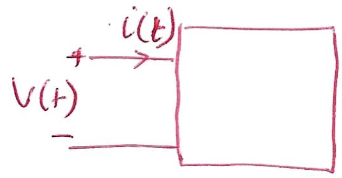
Chapter 10:- Sinusoidal steady state power calculations

10.1 Instantaneous power :-

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$



$$p(t) = v(t)\, i(t) \qquad i(t) = I_m \cos(\omega t)$$

$$v(t) = V_m \cos(\omega t + \theta_v - \theta_i)$$

$$= V_m I \cos(\omega t + \theta_v - \theta_i) \cos(\omega t)$$

using the identities as in the slides

$$p(t) = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \cos(2\omega t) - \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \sin(2\omega t)$$

$$p(t) = P + P \cos(2\omega t) + Q \sin(2\omega t)$$

10.2 Average and Reactive power

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$ average power (Real power)

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i)$$ reactive power

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$X_L = \omega L$$

$$X_C = \frac{-1}{\omega C}$$

for Resistor

$\theta_v - \theta_i = 0$ so $P = \frac{V_m I_m}{2} = V_{rms} I_{rms} = I_{rms}^2 R$ W

$$= \frac{I_m^2}{2} R$$

$Q = 0$ VAR

for Inductor

$\theta_v - \theta_i = 90^\circ$ so $P = 0$ W

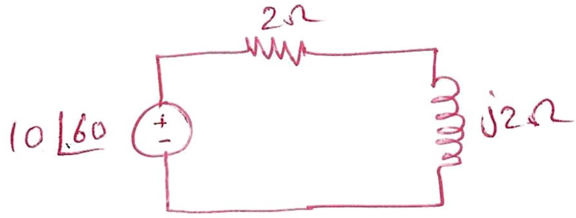
$Q = \frac{V_m I_m}{2} = V_{rms} I_{rms} = I_{rms}^2 X_L$ VAR

for Capacitor

$\theta_v - \theta_i = -90$ so $P = 0$ W

$Q = -\frac{V_m I_m}{2} = -V_{rms} I_{rms} = I_{rms}^2 X_C$ VAR

Example:- Find the average power absorbed by each element



$$I = \frac{10\angle 60}{2+j2} = 3.53\angle 15^\circ \text{ A}$$

$$P_{av} = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$$

$$P_{av_{j2\Omega}} = 0$$

$$P_{av_{2\Omega}} = \frac{I_m^2 R}{2} = I_{rms}^2 R$$

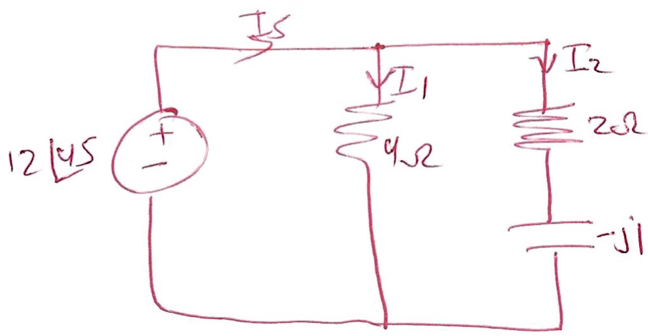
$$= \frac{(3.53)^2 \cdot 2}{2} = 12.5 \text{ W}$$

