Chapter 4 :- Techniques of Circuit analysis

* Cramer's rule : To solve simultaneous linear equations

$$25i_1 - 5i_2 - 20i_3 = 50$$

$$-5i_1 + 10i_2 - 4i_3 = 0$$

$$-5i_1 - 4i_2 + 9i_3 = 0$$

$$\begin{bmatrix} 25 & -5 & -20 \\ -5 & 10 & -4 \\ -5 & -4 & 9 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 25 & -5 & -20 \\ -5 & 10 & -4 \\ -5 & -4 & 9 \end{bmatrix} \quad |D| = 25[(10)(9) - (-4)(-4)] - (-5)[(-5)(9) - (-4)(-5)] + (-20)[(-5)(-4) - (-5)(10)]$$

$$= 25[90 - 16] + 5[-45 - 20] - 20[20 + 50]$$

$$= 125$$

$$D_{i_1} = \begin{bmatrix} 50 & -5 & -20 \\ 0 & 10 & -4 \\ 0 & -4 & 9 \end{bmatrix} \quad |D_{i_1}| = 50[90 - 16] - (-5)[0] + (-20)[0]$$

$$= 3700$$

$$D_{i_2} = \begin{bmatrix} 25 & 50 & -20 \\ -5 & 0 & -4 \\ -5 & 0 & 9 \end{bmatrix} \quad |D_{i_2}| = 25[0] - 50[-45 - 20] + (-20)[0]$$

$$= 3250$$

$$D_{i_3} = \begin{bmatrix} 25 & -5 & 50 \\ -5 & 10 & 0 \\ -5 & -4 & 0 \end{bmatrix} \quad |D_{i_3}| = 25[0] - (-5)[0] + 50[20 + 50]$$

$$= 3500$$

$$i_1 = \frac{|D_{i_1}|}{|D|} = \frac{3700}{125} = 29.6$$

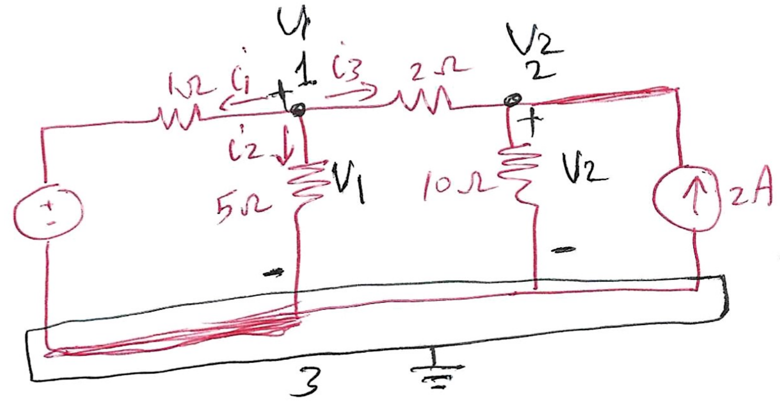
$$i_2 = \frac{|D_{i_2}|}{|D|} = \frac{3250}{125} = 26A$$

$$i_3 = \frac{|D_{i_3}|}{|D|} = \frac{3500}{125} = 28A$$

4.2 :- Introduction to the Node-Voltage Method

Node-Voltage Method is applicable to both planar and nonplanar Circuits.

For the circuit shown, we can summarize the node-voltage methods as shown



① Identify all essentials nodes (Do not select the non essentials nodes)

② select one of the essentials nodes (1, 2 or 3) as a reference node.

③ label all nonreference nodes with alphabetical label as V_1, V_2

④ write KCL equation on all labels non reference nodes as shown

KCL at node 1:-

$$i_1 + i_2 + i_3 = 0 \Rightarrow \frac{V_1 - 10}{1} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = 0 \quad \text{---(1)}$$

① $i_1 + 10 - V_1 = 0$

$i_1 = \frac{V_1 - 10}{1}$

KCL at node 2:-

$$\frac{V_2 - V_1}{2} + \frac{V_2}{10} - 2 = 0 \quad ---(2)$$

we can solve the two equations

$$\frac{V_1 - 10}{} + \frac{V_1}{5} + \frac{V_1}{2} - \frac{V_2}{2} = 0 \Rightarrow \frac{17V_1}{10} - \frac{V_2}{2} = 10 \Rightarrow V_1 = \frac{100 + 5V_2}{17}$$

$$\frac{V_2}{2} - \frac{V_1}{2} + \frac{V_2}{10} - 2 = 0 \Rightarrow \frac{6V_2}{10} - \frac{V_1}{2} = 2$$

$$6V_2 - 5V_1 = 20$$

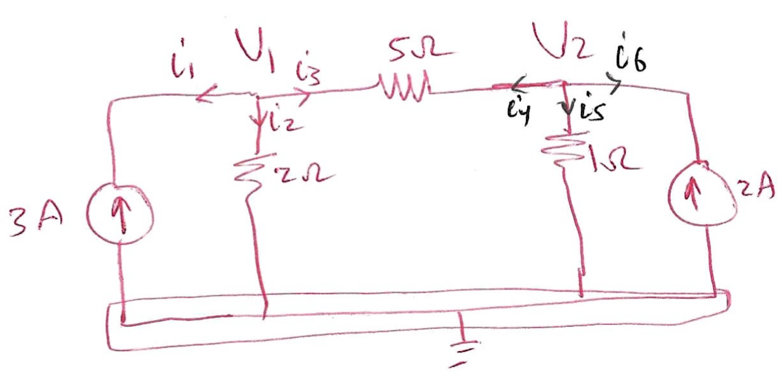
$$6V_2 - 5\left(\frac{100 + 5V_2}{17}\right) = 20$$

$$102V_2 - 500 - 25V_2 = 340$$

$$77V_2 = 840$$

$$V_2 = \frac{840}{77} = 10.9\,V \Longrightarrow V_1 = \frac{100 + 5(10.9)}{17} = 9.09\,V$$

Example:-



KCL at node 1:-

$$i_1 + i_2 + i_3 = 0$$

$$-3 + \frac{V_1}{2} + \frac{V_1 - V_2}{5} = 0 \quad ---(1)$$

KCL at node 2:-

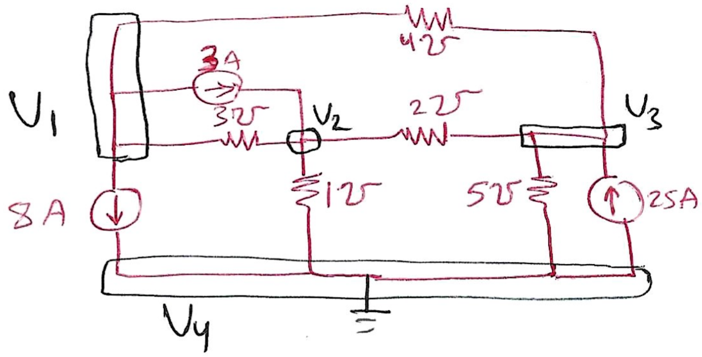
$$i_4 + i_5 + i_6 = 0$$

$$\frac{V_2 - V_1}{5} + \frac{V_2}{1} - 2 = 0$$

$V_1 = 5V, \; V_2 = 2.5V$

Example:-

KCL at node 1:



$$8 + 3(V_1 - V_2) + 3 + 4(V_1 - V_3) = 0$$

$$7V_1 - 3V_2 - 4V_3 = -11 \quad ---(1)$$

KCL at node 2:-

$$-3 + 3(V_2 - V_1) + 1V_2 + 2(V_2 - V_3) = 0$$

$$-3V_1 + 6V_2 - 2V_3 = 3 \quad ---(2)$$

KCL at node 3:-

$$4(V_3 - V_1) + 2(V_3 - V_2) + 5(V_3) - 25 = 0$$

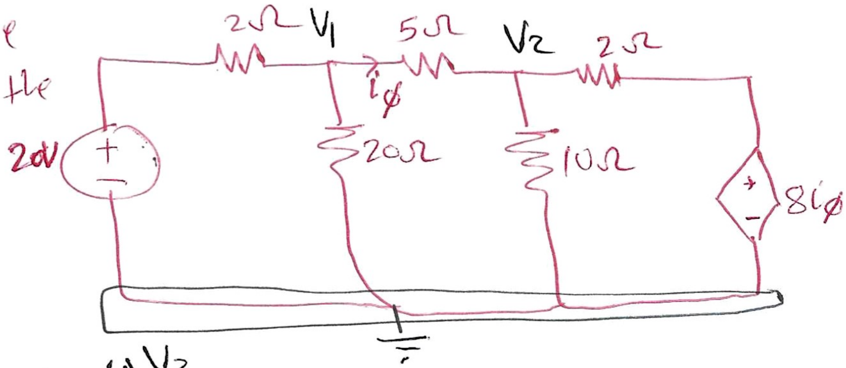
$$-4V_1 - 2V_2 + 11V_3 = 25 \quad ---(3)$$

solving:-

$V_1 = 1V$; $V_2 = 2V$; $V_3 = 3V$

4.3: The node-Voltage method and dependent Sources

Example: Use the node-Voltage to find the power dissipated in the 5Ω resistor.



KCL at node 1:-

$$\frac{V_1 - 20}{2} + \frac{V_1}{20} + \frac{V_1 - V_2}{5} = 0 \quad --(1) \; V_3$$

KCL at node 2:-

$$\frac{V_2 - V_1}{5} + \frac{V_2}{10} + \frac{V_2 - 8i_\phi}{2} = 0 \quad --(2)$$

$$i_\phi = \frac{V_1 - V_2}{5}$$

$$\frac{15 V_1}{20} - \frac{V_2}{5} = 10 \quad ---(1)$$

$$0.75 V_1 - 0.2 V_2 = 10 \quad ---(1)$$

$$\frac{V_2 - V_1}{5} + \frac{V_2}{10} + \frac{V_2 - 8 i_\phi}{2} = 0 \quad ---(2)$$

$$2V_2 - 2V_1 + V_2 + 5(V_2 - 8 i_\phi) = 0$$

$$2V_2 - 2V_1 + V_2 + 5V_2 - 8(V_1 - V_2) = 0$$

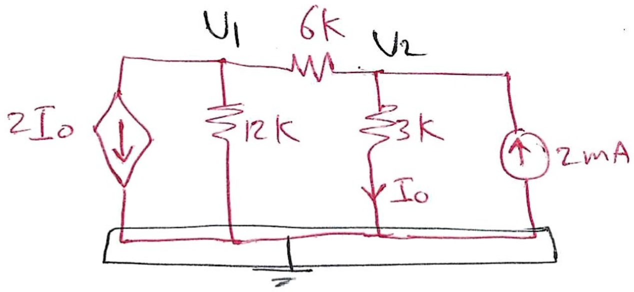
$$-10V_1 + 16 V_2 = 0 \quad ---(2)$$

solving:-

$$V_1 = 16V \; ; \; V_2 = 10\,V$$

$$i_\phi = \frac{16 - 10}{5} = 1.2\,A \Rightarrow P_{5\Omega} = (1.2)^2(5) = 7.2\,W$$

Example:



KCL at node 1:-

$$2I_o + \frac{V_1}{12K} + \frac{V_1 - V_2}{6K} = 0$$

$$I_o = \frac{V_2}{3K}$$

$$\frac{3V_1}{12K} + \frac{3V_2}{6K} = 0$$

KCL at node 2:-

$$\frac{V_2 - V_1}{6K} + \frac{V_2}{3K} - 2mA = 0$$

$$\frac{3V_2}{6K} - \frac{V_1}{6K} - 2mA = 0$$

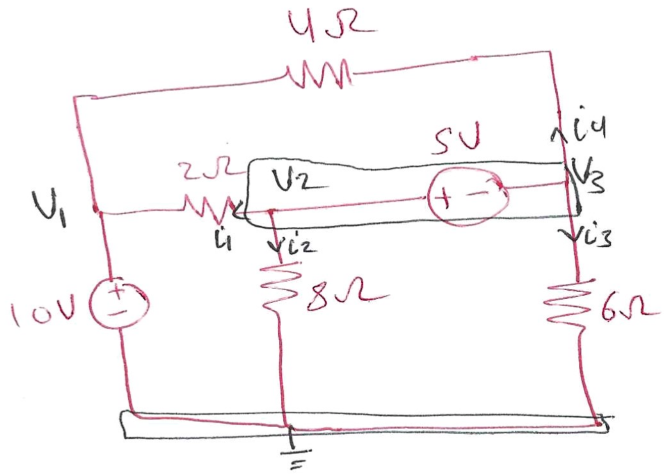
solving:-

$$V_1 = -\frac{24}{5}\,V$$

$$V_2 = \frac{12}{5}\,V$$

4.4: The node-Voltage method: Some special Cases:

Example:-



(1) The 10V Voltage source is Connected between non reference and reference node

So $V_1 = 10V$

No need to write an equation fore node 1

(2) The 5V Voltage source is Connected between two non reference nodes so node 2 and node 3 called supper node

1 equation is written for the supper node as:-

$i_1 + i_2 + i_3 + i_4 = 0$

$$\frac{V_2 - V_1}{2} + \frac{V_2}{8} + \frac{V_3}{6} + \frac{V_3 - V_1}{4} = 0$$

$$\frac{V_2 - 10}{2} + \frac{V_2}{8} + \frac{V_3}{6} + \frac{V_3 - 10}{4} = 0$$

$15V_2 + 10V_3 - 180 = 0 \quad ---(1)$

and we have $V_2 - V_3 = 5 \Rightarrow V_2 = 5 + V_3$

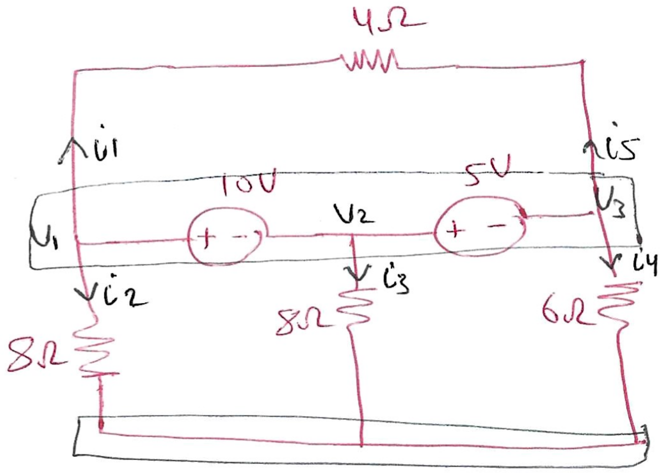
$15(5 + V_3) + 10V_3 - 180 = 0$

$25V_3 = 180 - 75$

$= 105 \Rightarrow V_3 = 4.2$

$V_2 = 9.2$

Example:



