سارة بكار

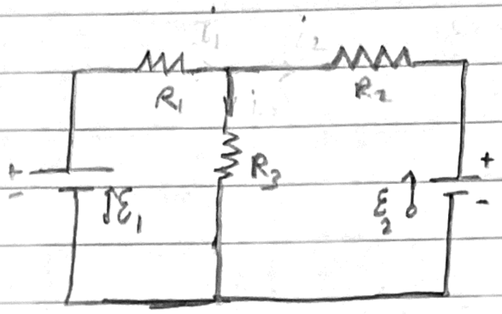
Principles of physics (10th eddition)

phy 132

CH 27 : circuits

Problems : 3, 8, 15, 18, 30, 62, 69

P_3: In Fig 27-18 $\mathcal{E}_1 = 1.00\ V$, $\mathcal{E}_2 = 3.00\ V$, $R_1 = 4.00\ \Omega$, $R_2 = 2.00\ \Omega$, $R_3 = 5.00\ \Omega$ and both batteries are ideal. What is the rate at which energy is dissipated in (a) R_1 (b) R_2 and (c) R_3? What is the power of (d) battery 1 and (e) battery 2? Is energy being absorbed or provided in (f) battery 1 and (g) battery 2?

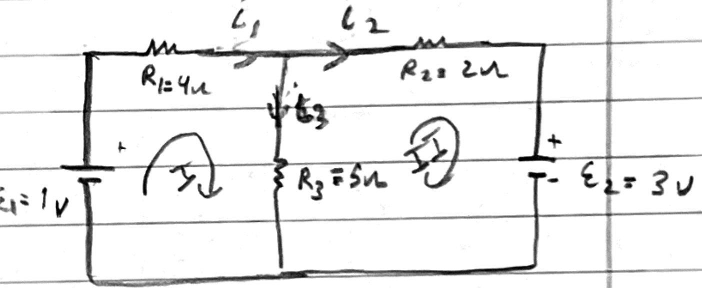


sol:

$\Sigma i = 0$

$i_1 = i_2 + i_3 \rightarrow (1)$

$\Sigma V_i = 0$



for (I): $\mathcal{E}_1 - i_1 R_1 - i_3 R_3 = 0$

$1 - 4i_1 - 5i_3 = 0 \rightarrow (2)$

for (II): $i_3 R_3 - i_2 R_2 - \mathcal{E}_2 = 0$

$5i_3 - 2i_2 - 3 = 0 \rightarrow (3)$

sub (1) in (2)

$1 - 4(i_2 + i_3) - 5i_3 = 0$

$1 - 4i_2 - 4i_3 - 5i_3 = 0$

$1 - 4i_2 - 9i_3 = 0$ —— (4)

سارة نجار حل المعادلتين ③ و ④ بطريقة الحذف

$$5i_3 - 2i_2 - 3 = 0 \quad (3)$$

$$1 - 4i_2 - 9i_3 = 0 \quad (4)$$

اضرب طرفي المعادلة ③ بـ -2

$$-10i_3 + 4i_2 + 6 = 0 \quad (5)$$

اجمع المعادلتين ④ مع ⑤

$$1 - 4i_2 - 9i_3 - 10i_3 + 4i_2 + 6 = 0$$

$$7 - 19i_3 = 0$$

$$i_3 = \frac{7}{19} \text{ A}$$

$$\boxed{i_3 = 0.368 \text{ A}}$$

sub i_3 in eq ④

$$1 - 4i_2 - 9(0.368) = 0$$

$$4i_2 = 1 - 9(0.368)$$

$$4i_2 = -2.315$$

$$\boxed{i_2 = -0.579 \text{ A}}$$

يمشي عكس الاتجاه الذي فرضناه

$$i_1 = i_2 + i_3$$

$$i_1 = -0.579 + 0.368$$

$$\boxed{i_1 = -0.211 \text{ A}}$$

أيضاً عكس الاتجاه المفترض

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a) $P_1 = i_1^2 R_1 = (-0.211)^2 (4) = 0.178$ watt

b) $P_2 = i_2^2 R_2 = (-0.579)^2 (2) = 0.670$ watt

c) $P_3 = i_3^2 R_3 = (0.368)^2 (5) = 0.677$ watt

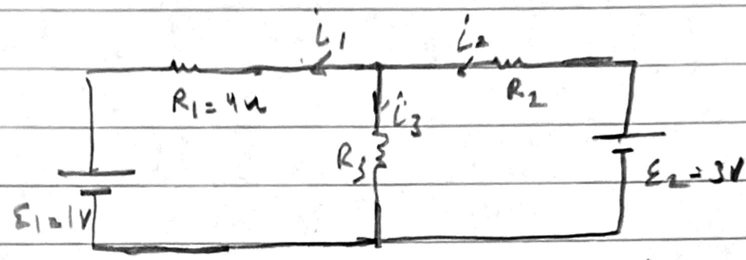
أدق or $\left(\frac{7}{19}\right)^2 (5) = 0.6786$ watt

