

Chapter 1: Measurement

- Physical quantities:

1. **Base quantities:** $\left\{ \begin{array}{l} \text{Length; meter (m)} \\ \text{Time; second (s)} \\ \text{Mass; kilogram (kg)} \end{array} \right\}$

2. **Derived quantities:** defined in terms of the base quantities and their standards and units.

Velocity m/s

Force = mass x acceleration

$$1 N = 1 Kg \frac{m}{s^2}$$

Joules (work-energy): $1 J = 1 kg m^2/s^2$

Watts (power): $1 W = 1 J/s = 1 kg m^2/s^3$

- The International System of Units (SI):

Quantity	Unit Name	Unit Symbol
Length	meter	m
Time	second	s
Mass	kilogram	kg

Base Standards, which must be both accessible and invariable (established by international agreement)

To express very large and very small quantities, we use Scientific Notation (powers of ten)

$$36900000 m = 3.69 \times 10^7 m$$

$$0.000035 s = 3.5 \times 10^{-5} m$$

- Prefixes for SI unit:

$$1 \text{ m} = 100 \text{ cm} = 10^2 \text{ cm} \rightarrow \rightarrow 1 \text{ cm} = 10^{-2} \text{ m}$$

Table 1-2 Prefixes for SI Units

Factor	Prefix ^a	Symbol
10^{24}	yotta-	Y
10^{21}	zetta-	Z
10^{18}	exa-	E
10^{15}	peta-	P
10^{12}	tera-	T
10^9	giga-	G
10^6	mega-	M
10^3	kilo-	k
10^2	hecto-	h
10^1	deka-	da
10^{-1}	deci-	d
10^{-2}	centi-	c
10^{-3}	milli-	m
10^{-6}	micro-	μ
10^{-9}	nano-	n
10^{-12}	pico-	p
10^{-15}	femto-	f
10^{-18}	atto-	a
10^{-21}	zepto-	z
10^{-24}	yocto-	y

$$1 \text{ km} = 10^3 \text{ m} = 1000 \text{ m}$$

$$2 \text{ ms} = 2 \times 10^{-3} \text{ s} = 0.002 \text{ s}$$

- Conversion of units (**Changing between Units**):

By using chain-link Conversions in which the original data are multiplied successively by conversion factors written as unity and the units are manipulated like algebraic quantities until only the desired units remain.

A conversion factor is a ratio of units that is equal to unity.

$$1 \text{ m} = 100 \text{ cm} = 10^2 \text{ cm} \rightarrow \rightarrow \frac{1 \text{ m}}{10^2 \text{ cm}} = \frac{10^2 \text{ cm}}{1 \text{ m}} = 1$$

Example:

Car velocity is $v_{car} = 72 \text{ km/h}$ and bike velocity is $v_{bike} = 25 \text{ m/s}$, which is faster?

$$v_{car} = \left(72 \frac{\text{km}}{\text{h}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) \left(\frac{1000 \text{ m}}{1 \text{ km}}\right) = 20 \frac{\text{m}}{\text{s}}$$

So bike is faster than the car

- Estimation: التقدير

How Many Times Does Your Heart Beat in a Lifetime?

$$\text{---Heart rate} = 60 \text{ beat per a minute} = \frac{60 \text{ beat}}{\text{min}}$$

Average age = 70 years

$$70 \text{ years} \left(\frac{365 \text{ day}}{1 \text{ year}}\right) \left(\frac{24 \text{ h}}{1 \text{ day}}\right) \left(\frac{60 \text{ min}}{1 \text{ h}}\right) \left(\frac{60 \text{ beat}}{\text{min}}\right) = 2.2 \times 10^9 \text{ beats}$$

$$\text{---Heart rate} = 80 \text{ beat per a minute} = \frac{80 \text{ beat}}{\text{min}}$$

Average age = 80 years

$$80 \text{ years} \left(\frac{365 \text{ day}}{1 \text{ year}}\right) \left(\frac{24 \text{ h}}{1 \text{ day}}\right) \left(\frac{60 \text{ min}}{1 \text{ h}}\right) \left(\frac{80 \text{ beat}}{\text{min}}\right) = 3.3 \times 10^9 \text{ beats}$$

- Standards of Length, Time and Mass:

- **Length** (meter): In the past, 1 meter has been defined by:

1. One ten-millionth of the distance from the North pole to the equator
2. A platinum-iridium **standard meter bar** kept in France
3. 1 650 763.73 wavelengths of an emission line of Kr-86

Today,



The meter is the length of the path traveled by light in a vacuum during a time interval of 1/299 792 458 of a second.