we have supper supper node

so one equation

$$i_1 + i_2 + i_3 + i_4 + i_5 = 0$$

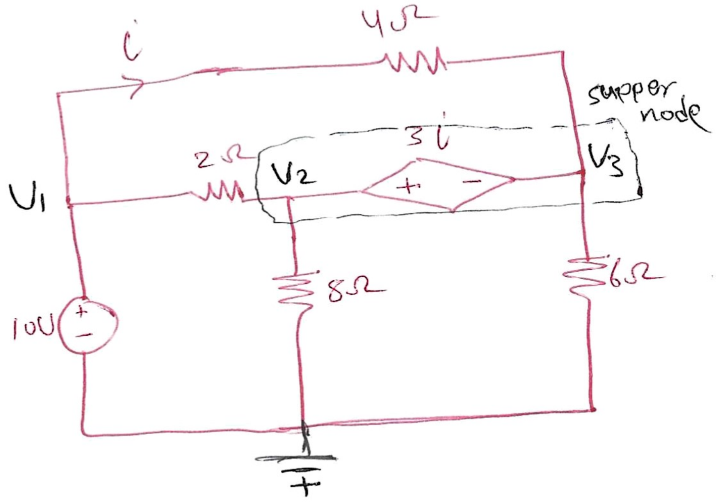
$$\cancel{\frac{V_1 - V_3}{4}} + \frac{V_1}{8} + \frac{V_2}{8} + \frac{V_3}{8} + \cancel{\frac{V_3 - V_1}{4}} = 0$$

$$\frac{V_1}{8} + \frac{V_2}{8} + \frac{V_3}{8} = 0$$

and we have ① $V_1 - V_2 = 10$

② $V_2 - V_3 = 5$

Example:



$$V_1 = 10V$$

$$\frac{V_2 - 10}{2} + \frac{V_2}{8} + \frac{V_3}{6} + \frac{V_3 - 10}{4} = 0$$

$$V_2 - V_3 = 3i$$

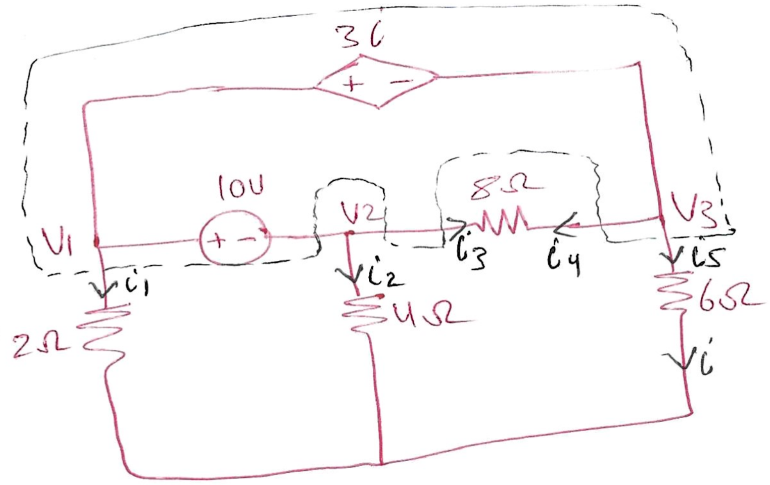
$$= 3\left(\frac{V_1 - V_3}{4}\right)$$

$$= \frac{3(10 - V_3)}{4}$$

$$4V_2 - 4V_3 = 30 - 3V_3$$

$$V_3 = 4V_2 - 30$$

Example:-



Supper-supper node

$i_1 + i_2 + i_3 + i_4 + i_5 = 0$

$$\frac{V_1}{2} + \frac{V_2}{4} + \frac{V_2 - \cancel{V_3}}{8} + \frac{\cancel{V_3} - V_2}{8} + \frac{V_3}{6} = 0$$

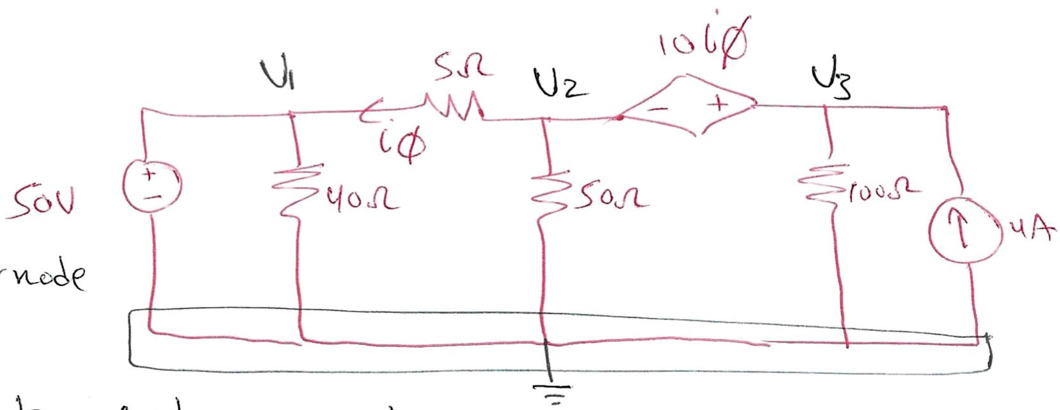
$$\frac{V_1}{2} + \frac{V_2}{4} + \frac{V_3}{8} = 0 \quad ---(1)$$

$V_1 - V_2 = 10 \quad ---(2)$

$$V_1 - V_3 = 3i = 3\left(\frac{V_3}{6}\right) = \frac{V_3}{2}$$

$\therefore V_1 = 1.5 V_3 \quad ---(3)$

Example:-



$V_1 = 50V$

V_2 and V_3 are suppernode

$$\frac{V_2 - 50}{5} + \frac{V_2}{50} + \frac{U_3}{100} + (-4) = 0 \quad ---(1)$$

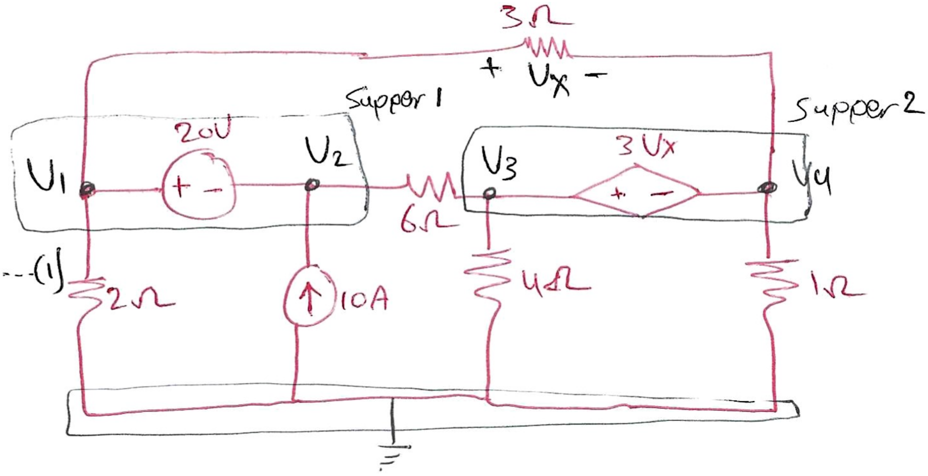
$$U_3 - U_2 = 10 i_\phi = 10\left(\frac{V_2 - 50}{5}\right) = 2U_2 - 100$$

$V_3 = 3V_2 - 100 \quad ---(2)$

Example:

Supper node 1: V_1 and V_2

$$\frac{V_1}{2} + \frac{V_1 - V_4}{3} - 10 + \frac{V_2 - V_3}{6} = 0 \quad \ldots (1)$$



Suppernode 2: V_3 and V_4

$$\frac{V_3 - V_2}{6} + \frac{V_3}{4} + \frac{V_4}{1} + \frac{V_4 - V_1}{3} = 0 \quad \ldots (2)$$

we have also:

$$V_1 - V_2 = 20V \quad \ldots (3)$$

$$V_3 - V_4 = 3V_x \quad \ldots (4)$$

$$= 3(V_1 - V_4)$$

$$= 3V_1 - 3V_4$$

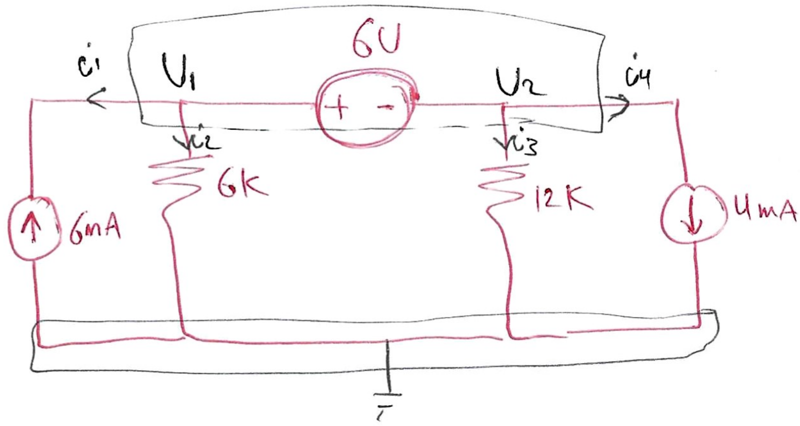
$$\Rightarrow V_3 = 3V_1 - 2V_4$$

Example:

Supper node: V_1 and V_2

$$i_1 + i_2 + i_3 + i_4 = 0$$

$$-6mA + \frac{V_1}{6k} + \frac{V_2}{12k} + 4mA = 0$$



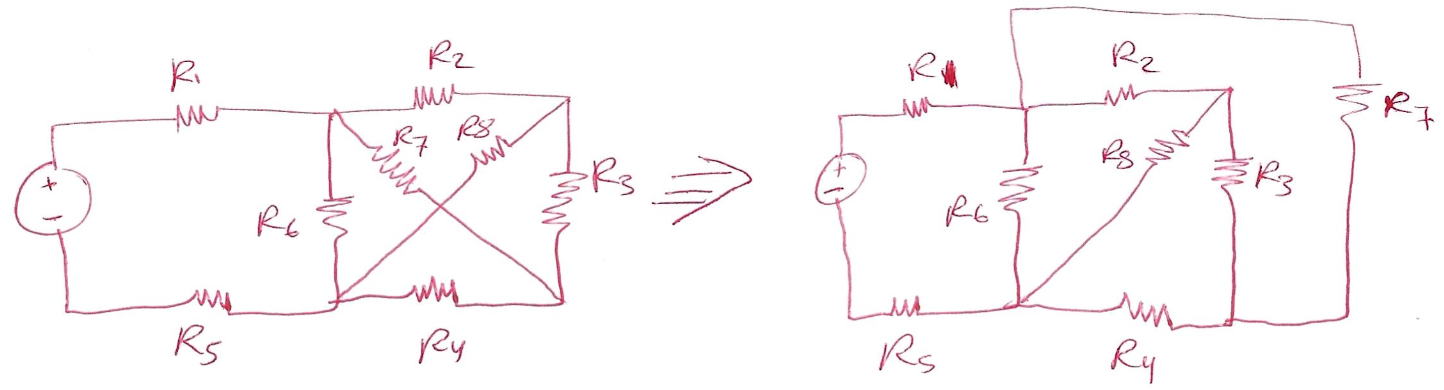
and we have

$$V_1 - V_2 = 6$$

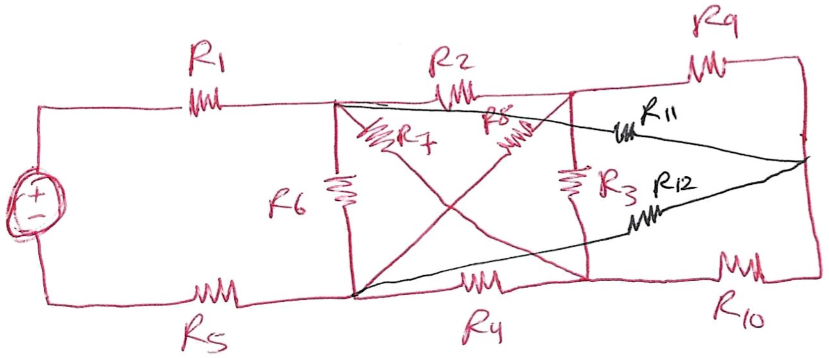
$$V_1 = 6 + V_2$$

4.5: Introduction to Mesh Current Method

* planar circuit: a circuit that can be drawn on a plane with no crossing branches as shown



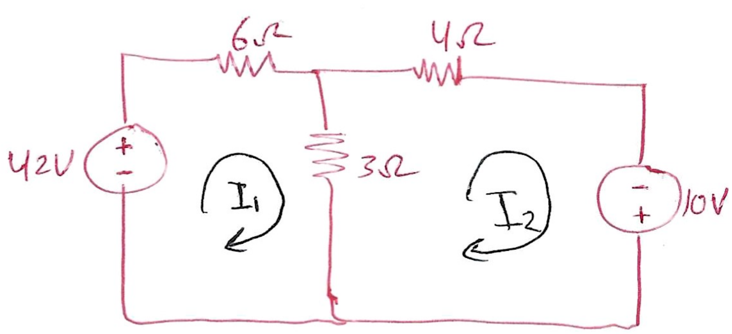
So planar



non planar

Mesh current method is valid for planar circuits only

Example:



KVL for mesh 1

$-42 + I_1(6+3) - I_2(3) = 0$

KVL for mesh 2

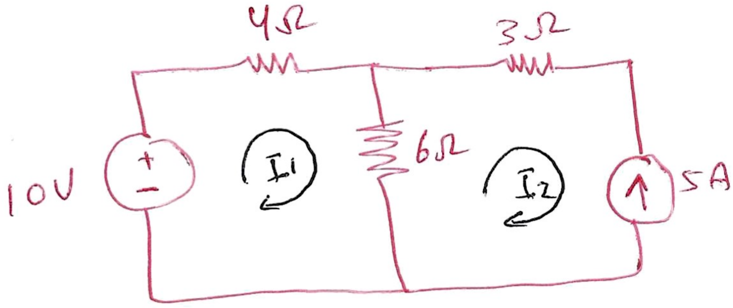
$-10 + I_2(4+3) - I_1(3) = 0$

solving for I_1 and I_2

$I_1 = 6A$ $I_2 = 4A$

Examples:-

from the circuit $I_2 = -5A$

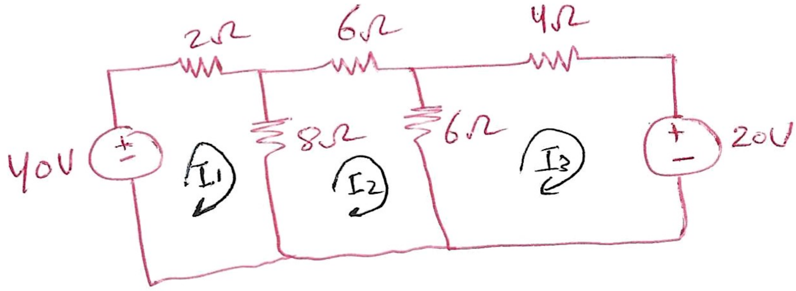


KVL for mesh ①

$$-10 + I_2(4+6) - I_2(6) = 0$$

$$\Rightarrow I_1 = -2A$$

Example:-



KVL for mesh ①

$$-40V + I_1(2+8) - I_2(8) = 0$$

KVL for mesh ②

$$I_2(8+6+6) - I_1(8) - I_3(6) = 0$$

KVL for mesh ③

$$20 + I_3(6+4) - I_2(6) = 0$$

$$10 I_1 - 8 I_2 + 0 I_3 = 40$$

$$-8 I_1 + 20 I_2 - 6 I_3 = 0$$

$$0 I_1 - 6 I_2 + 10 I_3 = -20$$

$$\begin{bmatrix} 10 & -8 & 0 \\ -8 & 20 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 40 \\ 0 \\ -20 \end{bmatrix}$$

