14

Introduction to Frequency-Selective Circuits

Assessment Problems

AP 14.1

$$f_c = 8\,\text{kHz}, \quad \omega_c = 2\pi f_c = 16\pi\,\text{krad/s}$$

$$\omega_c = \frac{1}{RC}; \qquad R = 10\,\text{k}\Omega;$$

$$\therefore \quad C = \frac{1}{\omega_c R} = \frac{1}{(16\pi \times 10^3)(10^4)} = 1.99\,\text{nF}$$

AP 14.2 **[a]** $\omega_c = 2\pi f_c = 2\pi(2000) = 4\pi\,\text{krad/s}$

$$L = \frac{R}{\omega_c} = \frac{5000}{4000\pi} = 0.40\,\text{H}$$

[b] $H(j\omega) = \dfrac{\omega_c}{\omega_c + j\omega} = \dfrac{4000\pi}{4000\pi + j\omega}$

When $\omega = 2\pi f = 2\pi(50{,}000) = 100{,}000\pi$ rad/s

$$H(j100{,}000\pi) = \frac{4000\pi}{4000\pi + j100{,}000\pi} = \frac{1}{1 + j25} = 0.04\underline{/87.71^\circ}$$

$$\therefore \quad |H(j100{,}000\pi)| = 0.04$$

[c] $\therefore \quad \theta(100{,}000\pi) = -87.71^\circ$

AP 14.3

$$\omega_c = \frac{R}{L} = \frac{5000}{3.5 \times 10^{-3}} = 1.43\,\text{Mrad/s}$$

AP 14.4 **[a]** $\omega_c = \dfrac{1}{RC} = \dfrac{10^6}{R} = \dfrac{10^6}{100} = 10\,\text{krad/s}$

[b] $\omega_c = \dfrac{10^6}{5000} = 200\,\text{rad/s}$

[c] $\omega_c = \dfrac{10^6}{3 \times 10^4} = 33.33\,\text{rad/s}$

AP 14.5 Let Z represent the parallel combination of $(1/SC)$ and R_L. Then

$$Z = \frac{R_L}{(R_LCs + 1)}$$

$$\text{Thus} \quad H(s) = \frac{Z}{R+Z} = \frac{R_L}{R(R_LCs+1)+R_L}$$

$$= \frac{(1/RC)}{s + \frac{R+R_L}{R_L}\left(\frac{1}{RC}\right)} = \frac{(1/RC)}{s + \frac{1}{K}\left(\frac{1}{RC}\right)}$$

$$\text{where} \quad K = \frac{R_L}{R+R_L}$$

AP 14.6

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(24\pi \times 10^3)^2(0.1 \times 10^{-6})} = 1.76\,\text{mH}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o}{R/L} \quad \text{so} \quad R = \frac{\omega_o L}{Q} = \frac{(24\pi \times 10^3)(1.76 \times 10^{-3})}{6} = 22.10\,\Omega$$

AP 14.7

$$\omega_o = 2\pi(2000) = 4000\pi\ \text{rad/s};$$

$$\beta = 2\pi(500) = 1000\pi\ \text{rad/s}; \qquad R = 250\,\Omega$$

$$\beta = \frac{1}{RC} \quad \text{so} \quad C = \frac{1}{\beta R} = \frac{1}{(1000\pi)(250)} = 1.27\,\mu\text{F}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{10^6}{(4000\pi)^2(1.27)} = 4.97\,\text{mH}$$

AP 14.8

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(10^4\pi)^2(0.2 \times 10^{-6})} = 5.07\,\text{mH}$$

$$\beta = \frac{1}{RC} \quad \text{so} \quad R = \frac{1}{\beta C} = \frac{1}{400\pi(0.2 \times 10^{-6})} = 3.98\,\text{k}\Omega$$

AP 14.9

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(400\pi)^2(0.2 \times 10^{-6})} = 31.66\,\text{mH}$$

$$Q = \frac{f_o}{\beta} = \frac{5 \times 10^3}{200} = 25 = \omega_o RC$$

$$\therefore \quad R = \frac{Q}{\omega_o C} = \frac{25}{(400\pi)(0.2 \times 10^{-6})} = 9.95\,\text{k}\Omega$$

AP 14.10

$$\omega_o = 8000\pi\,\text{rad/s}$$

$$C = 500\,\text{nF}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = 3.17\,\text{mH}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

$$\therefore \quad R = \frac{1}{\omega_o CQ} = \frac{1}{(8000\pi)(500)(5 \times 10^{-9})} = 15.92\,\Omega$$