- **Time**

One second is the time taken for the cesium atom $^{133}_{55}\text{Cs}$ to perform 9192631770 oscillations to emit radiation of a specific wavelength

- **Mass**

One kilogram is defined as the mass of a platinum-iridium alloy cylinder kept at the International Bureau of Weights and Measures in France

A second standard mass: **The atomic mass unit (u)**

- 1 atom of Carbon-12 is assigned a mass 12 u
- Used for measuring masses of atoms and molecules
- $1 \text{ u} = 1.660\,538\,86 \times 10^{-27} \text{ kg}$

- **Density is the mass per unit volume**

$$\rho = \frac{m}{V}$$

Problem 1: Earth is approximately a sphere of radius 6.37×10^6 m. What are (a) its circumference in kilometers, (b) its surface area in square kilometers, and (c) its volume in cubic kilometers?

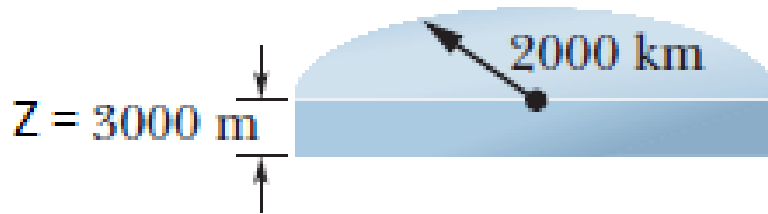
$$R = 6.37 \times 10^6 \text{ m} = 6.37 \times 10^3 \text{ km}$$

a) Circumference: $C = 2\pi R = 2\pi(6.37 \times 10^3 \text{ km}) = 4.00 \times 10^4 \text{ km}$

b) Surface area of the Earth: $A = 4\pi R^2 = 4\pi(6.37 \times 10^3 \text{ km})^2 = 5.10 \times 10^8 \text{ km}^2$

c) Volume of the Earth: $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(6.37 \times 10^3 \text{ km})^3 = 1.08 \times 10^{12} \text{ km}^3$

Problem 9: Antarctica is roughly semicircular, with a radius of 2000 km. The average thickness of its ice cover is 3000 m. How many cubic centimeters of ice does Antarctica contain? (Ignore the curvature of Earth.)



The volume of ice is given by the product of the semicircular surface area and the thickness.

The area of the semicircle: $A = \frac{\pi r^2}{2}$; r is the radius.

The volume of the ANTARTICA: $V = \frac{\pi r^2}{2} Z$

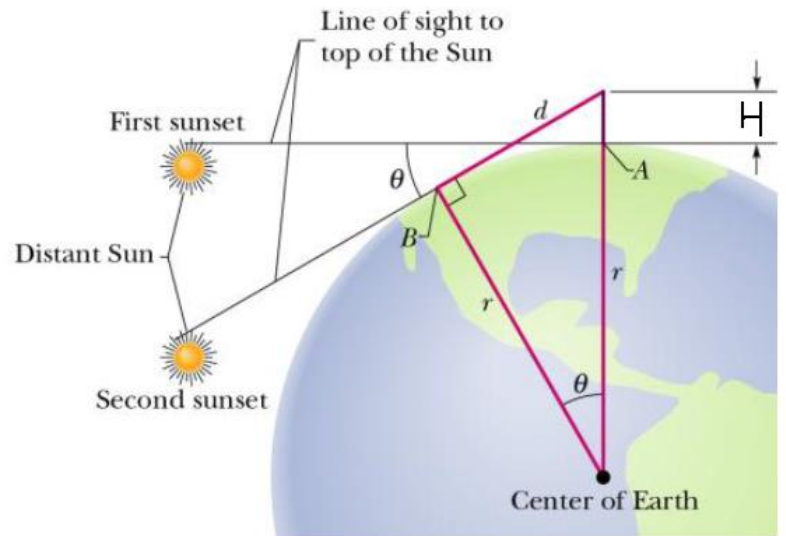
$$r = 2000 \text{ Km} = 2000 \text{ Km} \left(\frac{10^3 \text{ m}}{1 \text{ Km}} \right) \left(\frac{10^2 \text{ cm}}{1 \text{ m}} \right) = 2000 \times 10^5 \text{ cm} = 2 \times 10^8 \text{ cm}$$

$$z = 3000 \text{ m} = 3000 \text{ m} \left(\frac{10^2 \text{ cm}}{1 \text{ m}} \right) = 3 \times 10^5 \text{ cm}$$

$$V = \frac{\pi r^2}{2} Z = \frac{\pi}{2} (2 \times 10^8 \text{ cm})^2 3 \times 10^5 \text{ cm} = 1.9 \times 10^{22} \text{ cm}^3$$