Solving:-

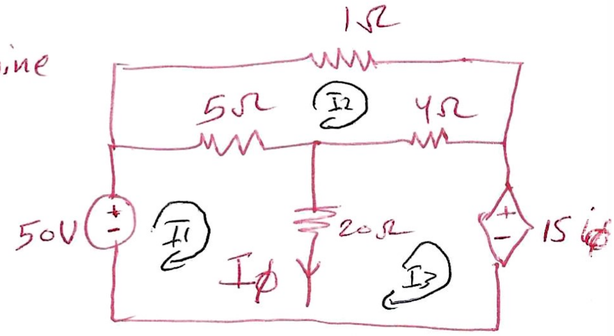
$$I_1 = 5.6 A$$

$$I_2 = 2 A$$

$$I_3 = -0.8 A$$

4.6 The mesh Current method and dependent sources

Example:- use the mesh-current method to determine dissipated in the 4Ω resistor



mesh (1)

$-50 + I_1(5+20) - I_2(5) - I_3(20) = 0$

$25 I_1 - 5 I_2 - 20 I_3 = 50 \quad ---(1)$

mesh (2)

$I_2(5+4+1) - I_1(5) - I_3(4) = 0$

$-5 I_1 + 10 I_2 - 4 I_3 = 0 \quad ---(2)$

mesh (3)

$I_3(20+4) - I_2(4) - I_1(20) + 15 I_\phi = 0$

$I_\phi = I_1 - I_3$

$24 I_3 - 4 I_2 - 20 I_1 + 15(I_1 - I_3) = 0$

$-5 I_1 - 4 I_2 + 9 I_3 = 0 \quad ---(3)$

$$\begin{bmatrix} 25 & -5 & -20 \\ -5 & 10 & -4 \\ -5 & -4 & 9 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix}$$

$I_1 = 29.6\ A$

$I_2 = 26\ A$

$I_3 = 28\ A$

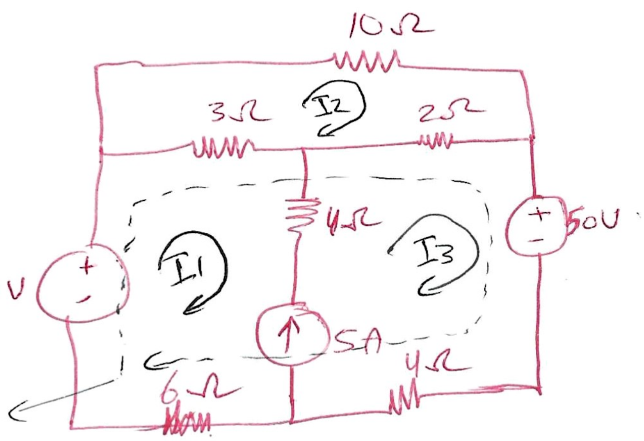
to Calculate the power dissipated in the 4Ω we need only I_2 and I_3

$P_{4\Omega} = 4(I_3 - I_2)^2 = 4(28-26)^2 = 16\ W$

4.7 The mesh current method, Some special Cases:-

Supper mesh:-

I_1 and I_3 are supper mesh



$I_3 - I_1 = 5\ A \quad ----(1)$

$-100 + I_1(3+6) + I_3(2+4) + 50 - I_2(3+2) = 0$

$9 I_1 - 5 I_2 + 6 I_3 - 50 = 0 \quad ---(2)$

mesh (2)

$I_2(3+10+2) - I_1(3) - I_3(2) = 0$

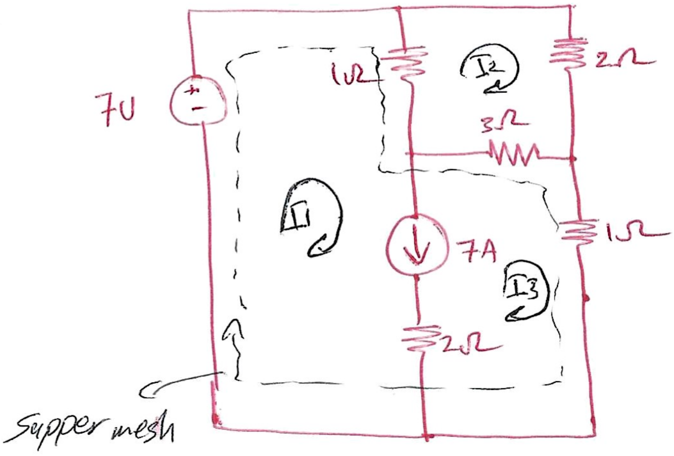
$-3 I_1 + 15 I_2 - 2 I_3 = 0 \quad ----(3)$

Solving:- $I_1 = 1.75 A,\ I_2 = 1.25 A,\ I_3 = 6.75 A$

Example:-

Supper mesh

I_1 and I_3 are supper mesh



$$-7V + I_1(1) + I_3(3+1) - I_2(1+3) = 0$$

$$I_1 - 4I_2 + 4I_3 = 7 \quad ---(1)$$

$$I_1 - I_3 = 7\,A \quad ----(2)$$

mesh (2)

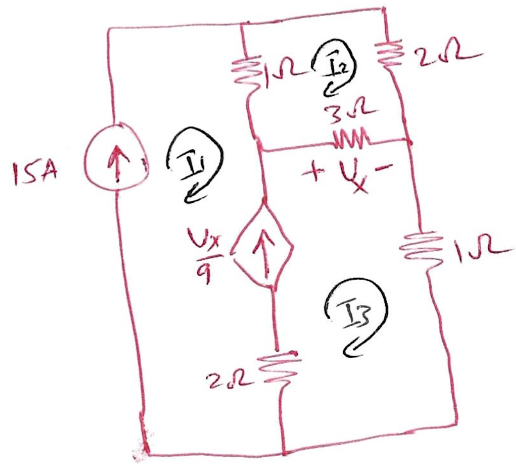
$$I_2(1+2+3) - I_1(1) - I_3(3) = 0$$

$$-I_1 + 6I_2 - 3I_3 = 0 \quad ----(3)$$

solving:- $I_1 = 9A$, $I_2 = 2.5\,A$, $I_3 = 2\,A$

Example:-

I_1 and I_3 are supper mesh



but $I_1 = 15\,A \quad ---(1)$

$$I_3 - I_1 = \frac{V_x}{9} \;,\quad V_x = 3(I_3 - I_2)$$

$$\therefore\; I_3 - 15 = \frac{3(I_3 - I_2)}{9} \quad ---(2)$$

mesh (2)

$$I_2(1+2+3) - I_1(1) - I_3(3) = 0$$

Solving:-

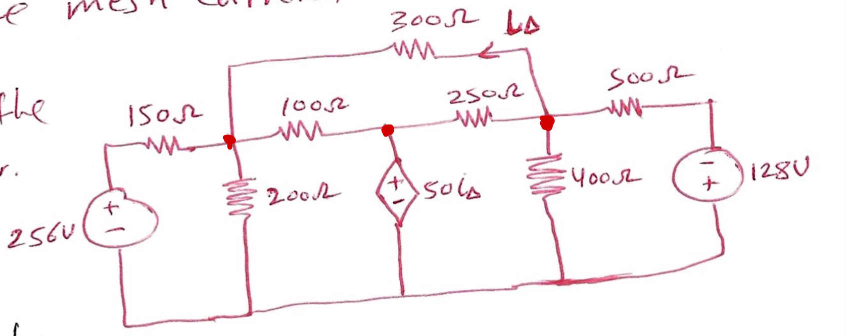
$I_1 = 15A$

$I_2 = 11\,A$

$I_3 = 17A$

4.8 The node-Voltage Versus the mesh Current

Example: For the Circuit shown, find the power dissipated in the 300 Ω resistor.



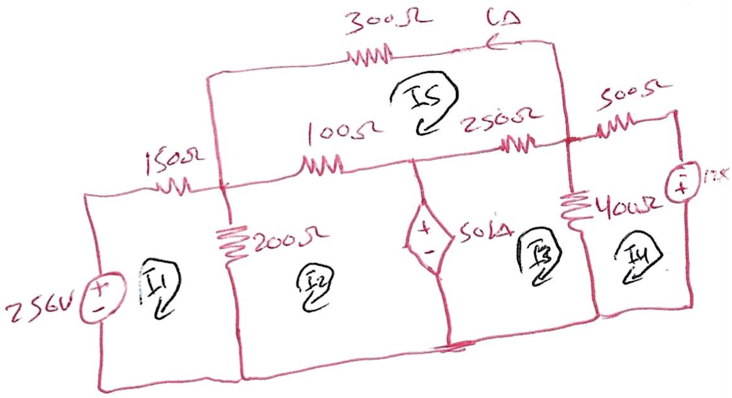
The Circuit shown Could be solved by both mesh and nodal methods

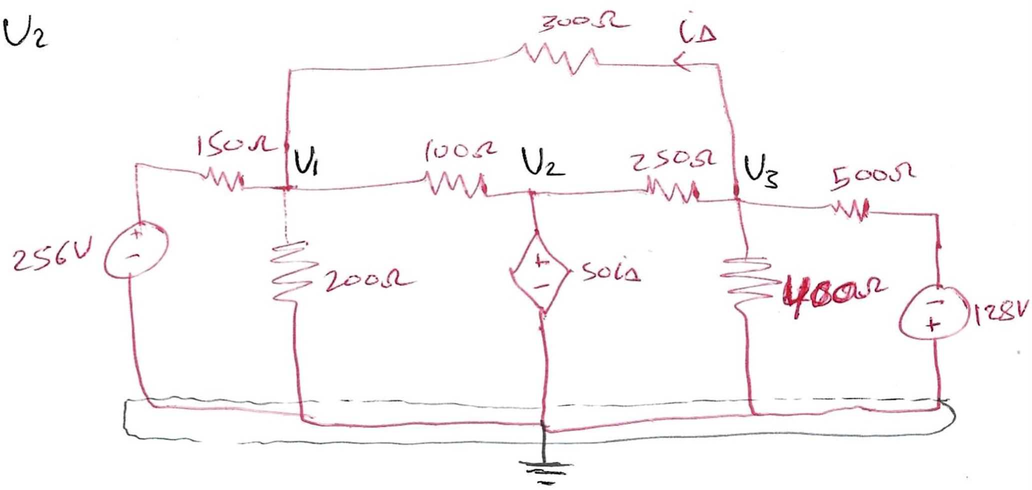
using mesh 5 equations are required,

however, using nodal only 2 equations are required

we need to solve for

V_1 and V_3 because V_2 is Known





$$\frac{V_1-256}{150}+\frac{V_1}{200}+\frac{V_1-V_2}{100}+\frac{V_1-V_3}{300}=0 \quad ---(1)$$

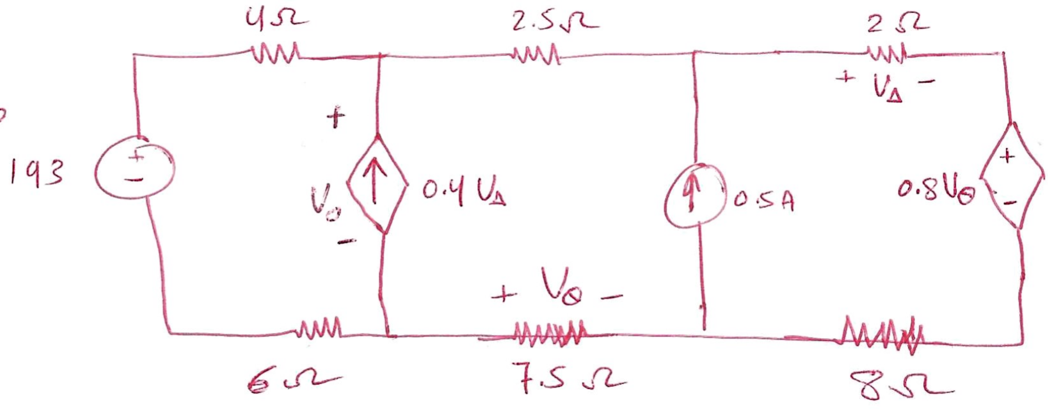
$$\frac{V_3-V_1}{300}+\frac{V_3-V_2}{250}+\frac{V_3}{400}+\frac{V_3+128}{500}=0 \quad ---(2)$$

$$V_2=50 i_\Delta=\frac{50(V_3-V_1)}{300}=\frac{V_3-V_1}{6}$$

Solving:- $V_1=62.5\,V$, $V_3=-8\,V$

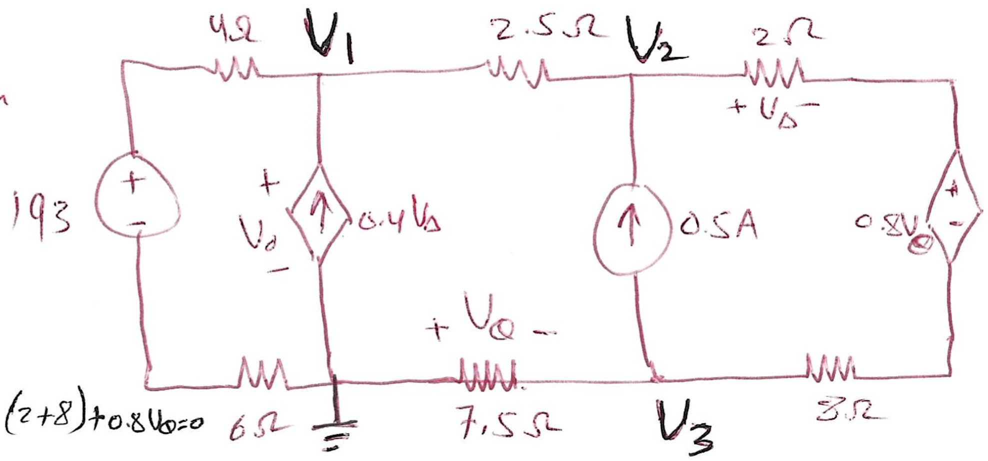
$$P_{300\Omega}=\frac{(62.5--8)^2}{300}=16.5675\ W$$

Example: For the circuit shown, find V_o



using nodal analysis we need 3 equations to solve for V_o.

However, using mesh Analysis only 1 equation is required.