AP 14.11

$$\omega_o = 2\pi f_o = 2\pi(20{,}000) = 40\pi\,\text{krad/s}; \qquad R = 100\,\Omega; \qquad Q = 5$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o}{(R/L)} \quad \text{so} \quad L = \frac{QR}{\omega_o} = \frac{5(100)}{(40\pi \times 10^3)} = 3.98\,\text{mH}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad C = \frac{1}{\omega_o^2 L} = \frac{1}{(40\pi \times 10^3)^2(3.98 \times 10^{-3})} = 15.92\,\text{nF}$$

Problems

P 14.1 **[a]** $\omega_c = \frac{R}{L} = \frac{1200}{50 \times 10^{-3}} = 24\,\text{krad/s}$

$$\therefore \quad f_c = \frac{\omega_c}{2\pi} = \frac{24{,}000}{2\pi} = 3819.72\,\text{Hz}$$

[b] $H(s) = \frac{\omega_c}{s + \omega_c} = \frac{24{,}000}{s + 24{,}000}$

$$H(j\omega) = \frac{24{,}000}{24{,}000 + j\omega}$$

$$H(j\omega_c) = \frac{24{,}000}{24{,}000 + j24{,}000} = 0.7071\underline{/-45^\circ}$$

$$H(j0.125\omega_c) = \frac{24{,}000}{24{,}000 + j3000} = 0.9923\underline{/-7.125^\circ}$$

$$H(j8\omega_c) = \frac{24{,}000}{24{,}000 + j192{,}000} = 0.124\underline{/-82.875^\circ}$$

[c] $v_o(t)|_{\omega_c} = 14.142\cos(24{,}000t - 45^\circ)\,\text{V}$

$$v_o(t)|_{0.125\omega_c} = 19.846\cos(3000t - 7.125^\circ)\,\text{V}$$

$$v_o(t)|_{8\omega_c} = 2.48\cos(192{,}000t - 82.875^\circ)\,\text{V}$$

P 14.2 **[a]** $\omega_c = \frac{1}{RC} = \frac{1}{(160)(5 \times 10{-}6)} = 1250\,\text{rad/s}$

$$f_c = \frac{\omega_c}{2\pi} = 198.94\,\text{Hz}$$

[b] $H(j\omega) = \frac{\omega_c}{s + \omega_c} = \frac{1250}{s + 1250}$

$$H(j\omega) = \frac{1250}{1250 + j\omega}$$

$$H(j\omega_c) = \frac{1250}{1250 + j1250} = 0.7071\underline{/-45^\circ}$$

$$H(j0.1\omega_c) = \frac{1250}{1250 + j125} = 0.9950\underline{/-5.71^\circ}$$

$$H(j10\omega_c) = \frac{1250}{1250 + j12{,}500} = 0.0995\underline{/-84.29^\circ}$$

[c] $v_o(t)|_{\omega_c} = 25(0.7071)\cos(1250t - 45^\circ)$

$$= 17.68\cos(1250t - 45^\circ)\,\text{mV}$$

$$v_o(t)|_{0.1\omega_c} = 25(0.9950)\cos(125t - 5.71^\circ)$$

$$= 24.88\cos(125t - 5.71^\circ)\,\text{mV}$$

$$v_o(t)|_{10\omega_c} = 25(0.0995)\cos(12{,}500t - 84.29^\circ)$$

$$= 2.49\cos(12{,}500t - 84.29^\circ)\,\text{mV}$$

P 14.3 **[a]** $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{sL + R + R_l} = \dfrac{(R/L)}{s + (R + R_l)/L}$

[b] $H(j\omega) = \dfrac{(R/L)}{\left(\frac{R+R_l}{L}\right) + j\omega}$

$$|H(j\omega)| = \frac{(R/L)}{\sqrt{\left(\frac{R+R_l}{L}\right)^2 + \omega^2}}$$

$|H(j\omega)|_{\max}$ occurs when $\omega = 0$

[c] $|H(j\omega)|_{\max} = \dfrac{R}{R + R_l}$

[d] $|H(j\omega_c)| = \dfrac{R}{\sqrt{2}(R + R_l)} = \dfrac{R/L}{\sqrt{\left(\frac{R+R_l}{L}\right)^2 + \omega_c^2}}$

$$\therefore \quad \omega_c^2 = \left(\frac{R + R_l}{L}\right)^2; \qquad \therefore \quad \omega_c = (R + R_l)/L$$