$\simeq 0.679$ watt

d) P for $\mathcal{E}_1 = i_1 \mathcal{E}_1 = (-0.211)(1) = -0.211$ watt

e) P for $\mathcal{E}_2 = i_2 \mathcal{E}_2 = (0.579)(5) = 1.737 \simeq 1.74$ watt

لانه حسب الاشارات اتجاه التيارات كالتالي



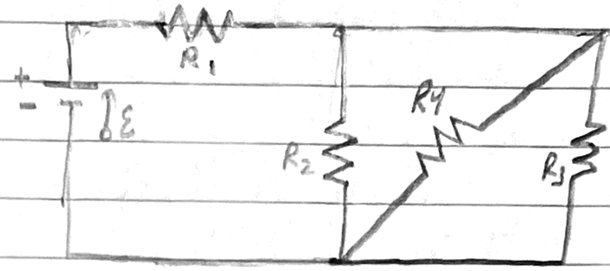
اتجاه i_2 مع اتجاه البطارية $\mathcal{E}_2$ اي انها المصدر للتيار وليست مستنفدة

f) bettary 1 absorbing

g) bettary 2 providing

ساره كرار

P8: In Fig 27-23, $R_1 = 100\,\Omega$, $R_2 = R_3 = 50.0\,\Omega$, $R_4 = 75.0\,\Omega$ and the ideal battery has emf $\mathcal{E} = 12.0\,V$ (a) what is the equivalent resistance? What is i in (b) resistance 1 (c) resistance 2 (d) resistance 3 and (e) resistance 4?

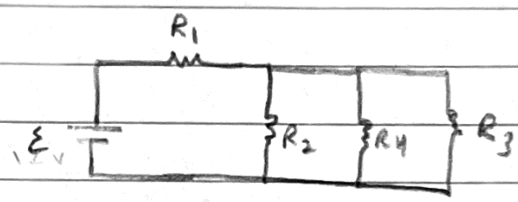


sol:

a) R_2, R_3, R_4 in parallel

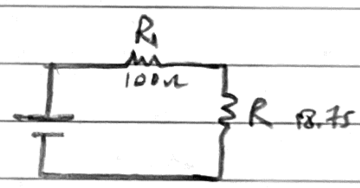
$$\frac{1}{R} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$= \frac{1}{75} + \frac{1}{50} + \frac{1}{50}$$



$$\frac{1}{R} = 0.05\overline{3}$$

$$R = 18.75\,\Omega$$



R_1, R in series

$$R_{eq} = R_1 + R$$

$$= 100 + 18.75$$

$$R_{eq} = 118.75\,\Omega$$

b) $i_{tot} = \frac{V}{R_{eq}} = \frac{12}{118.75} = 0.101\,A$

$i_{R_1} = i_{tot} = 0.101\,A$

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c) $V_R = V_{R_2} = V_{R_3} = V_{R_4}$

$V_R = iR$
$= 0.101 \times 18.75$
$= 1.89$ volt

$i_2 = \frac{V_{R_2}}{R_2} = \frac{1.89}{50} = 0.0378$ A

d) $i_3 = \frac{V_{R_3}}{R_3} = \frac{1.89}{50} = 0.0378$ A

or $i_2 = i_3 = 0.0378$ A since they have the same V, R

e) $i_4 = \frac{V_{R_4}}{R_4} = \frac{1.89}{75} = 0.0252$ A

P15: What multiple of time constant τ gives the time taken by an initially uncharged capacitor in an RC series circuit to be charged to 89.0% of its final charge?

sol:
During charging the charge on the positive plate of the capacitor is given by

$$q = C\varepsilon(1 - e^{-t/\tau})$$

$$q = q_0(1 - e^{-t/\tau})$$, q_0 : equilibrium or final charge

$$0.89\, q_0 = q_0(1 - e^{-t/\tau})$$

$$0.89 = (1 - e^{-t/\tau})$$
$$e^{-t/\tau} = 1 - 0.89$$
$$e^{-t/\tau} = 0.11$$

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Taking the natural logarithm of both sides, we obtain

$$\ln(e^{-t/\tau}) = \ln(0.11)$$

$$\frac{-t}{\tau} = -2.207$$

$$\frac{t}{\tau} = 2.207$$

$$t = 2.207\tau$$

$$t \simeq 2.21\tau$$

So t is 2.21 multiple of time constant τ

P18: A capacitor with an initial potential difference of 80.0 V is discharged through a resistor when a switch between them is closed at t = 0. At t = 10.0 s, the potential difference across the capacitor is 1.00 V. (a) What is the time constant of the circuit? (b) What is the potential difference across the capacitor at t = 17.0 s?