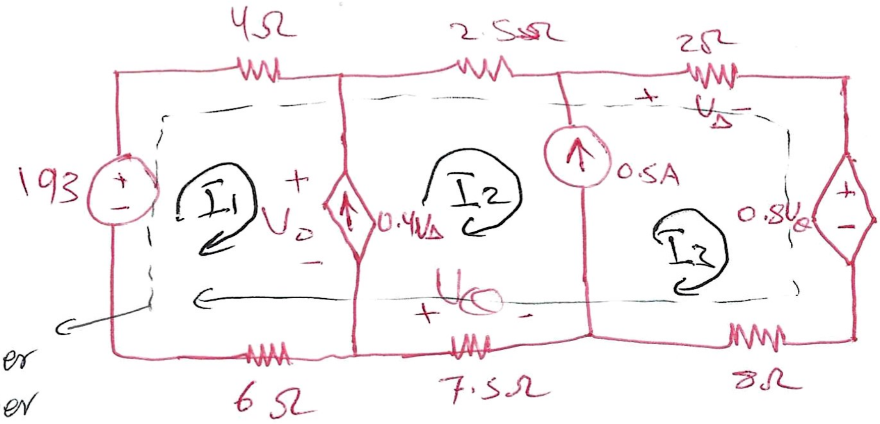
supper mesh

$$-193 + I_1(4+6) + I_2(2.5+7.5) + I_3(2+8) + 0.8V_\Theta = 0$$

$$I_2 - I_1 = 0.4\,V_\Delta$$
$$= 0.4(2I_3)$$
$$= 0.8\,I_3$$

$$I_3 - I_2 = 0.5\,A$$

$$V_\Theta = -(7.5)\,I_2$$



supper supper mesh

$$I_2 = I_3 - 0.5$$

$$I_3 = \frac{I_2 - I_1}{0.8}, \quad I_1 = I_2 - 0.8 I_3$$

$$-193 + 10I_1 + 10I_2 + 10I_3 + 0.8(-7.5)\,I_2 = 0$$

$$-193 + 10I_1 + 10I_3 - 5 + 10I_3 - 6I_3 + 3 = 0$$

$$-195 + 10(I_2 - 0.8I_3) + 14I_3 = 0$$

$$-200 + 16I_3 = 0 \Rightarrow I_3 = 12.5A, \quad I_2 = 12A, \quad I_1 = 2A$$

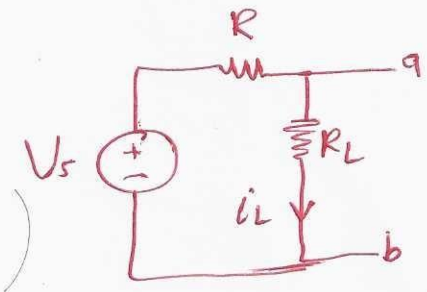
KVL

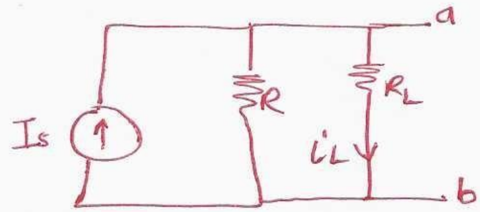
$$-193 + 10I_1 + V_o = 0$$

$$\Rightarrow V_o = 193 - (10)(2) = 173V$$

4.9 Source Transformation:

Source transformation: will allow the transformations of a Voltage Source in Series with a risistor to a current source in parallel with resistor



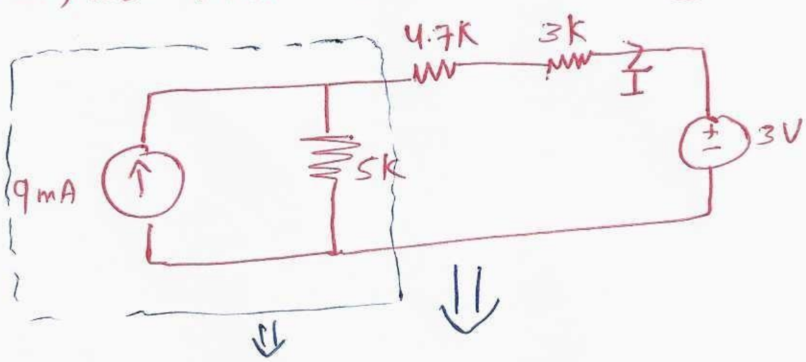


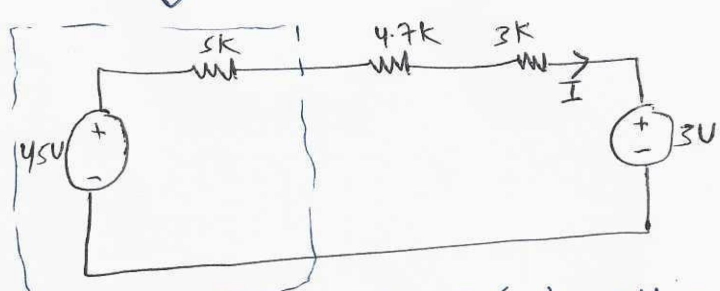
$$i_L = \frac{V_s}{R+R_L}$$

$$i_L = \frac{R\,I_s}{R+R_L}$$

$$\Rightarrow \frac{V_s}{R+R_L} = \frac{R\,I_s}{R+R_L} \quad \therefore \quad V_s = R\,I_s \quad \text{or} \quad I_s = \frac{V_s}{R}$$

Examples- Find the current I using source transformation.



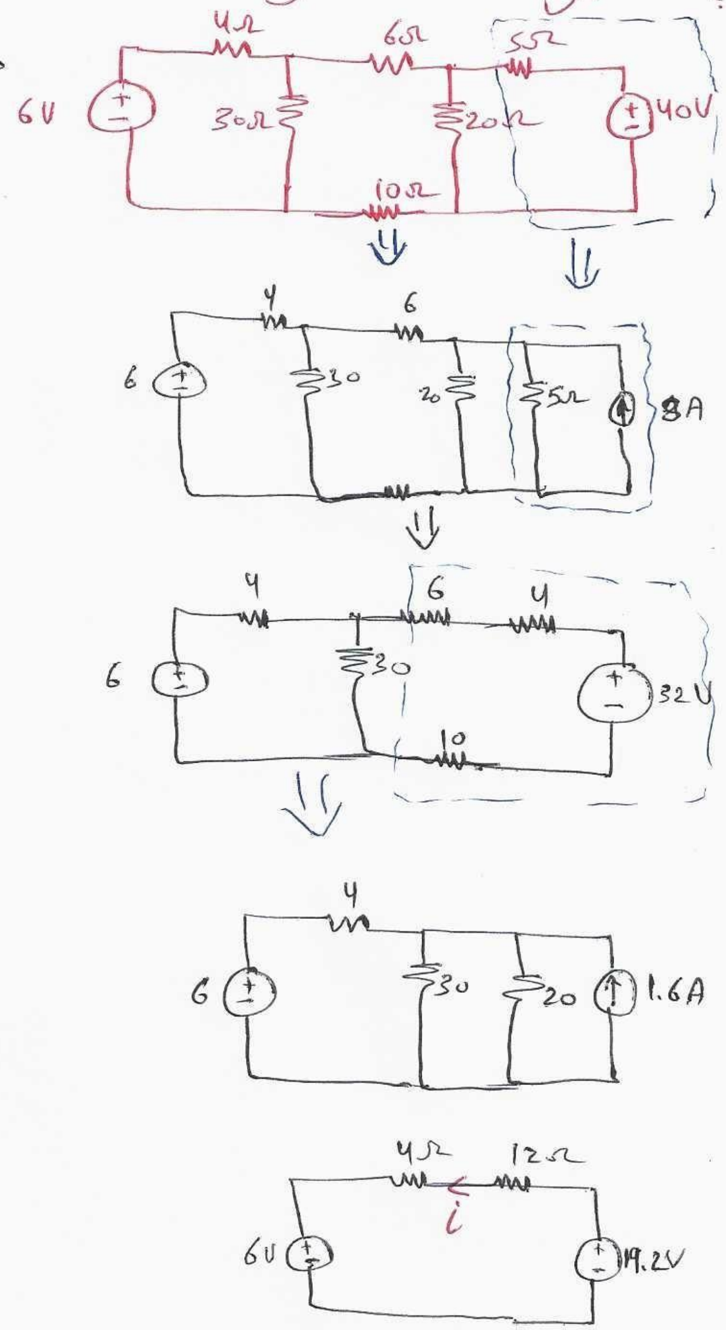


$$-45 + I(5k) + I(4.7k) + I(3k) + 3V = 0$$

$$I = \frac{45-3}{5k+4.7k+3k} = 3.3\ mA$$

Example:- a) Find the power associated with the 6V source?

b) state whether the 6V source is absorbing or delivering power?



we are going to use source transformation to reduce the circuit, however, note that we will not alter or transfere the 6V source because it is the objective

$\frac{40}{5} = 8A$

$5 // 20 = \frac{5 \times 20}{25} = 4\Omega$

$4 \times 8 = 32\ V$

$4 + 6 + 10 = 20\Omega$

$\frac{32}{20} = 1.6A$

$20 // 30 = \frac{20 \times 30}{50} = 12\Omega$

$12 \times 1.6 = 19.2\ V$

$$-19.2 + i(4+12) + 6 = 0$$

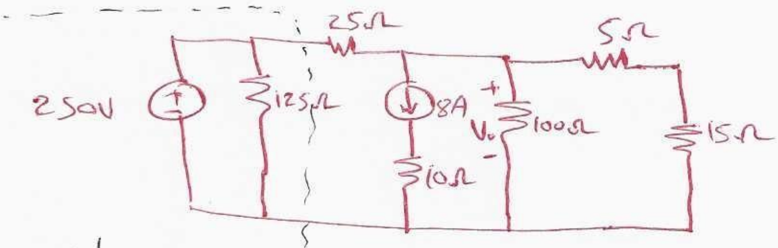
$$i = \frac{19.2 - 6}{16} = 0.825A$$

$$P_{6V} = Vi = (6)(0.825) = 4.95\ W$$

The 6V source is absorbing power.

Example: use Source transformations to find the Voltage V_o?

our objective is V_o

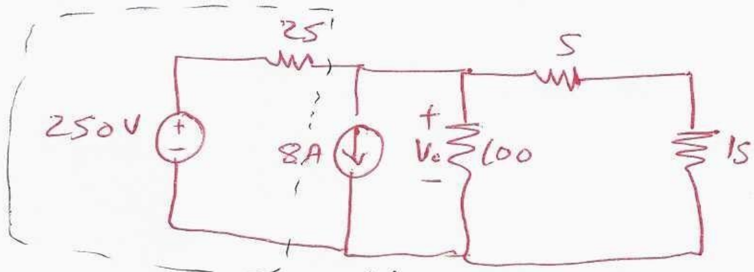


ما دام اهتمامنا بعد هذه النقطة
ممكن إزالة 125Ω

Since the 125Ω resistor connected across or in parallel to the 250 V source then we can remove it without altering any Voltage or current in the circuit except the 250 V current which is not an objective any how. Therfore, we remove the 125Ω

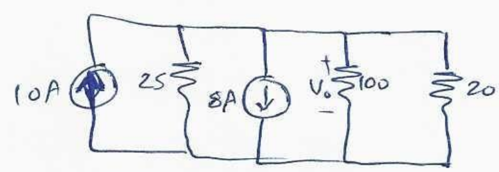
similarly, the 10 Ω resistor is connected in series with the 8A source, then we can remove it without altering any Voltage or current in the circuit.

So the circuit become ⟹

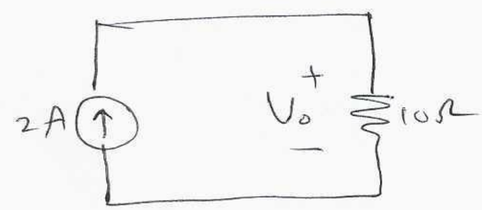


$$\frac{250}{25} = 10A$$

$$20 \parallel 100 \parallel 25 = 10\Omega$$



$$V_o = (2)(10) = 20V$$

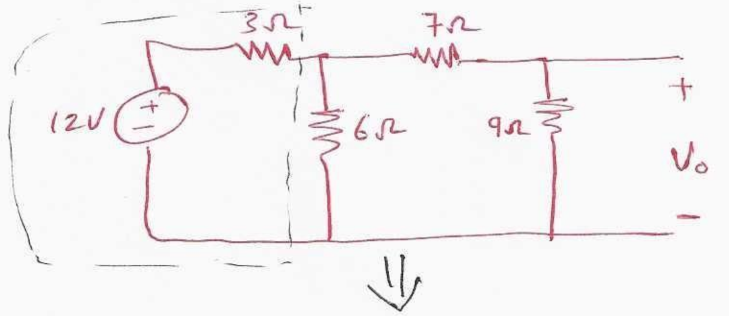


* Dependent source

$$V_s = I_s R$$

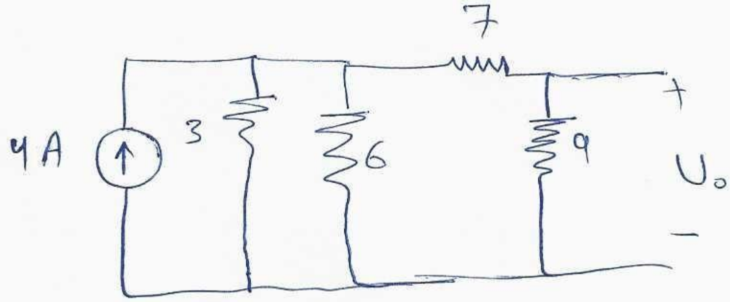
$$I_s = \frac{V_s}{R}$$

Example: Find V_o using source transformation



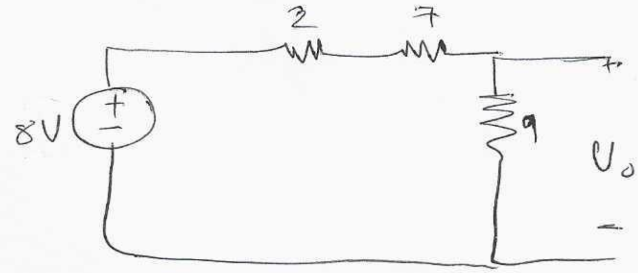
$\frac{12}{3} = 4A$

$3 \parallel 6 = \frac{3 \times 6}{9} = 2\,\Omega$



$2 \times 4 = 8\,V$

$2 + 7 = 9\,\Omega$



$V_o = \frac{9}{9+9}\,8 = 4\,V$

4.13 Superposition:

whenever a linear system is excited or driven by more than one independent source of energy, the total response is the sum of the individual responses by the independent sources.

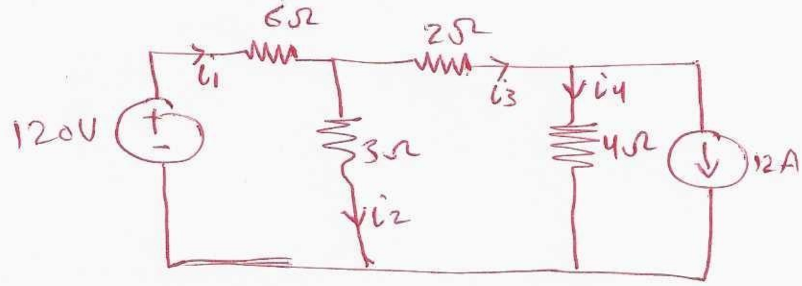
Dependent sources are left intact because they are controlled by circuit variables.

1- Turn off all independent sources except one source. Find the output (voltage or current) due to that source using nodal, mesh, Kirchhoff ----

2- Repeat step 1 for each of the other independent sources

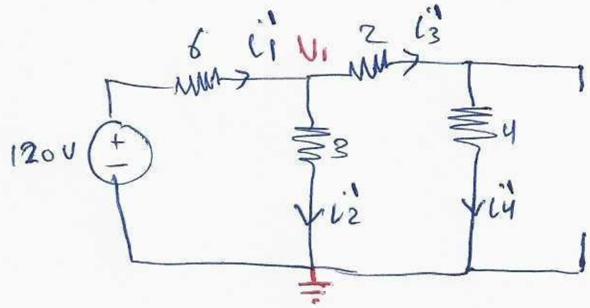
3- Find the total contribution by adding algebraically all the contributions due to each independent sources.