[e] $\omega_c = \dfrac{1200 + 300}{0.05} = 30{,}000$ rad/s

$$H(j\omega) = \frac{24{,}000}{30{,}000 + j\omega}$$

$$H(j0) = 0.8$$

$$H(j30{,}000) = \frac{0.8}{\sqrt{2}}\underline{/-45^\circ} = 0.5657\underline{/-45^\circ}$$

$$H(j6000) = \frac{24{,}000}{30{,}000 + j6000} = 0.7845\underline{/-11.31^\circ}$$

$$H(j150{,}000) = \frac{24{,}000}{30{,}000 + j150{,}000} = 0.1569\underline{/-78.69^\circ}$$

P 14.4 **[a]** Let $Z = \dfrac{R_L(1/sC)}{R_L + 1/sC} = \dfrac{R_L}{R_LCs + 1}$

Then $$H(s) = \frac{Z}{Z+R}$$

$$= \frac{R_L}{RR_LCs + R + R_L}$$

$$= \frac{(1/RC)}{s + \left(\dfrac{R+R_L}{RR_LC}\right)}$$

[b] $$|H(j\omega)| = \frac{(1/RC)}{\sqrt{\omega^2 + [(R+R_L)/RR_LC]^2}}$$

$|H(j\omega)|$ is maximum at $\omega = 0$.

[c] $$|H(j\omega)|_{\max} = \frac{R_L}{R+R_L}$$

[d] $$|H(j\omega_c)| = \frac{R_L}{\sqrt{2}(R+R_L)} = \frac{(1/RC)}{\sqrt{\omega_C^2 + [(R+R_L)/RR_LC]^2}}$$

$$\therefore \quad \omega_c = \frac{R+R_L}{RR_LC} = \frac{1}{RC}\,(1 + (R/R_L))$$

[e] $$\omega_c = \frac{1}{(160)(5\times10^{-6})}[1 + (160/320)] = 1250(1+0.5) = 1875 \text{ rad/s}$$

$$H(j0) = \frac{1250}{1875} = 0.667\underline{/0^\circ}$$

$$H(j\omega_c) = \frac{1250}{1875 + j1875} = 0.4714\underline{/-45^\circ}$$

$$H(j0.2\omega_c) = \frac{1250}{1875 + j375} = 0.6537\underline{/-11.31^\circ}$$

$$H(j5\omega_c) = \frac{1250}{1875 + j9375} = 0.1307\underline{/-78.69^\circ}$$

P 14.5 **[a]** $Z_L = j\omega L = j0L = 0$ so it is a short circuit.

At $\omega = 0$, $V_o = V_i$

[b] $Z_L = j\omega L = j\infty L = \infty$ so it is an open circuit.

At $\omega = \infty$, $V_o = 0$

[c] This is a low pass filter, with a gain of 1 at low frequencies and a gain of 0 at high frequencies.

[d] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{R + sL} = \dfrac{R/L}{s + R/L}$

[e] $\omega_c = \dfrac{R}{L} = \dfrac{330}{0.01} = 33$ krad/s

P 14.6 **[a]** $H(s) = \dfrac{V_o}{V_i} = \dfrac{R\|R_L}{R\|R_L + sL} = \dfrac{\dfrac{R}{L}\left(\dfrac{R_L}{R+R_L}\right)}{s + \dfrac{R}{L}\left(\dfrac{R_L}{R+R_L}\right)}$

[b] $\omega_{c(UL)} = \dfrac{R}{L}; \qquad \omega_{c(L)} = \dfrac{R}{L}\left(\dfrac{R_L}{R+R_L}\right)$ so the cutoff frequencies are different.

$H(0)_{(UL)} = 1; \qquad H(0)_{(L)} = 1$ so the passband gains are the same.

