سرعبار

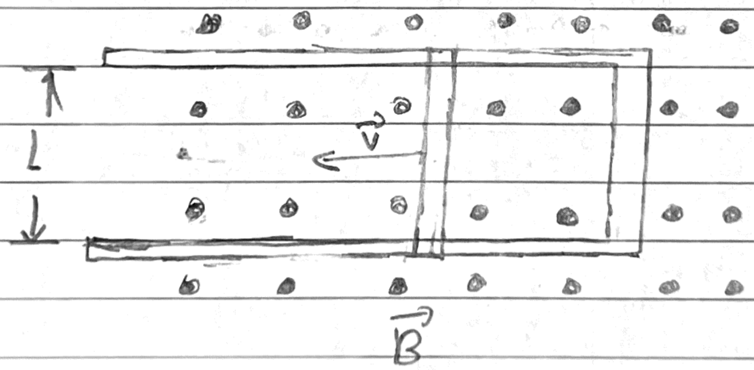
Principles of physics (10th eddition)

phy 132

CH30: Induction and Inductance

Problems: 1, 10, 16, 18, 23, 31, 43, 77

P1: In Fig. 30-21 a metal rod is forced to move with constant velocity $\vec{v}$ along two parallel metal rails, connected with a strip of metal at one end, A magnetic field of magnitude B = 0.125 T points out of the page. (a) if the rails are separated by L = 25.0 cm and the speed of the rod is 38 cm/s, What emf is generated? (b) If the rod has a resistance of 18 Ω and the rails and connector have negligible resistance, What is the current in the rod? (C) At What rate is energy being transferred to thermal energy? (d) What is the magnitude of the leftward force that causes the rod to move?



$B = 0.125T \quad L = 25\,cm = 25 \times 10^{-2}\,m \quad v = 38\,cm/s = 0.38\,m/s$

Sol: (a) $\mathcal{E} = \frac{d\Phi_B}{dt}$ but $\Phi_B = B.A$

$= (B.Lx)$

$= BLx\cos 0$

$= BLx$

$\mathcal{E} = \frac{d}{dt} BLx$

$\mathcal{E} = BL\frac{dx}{dt}$

$\mathcal{E} = BLV$

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$\mathcal{E} = BLv = (0.125)(25 \times 10^{-2})(0.38)$

$= 0.011875 \text{ Volt}$

$= 11.9 \times 10^{-3} \text{ Volt}$

$= 11.9 \text{ mV}$

b) $R = 18\,\Omega$

$i = \frac{V}{R} = \frac{0.011875}{18} = 6.597 \times 10^{-4} \text{ A}$

$= 0.6597 \times 10^{-3} \text{ A}$

$= 0.6597 \text{ mA}$

$= 0.66 \text{ mA}$

c) $P = i^2 R$

$= (0.6597 \times 10^{-3})^2 (18)$

$= 7.83 \times 10^{-6} \text{ watt}$

$= 7.83 \ \mu\text{W}$

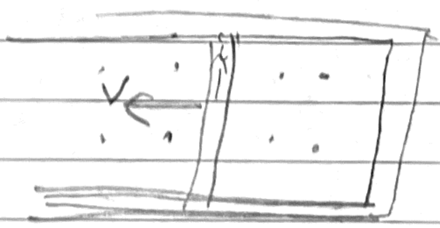
d) $F = iLB$

$= 0.6597 \times 10^{-3} \times 25 \times 10^{-2} \times 0.125$

$= 2.06 \times 10^{-5} \text{ N}$

$= 20.6 \times 10^{-6} \text{ N}$

$= 20.6 \ \mu\text{N}$



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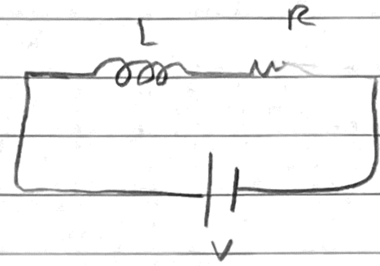
? P10: At $t=0$, a battery is connected to a series arrangment of a resistor and an inductor. At what multiple of the inductive time constant will the energy stored in the inductor's magnetic field be 0.250 of its steady-state value?