Example: Consider the following circuit, use the principle of superposition to find the branch currents i_1, i_2, i_3 and i_4



① Voltage source is active and deactivate the current source by opening it

using nodal, we can solve for V_1



$$\frac{V_1 - 120}{6} + \frac{V_1}{3} + \frac{V_1}{2+4} = 0$$

$$V_1 - 120 + 2V_1 + V_1 = 0$$

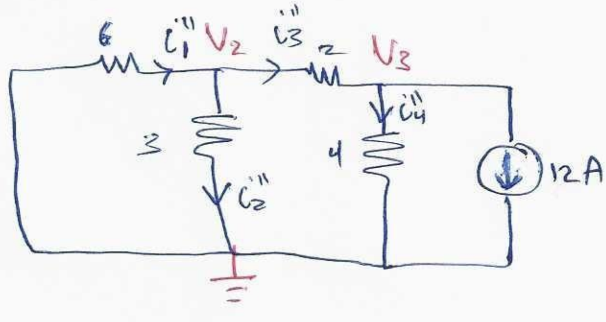
$$V_1 = \frac{120}{4} = 30V$$

So $i_1' = \frac{120-30}{6} = 15A$, $i_2' = \frac{30}{3} = 10A$, $i_3' = i_4' = \frac{30}{6} = 5A$

② Current source is active and deactivate the voltage source by shorting it

Using Nodal

at V_2:



$$\frac{V_2}{6} + \frac{V_2}{3} + \frac{V_2 - V_3}{2} = 0 \quad \text{---(1)}$$

$$V_2 + 2V_2 + 3V_2 - 3V_3 = 0$$

$$V_2 = \frac{V_3}{2}$$

at V_3

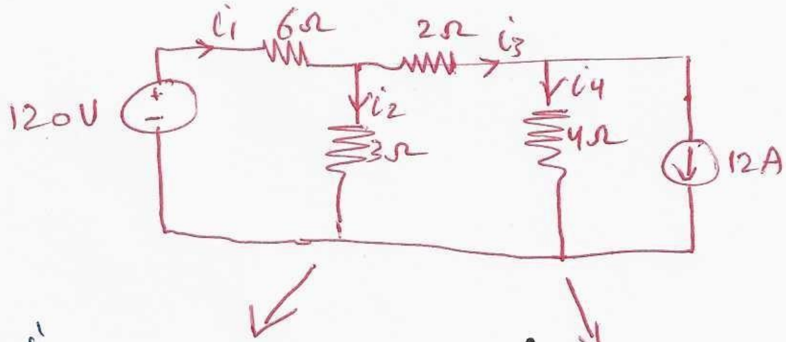
$$\frac{V_3 - V_2}{2} + \frac{V_3}{4} + 12 = 0 \quad \text{---(2)}$$

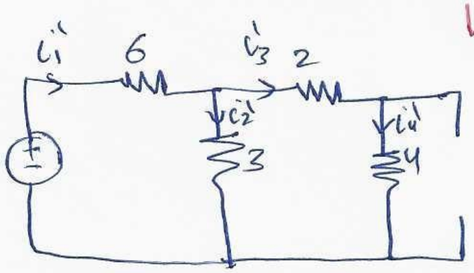
$$4V_3 - 4V_2 + 2V_3 + 96 = 0$$

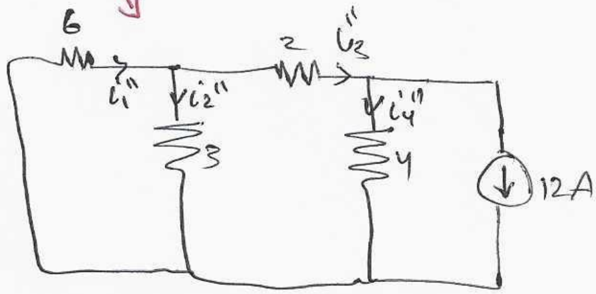
$$4V_3 - 4\left(\frac{V_3}{2}\right) + 2V_3 + 96 = 0$$

$$4V_3 = -96 \Rightarrow V_3 = \frac{-96}{4} = -24 \Rightarrow V_2 = -12$$

So $i_1'' = \frac{-V_2}{6} = \frac{12}{6} = 2A$, $i_2'' = \frac{V_2}{3} = \frac{-12}{3} = -4$, $i_3'' = \frac{V_2 - V_3}{2} = \frac{-12+24}{2} = 6A$, $i_4'' = \frac{V_3}{4} = -6$







Now

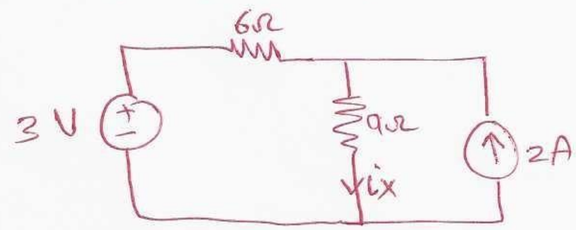
$$i_1 = i_1' + i_1'' = 15 + 2 = 17\ A$$

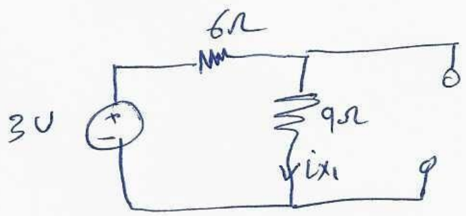
$$i_2 = i_2' + i_2'' = 10 + (-4) = 6\ A$$

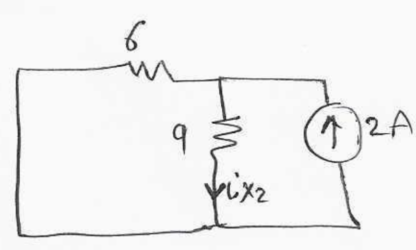
$$i_3 = i_3' + i_3'' = 5 + 6 = 11\ A$$

$$i_4 = i_4' + i_4'' = 5 + (-6) = -1\ A$$

Example:- use superposition to solve for i_x







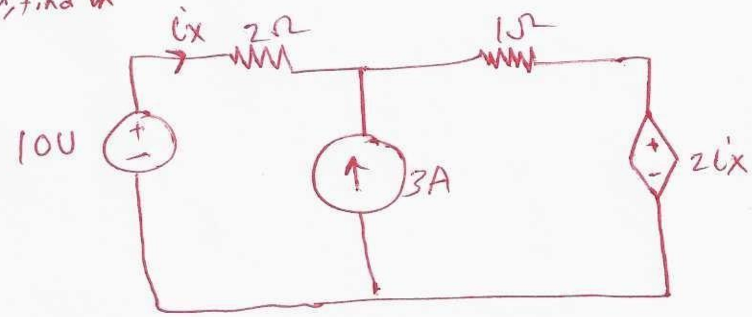
$$i_{x_1} = \frac{3}{15} = 0.2\ A$$

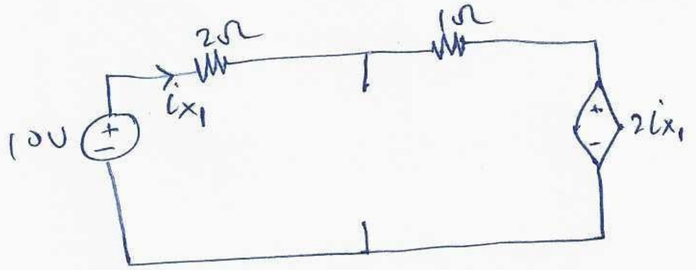
$$i_{x_2} = \frac{6}{6+9}(2) = 0.8\ A$$

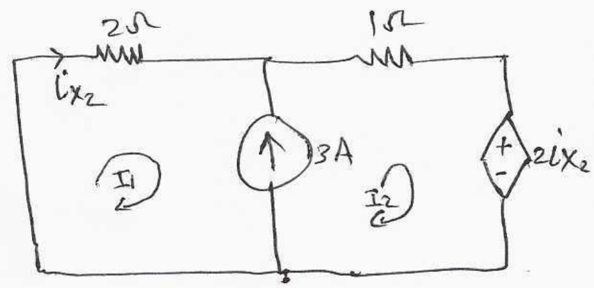
$$i_x = i_{x_1} + i_{x_2} = 0.2 + 0.8 = 1\ A$$

Superposition with dependent Source:

Example: Using superposition, find i_x







$-10 + 3i_{x_1} + 2i_{x_1} = 0$

$\therefore i_{x_1} = 2A$

$I_2 - I_1 = 3A \quad , \quad I_1 = i_{x_2}$

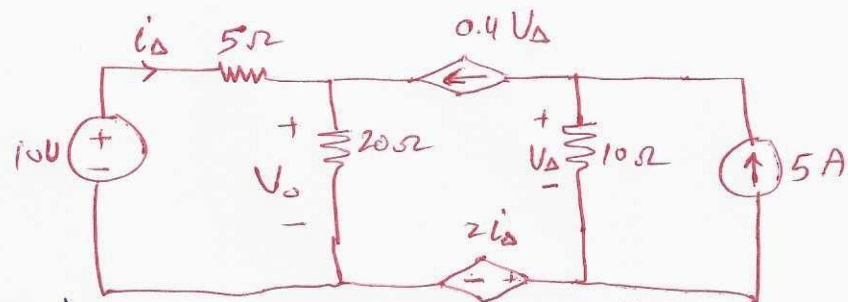
$I_1(2) + I_2(1) + 2i_{x_2} = 0$

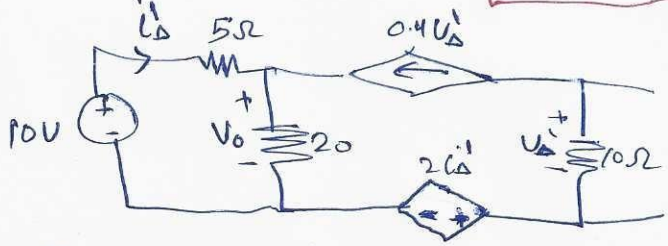
$2i_{x_2} + (3 + i_{x_2}) + 2i_{x_2} = 0$

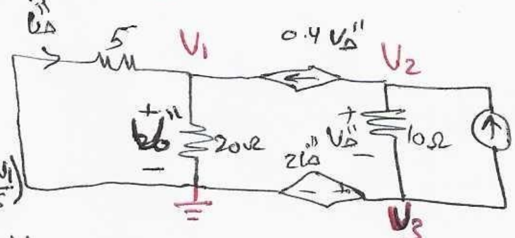
$\therefore i_{x_2} = -0.6A$

$\Rightarrow i_x = i_{x_1} + i_{x_2} = 2A + (-0.6A) = 1.4A$

Example:







$V_1 = V_o''$

$V_2 = V_\Delta'' + V_3$

$V_3 = 2i_\Delta'' = 2\left(\frac{-V_1}{5}\right)$

at V_1: $\frac{V_1}{20} + \frac{V_1}{5} - 0.4V_\Delta'' = 0$ ---(1)

$5V_1 - 8V_2 + 8V_3 = 0$

at V_2:

$0.4V_\Delta'' + \frac{V_2 - V_3}{10} - 5 = 0$ ---(2)

From the circuit

$V_\Delta' = (0.4V_\Delta')(10) = 4V_\Delta'$

This valid only if $V_\Delta' = 0$

So $V_o' = \frac{20}{25}(10) = 8V$

$$5V_1 - 8V_2 + 8V_3 = 0$$

$$V_3 = \frac{-2V_1}{5}$$

$$5V_1 - 8V_2 + 8\left(\frac{-2V_1}{5}\right) = 0$$

$$25V_1 - 40V_2 - 16V_1 = 0$$

$$9V_1 - 40V_2 = 0 \Rightarrow V_2 = \frac{9V_1}{40}$$

from (2)

$$0.4V_\Delta'' + \frac{V_2 - V_3}{10} - 5 = 0$$

$$0.4(V_2 - V_3) + \frac{V_2 - V_3}{10} - 5 = 0$$

$$4V_2 - 4V_3 + V_2 - V_3 - 5 = 0$$

$$5V_2 - 5\left(\frac{-2V_1}{5}\right) - 5 = 0$$

$$5V_2 + 2V_1 - 5 = 0$$

$$5\left(\frac{9V_1}{40}\right) + 2V_1 - 5 = 0$$

$$9V_1 + 16V_1 - 400 = 0$$

$$V_1 = \frac{400}{25} = 16V = V_o''$$

$$\therefore V_o = V_o' + V_o''$$

$$= 8 + 16 = 24V$$

4.10 Thevenin and Norton Equivalents

Thevenin's theorem:-

A linear two terminals circuit can be replaced by an equivalent circuit consisting of voltage source V_{th} in series with a resistor R_{th} where V_{th} is the open circuit voltage at the terminals and R_{th} is the input or equivalent resistance at the terminals when the independent sources are killed

Norton's theorem:-

A linear two terminals circuit can be replaced by an equivalent circuit of a current source I_N in parallel with a resistor R_N, where I_N is the short circuit current through the terminals

To find R_{th} or R_N

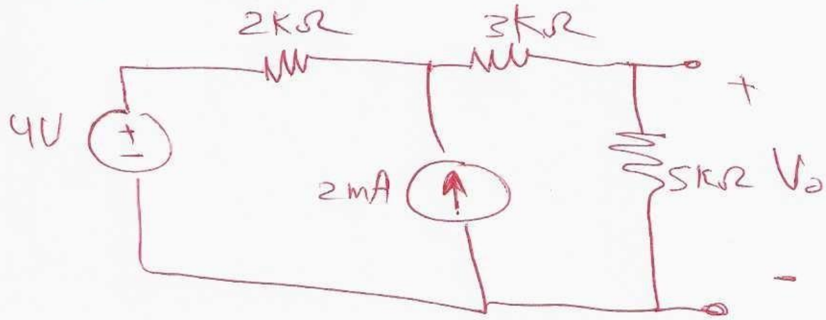
Case I:- if the circuit has no dependent sources, kill all independent sources and apply series and parallel combination.

Case II:- if the circuit has dependent sources

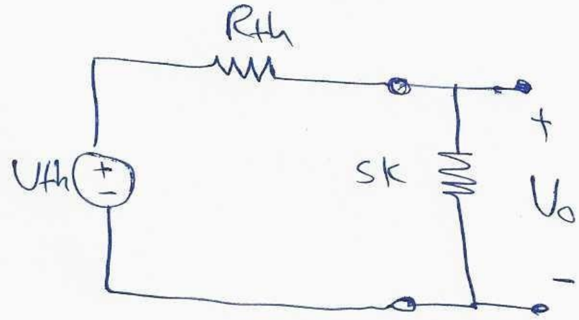
① Either by $\frac{V_{th}}{I_N} = R_{th}$

② or by applying voltage source V_T or current source I_T and obtain $R_{th} = \frac{V_T}{I_T}$ (kill all independent sources)

Example:- Find V_o using thevenin's theorem



To find V_{th}, we need to calculate $V_{o.c}$, we rplace the 5KΩ by an open circuit.