[c] $\omega_{c(UL)} = 33{,}000$ rad/s

$\omega_{c(L)} = 33{,}000 - 0.05(33{,}000) = 31{,}350$ rad/s

$31{,}350 = \dfrac{330}{0.01}\left(\dfrac{R_L}{330 + R_L}\right)$ so $\dfrac{R_L}{330 + R_L} = 0.95$

$\therefore \quad 0.05R_L = 313.5$ so $R_L \geq 6270\,\Omega$

P 14.7 **[a]** $\dfrac{R}{L} = 10{,}000\pi$ rad/s

$R = (0.001)(10{,}000)(\pi) = 31.42\,\Omega$

[b] $R_e = 31.42\|68 = 21.49\,\Omega$

$\omega_{\text{loaded}} = \dfrac{R_e}{L} = 21{,}488.34$ rad/s

$\therefore \quad f_{\text{loaded}} = 3419.98\,\text{Hz}$

[c] The $33\,\Omega$ resistor in Appendix H is closest to the desired value of $31.42\,\Omega$. Therefore,

$\omega_c = 33\,\text{krad/s}$ so $f_c = 5252.11\,\text{Hz}$

P 14.8 **[a]** $f_c = \dfrac{1600}{2\pi} = 254.65$ Hz

[b] $1600 = \dfrac{R}{0.01}$ so $R = 1600(0.01) = 16\,\Omega$

[c] $\omega_c' = 1600 - 160 = 1440$ rad/s

$R' = 1440(0.01) = 14.4\,\Omega$

$16||R_{\text{L}} = 14.4$ so $\dfrac{16R_{\text{L}}}{16 + R_{\text{L}}} = 14.4$

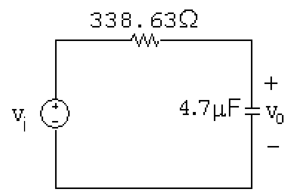
Solving, $R_{\text{L}} = 144\,\Omega$

[d] The magnitude of $H(j\omega)$ when $\omega = 0$ is still 1. The resistive load affects the cutoff frequency but not the magnitude of this low-pass filter.

P 14.9 **[a]** $\omega_c = 2\pi(100) = 628.32$ rad/s

[b] $\omega_c = \dfrac{1}{RC}$ so $R = \dfrac{1}{\omega_c C} = \dfrac{1}{(628.32)(4.7\times10^{-6})} = 338.63\,\Omega$

[c]



[d] $$H(s) = \frac{V_o}{V_i} = \frac{1/sC}{R + 1/sC} = \frac{1/RC}{s + 1/RC} = \frac{628.32}{s + 628.32}$$

[e] $$H(s) = \frac{V_o}{V_i} = \frac{(1/sC)\|R_L}{R + (1/sC)\|R_L} = \frac{1/RC}{s + \left(\dfrac{R+R_L}{R_L}\right)1/RC} = \frac{628.32}{s + 2(628.32)}$$

[f] $\omega_c = 2(628.32) = 1256.64$ rad/s

[g] $H(0) = 1/2$

P 14.10 **[a]** $$f_c = \frac{\omega_c}{2\pi} = \frac{50{,}000}{2\pi} = \frac{50}{2\pi}\times10^3 = 7957.75\,\text{Hz}$$

[b] $$\frac{1}{RC} = 50\times10^3$$

$$R = \frac{1}{(50\times10^3)(0.5\times10^{-6})} = 40\,\Omega$$

[c] With a load resistor added in parallel with the capacitor the transfer function becomes

$$H(s) = \frac{R_L\|(1/sC)}{R + R_L\|(1/sC)} = \frac{R_L/sC}{R[R_L + (1/sC)] + R_L/sC}$$

$$= \frac{R_L}{RR_LsC + R + R_L} = \frac{1/RC}{s + [(R+R_L)/RR_LC]}$$

This transfer function is in the form of a low-pass filter, with a cutoff frequency equal to the quantity added to s in the denominator. Therefore,

$$\omega_c = \frac{R+R_L}{RR_LC} = \frac{1}{RC}\left(1 + \frac{R}{R_L}\right)$$

$$\therefore\quad \frac{R}{R_L} = 0.05 \qquad \therefore\quad R_L = 20R = 800\,\Omega$$

[d] $H(j0) = \dfrac{R_L}{R + R_L} = \dfrac{800}{840} = 0.9524$

P 14.11 **[a]** $\dfrac{1}{RC} = \dfrac{1}{(20)(80 \times 10^{-6})} = 625\,\text{rad/s}$

$$f_c = \frac{625}{2\pi} = 99.47\,\text{Hz}$$

[b] $H(s) = \dfrac{s}{s + \omega_c} \qquad \therefore \qquad H(j\omega) = \dfrac{j\omega}{625 + j\omega}$

$$H(j\omega_c) = H(j6250) = \frac{j625}{625 + j625} = 0.7071\underline{/45^\circ}$$

$$H(j0.125\omega_c) = H(j78.125) = \frac{j78.125}{625 + j78.125} = 0.124\underline{/82.87^\circ}$$

$$H(j8\omega_c) = H(j5000\omega_c) = \frac{j5000}{625 + j5000} = 0.9923\underline{/7.125^\circ}$$