$$P_{av_{V_s}} = -\frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$$

$$= -\frac{(10)(3.53)}{2}\cos(60-15)$$

$$= -12.5 \text{ watt} \quad \text{supplied}$$

Example:- Determine the average power absorbed by each Resistor



$$I_1 = \frac{12\angle 45}{4\Omega} = 3\angle 45 \text{ A}$$

$$I_2 = \frac{12\angle 45}{2-j1} = 5.36\angle 71.57 \text{ A}$$

$$I_s = I_1 + I_2 = 8.15\angle 62.1^\circ \text{ A}$$

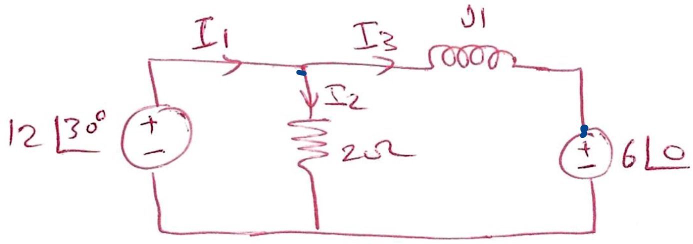
$$P_{4\Omega} = \frac{I_{m_1}^2 \cdot 4}{2} = 18 \text{ W}$$

$$P_{2\Omega} = \frac{I_{m2}^2 \cdot 2}{2} = 28.7 \text{ W}$$

supplied

$$P_{av_{V_s}} = -\frac{(12)(8.16)}{2}\cos(45-62.1) = -46.7 \text{ W} = -\left(\overset{18}{P_{av_{4\Omega}}} + \overset{28.7}{P_{av_{2\Omega}}} + \overset{0}{P_{av_{-j1}}}\right)$$

Example:- Determine average power supplied or absorbed by each element



$$I_2 = \frac{12\angle 30}{2} = 6\angle 30 \ A$$

$$I_3 = \frac{12\angle 30° - 6\angle 0}{j1} = 7.43\angle -36.19° \ A$$

$$I_1 = I_2 + I_3 = 11.29\angle -7.07° \ A$$

$$P_{2\Omega} = \frac{I_{2m}^2 \cdot 2}{2} = 36 \ W$$

$$P_{12\angle 30°} = -\frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

$$= -\frac{(12)(11.29)}{2} \cos(30 - -7.07)$$

$$= -54 \ W \text{ supply}$$

$$P_{6\angle 0} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

$$= \frac{(6)(7.43)}{2} \cos(0 - -36.19)$$

$$= 18 \ W \text{ absorbed}$$

* $V_{rms} = \frac{V_m}{\sqrt{2}}$

$P_{av} = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$ W

$P_{apparent} = V_{rms} I_{rms}$ VA

PF = Power factor = $\cos(\theta_v - \theta_i) \Rightarrow P_{av} = P_a \cdot pf$

For Resistor

$\theta_v - \theta_i = 0$

$\therefore pf = 1$

For Inductor

$\theta_v - \theta_i = 90$ $\qquad \theta_i < \theta_v$

$\therefore pf = 0$

For Capacitor

$\theta_v - \theta_i = -90$ $\qquad \theta_i > \theta_v$

$\therefore pf = 0$

For Inductive Load

$90^\circ > \theta_v - \theta_i > 0$

$1 > pf > 0$ lagging Power factor

Current lags Voltage

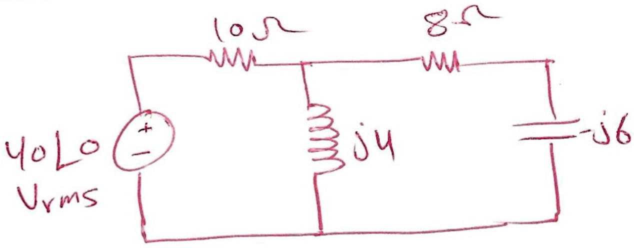
For Capacitive Load

$-90 < \theta_v - \theta_i < 0$

$1 > pf > 0$ Leading power factor

Current leads Voltage

Example :- Calculate the power factor seen by the source and the average power supplied by the source