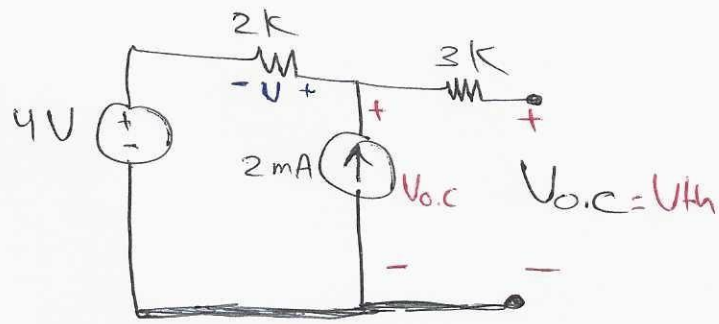


$-4 + (-V) + V_{o.c} = 0$

$-4 + (-(2k)(2m)) + V_{o.c} = 0$

$-4 - 4 + V_{o.c} = 0$

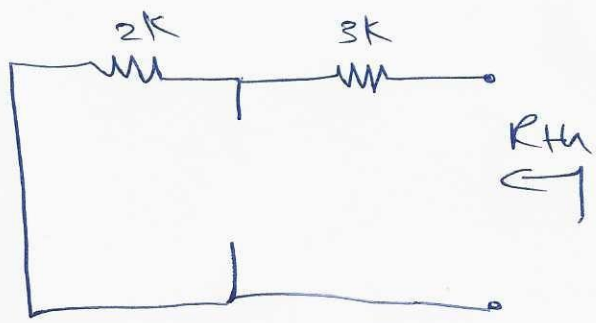
$\therefore V_{o.c} = 8V = V_{th}$

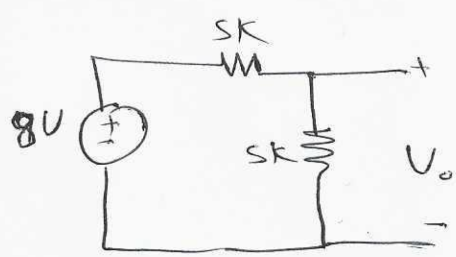


To find Rth, kill all independant sources

$R_{th} = 2K + 3K = 5K\Omega$

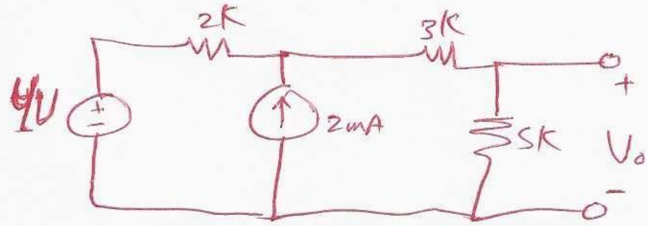
$\therefore V_o = \frac{5}{5+5}(8) = 4V$





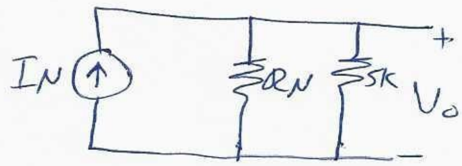
Example: Find V_o using Norton's theorem

To find I_N, we replace the 5kΩ by a short circuit

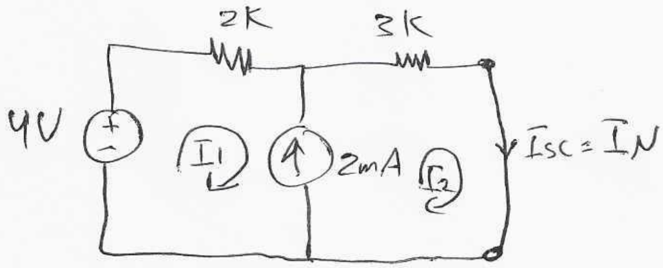


$$I_2 - I_1 = 2mA \Rightarrow I_2 = 2mA + I_1$$

$$I_2 = I_{s.c} = I_N$$



$$-4 + 2K\, I_1 + 3K\, I_2 = 0$$



$$-4 + 2000 I_1 + 3000(2mA + I_1) = 0$$

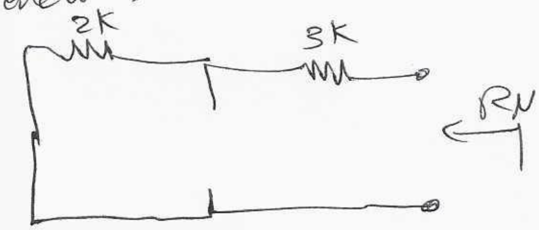
$$-4 + 2000 I_1 + 6 + 3000 I_1 = 0$$

$$5000 I_1 = -2$$

$$\therefore I_1 = \frac{-2}{5000} = 0.4mA$$

$$\Rightarrow I_2 = I_{s.c} = I_N = 2mA + (-0.4mA) = 1.6mA$$

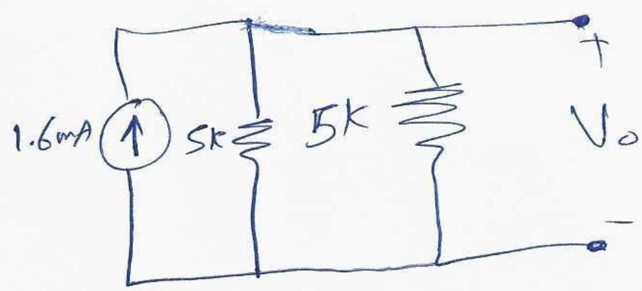
To find R_N, kill all independent sources



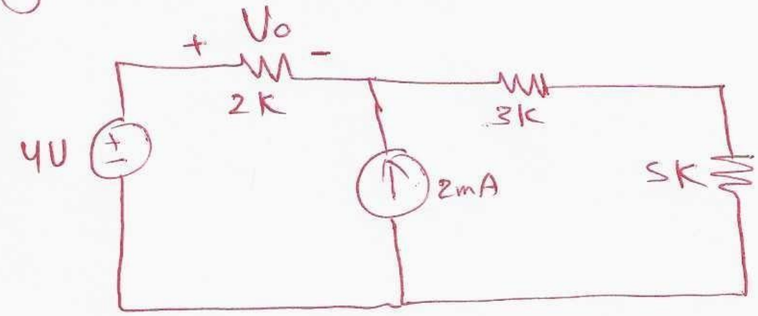
$$R_N = 2 + 3 = 5k\Omega$$

$$V_o = (1.6mA)(5 // 5)$$

$$= 4V$$

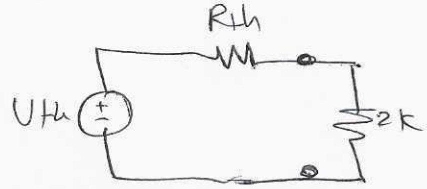


Example: Find V_o using thevenin's theorem



To find V_{th}, replace the $2k\Omega$ resistor by an open circuit

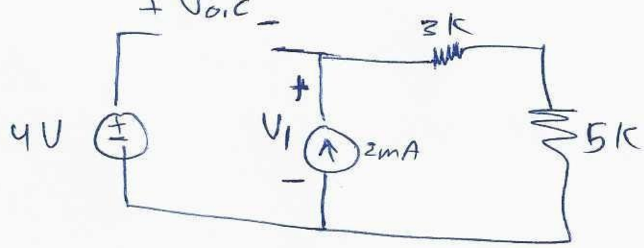
$-4V + V_{o.c} + V_1 = 0$



$-V_1 + 2mA(3+5\ k\Omega) = 0$

$-V_1 + 16\ V = 0$

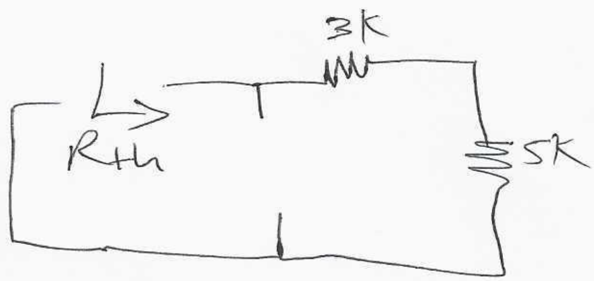
$\therefore V_1 = 16\ V$



$\Rightarrow V_{o.c} = -16 + 4 = -12V$

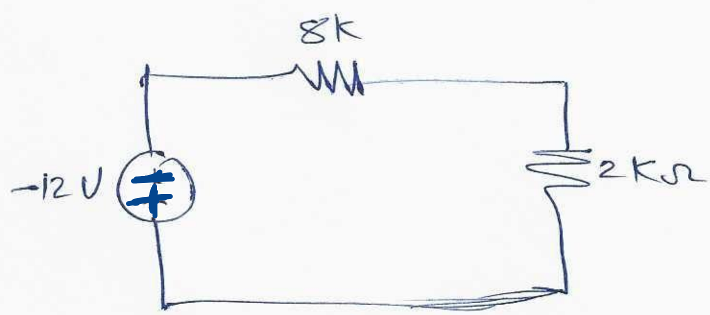
To find R_{th}, kill all independent sources

$R_{th} = 3 + 5 = 8K$

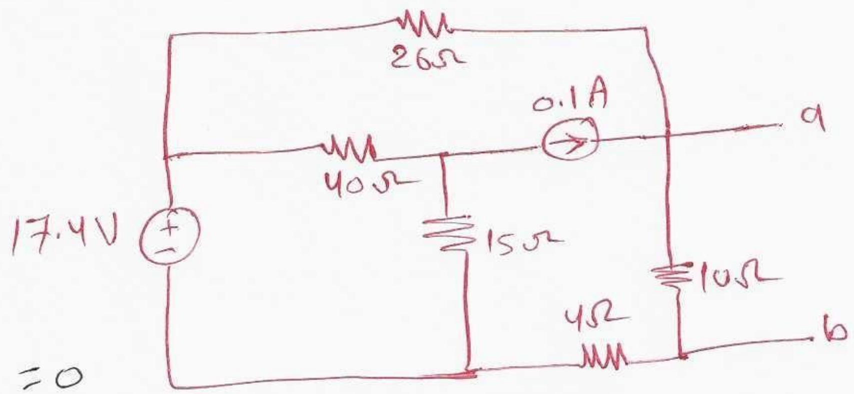


$V_o = -12\left(\frac{2}{2+8}\right)$

$= -2.4V$



Example :- for the Circuit shown, find thevenin equivalent Circuit with respect to the terminals a and b

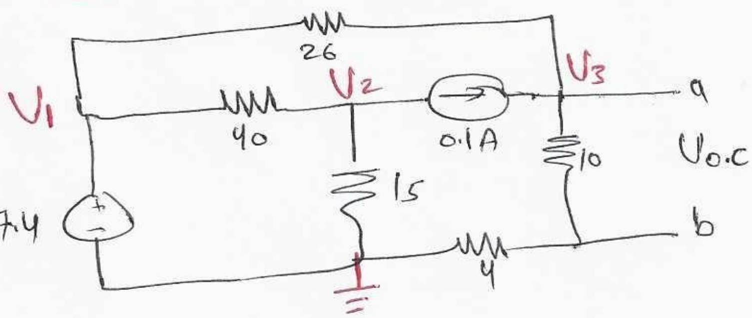


$$V_{oc} = V_3\left(\frac{10}{10+4}\right)$$

$V_1 = 17.4\,V$

at V_2 :-

$$\frac{V_2 - 17.4}{40} + \frac{V_2}{15} + 0.1 = 0$$



at V_3 :-

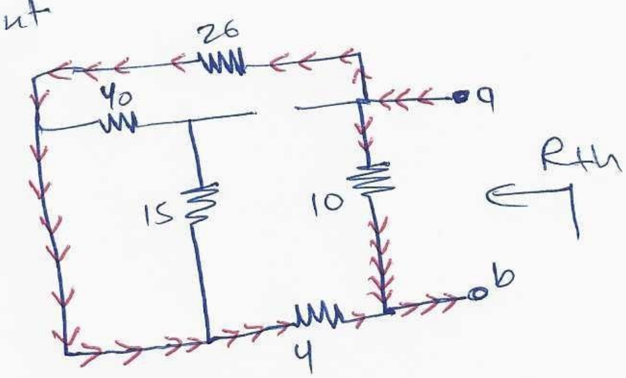
$$-0.1 + \frac{V_3}{14} + \frac{V_3 - 17.4}{26} = 0$$

$$-36.4 + 26\,V_3 + 14\,V_3 - 243.6 = 0$$

$$40\,V_3 = 280 \Rightarrow V_3 = 7\,V$$

$$\therefore V_{th} = V_{o.c} = 7\left(\frac{10}{14}\right) = 5\,V$$

To find R_{th}, we kill all independent Sources as :-

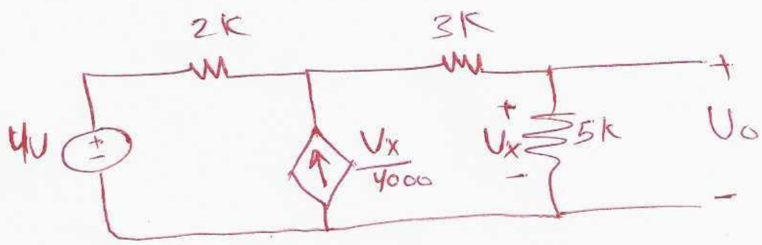


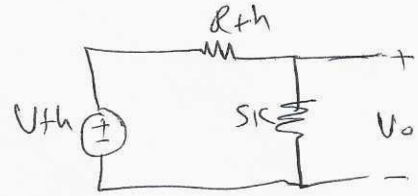
$$(26+4) \,//\, (10) =$$

$$(30 \,//\, 10) = \frac{30 \times 10}{40} = 7.5\,\Omega$$

Example: Find V_o using thevenin's theorem

To find V_{th}, Replace the $5K\Omega$ by an open Circuit





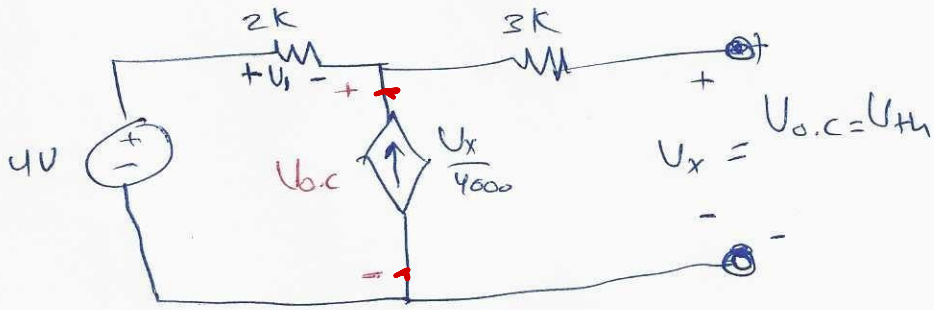
$$-4 + V_1 + V_{o.c} = 0$$

$$-4 + \left(-2K\left(\frac{V_x}{4000}\right)\right) + V_{oc} = 0$$

$$-4 - \frac{1}{2} V_x + V_x = 0$$

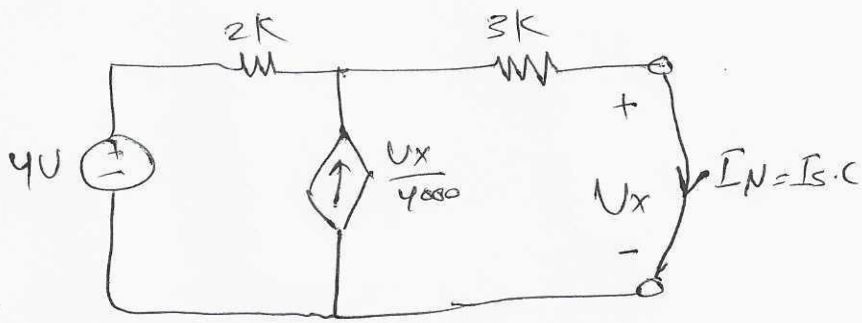
$$\frac{1}{2} V_x = 4V$$

$$V_x = V_{o.c} = V_{th} = 8V$$



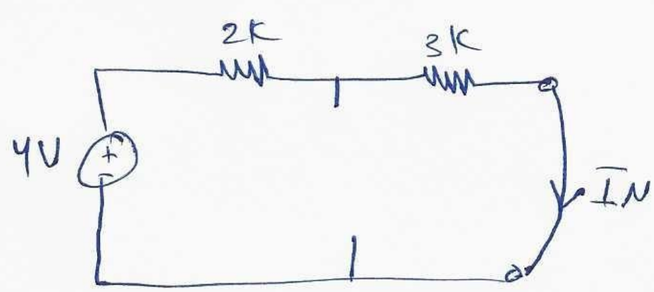
To find Rth, we Can not use series parallel Combination due to dependent Sources but we can solve for Rth by two Cases:

(1) $R_{th} = \frac{V_{th}}{I_N} = \frac{V_{o.c}}{I_{s.c}}$



$V_x = 0$

$\Rightarrow$ dependent current source $\frac{V_x}{4000} = 0 \Rightarrow$ open Circuit



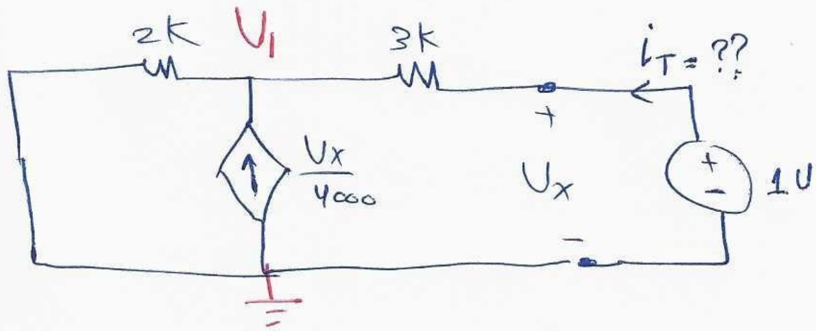
$$-4 + I_N(2K + 3K) = 0$$

$$I_N = \frac{4}{5K} = 0.8mA$$

$$\therefore R_{th} = \frac{8V}{0.8mA} = 10K\Omega$$

② $R_{th} = \frac{V_T}{I_T}$ (kill all independent sources)

To find i_T, we solve for V_1



$$\frac{V_1}{2K} - \frac{V_x}{4000} + \frac{V_1 - 1}{3000} = 0$$

$V_x = 1V$

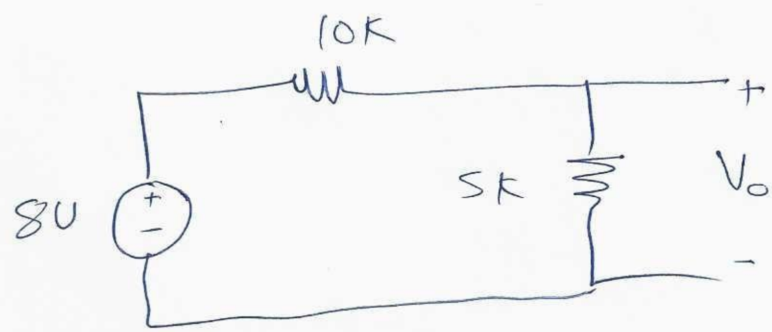
$$\frac{V_1}{2000} - \frac{1}{4000} + \frac{V_1 - 1}{3000} = 0$$

$$6V_1 - 3 + 4V_1 - 4 = 0$$

$$V_1 = \frac{7}{10}$$

$$\therefore i_T = \frac{1 - 0.7}{3000} = \frac{0.3}{3000} = 0.1\ mA$$

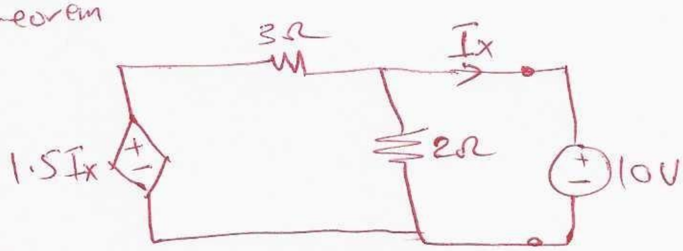
$$\Rightarrow R_{th} = \frac{1}{0.1mA} = 10\ K\Omega$$



$$V_o = \frac{5}{5+10}(8)$$

$$= \frac{8}{3}\ V$$

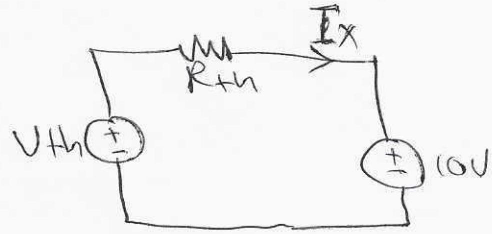
Example:- Find I_x using thevenin's theorem



To find V_{th}

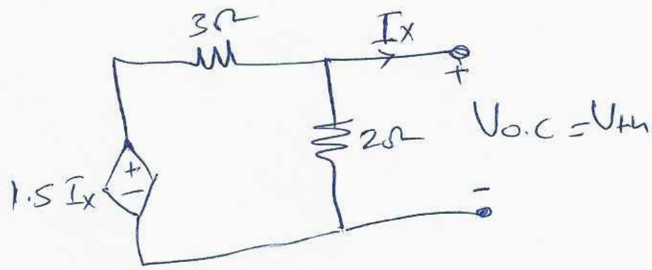
Since there is no independent sources

$\therefore V_{th} = 0$



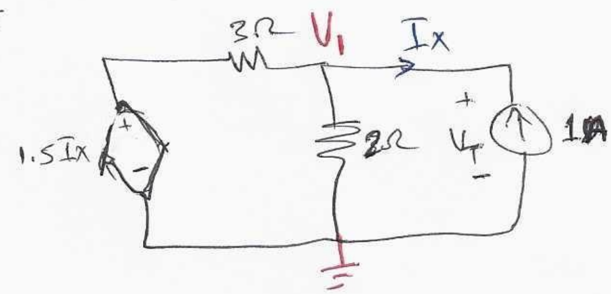
to find R_{th} we can't use $\frac{V_{th}}{I_N} = \frac{0}{0}$

so we use $\frac{V_T}{I_T}$



so we either apply a test voltage or a test source

so we solve to V_1 which is equal to V_T



$$\frac{V_1 - 1.5 I_x}{3} + \frac{V_1}{2} - 1 = 0$$

$I_x = -1A$

$$\frac{V_1 - 1.5(-1)}{3} + \frac{V_1}{2} - 1 = 0$$

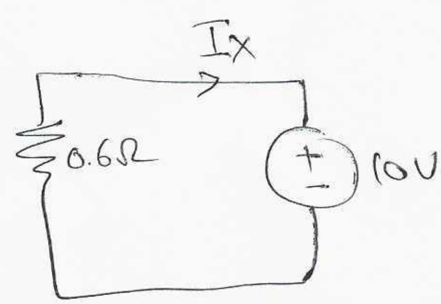
$$\frac{V_1 + 1.5}{3} + \frac{V_1}{2} - 1 = 0$$

$$2V_1 + 3 + 3V_1 - 6 = 0$$

$$5V_1 = 3 \Rightarrow V_1 = 0.6\,V$$

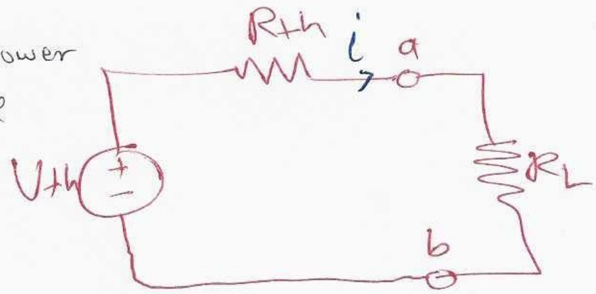
$$\therefore R_{th} = \frac{V_T}{I_T} = \frac{0.6}{1} = 0.6\Omega$$

$$\therefore I_x = -\frac{10}{0.6} = -16.67A$$



4.12 Maximum Power Transfer:-

A load resistance will recieve maximum power from a circuit when the resistance of the load is exactly the same as the thevenin's resistance.



$$P = i^2 R_L = \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 R_L$$

$$\frac{dp}{dt} = V_{th}^2 \left[\frac{(R_{th}+R_L)^2 - R_L\, 2(R_{th}+R_L)}{(R_{th}+R_L)^4}\right]$$

p is maximum when the derivative is zero and

$$(R_{th}+R_L)^2 - R_L \cdot 2(R_{th}+R_L) = 0$$

$$(R_{th}+R_L)\left[(R_{th}+R_L) - 2R_L\right] = 0$$

$$\therefore\ R_{th} + R_L - 2R_L = 0$$

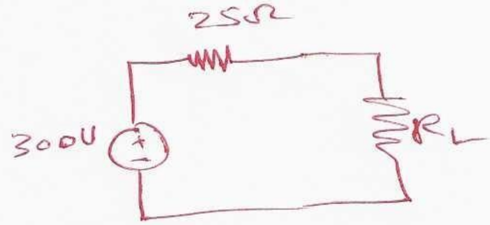
$$R_{th} = R_L$$

$$P_{max} = \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 R_L \quad , \quad R_L = R_{th}$$

$$= \frac{V_{th}^2}{(R_{th}+R_{th})^2} R_{th} = \frac{V_{th}^2}{(2R_{th})^2} R_{th} = \frac{V_{th}^2}{4R_{th}}$$

Example :- what is the Value of R_L that will absorb the maximum power.

R_L	$P(\omega)$
15	843.75
20	888.88
25	900
30	892.56
35	875

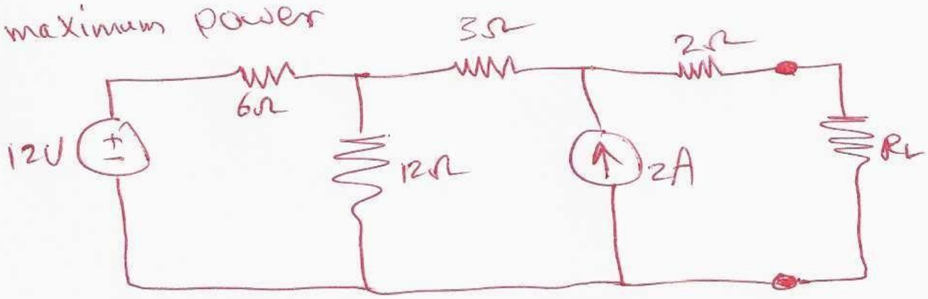


$\therefore$ for $R_L = R_{th} = 25\Omega$

P will be the maximum

Example :- Find the Value of RL for maximum power transfer in the circuit shown

- Find the maximum power

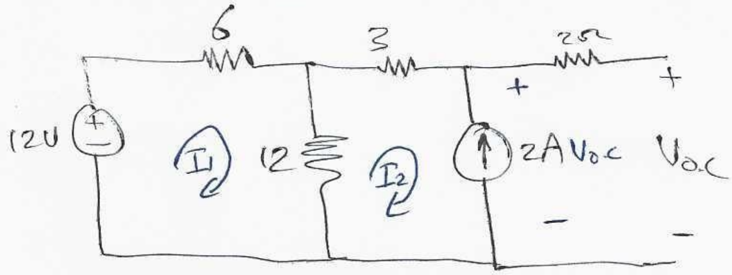


To find V_{th}

for mesh 2 :-

$I_2 = -2A$

for mesh 1 :-



$-12 + I_1(6+12) - I_2(12) = 0$

$-12 + 18I_1 + 24 = 0$

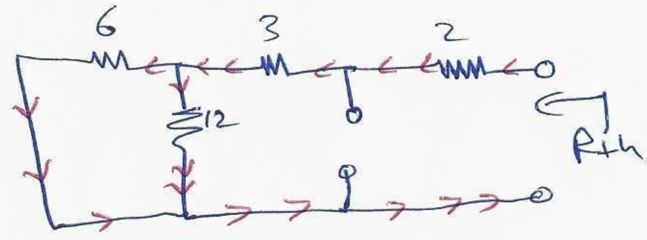
$I_1 = \frac{-12}{18} = \frac{-2}{3} A$

$-12 + 6I_1 + 3I_2 + V_{o.c} = 0$

$-12 + 6\left(\frac{-2}{3}\right) + 3(-2) + V_{o.c} = 0$

$V_{o.c} = 22V = V_{th}$

To find R_{th}



$R_{th} = (6 \parallel 12) + 3 + 2 =$

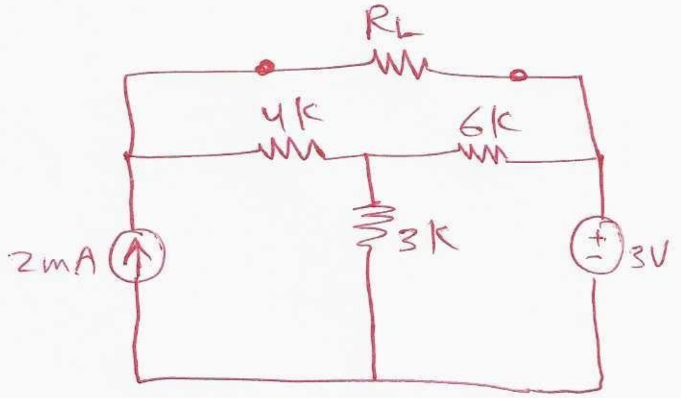
$4 + 3 + 2 = 9\Omega$

$\therefore R_L = R_{th} = 9\Omega$

$\therefore P \ldots \frac{V_{th}}{} \ldots \frac{(22)^2}{} = 13.44\ W$

Example: - Find the value of R_L for maximum power transfer in the circuit shown.

- Find the maximum power



To find V_{th}

for mesh (1)

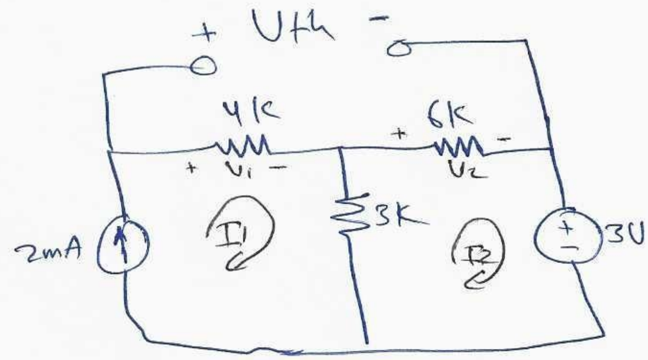
$I_1 = 2mA$

for mesh (2)

$I_2(3+6)k - I_1(3k) + 3V = 0$

$9000 I_2 - 6 + 3 = 0$

$I_2 = \frac{1}{3} mA$



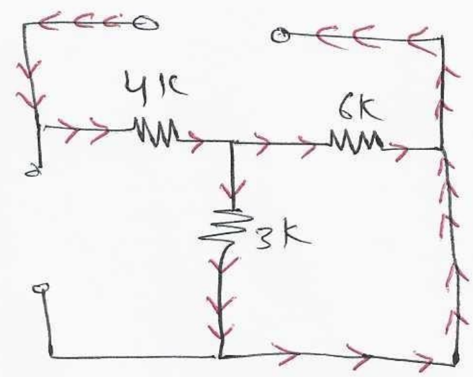
$V_{th} - V_2 - V_1 = 0$

$V_{th} - (6k)(\frac{1}{3}mA) - (4k)(2mA) = 0$

$V_{th} - 2 - 8 = 0$

$V_{th} = 10\,V$

To find R_{th}



$R_{th} = 4 + (3 // 6)$

$= 4K + 2K = 6\,K\Omega$

$\therefore\ R_L = R_{th} = 6\,K\Omega$

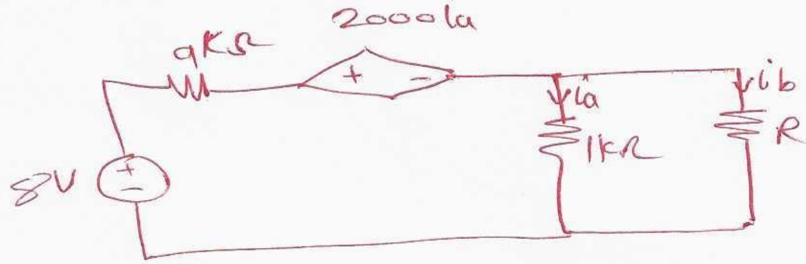
$$P_{L\,max} = \frac{V_{th}^2}{4R_{th}} = \frac{(10)^2}{4(6)} = \frac{25}{6}\ mW$$

Example:- For the Circuit shown below, the Current i_a has units of Amps. Determine:-

① - The resistance R that will Cause the current i_b in the Circuit to be 4mA.