[c] $v_o(t)|_{\omega_c} = (0.7071)(75)\cos(625t + 45^\circ)$

$$= 53.03\cos(625t + 45^\circ)\,\text{mV}$$

$$v_o(t)|_{0.125\omega_c} = (0.124)(75)\cos(78.125t + 82.87^\circ)$$

$$= 9.3\cos(78.125t + 82.87^\circ)\,\text{mV}$$

$$v_o(t)|_{8\omega_c} = (0.9923)(75)\cos(5000t + 7.125^\circ)$$

$$= 74.42\cos(5000t + 7.125^\circ)\,\text{mV}$$

P 14.12 **[a]** $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{R + R_c + (1/sC)}$

$$= \frac{R}{R + R_c} \cdot \frac{s}{[s + (1/(R + R_c)C)]}$$

[b] $H(j\omega) = \dfrac{R}{R + R_c} \cdot \dfrac{j\omega}{j\omega + (1/(R + R_c)C)}$

$$|H(j\omega)| = \frac{R}{R + R_c} \cdot \frac{\omega}{\sqrt{\omega^2 + \frac{1}{(R+R_c)^2C^2}}}$$

The magnitude will be maximum when $\omega = \infty$.

[c] $|H(j\omega)|_{\max} = \dfrac{R}{R + R_c}$

[d] $|H(j\omega_c)| = \dfrac{R\omega_c}{(R+R_c)\sqrt{\omega_c^2 + [1/(R+R_c)C]^2}}$

$$\therefore \quad |H(j\omega)| = \frac{R}{\sqrt{2}(R+R_c)} \qquad \text{when}$$

$$\therefore \quad \omega_c^2 = \frac{1}{(R+R_c)^2C^2}$$

$$\text{or } \omega_c = \frac{1}{(R+R_c)C}$$

[e] $\omega_c = \dfrac{1}{(25)(80\times 10^{-6})} = 500$ rad/s

$$\frac{R}{R+R_c} = \frac{20}{25} = 0.8$$

$$\therefore \qquad H(j\omega) = \frac{0.8j\omega}{500+j\omega}$$

$$H(j\omega_c) = \frac{(0.8)j500}{500+j500} = 0.5657\underline{/45^\circ}$$

$$H(j0.125\omega_c) = \frac{(0.8)j62.5}{500+j62.5} = 0.099\underline{/82.87^\circ}$$

$$H(j8\omega_c) = \frac{(0.8)j4000}{500+j4000} = 0.7938\underline{/7.125^\circ}$$

P 14.13 **[a]** $\omega_c = \dfrac{1}{RC} = 2\pi(300) = 600\pi$ rad/s

$$\therefore \quad R = \frac{1}{\omega_c C} = \frac{1}{(600\pi)(100\times 10^{-9})} = 5305.16\,\Omega = 5.305\,\text{k}\Omega$$

[b] $R_e = 5305.16\|47{,}000 = 4767.08\,\Omega$

$$\omega_c = \frac{1}{R_eC} = \frac{1}{(4767.08)(100\times 10^{-9})} = 2097.7 \text{ rad/s}$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{2097.7}{2\pi} = 333.86\,\text{Hz}$$

P 14.14 **[a]** For $\omega = 0$, the inductor behaves as a short circuit, so $V_o = 0$.
For $\omega = \infty$, the inductor behaves as an open circuit, so $V_o = V_i$.
Thus, the circuit is a high-pass filter.

[b] $H(s) = \dfrac{sL}{R+sL} = \dfrac{s}{s+R/L} = \dfrac{s}{s+15{,}000}$

[c] $\omega_c = \dfrac{R}{L} = 15{,}000$ rad/s

[d] $|H(jR/L)| = \left|\dfrac{jR/L}{jR/L + R/L}\right| = \left|\dfrac{j}{j+1}\right| = \dfrac{1}{\sqrt{2}}$

P 14.15 **[a]** $H(s) = \dfrac{V_o}{V_i} = \dfrac{R_L \| sL}{R + R_L \| sL} = \dfrac{s\left(\dfrac{R_L}{R+R_L}\right)}{s + \dfrac{R}{L}\left(\dfrac{R_L}{R+R_L}\right)}$

$$= \frac{\frac{1}{2}s}{s + \frac{1}{2}(15{,}000)}$$

[b] $\omega_c = \dfrac{R}{L}\left(\dfrac{R_L}{R+R_L}\right) = \dfrac{1}{2}(15{,}000) = 7500$ rad/s

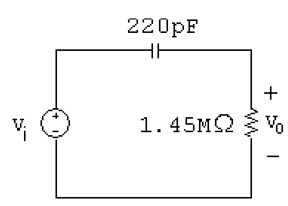
[c] $\omega_{c(L)} = \dfrac{1}{2}\omega_{c(UL)}$

[d] The gain in the passband is also reduced by a factor of 1/2 for the loaded filter.

