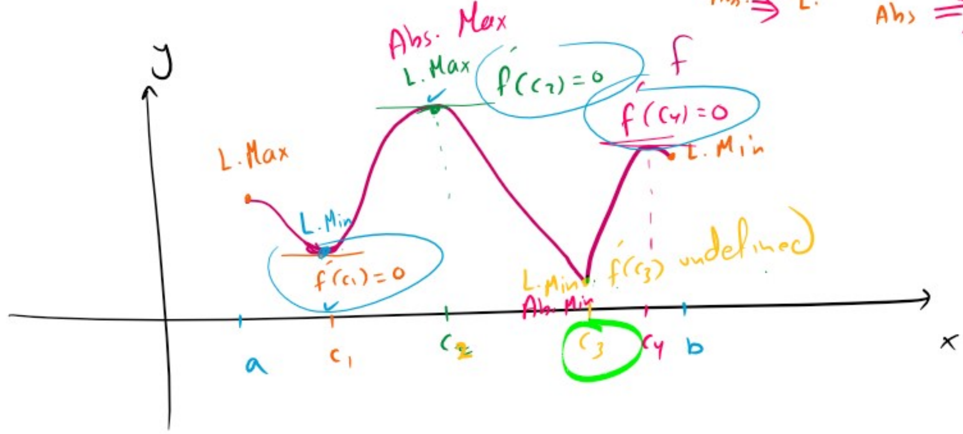
EV's القيم القصوى
نسبية

CP's → Critical Points ⇓ نقاط داخلية

$f' = 0$ or f' undefined

EP → End points

EV.
- Max: Abs. ⇒ L.
- Min: Abs ⇒ L.



f has L. Max of $f(a)$ at $x = a$
f " L. Min of $f(c_1)$ at $x = c_1$
f " L. Max (Abs) of $f(c_2)$ at $x = c_2$
f " L. Min (Abs) of $f(c_3)$ at $x = c_3$
f " L. Min of $f(b)$ at $x = b$
f " L. Max of $f(c_4)$ at $x = c_4$

Th If f diff at c and f has EV at c then $f'(c) = 0$

Exp $f(x) = x^2$ on $[-2, 3]$
Find EV's

Find EV's

check CP's and check Endpoint

[illegible]

$f' = 2x = 0 \Rightarrow \boxed{x=0} \in [-2,3]$

$\Rightarrow x=0$ is the only CP

$\Rightarrow f(0) = 0^2 = 0$

$(0,0) \rightarrow$ Abs. Min $\Rightarrow$ L. Min

Endpoints $(-2, f(-2)) = (-2, (-2)^2) = (-2, 4)$

$(3, f(3)) = (3, 3^2) = (3, 9) \rightarrow$ Abs. Max

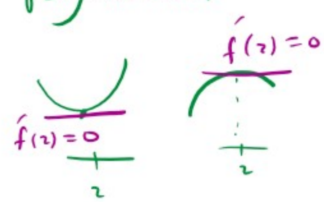
$\Rightarrow$ L. Max

Exp Suppose $f(x) = \alpha x^3 - 12x$ has EV at $x=2$

Find α

f cont. $\Rightarrow$ f diff since f is polynomial

$\Rightarrow f'(2) = 0$



$f'(x) = 3\alpha x^2 - 12$

$f'(2) = 3\alpha(2)^2 - 12 = 0$

$3\alpha(4) - 12 = 0$

$12\alpha - 12 = 0$

$12\alpha = 12$

$\boxed{\alpha = 1}$

$\cup \quad \cap$

Th says if f diff and has EV at $c \Rightarrow f'(c) = 0$

The converse of this Th is not true

The conferse of this Th is not true

if $f'(c) = 0$, then f may not have EV at c

Exp $f(x) = x^3$

$f'(x) = 3x^2 = 0$

$3x^2 = 0$

$\boxed{x = 0}$

$f'(0) = 0 \not\Rightarrow f$ has EV

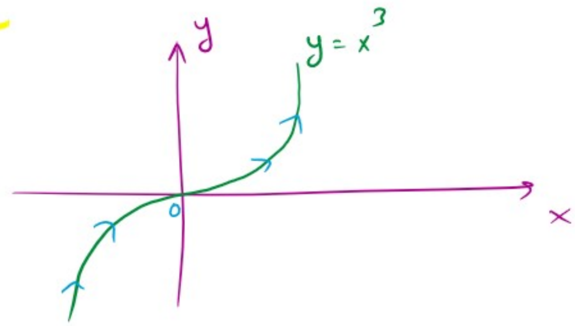
$f(0)$ is not EV

$\cap \ \cup \ \wedge \ \vee$

f

f': $+ + + \overset{0}{|} + + +$ (at 0)