ANTARTICA contains $1.9 \times 10^{22} \text{ cm}^3$ of ice

Problem 19: Suppose that, while lying on a beach near the equator watching the Sun set over a calm ocean, you start a stopwatch just as the top of the Sun disappears. You then stand, elevating your eyes by a height $H = 1.70$ m, and stop the watch when the top of the Sun again disappears. If the elapsed time is $t = 11.1$ s, what is the radius r of Earth?



$$\frac{\theta}{360^\circ} = \frac{t}{24 \text{ h}}$$

$$\theta = \frac{11.1 \text{ s}}{(24 \text{ h}) \left(\frac{3600 \text{ s}}{1 \text{ h}} \right)} (360^\circ) = 0.04625^\circ$$

Let d be the distance from point B to your eyes.

$$\tan \theta = \frac{d}{r} \quad \rightarrow \rightarrow \rightarrow \quad r^2 \tan^2 \theta = d^2$$

By Pythagorean Theorem,

$$d^2 + r^2 = (r + H)^2 = H^2 + r^2 + 2rH$$

$$d^2 + r^2 \cong r^2 + 2rH, \text{ since } r \gg H$$

$$d^2 \cong 2rH$$

$$r^2 \tan^2 \theta = 2rH$$

$$r = \frac{2H}{\tan^2 \theta} = \frac{2(1.70 \text{ m})}{\tan^2(0.04625^\circ)} = 5.21 \times 10^6 \text{ m}$$

Problem 27: Iron has a density of 7.87 g/cm^3 , and the mass of an iron atom is $9.27 \times 10^{-26} \text{ kg}$. If the atoms are spherical and tightly packed, (a) what is the volume of an iron atom and (b) what is the distance between the centers of adjacent atoms?

$$\rho = \frac{m}{V}$$

$$1 \text{ m} = 10^2 \text{ cm} \rightarrow 1 \text{ m}^3 = 10^{2^3} \text{ cm}^3 = 10^6 \text{ cm}^3$$

$$\rho = \left(7.87 \frac{\text{g}}{\text{cm}^3}\right) \left(\frac{1 \text{ kg}}{1000 \text{ g}}\right) \left(\frac{10^6 \text{ cm}^3}{1 \text{ m}^3}\right) = 7870 \text{ kg/m}^3$$

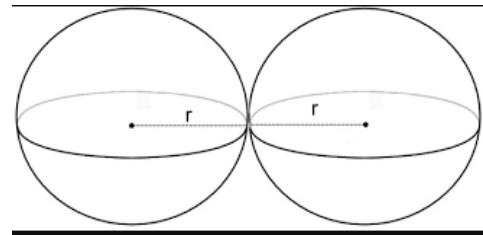
a) The volume of an iron atom is

$$V = \frac{m}{\rho} = \frac{9.27 \times 10^{-26} \text{ kg}}{7870 \text{ kg/m}^3} = 1.18 \times 10^{-29} \text{ m}^3$$

b) The distance between the centers of adjacent atoms is twice the radius of the atom

The volume of the sphere $V = \frac{4}{3}\pi r^3$

$$r = \left(\frac{3}{4\pi}V\right)^{1/3} = 1.41 \times 10^{-10} \text{ m}$$



The distance between the centers of adjacent atoms = $2r$

$$= 2(1.41 \times 10^{-10} \text{ m}) = 2.82 \times 10^{-10} \text{ m}$$

- **Dimensional Analysis**

Example: A particle moves with a constant speed v in a circular orbit of radius r , see the below figure. Given that the magnitude of the acceleration a is proportional to some power of r , say r^m , and some power of v , say v^n , then determine the powers of r and v ?

$$a = k r^m v^n, k \text{ is constant}$$

Length L

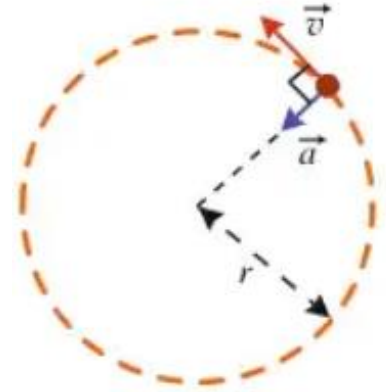
Mass M

Time T

--- Acceleration $\frac{L}{T^2}$

Radius L

Velocity $\frac{L}{T}$



$$a = k r^m v^n$$

$$\frac{L}{T^2} = L^m \left(\frac{L}{T}\right)^n = \frac{L^{m+n}}{T^n}$$

$$m + n = 1$$

$$n = 2 \rightarrow m = -1$$

So we can conclude that

$$a = k r^{-1} v^2 = \frac{v^2}{r}$$