P 14.16 **[a]** $\omega_c = 2\pi(500) = 3141.59$ rad/s

[b] $\omega_c = \dfrac{1}{RC}$ so $R = \dfrac{1}{\omega_c C} = \dfrac{1}{(3141.59)(220 \times 10^{-12})} = 1.45\,\text{M}\Omega$

[c]



[d] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{R + 1/sC} = \dfrac{s}{s + 1/RC} = \dfrac{s}{s + 3141.59}$

[e] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R\|R_L}{R\|R_L + (1/sC)} = \dfrac{s}{s + \left(\dfrac{R+R_L}{R_L}\right)1/RC} = \dfrac{s}{s + 2(3141.59)}$

[f] $\omega_c = 2(3141.59) = 6283.19$ rad/s

[g] $H(\infty) = 1$

P 14.17 **[a]** $R = \omega_c L = (1500 \times 10^3)(100 \times 10^{-6}) = 150\,\Omega$ (a value from Appendix H)

[b] With a load resistor in parallel with the inductor, the transfer function becomes

$$H(s) = \frac{sL\|R_L}{R + sL\|R_L} = \frac{sLR_L}{R(sL + R_L) + sLR_L} = \frac{s[R_L/(R+R_L)]}{s + [RR_L/(R+R_L)]}$$

This transfer function is in the form of a high-pass filter whose cutoff frequency is the quantity added to s in the denominator. Thus,

$$\omega_c = \frac{RR_L}{L(R+R_L)}$$

Substituting in the values of R and L from part (a), we can solve for the value of load resistance that gives a cutoff frequency of 1200 krad/s:

$$\frac{150R_L}{100\times 10^{-6}(150+R_L)} = 1200\times 10^3 \qquad \text{so} \qquad R_L = 600\,\Omega$$

The smallest resistor from Appendix H that is larger than $600\,\Omega$ is $680\,\Omega$.

P 14.18 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(10\times 10^{-3})(10\times 10^{-9})} = 10^{10}$

$\omega_o = 10^5$ rad/s $= 100$ krad/s

[b] $f_o = \dfrac{\omega_o}{2\pi} = \dfrac{10^5}{2\pi} = 15.9$ kHz

[c] $Q = \omega_o RC = (100\times 10^3)(8000)(10\times 10^{-9}) = 8$

[d] $\omega_{c1} = \omega_o\left[-\dfrac{1}{2Q} + \sqrt{1+\left(\dfrac{1}{2Q}\right)^2}\right] = 10^5\left[-\dfrac{1}{16} + \sqrt{1+\dfrac{1}{256}}\right] = 93.95$ krad/s

[e] $\therefore\quad f_{c1} = \dfrac{\omega_{c1}}{2\pi} = 14.95$ kHz

[f] $\omega_{c2} = \omega_o\left[\dfrac{1}{2Q} + \sqrt{1+\left(\dfrac{1}{2Q}\right)^2}\right] = 10^5\left[\dfrac{1}{16} + \sqrt{1+\dfrac{1}{256}}\right] = 106.45$ krad/s

[g] $\therefore\quad f_{c2} = \dfrac{\omega_{c2}}{2\pi} = 16.94$ kHz

[h] $\beta = \dfrac{\omega_o}{Q} = \dfrac{10^5}{8} = 12.5$ krad/s or 1.99 kHz

P 14.19 $\omega_o = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{(121)(100)} = 110$ krad/s

$$f_o = \frac{\omega_o}{2\pi} = 17.51\text{ kHz}$$

$$\beta = 121 - 100 = 21\text{ krad/s} \quad \text{or} \quad 2.79\text{ kHz}$$

$$Q = \frac{\omega_o}{\beta} = \frac{110}{21} = 5.24$$

P 14.20 $\beta = \dfrac{\omega_o}{Q} = \dfrac{50{,}000}{4} = 12.5\,\text{krad/s}; \qquad \dfrac{12{,}500}{2\pi} = 1.99\,\text{kHz}$