$Z = 10 + j4 // (8 - j6)$

$= 12.69 \angle 20.62 \ \Omega$

$I_s = \dfrac{40\angle 0}{12.69\angle 20.62} = 3.152 \angle -20.62 \ A_{rms}$

$\theta_v = 0^\circ$

$\theta_i = -20.62^\circ$

$$pf = \cos(\theta_v - \theta_i)$$

$$= \cos(20.62)$$

$$= 0.936 \text{ Lagging} \quad (\text{Inductive load})$$

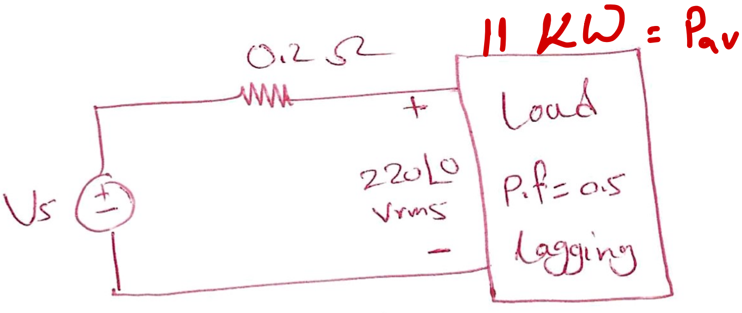
The average power supplied by the source equals to the average power absorbed by the circuit

$$P_{av_{source}} = -(40)(3.152)\cos(0-(-20.62))$$

$$= -118 \text{ W supplied.}$$

Example: An Industrial load consumes 11 KW at 0.5 pf lagging from a 220 Vrms line. The transmission line resistance from the power company to the plant is 0.2 Ω

1) Determin the power average that must be supplied by the power company



$$P_{av_{Load}} = V_{rms} I_{rms} \, P.f$$

$$\therefore I_{rms} = \frac{P_{av_{Load}}}{V_{rms} \cdot Pf}$$

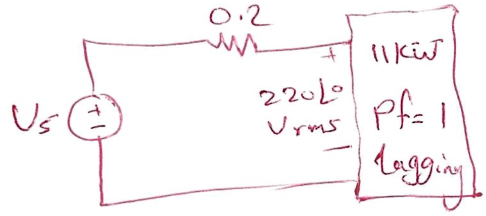
$$= \frac{11 KW}{(220)(0.5)} = 100 \text{ Arms}$$

$$P_{Loss} = (I_{rms}^2)(0.2) = 2 KW$$

$$\therefore P_{sup} = P_{av_{Load}} + P_{av_{Loss}}$$

$$= 11 + 2 = 13 KW$$

2) Repeat ① If the power factor is changed to unity



$P_{av\,load} = V_{rms} I_{rms} \cdot pf$

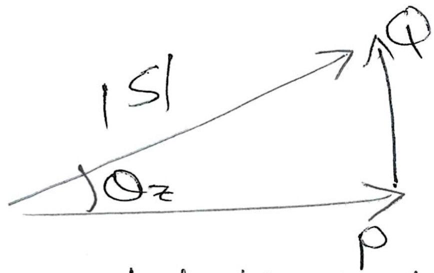
$\therefore I_{rms} = \dfrac{P_{av}}{V_{rms}\ pf} = 50\ A_{rms}$