$U_B(t) = 0.25\,(U_B)_s$ — ① The energy stored in the inductor at any time t will be 0.25 U_B steady state

* The energy stored in the inductor is given by

$$U_B(t) = \frac{1}{2} L i^2 \quad \text{— ②}$$

* The i value for RL circuit

$$i = i_0 (1 - e^{-t/\tau}) \longrightarrow ③$$

, τ: inductive time constant

$$\tau = \frac{L}{R}$$

sub ③ in ②

$$U_B(t) = \frac{1}{2} L \left(i_0 (1 - e^{-t/\tau}) \right)^2$$

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$$U_B(t) = \frac{1}{2} L i_0^2 (1 - e^{-t/\tau})^2 \quad \text{—— (4)}$$

steady state happens at $t = \infty$

$$(U_B)_s (t \to \infty) = \frac{1}{2} L i_0^2 (1 - e^{-\infty})$$

$$(U_B)_s = \frac{1}{2} L i_0^2 \rightarrow (5)$$

from eq (1)

$$U_B(t) = 0.25 (U_B)_s$$

⇓ from eq (4) ⇓ from eq (5)

$$\frac{1}{2} L i_0^2 (1 - e^{-t/\tau})^2 = 0.25 \times \frac{1}{2} L i_0^2$$

$$(1 - e^{-t/\tau})^2 = 0.25$$

$$1 - e^{-t/\tau} = 0.5$$

$$1 - 0.5 = e^{-t/\tau}$$

$$0.5 = e^{-t/\tau}$$

$$\ln 0.5 = \frac{-t}{\tau}$$

$$-0.693 = \frac{-t}{\tau}$$

$$\Rightarrow \boxed{t = 0.693\,\tau}$$

ساعيا

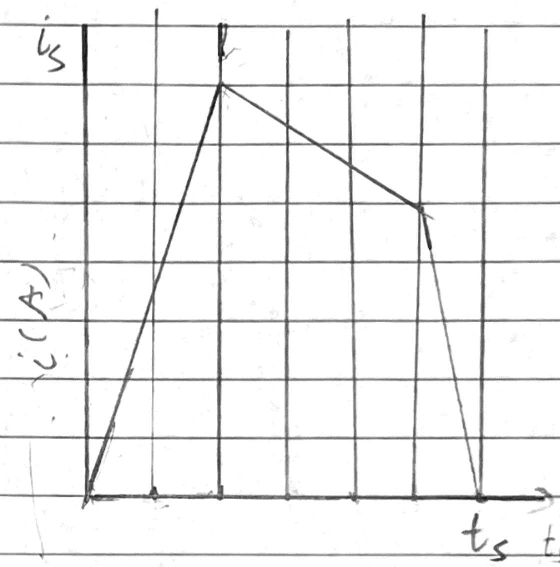
P16: The current i through a 4.6 H inductor varies with time t as shown by the graph of Fig 30-27, where the vertical axis scale is set by $i_s = 16$ A and the horizantal axis scale is set by $t_s = 6.0$ ms. The inductor has a resistance of 12 Ω. Find the magnitude of the induced emf $\mathcal{E}$ during time intervales (a) 0 to 2 ms, (b) 2 ms to 5 ms and (C) 5 ms to 6 ms (Ignore the ~~behavior~~ behavior at the ends of the intervals)

Sol: $L = 4.6$ H

$i_s = 16$ so each scale is 2A

$t_s = 6$ ms so each scale is 1ms

$R = 12\ \Omega$



a) $\mathcal{E}_L = -L \dfrac{di}{dt} \Rightarrow \mathcal{E}_L = -L \dfrac{\Delta i}{\Delta t}$

magnitude $\Rightarrow |\mathcal{E}_L|$

t 0ms → 2ms ⇒ (0, 0) (2, 14) from graph

$\mathcal{E} = \dfrac{(4.6)(14-0)}{2\times10^{-3}} = 32200 = 3.22\times10^{4}$ V

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b) t 2 ms → 5 ms

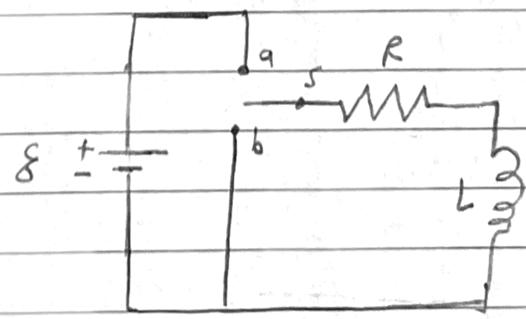
(2, 14) → (5, 10)
msec A

$$\mathcal{E} = \frac{(4.6)(10-14)}{(5-2)\times10^{-3}} = 6133.3 \text{ V}$$

c) 5 ms → 6 ms (5, 10) → (6, 0)

$$\mathcal{E} = \frac{(4.6)(0-10)}{(6-5)\times10^{-3}} = 46000 = 4.6\times10^{4} \text{ Volt}$$