$(0,0)$ is CP



Q. How to classify the EV's

A: We can use FDT, SDT

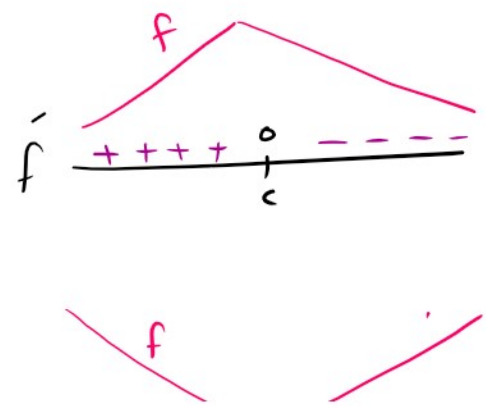
EV's: Max → Abs. → L. ; Min → Abs. → L.

Th FDT: (First Derivative Test)

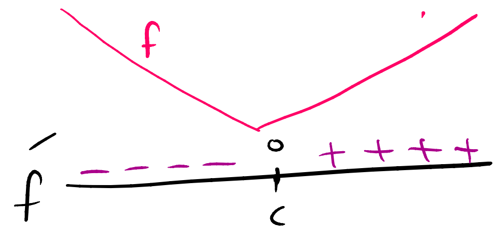
Assume f has CP at c

f' exists about c

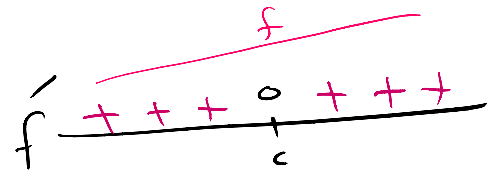
[1] If f' changes sign from $+$ to $-$ at $x = c$ then $f(c)$ is L. Max

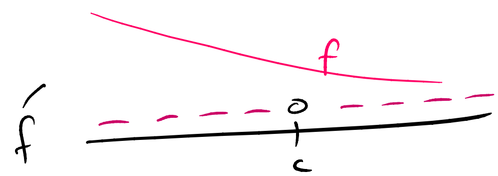


then $f(c)$ is L.



② If f' changes sign from $-$ to $+$ at $x=c$
then $f(c)$ is L. Min





③ If f' does not change sign at $x=c$
then f has no EV's

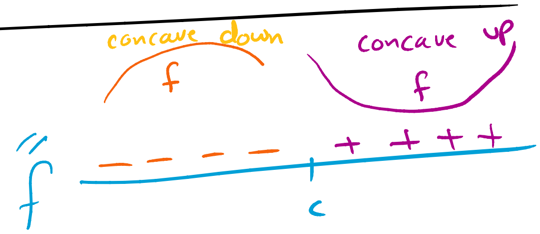
Th (SDT : Second Derivative Test)
Assume $f'(c)=0$ and f'' is cont. about c

① If $f''(c) < 0$ then $f(c)$ is L. Max

② If $f''(c) > 0$ then $f(c)$ is L. Min

③ If $f''(c) = 0$ then Test fail

Remark ① If $f''(x) \geq 0$ $\forall x \in I$



then f concave up on I

② If $f''(x) \leq 0$

② If $f''(x) \le 0$
$\forall x \in I$
then f concave down

Def A point where f has tangent and f changes concavity is called inflection point

To find inflection point $\Rightarrow$ we start where $f''(x) = 0$ and check

Exp $f(x) = x^4 - 4x^3 + 10$ Find

① $D(f) = \mathbb{R} = (-\infty, \infty)$

② CP's $\Rightarrow f'(x) = 4x^3 - 12x^2$

$f'(x) = 0 \Rightarrow 4x^2[x - 3] = 0$

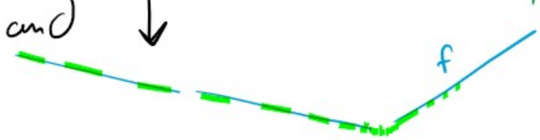
$\boxed{x = 0}$ or $\boxed{x = 3}$

$\in D(f)$ $\Downarrow$ $f(0) = 10$

$\in D(f)$ $\Downarrow$ $f(3) = 3^4 - 4(3)^3 + 10$
$= 81 - 4(27) + 10$
$= -17$

(0, 10)
(3, -17)
$\rightarrow$ Critical Point

③ intervals where f $\uparrow$ and $\downarrow$

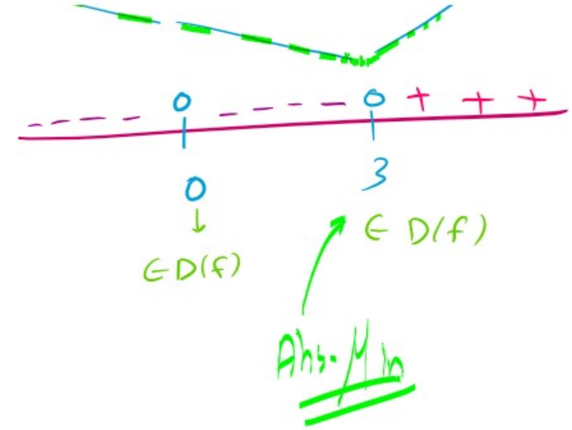


$f'(x) = 4x^3 - 12x^2$

$f'(x) = 4x^3 - 12x^2$

$= 4x^2 [x - 3]$

[illegible]