$P_{av.\,loss} = I_{rms}^2 R = (50)^2 (0.2) = 0.5\ KW$

$P_{av\,sup.} = 0.5 + 11 = 11.5\ KW$

10.4 Complex Power

The complex power is the complex sum of the real power and the ractive power



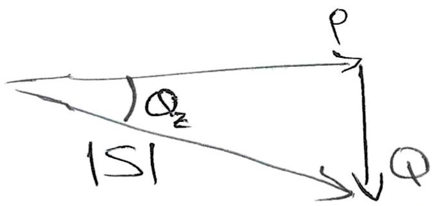
inductive load

$S = P + jQ$

S: complex power

P: average power

Q: realtive power



Capacitive Load

$\theta = \tan^{-1}\dfrac{Q}{P} = \cos^{-1}(p.f)$

$= \theta_z = \theta_v - \theta_i$

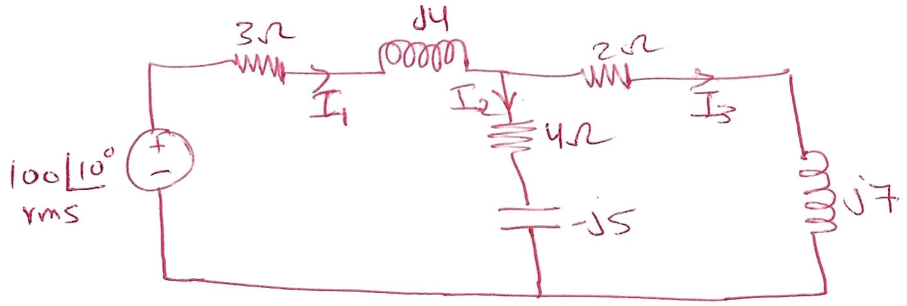
$Q = p\tan\theta \Rightarrow P = \dfrac{Q}{\tan\theta}$

$|S|$: apperant power $= \sqrt{P^2 + Q^2}$

$= V_{rms} I_{rms} \angle \theta_v - \theta_i$

$= V_{rms} I_{rms}^*$ wher I_{rms}^* is the conjugate of the rms phasor

Example :- what are the VARs consumed by the Circuit?



$$Q = V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)$$

$$I_1 = \frac{V_s}{Z}$$

$$Z = 3 + j4 + (4 - j5) \,//\, (2 + j7)$$

$$= 10.35 + j\,4.55 = 11.3\,\angle 23.7^\circ$$

$$\therefore I_1 = \frac{100\angle 10^\circ}{11.3\angle 23.7^\circ} = 8.84\,\angle -13.7^\circ \; A_{rms}$$