P18: The switch in Fig 30-15 is closed on (a) time $t = 0$. what is the ratio $\mathcal{E}_L/\mathcal{E}$ of the inductor's self induced emf to the battery's emf (a) just after $t=0$ and (b) $t = 3.50\,\tau_L$? (c) At what multiple of τ_L will $\mathcal{E}_L/\mathcal{E} = 0.250$



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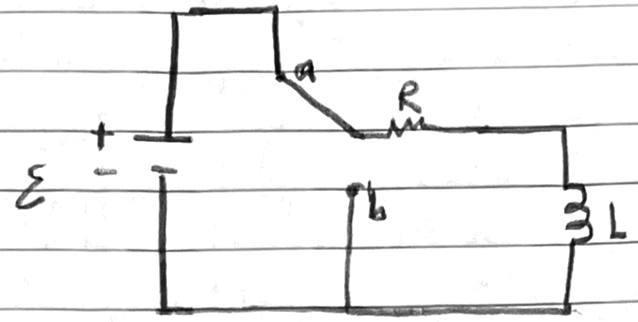
Sol:



Charging

$$-iR - L\frac{di}{dt} + \varepsilon = 0$$

$$L\frac{di}{dt} + Ri = \varepsilon$$

$$\Rightarrow \quad i = \frac{\varepsilon}{R}\left(1 - e^{-tR/L}\right) \quad , I_L = \frac{\varepsilon}{R}$$

$$V_L \text{ or } \varepsilon_L = -L\frac{di}{dt}$$

$$|\varepsilon_L| = \left|-L\frac{di}{dt}\right| = L\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{\varepsilon}{R}\left(0 - -\frac{R}{L}e^{-tR/L}\right)$$

$$= \frac{\varepsilon}{R} \cdot \frac{R}{L} e^{-tR/L}$$

$$\frac{di}{dt} = \frac{\varepsilon}{L} e^{-tR/L}$$

$$\Rightarrow |\varepsilon_L| = L\frac{di}{dt} = L\frac{\varepsilon}{L}e^{-tR/L} \Rightarrow \boxed{|\varepsilon_L| = \varepsilon e^{-tR/L}}$$

ساره بكار

a) at $t=0$ $\quad \mathcal{E}_L = \mathcal{E} e^0$

$$\frac{\mathcal{E}_L}{\mathcal{E}} = e^0$$

$$\frac{\mathcal{E}_L}{\mathcal{E}} = 1$$

b) at $t = 3.50\tau_L$

$$\mathcal{E}_L = \mathcal{E} e^{-tR/L} \quad , \quad \tau_L = \frac{L}{R}$$

$$\mathcal{E}_L = \mathcal{E} e^{-t/\tau_L}$$

$$\mathcal{E}_L(t = 3.5\tau_L) = \mathcal{E} e^{3.5\tau_L/\tau_L}$$

$$\frac{\mathcal{E}_L}{\mathcal{E}} = e^{-3.5}$$

$$\frac{\mathcal{E}_L}{\mathcal{E}} = 0.03$$

c) $\frac{\mathcal{E}_L}{\mathcal{E}} = 0.250$ $\quad t = ?$

$$\Rightarrow \mathcal{E}_L = \mathcal{E} e^{-t/\tau_L}$$

$$\frac{\mathcal{E}_L}{\mathcal{E}} = e^{-t/\tau_L} = 0.25$$

$$e^{-t/\tau_L} = 0.25 \Rightarrow \frac{-t}{\tau_L} = \ln 0.25$$

$$t/\tau_L = 1.386 \Rightarrow t \simeq 1.39\tau_L$$

سماوي

P23: The current in an RL circuit drops from 2.30A to 3.40 mA in 0.025 sec following Removal of the battery from the circuit. If L is 10 H, find Resistance R in the circuit.