Sol: a) $\mathcal{E}_0 = 80.0\ V$ At $t = 10$ $\mathcal{E} = 1$ volt

$$q = q_0 e^{-t/RC}$$ for discharging capacitor

$$\frac{q}{c} = \frac{q_0}{c} e^{-t/\tau}$$

$$\mathcal{E} = \mathcal{E}_0 e^{-t/\tau}$$

$$1 = 80 e^{-10/\tau}$$

$$\frac{1}{80} = e^{-10/\tau}$$

$$0.0125 = e^{-10/\tau}$$

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$$\ln(0.0125) = \ln e^{-10/\tau}$$

$$-4.38 = \frac{-10}{\tau}$$

$$4.38 = \frac{10}{\tau}$$

$$\tau = \frac{10}{4.38}$$

$$\tau = 2.28 \text{ se}$$

b) $\mathcal{E}(t = 17\,\text{sec}) = \mathcal{E}_0 e^{-t/\tau}$

$$\mathcal{E}(t=17) = 80 e^{-17/2.28}$$

$$= 80 e^{-7.46}$$

$$\mathcal{E} = 0.0462 \text{ Volt}$$

P80: A capacitor with intial charge q_0 is discharged through a resistor. What multiple of time constant τ gives the time the capacitor takes to lose (a) the first 25% of the charge and b) 50% of its charge?

sol: during the discharging, the charge stored in the capacitor at time t is given by

$$q = q_0 e^{-t/\tau}$$

a) When 25% of the charge is lost then the charge in the capacitor is 100% - 25% = 75% of the charge

$$q = q_0 - 0.25 q_0 = 0.75 q_0$$

$$\Rightarrow 0.75 q_0 = q_0 e^{-t/\tau}$$

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$0.75 = e^{-t/\tau}$

$\ln 0.75 = \frac{-t}{\tau}$

$-0.288 = \frac{-t}{\tau}$

$0.288 = \frac{t}{\tau} \Rightarrow t = 0.288\,\tau$

b) $q = q_0 - 0.50 q_0 = 0.50\, q_0$

$q = q_0 e^{-t/\tau}$

$0.50 q_0 = q_0 e^{-t/\tau}$

$0.5 = e^{-t/\tau}$

$\ln(0.5) = \frac{-t}{\tau}$

$-0.693 = \frac{-t}{\tau}$

$0.693 = \frac{t}{\tau} \Rightarrow t = 0.693\,\tau$

P62: A 1.0 μF capacitor with an initial stored energy of 0.60 J is discharged through a 1.0 $M\Omega$ resistor. (a) What is the initial charge on the capacitor? (b) What is the current through the resistor when the discharge starts? Find an expression that gives, as a function of time t (c) the potential difference V_C across the capacitor (d) the potential difference V_R across the resistor and (e) the rate at which thermal energy is produced in the resistor?

سارة بكار

sol: $C = 1.0\ \mu F$, $E = 0.60 J$, $R = 1.0\ M\Omega$

a) $E = \frac{q_0^2}{2C}$, q_0 : intial charge on the Capacitor

$q_0^2 = 2EC$

$q_0 = \sqrt{2EC}$

$= \sqrt{2 \times 0.60 \times 1\times10^{-6}}$

$q_0 = 1.095 \times 10^{-3}\ C$

b) $q = q_0 e^{-t/RC}$

$i = \frac{dq}{dt} = \frac{-q_0}{RC} e^{-t/RC}$

$i = \frac{-q_0}{\tau} e^{-t/RC}$

$i = -i_0 e^{-t/RC}$

$i_0 = \frac{q_0}{\tau} = \frac{1.095\times10^{-3}}{1\times10^{6} \times 1\times10^{-6}} = 1.095\times10^{-3} A$ (The current through the resistore when discharge start)

$\Rightarrow i = -1.095\times10^{-3} e^{-t/\tau}$

but $\tau = RC = 1\times10^{6} \times 1\times10^{-6}$

$\Rightarrow$ $\boxed{i = -1.095\times10^{-3} e^{-t}}$

The expression as a function of time t

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c) $V_c = V_0 \, e^{-t/RC}$ or $\mathcal{E} = \mathcal{E}_0 \, e^{-t/RC}$

$$= \frac{q_0}{C} \, e^{-t/RC}$$

$$= \frac{1.095 \times 10^{-3}}{1 \times 10^{-6}} \, e^{-t/(1\times10^{-6} \times 1\times10^{6})}$$