$f \uparrow$ on $[3, \infty)$

$f \downarrow$ on $(-\infty, 3]$

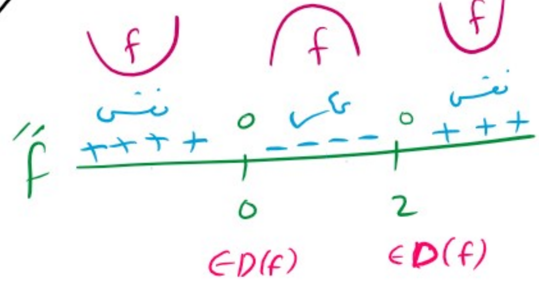
④ intervals of canavity

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

$f''(x) = 0 \Rightarrow 12x(x-2) = 0$

$\boxed{x = 0}, \boxed{x = 2}$



f is concave up on $(-\infty, 0] \cup [2, \infty)$

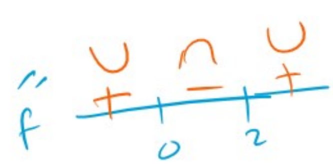
f is concave down on $[0, 2]$

⑤ inflection points

$f'' = 0 \Rightarrow x = 0 \in D(f)$

$x = 2 \in D(f)$

change concavity



$f'(x) = 4x^3 - 12x^2$

$f'(0)$

$f'(2)$

$= (0, 10)$

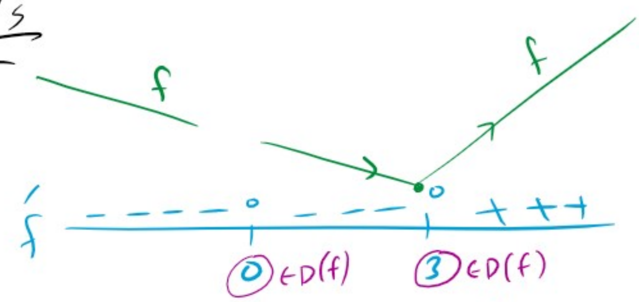
inflection point $(0, f(0)) = (0, 10)$

$(2, f(2)) = (2, -6)$

$f(2) = 2^4 - 4(2^3) + 10$
$= 16 - 4(8) + 10$
$= -6$

⑥ EV's

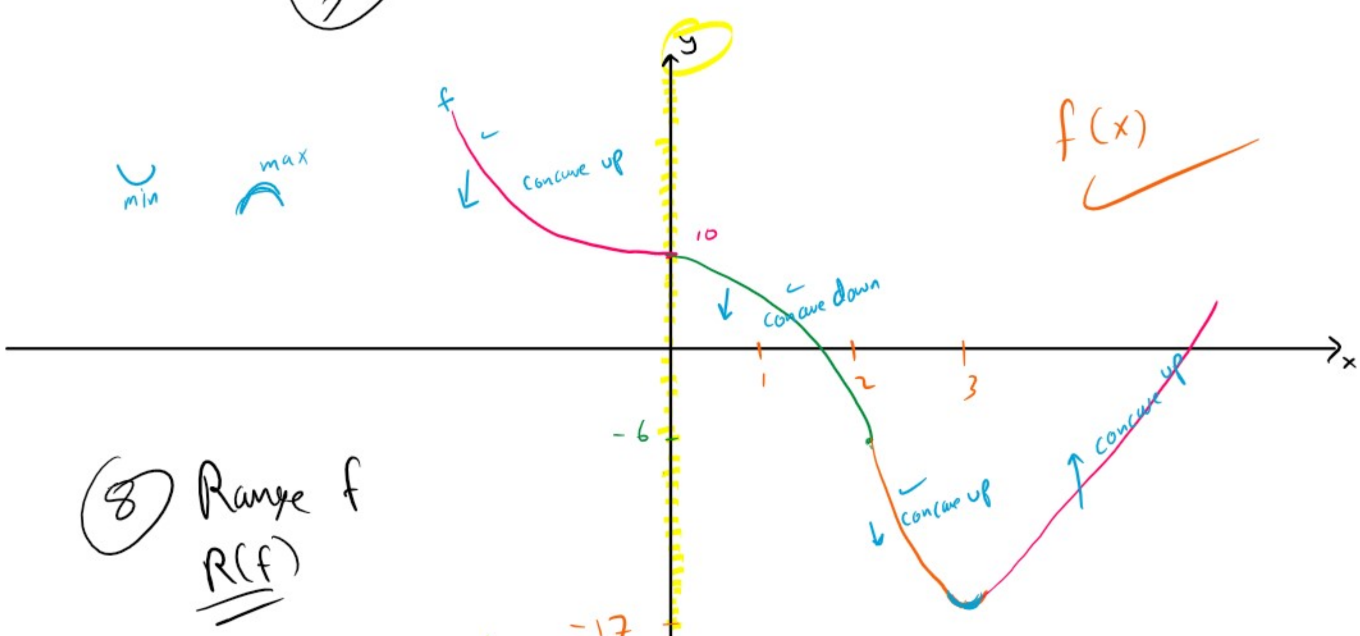
f'