sol:

$$i = i_0 \, e^{-t/\tau_L}$$

$$\frac{i}{i_0} = e^{-t/\tau_L}$$

$$\ln\left(\frac{i}{i_0}\right) = \frac{-t}{\tau_L}$$

$$\tau_L = \frac{-t}{\ln\left(\frac{i}{i_0}\right)}$$

$$= - \frac{0.025}{\ln\left(3.4 \times 10^{-3} / 2.3\right)}$$

$$= 3.836 \times 10^{-3} \text{ sec}$$

$$\tau_L = \frac{L}{R} \Rightarrow R = \frac{L}{\tau_L} = \frac{10}{3.836 \times 10^{-3}}$$

$$= 2606.7 \, \Omega$$

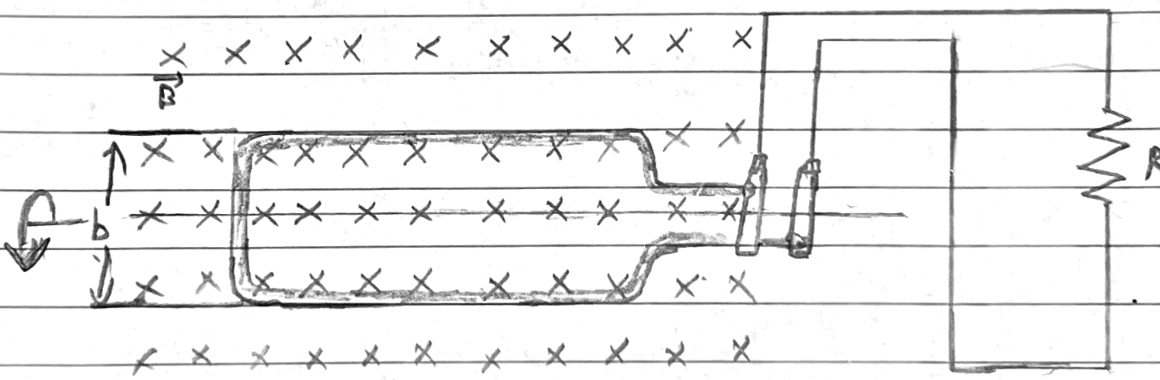
$$\simeq 2.6 \, k\Omega$$

سا مزار

P31: A rectangular coil of N turns and of length a and width b is rotated at frequency f in a uniform magnetic field $\vec{B}$, as indicated in Fig 30-34. The coil is connected to co-rotating cylinders, against which metal brushes slide to make contact (a) Show that the emf induced in the coil is given (as a function of time t) by

$$\mathcal{E} = 2\pi f NabB \sin(2\pi f t) = \mathcal{E}_0 \sin(2\pi f t)$$

This is the principle of the commercial alternating-current generator (b) What value of Nab gives an emf with $\mathcal{E}_0 = 220\ V$ when the loop is rotated at 60.0 rev/s in a uniform magnetic field of 0.400 T?



Sol:

a) $\Phi_B = B \cdot A$

$\Phi_B = BA\cos\theta$

* since the coil is rotating steadily, θ increases linearly with time. Thus, $\theta = \omega t = 2\pi f t$

* the area of the coil $A = ab$

سارة بكار

From Faraday's law

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -N \frac{d(BA\cos\theta)}{dt}$$