و $\tau = RC = 1$

$$\boxed{V_c = 1.095 \times 10^{3} \, e^{-t}}$$

d) $V_R = -\frac{q_0}{C} \, e^{-t/RC}$

$$\Rightarrow \boxed{V_R = -1.095 \times 10^{3} \, e^{-t}}$$

e) $P = i^2 R$

$$= \left(-1.095 \times 10^{-3} \, e^{-t}\right)^2 \times 1 \times 10^{6}$$

$$= 1.199 \times 10^{-6} \, e^{-2t} \times 10^{6}$$

$$\boxed{P = 1.199 \, e^{-2t}}$$

OR $P = i^2 R$

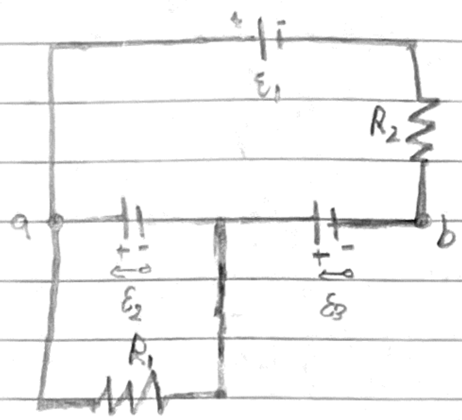
$$= \left(-\frac{q_0}{RC} \, e^{-t/RC}\right)^2 R$$

$$= \frac{q_0^2}{R^2 C^2} \, R \, e^{-2t/RC}$$

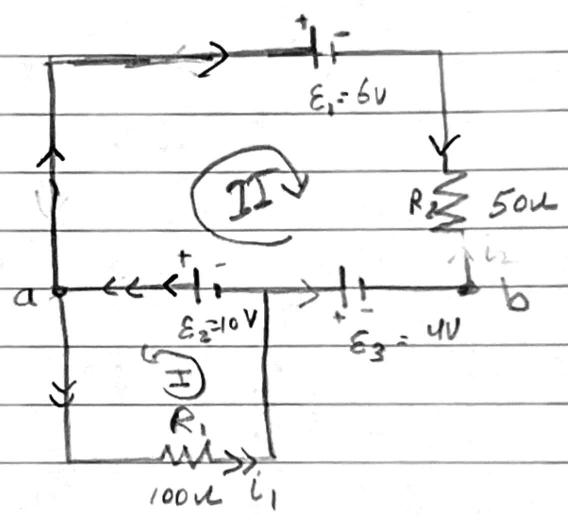
$$P = \frac{q_0^2}{RC^2} \, e^{-2t/RC} = \frac{(1.095 \times 10^{-3})^2}{1 \times 10^{6} \times 1 \times 10^{-12}} \, e^{-2t} = 1,99 \, e^{-2t}$$

سارة بخار

P69: In Fig 27-60, $R_1 = 100\,\Omega$, $R_2 = 50\,\Omega$ and the ideal batteries have emfs $\mathcal{E}_1 = 6.0\,V$, $\mathcal{E}_2 = 10\,V$ and $\mathcal{E}_3 = 4.0\,V$. Find (a) the current in resistor 1, (b) the current in resistor 2 and (c) the potential difference between points a and b



Sol:



a) I

$\mathcal{E}_2 - i_1 R_1 = 0$

$\mathcal{E}_2 = i_1 R_1$

$i_1 = \frac{\mathcal{E}_2}{R_1}$

$i_1 = \frac{10}{100}$

$i_1 = 0.1\,A$

b) II

$\mathcal{E}_2 - \mathcal{E}_1 - i_2 R_2 + \mathcal{E}_3 = 0$

$\mathcal{E}_2 - \mathcal{E}_1 + \mathcal{E}_3 = i_2 R_2$

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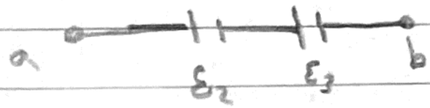
$$i_2 = \frac{\varepsilon_2 - \varepsilon_1 + \varepsilon_3}{R_2} = \frac{10 - 6 + 4}{50} = \frac{8}{50} = 0.16\ A$$

c) $V_a = V_b + \varepsilon_3 + \varepsilon_2$



$$V_a - V_b = \varepsilon_3 + \varepsilon_2$$
$$= 4 + 10$$
$$= 14\ \text{volt}$$