[\frac{R_8}{R_7}\right]f(t) - \left[\frac{R_3}{R_4R_1C_1}\right]\frac{dx}{dt} - \left[\frac{R_5 + R_6}{R_6R_1R_2C_1C_2}\right]x$$

Therefore $$M = \frac{R_7}{R_8}, \qquad D = \frac{R_3R_7}{R_8R_4R_1C_1} \quad \text{and} \quad K = \frac{R_7(R_5 + R_6)}{R_8R_6R_1R_2C_1C_2}$$

Box Number	Function
1	inverting and scaling
2	summing and inverting
3	integrating and scaling
4	integrating and scaling
5	inverting and scaling
6	noninverting and scaling

P 8.66 **[a]** $\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(5\times10^{-9})(2\times10^{-12})}} = 10^{10}$ rad/sec

$$\therefore \qquad f_0 = \frac{\omega_0}{2\pi} = \frac{10^{10}}{2\pi} = 1.59\times10^9 \text{ Hz} = 1.59 \text{ GHz}$$

[b] $I_0 = \frac{10}{25} = 0.4\,\text{A}$

$$w(0) = \frac{1}{2}LI_0^2 = \frac{1}{2}(5\times10^{-9})(0.4)^2 = 4\times10^{-10}\text{ J} = 0.4 \text{ nJ}$$

[c] Because the inductor and capacitor are assumed to be ideal, none of the initial energy will ever be dissipated, so for all $t \geq 0$ the 0.4 nJ will be continually exchanged between the inductor and capacitor.

P 8.67 **[a]** $\omega_0 = 2\pi f_0 = 2\pi(2\times10^9) = 4\pi\times10^9$rad/s

$$\omega_0^2 = \frac{1}{LC}$$

$$\therefore \qquad C = \frac{1}{L\omega_0^2} = \frac{1}{(10^{-9})(4\pi\times10^9)^2} = 6.33\times10^{-12} = 6.33 \text{ pF}$$

[b] $v_o(t) = \frac{V}{\omega_0 RC}\sin\omega_0 t = \frac{4}{(4\pi\times10^9)(10)(6.33\times10^{-12})}\sin 4\pi\times10^9 t$

$$= 5.03\sin 4\pi\times10^9 t\,\text{V}, \quad t \geq 0$$

P 8.68 **[a]** $\alpha = \frac{R}{2L} = \frac{0.01}{2(5\times10^{-9})} = 10^6$ rad/s

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(5\times10^{-9})(2\times10^{-12})}} = 10^{10} \text{ rad/s}$$

[b] $\omega_0^2 > \alpha^2$ so the response is underdamped

[c] $\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{(10^{10})^2 - (10^6)^2} \approx 10^{10}$ rad/s

$$f_0 = \frac{\omega_0}{2\pi} = \frac{10^{10}}{2\pi} = 1.59 \times 10^9 \text{ Hz} = 1.59 \text{ GHz}$$

Therefore the addition of the $10\,\text{m}\Omega$ resistance does not change the frequency of oscillation.

[d] Because of the added resistance, the oscillation now occurs within a decaying exponential envelope (see Fig. 8.9). The form of the exponential envelope is $e^{-\alpha t} = e^{-10^6 t}$. Let's assume that the oscillation is described by the function $Ke^{-10^6 t}\cos 10^{10}t$, where the maximum magnitude, K, exists at $t = 0$. How long does it take before the magnitude of the oscillation has decayed to $0.01K$?

$$Ke^{-10^6 t} = 0.01K \qquad \text{so} \qquad e^{-10^6 t} = 0.01$$

$$\therefore \qquad t = \frac{\ln 0.01}{-10^6} = 4.6 \times 10^{-6}\,\text{s} = 4.6\,\mu\text{s}$$

Therefore, the oscillations will persist for only $4.6\,\mu$s, due to the presence of a small amount of resistance in the circuit. This is why an LC oscillator is not used in the clock generator circuit.