$$\omega_{c2} = 50{,}000\left[\frac{1}{8} + \sqrt{1 + \left(\frac{1}{8}\right)^2}\right] = 56.64\,\text{krad/s}$$

$$f_{c2} = \frac{56.64\,\text{k}}{2\pi} = 9.01\,\text{kHz}$$

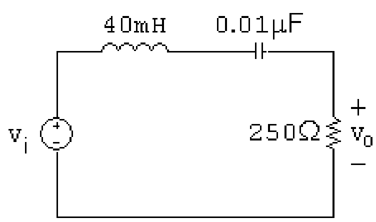
$$\omega_{c1} = 50{,}000\left[-\frac{1}{8} + \sqrt{1 + \left(\frac{1}{8}\right)^2}\right] = 44.14\,\text{krad/s}$$

$$f_{c1} = \frac{44.14\,\text{k}}{2\pi} = 7.02\,\text{kHz}$$

P 14.21 **[a]** $\omega_o = \sqrt{1/LC} \quad \text{so} \quad L = \dfrac{1}{\omega_o^2 C} = \dfrac{1}{(50{,}000)^2(0.01 \times 10^{-6})} = 40\,\text{mH}$

$$Q = \frac{\omega_o}{\beta} \quad \text{so} \quad \beta = \frac{\omega_o}{Q} = \frac{50{,}000}{8} = 6250 \text{ rad/s}$$

$$\beta = \frac{R}{L} \quad \text{so} \quad R = L\beta = (40 \times 10^{-3})(6250) = 250\,\Omega$$



[b] From part (a), $\beta = 6250$ rad/s. Then,

$$\omega_{c1,2} = \pm\frac{\beta}{2} + \sqrt{\frac{\beta}{2} + \omega_o^2} = \pm\frac{6250}{2} + \sqrt{\left(\frac{6250}{2}\right)^2 + 50{,}000^2} = \pm 3125 + 50{,}097.56$$

$$\omega_{c1} = 46{,}972.56 \text{ rad/s} \qquad \omega_{c2} = 53{,}222.56 \text{ rad/s}$$

P 14.22 $H(j\omega) = \dfrac{j\omega(6250)}{50{,}000^2 - \omega^2 + j\omega(6250)}$

[a] $H(j50{,}000) = \dfrac{j50{,}000(6250)}{50{,}000^2 - 50{,}000^2 + j(50{,}000)(6250)} = 1$

$$V_o = (1)V_i \quad \therefore \quad v_o(t) = 5\cos 50{,}000t\,\text{V}$$

[b] $H(j46{,}972.56) = \dfrac{j46{,}972.56(6250)}{50{,}000^2 - 46{,}972.56^2 + j(46{,}972.56)(6250)} = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}V_i \quad \therefore \quad v_o(t) = 3.54\cos(46{,}972.56t + 45^\circ)\,\mathrm{V}$

[c] $H(j53{,}222.56) = \dfrac{j53{,}222.56(6250)}{50{,}000^2 - 53{,}222.56^2 + j(53{,}222.56)(6250)} = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}V_i \quad \therefore \quad v_o(t) = 3.54\cos(53{,}222.56t - 45^\circ)\,\mathrm{V}$

[d] $H(j5000) = \dfrac{j5000(6250)}{50{,}000^2 - 5000^2 + j(5000)(6250)} = 0.0126\underline{/89.3^\circ}$

$V_o = 0.0126\underline{/89.3^\circ}V_i \quad \therefore \quad v_o(t) = 63.1\cos(5000t + 89.3^\circ)\,\mathrm{mV}$

[e] $H(j500{,}000) = \dfrac{j500{,}000(6250)}{50{,}000^2 - 500{,}000^2 + j(500{,}000)(6250)} = 0.0126\underline{/-89.3^\circ}$

$V_o = 0.0126\underline{/-89.3^\circ}V_i \quad \therefore \quad v_o(t) = 63.1\cos(500{,}000t - 89.3^\circ)\,\mathrm{mV}$

P 14.23 $H(s) = 1 - \dfrac{(R/L)s}{s^2 + (R/L)s + (1/LC)} = \dfrac{s^2 + (1/LC)}{s^2 + (R/L)s + (1/LC)}$

$$H(j\omega) = \frac{50{,}000^2 - \omega^2}{50{,}000^2 - \omega^2 + j\omega(6250)}$$