$(3, f(3)) = (3, -17)$ L Min (Abs. Min)

$f(3) = -17$

⑦ Sketch f



⑧ Range f

R(f)

$R(f) = [-17, \infty)$

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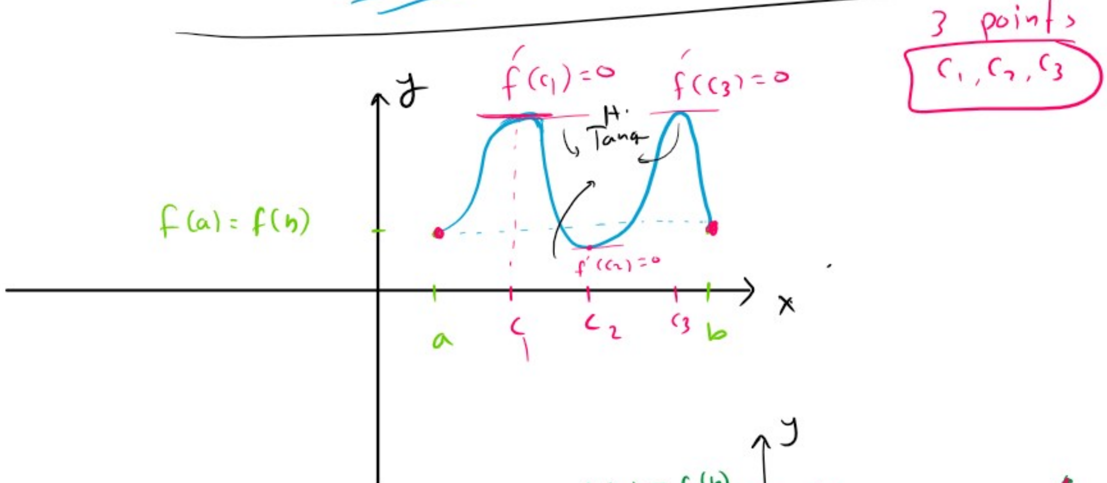
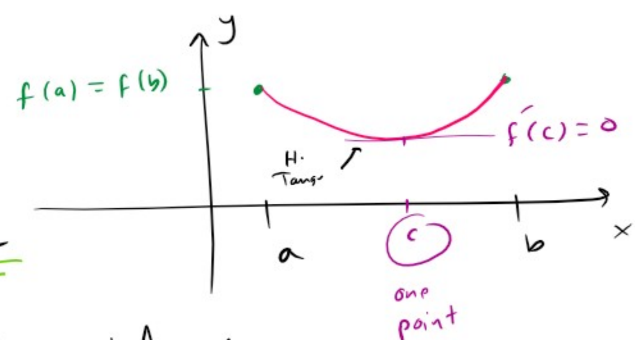
Th (Rolle's Th)

If
- f cont. on $[a,b]$
- f diff on (a,b) s.t $f(a) = f(b)$

Then $\exists$ at least one point $c \in (a,b)$ s.t $f'(c) = 0$





Exp $f(x) = \sin x + 2$

① Does f have horizontal tangent on $[0, 2\pi]$

✓ f cont. on $[0, 2\pi]$

✓ f diff on $(0, 2\pi)$

✓ $f(0) = f(2\pi)$

$\sin 0 + 2 = \sin 2\pi + 2$

$2 = 2$

$\frac{3\pi}{2}$

Roll

Roll

النتيجة

$\exists$ at least one point $c \in (0, 2\pi)$
such that $\boxed{f'(c) = 0}$

Yes $\exists$ Horizonal tangent

(2) Find point where f has H. tangent

$f'(x) = 0$

$\cos x + 0 = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \ldots \in (0, 2\pi)$

(3) sketch

$f(x) = \sin x + 2$

$[0, 2\pi]$