$$\therefore Q = (100)(8.84)\sin(10 - (-13.7))$$

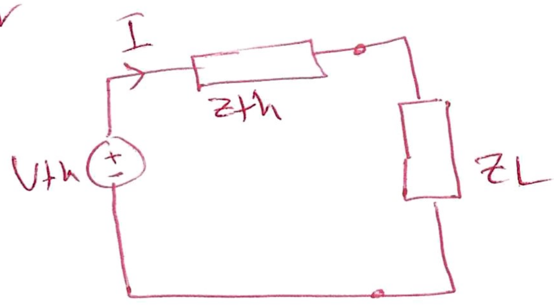
$$= 355 \text{ VAR}$$

$$P_{av_T} = P_{av_1} + P_{av_2} + \cdots + P_{av_n}$$

$$Q_T = Q_1 + Q_2 + \cdots + Q_n$$

$$S_T = S_1 + S_2 + \cdots + S_n$$

10.6 Maximum Power Transfer



For Maximum power transfer, ZL must equal to conjugate of the thevenin impedance

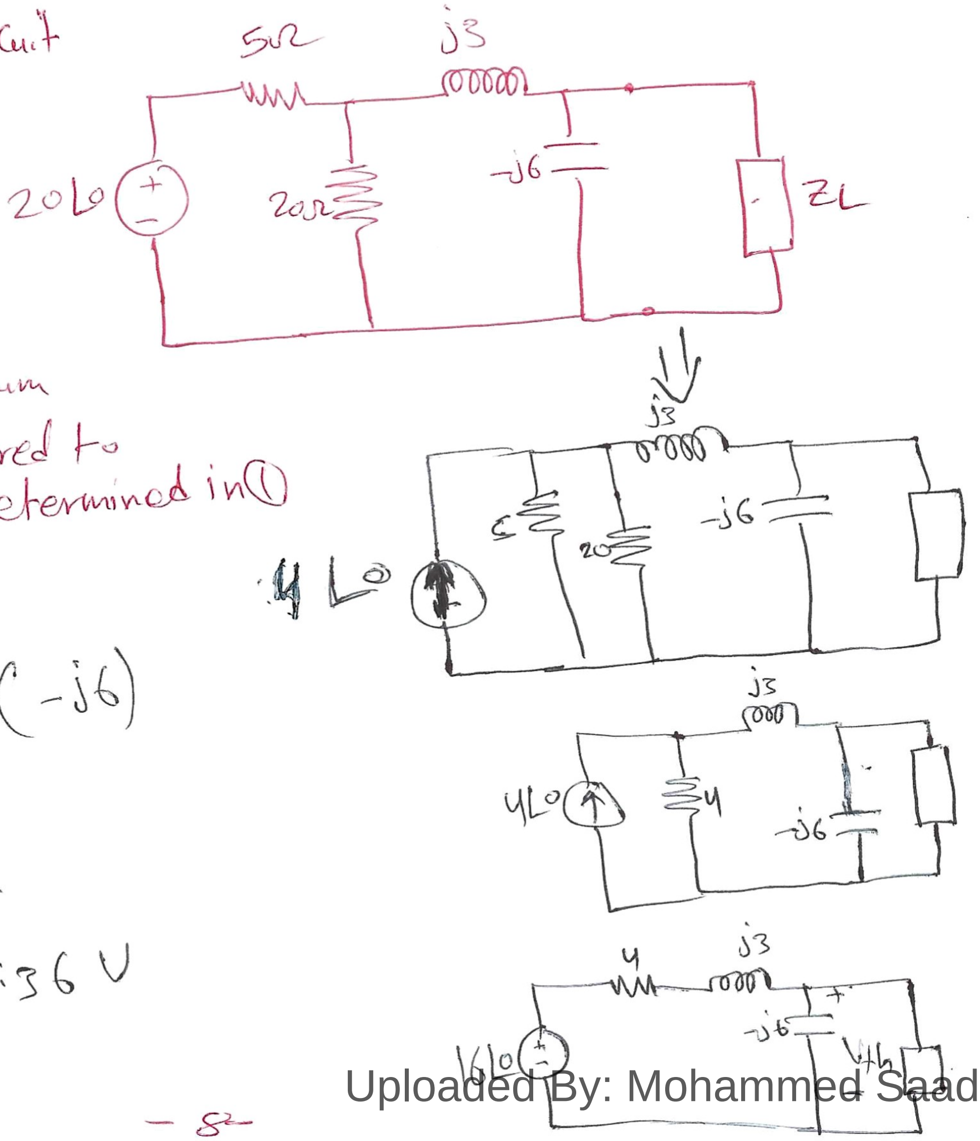
$$Z_L = Z_{th}^*$$

for maximum power

$$P_{max} = \frac{1}{4}\frac{V_{th\,rms}^2}{R_L} \qquad V_{rms}$$

$$= \frac{1}{8}\frac{V_{thm}^2}{R_L} \qquad V_m$$

Example : for the circuit shown, determine the impedance ZL that results in maximum power transfer to ZL



② what is the maximum average power transferred to the load impedance determined in ①

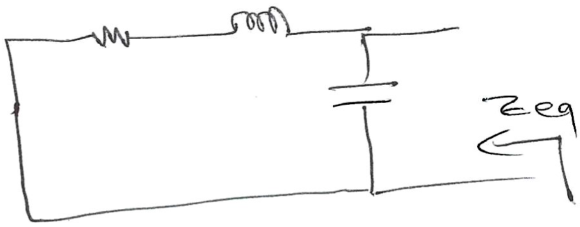
$$V_{th} = \frac{16\angle 0}{4+j3-j6}(-j6)$$