$$\mathcal{E} = -NBA \frac{d\cos(2\pi ft)}{dt}$$

$$\mathcal{E} = -NBA\,(-\sin(2\pi ft))\,(2\pi f)$$

$$\mathcal{E} = +NBab\,(2\pi f)\,\sin(2\pi ft)$$

$$\mathcal{E} = 2\pi f\,NabB\,\sin(2\pi ft)$$

$\mathcal{E}_0$ is the amplitude $= 2\pi f\,NabB$

$$\mathcal{E} = \mathcal{E}_0 \sin(2\pi ft)$$

b) $\mathcal{E}_0 = 220\,V$, $f = 60.0$ rev/s , $B = 0.500\,T$

$$\mathcal{E}_0 = 2\pi f\,NabB$$

$$Nab = \frac{\mathcal{E}_0}{2\pi fB}$$

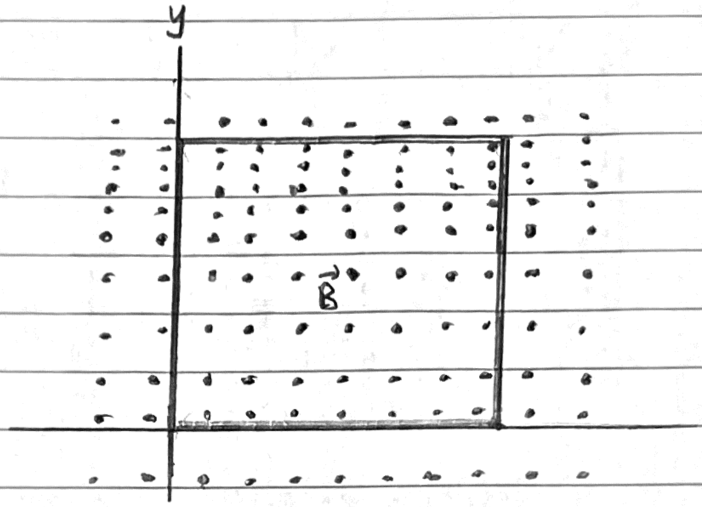
$$= \frac{220}{2(3.14)(60)(0.4)}$$

$= 1.459\ m^2$

$\simeq 1.46\ m^2$

سارد، بدا

P43: As seen in Fig 30-39, a square loop of wire has sides of length 3.0 cm. A magnetic field is directed out of the page; its magnitude is given by $B = 5.0 t^2 y$, where B is in teslas, t is in seconds, and y is in meters. At $t = 2.5$ sec, what are the a) magnitude b) direction of emf induced in the loop?



sol: $L = 3 \text{ cm} = 0.03 \text{ m}$

$B = 5t^2 y$

$\Phi_B = B \cdot A$

$d\Phi_B = B\,dA$ but $A = Ly$

$d\Phi_B = 5t^2 y\,dA$

$d\Phi_B = 5t^2 y L\,dy$

$\Rightarrow \Phi_B = \int 5t^2 y L\,dy$

$= 5Lt^2 \int_0^L y\,dy$

$= 5Lt^2 \left.\frac{y^2}{2}\right]_0^L$

سارەبار

$$\Phi_B = 5Lt^2\left(\frac{L^2}{2} - 0\right)$$

$$\Phi_B = \frac{5}{2}L^3t^2$$

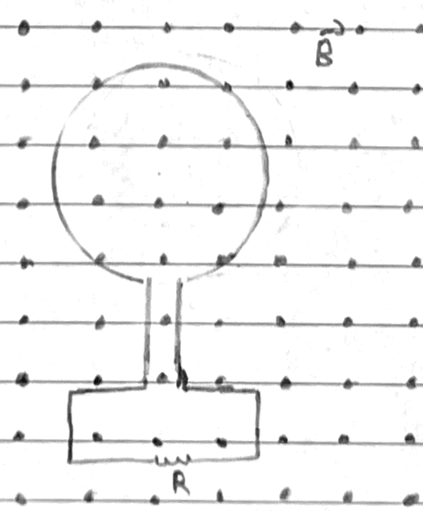
$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \frac{5}{2}\,2t\,L^3$$

$$|\mathcal{E}| = 5L^3t$$

$$\mathcal{E}(t=2.5) = 5\,(0.03)^3\,(2.5)$$
$$= 3.375\times10^{-4}\ \text{volt}$$
$$= 0.3375\times10^{-3}\ \text{volt}$$
$$\simeq 0.34\ \text{mvolt}$$

b) according to Lenz's law its direction is clockwise

P77: In Fig 30.58, the magnetic flux through the loop increases according to the relation $\Phi_B = 3.0t^2 + 7.0t$ where Φ_B is in milliwebers and t is in seconds (a) What is the magnitude of the emf induced in the loop when $t = 1.5$ sec? (b) Is the direction of the current through R to the right or left?



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a) $|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right|$

$= \frac{d}{dt}(3.0t^2 + 7.0t)$

$= 6t + 7$

$\varepsilon(t = 1.5) = 6(1.5) + 7$

$= 16$ mVolt

b) To left

حسب قاعدة اليد اليمنى المجال المغناطيسي يولد تيار عكس عقارب الساعة
لكن بما أن $\vec{B}$ تزداد فإنه حسب lnz سيتولد تيار يولد مجال
مغناطيسي عكس الاتجاه الأصلي بالتالي التيار لليسار

* Increasing the external field $\vec{B}$ induces a current with a field $\vec{B}_{ind}$ that opposes the change