② The resistance R that will Cause maximum power Transfer to it from the Circuit.

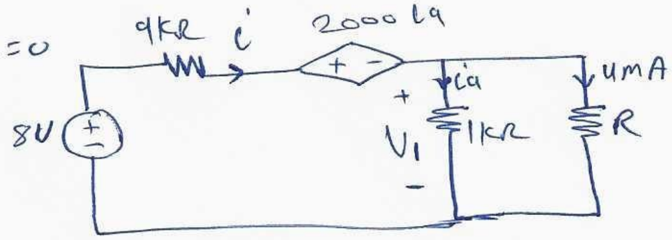
① to solve for R



$-8 + 9000i + 2000i_a + 1000i_a = 0$

$i = i_a + 4mA$

$-8 + 9000(i_a + 4mA) + 2000i_a + 1000i_a = 0$



$-8 + 9000i_a + 36 + 2000i_a + 1000i_a = 0$

$-8 + 12000i_a + 36 = 0$

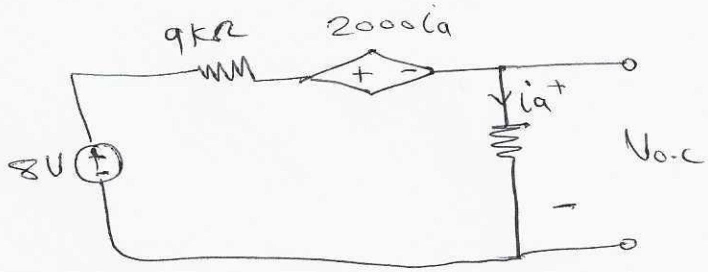
$i_a = \frac{-28}{12000} = -2.33\ mA$

$V_1 = i_a(1k\Omega) = -2.33\ V$

$R = \frac{V_1}{4mA} = \frac{-2.33V}{4mA} = -582.5\ \Omega$

② To find $R_{th} = \frac{V_{o.c}}{I_{sc}}$

to find $V_{o.c}$



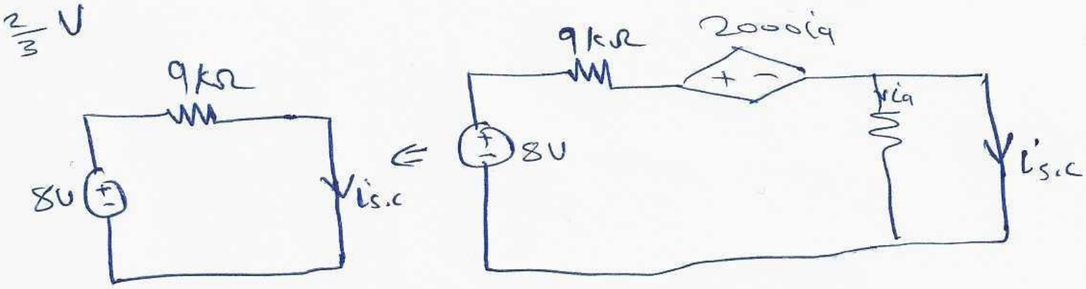
$-8V + 9000i_a + 2000i_a + 1000i_a = 0$

$-8 + 12000i_a = 0$

$i_a = \frac{8}{12}\ mA = \frac{2}{3}\ mA$

$V_{o.c} = 1000i_a = \frac{2}{3}\ V$

to find i_{sc}



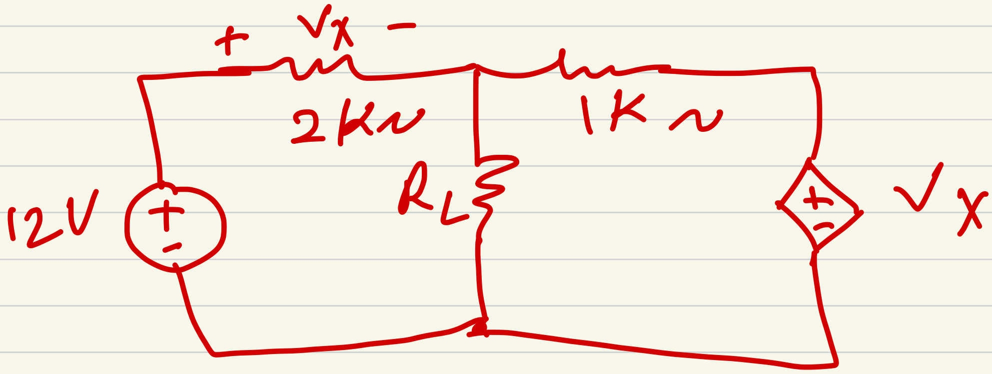
$i_a = 0$

$-8V + 9000i_{sc} = 0$

$i_{sc} = \frac{8}{9}\ mA$

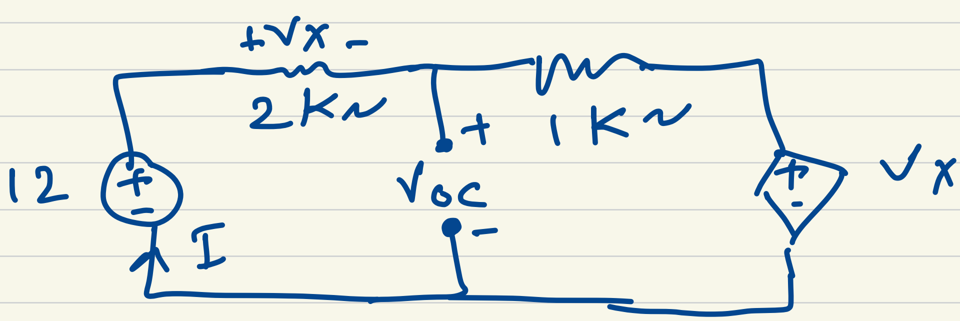
$\frac{2}{3} / \frac{8}{9} = \frac{18}{24}\ k\Omega = 0.75\ k\Omega = 750\ \Omega$

Find P_{max} on R_L



$$P = \frac{V_{TH}^2}{4R_L}$$

to find V_{TH}



$$V_x = I(2k) = 2I\,k$$

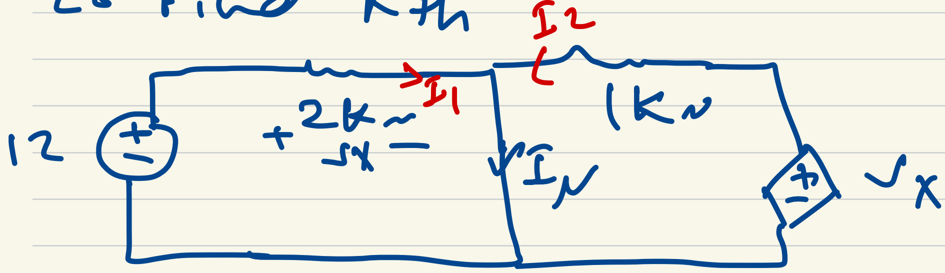
$$I = \frac{12 - V_x}{3k} = \frac{12 - 2I}{3} \rightarrow 3I = 12 - 2I$$

$$I = \frac{12}{5} = 2.4 \text{ mA}$$

$$-12 + 2(2.4) + V_{o.c} = 0$$

$$\boxed{V_{o.c} = 7.2 \text{ Volt}}$$

to find R_{th}



$$R_{th} = \frac{V_{th}}{I_N} = \frac{7.2}{18} = 0.4 \text{ K}\Omega$$

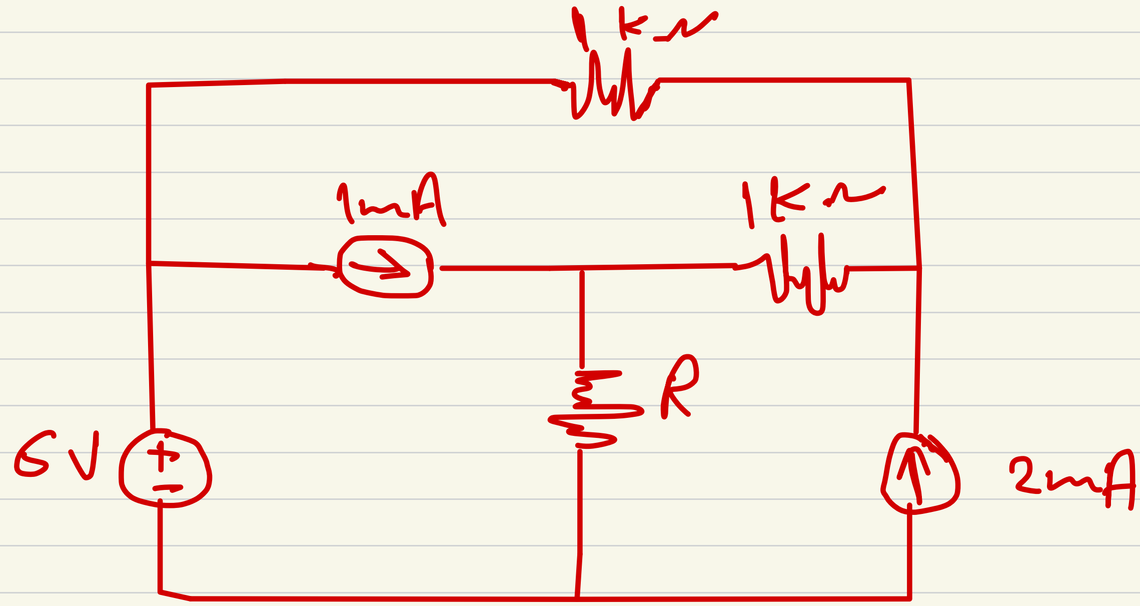
$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{(7.2)^2}{4(0.4)} = 32.4 \text{ mW.}$$

$$I_1 = \frac{12}{2} = 6 \text{ mA}$$

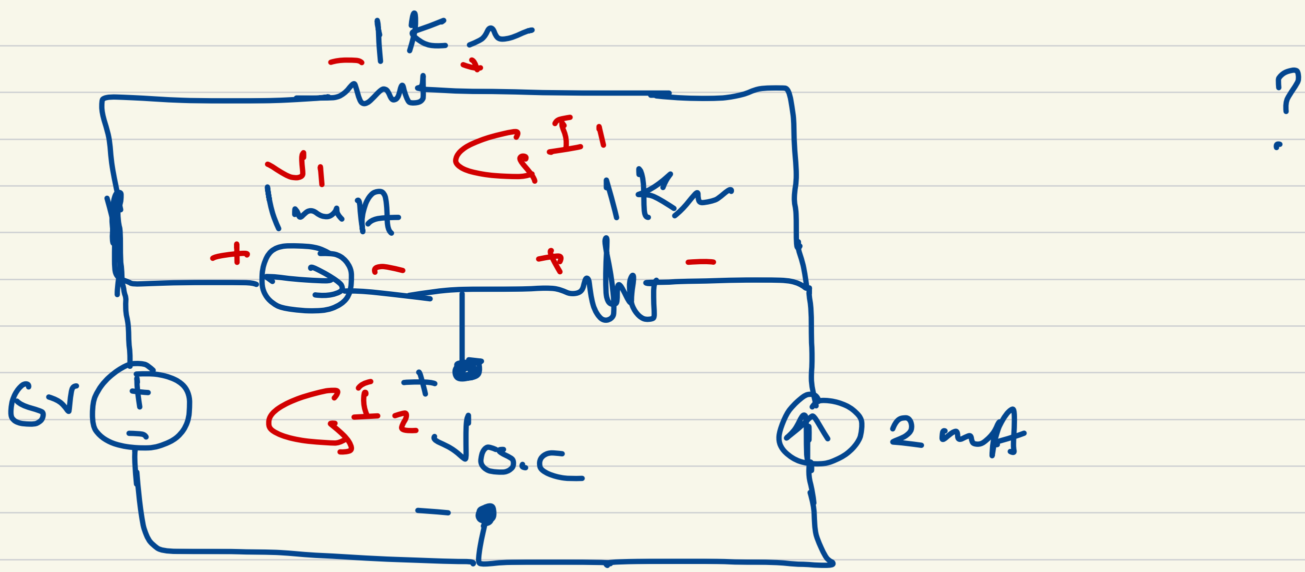
$$I_2 = \frac{V_x}{1} \leftarrow (V_x = 12) = 12 \text{ mA}$$

$$I_N = I_1 + I_2 = 18 \text{ mA}$$

Find maximum Power transfer on R



to Find V_{th}



$I_1 - I_2 = 1mA \rightarrow I_1 = 3\ mA$

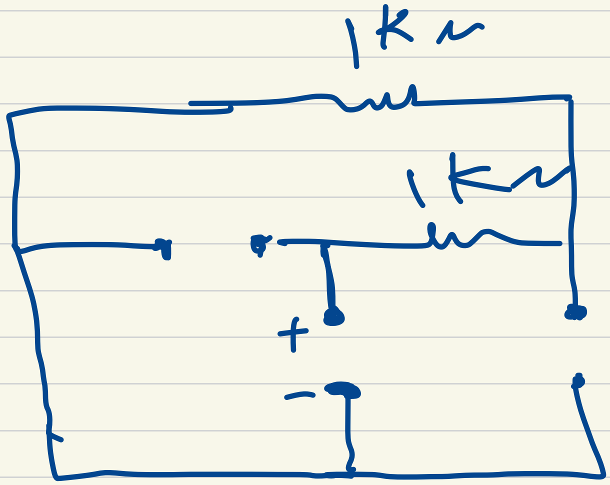
$I_2 = 2mA$

$V_1 + 1 + 3 = 0 \rightarrow V_1 = -4$ Volt

$V_{o.c} - 6 + V_1 = 0$

$V_{o.c} - 6 - 4 = 0 \rightarrow \boxed{V_{o.c} = 10\ Volt}$

to find R_{th}



$R_{th} = 1k\Omega + 1k\Omega = 2k\Omega$

then $P_{max} = \dfrac{V_{th}^2}{4R_{th}}$

$= \dfrac{(10)^2}{(4)(2)} = 12.5mW$