$$= 19.2\angle 53.13$$

$$= 11.52 - j15.36 \text{ V}$$

$$Z_{eq} = \frac{(-j6)(4+j3)}{4+3j-j6}$$

$$= 5.76 - j1.68\ \Omega$$



$\therefore$ for maximum power transfer $Z_L = 5.76 + j1.68\ \Omega$

$$P_{av\,maximum} = \frac{V_{th}^2}{8R_L}$$

$$= \frac{(19.2)^2}{8(5.76)}$$

$$= 8\ W$$

$$P_{max} = \frac{V_{th}^2}{4R_L} \quad V_{rms}$$

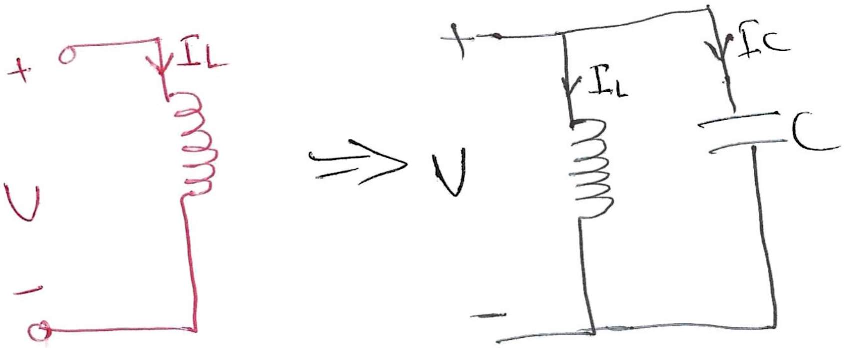
$$P_{max} = \frac{V_{th}^2}{8R_L} \quad V_m$$

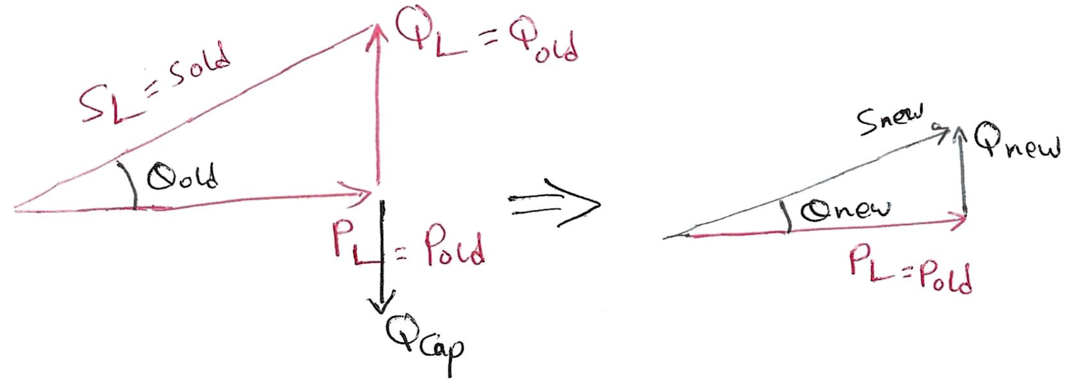
Power factor Correction

Power factor Correction is the process of increasing the power factor without altering the Voltage or the Current to the orginal Load. It is necessary for economics reasons.

