

Find the explicit solution of initial-value problem

$$y'' - 3y^2 = 0, \quad y(0) = 2, \quad y'(0) = 4.$$

Find the interval where the solution is defined.

$$\textcircled{1} \text{ let } v = y', \quad y'' = v' = \frac{dv}{dt} = \frac{dv}{dy} \frac{dy}{dt} = v \frac{dv}{dy}.$$

$$\textcircled{1} v \frac{dv}{dy} - 3y^2 = 0$$

$$\textcircled{1} \int v dv = \int 3y^2 dy$$

$$\textcircled{1} \frac{v^2}{2} = y^3 + C_1$$

$$\frac{v(0)^2}{2} = y(0)^3 + C_1 \Rightarrow \frac{16}{2} = 8 + C_1 \Rightarrow \boxed{C_1 = 0}$$

$$\textcircled{1} v = \frac{dy}{dt} = \sqrt{2} y^{3/2}$$

$$\textcircled{1} \int y^{-3/2} dy = \sqrt{2} \int dt$$

$$\textcircled{1} -2y^{-1/2} = \sqrt{2}t + C_2$$

$$-2y(0)^{-1/2} = C_2 \Rightarrow -2(2)^{-1/2} = C_2 \Rightarrow \boxed{C_2 = -\sqrt{2}}$$

$$\textcircled{1} y = 2(1-t)^{-2}, \quad t < 1.$$

Good Luck