To improve the power factor we must decrease the reactive power

For inductive Circuit, we add a Capacitor in parallel to the Load





$$S_{old} = P_{old} + Q_{old} \qquad S_{new} = P_{old} + Q_{new}$$

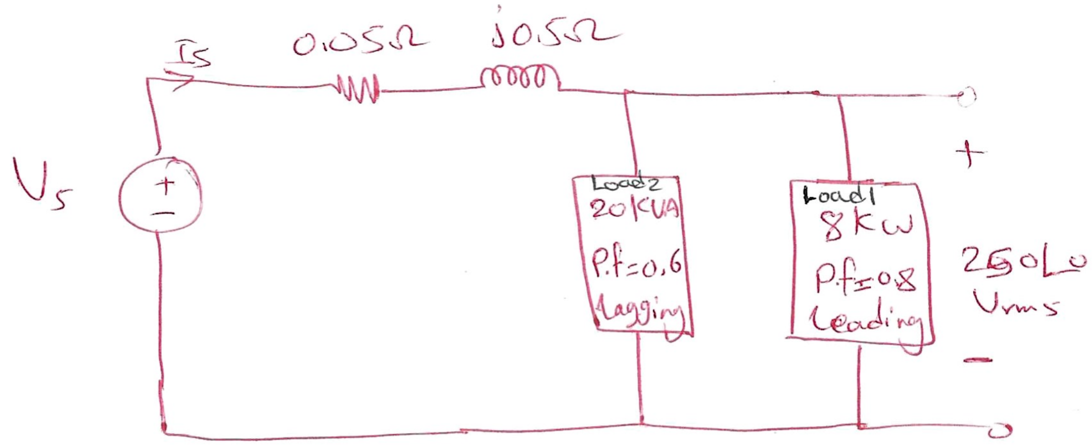
$$Q_{new} = Q_{old} + Q_{cap}$$

$$Q_{cap} = Q_{new} - Q_{old}$$

$$= -\omega C V_{rms}^2$$

$$\therefore C = -\frac{Q_{cap}}{\omega V_{rms}^2}$$

Example 8



1) Determine the Power factor of the two Loads in parallel

$P_{av} = 8\,KW$ given

$$\frac{Q}{P} = \tan\theta \qquad \theta = \cos^{-1} p.f = \cos^{-1}(0.8) \quad \text{Leading}$$

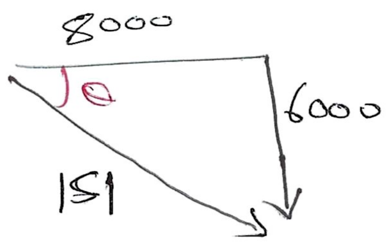
$$= 37^\circ$$

Capacitor

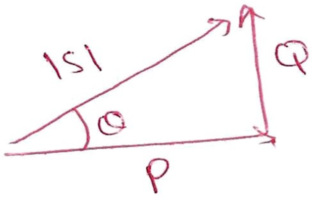
$$Q = -P \tan(\cos^{-1}(0.8)) = -6000\ VAR = -6000\ VAR$$

$\therefore S_1 = P + jQ$

$= 8000 - j6000$



For Load 2



$|S| = 20000\,VA$ given

$\theta = \cos^{-1}(pf) = \cos^{-1}(0.6)$

$P_{av_2} = |S| \cos\theta$

$= |S| . pf$

$= (20000)(0.6) = 12000\,W$

$Q = |S| \sin\theta$

$= P_{av_2} \tan(\cos^{-1}(pf))$

$= 12000 \tan(\cos^{-1}(0.6)) = 16000\,VAR$

$S_2 = 12000 + j16000\ VA$

$S_T = S_1 + S_2$

$= 20000 + j10000\ VA$

$= 22360 \angle 26.565^\circ$

$\therefore pf_{\text{two loads}} = \cos(26.565) = 0.8944$ lagging

2) Determine the apparent power required to supply the loads, the magnitude of the current I_s, the average power loss in the transmission line.

$S_T = S_{\text{two loads}} = 20000 + j\,10000$

$= 22360 \angle 26.565^\circ$

$S_T = V_{rms} I_{rms}^*$

$\therefore I_{rms}^* = \frac{S_T}{V_{rms}} = \frac{22360 \angle 26.565}{250 \angle 0} = 89.44 \angle 26.565$

$\therefore I_{rms} = 89.44 \angle -26.565$

$P_{apparent} = P_a = |S_T| = 22360 \text{ VA}$

$P_{\text{ave loss}} = I_s^2 (0.05)$

$= 400 \text{ W}$

3) Comput the Value of the Capacitor that would Correct the Power factor to 1 placed in Parallel with two Loads, $\omega = 377$ rad/sec

$\theta_{new} = \cos^{-1}(1) = 0^\circ$

$Q_{new} = 0$

$Q_{old} = 10000 \text{ VAR}$

$\therefore Q_{cap} = Q_{new} - Q_{old} = 0 - 10000 = -10000$

$C = \frac{-Q_{cap}}{\omega V_{rms}^2} = \frac{10000}{(377)(250)^2} = 424.4\ \mu F$