[a] $H(j50{,}000) = \dfrac{50{,}000^2 - 50{,}000^2}{50{,}000^2 - 50{,}000^2 + j(50{,}000)(6250)} = 0$

$V_o = (0)V_i \quad \therefore \quad v_o(t) = 0\,\mathrm{mV}$

[b] $H(j46{,}972.56) = \dfrac{50{,}000^2 - 46{,}972.56^2}{50{,}000^2 - 46{,}972.56^2 + j(46{,}972.56)(6250)} = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}V_i \quad \therefore \quad v_o(t) = 3.54\cos(46{,}972.56t - 45^\circ)\,\mathrm{V}$

[c] $H(j53{,}222.56) = \dfrac{50{,}000^2 - 53{,}222.56^2}{50{,}000^2 - 53{,}222.56^2 + j(53{,}222.56)(6250)} = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}V_i \quad \therefore \quad v_o(t) = 3.54\cos(53{,}222.56t + 45^\circ)\,\mathrm{V}$

[d] $H(j5000) = \dfrac{50{,}000^2 - 5000^2}{50{,}000^2 - 5000^2 + j(5000)(6250)} = 0.9999\underline{/-0.72^\circ}$

$V_o = 0.9999\underline{/-0.72^\circ}V_i \quad \therefore \quad v_o(t) = 4.9996\cos(5000t - 0.72^\circ)\,\mathrm{V}$

[e] $H(j500{,}000) = \dfrac{50{,}000^2 - 500{,}000^2}{50{,}000^2 - 500{,}000^2 + j(500{,}000)(6250)} = 0.9999\underline{/0.72^\circ}$

$$V_o = 0.9999\underline{/0.72^\circ}V_i \quad \therefore \quad v_o(t) = 4.9996\cos(500{,}000t + 0.72^\circ)\,\mathrm{V}$$

P 14.24 By definition $Q = \omega_o/\beta$ therefore $\beta = \omega_o/Q$. Substituting this expression into Eqs. 14.34 and 14.35 yields

$$\omega_{c1} = -\frac{\omega_o}{2Q} + \sqrt{\left(\frac{\omega_o}{2Q}\right)^2 + \omega_o^2}$$

$$\omega_{c2} = \frac{\omega_o}{2Q} + \sqrt{\left(\frac{\omega_o}{2Q}\right)^2 + \omega_o^2}$$

Now factor ω_o out to get

$$\omega_{c1} = \omega_o\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

$$\omega_{c2} = \omega_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

P 14.25 **[a]** $L = \dfrac{1}{\omega_o^2 C} = \dfrac{1}{(50\times 10^{-9})(20\times 10^3)^2} = 50\,\mathrm{mH}$

$$R = \frac{Q}{\omega_o C} = \frac{5}{(20\times 10^3)(50\times 10^{-9})} = 5\,\mathrm{k}\Omega$$

[b] $\omega_{c2} = \omega_o\left[\dfrac{1}{2Q} + \sqrt{1 + \left(\dfrac{1}{2Q}\right)^2}\right] = 20{,}000\left[\dfrac{1}{10} + \sqrt{1 + \dfrac{1}{100}}\right]$

$$= 22.10\,\mathrm{krad/s} \quad \therefore \quad f_{c2} = \frac{\omega_{c2}}{2\pi} = 3.52\,\mathrm{kHz}$$

$$\omega_{c1} = \omega_o\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 20{,}000\left[-\frac{1}{10} + \sqrt{1 + \frac{1}{100}}\right]$$

$$= 18.10\,\mathrm{krad/s} \quad \therefore \quad f_{c1} = \frac{\omega_{c1}}{2\pi} = 2.88\,\mathrm{kHz}$$

[c] $\beta = \dfrac{\omega_o}{Q} = \dfrac{20{,}000}{5} = 4000$ rad/s or $636.62\,\mathrm{Hz}$

P 14.26 **[a]** We need $\omega_c = 20{,}000$ rad/s. There are several possible approaches – this one starts by choosing $L = 1$ mH. Then,

$$C = \frac{1}{20{,}000^2(0.001)} = 2.5\,\mu\text{F}$$

Use the closest value from Appendix H, which is $2.2\,\mu$F to give

$$\omega_c = \sqrt{\frac{1}{(0.001)(2.2\times10^{-6})}} = 21{,}320 \text{ rad/s}$$

Then, $\quad R = \dfrac{Q}{\omega_o C} = \dfrac{5}{(21320)(2.2\times10^{-6})} = 106.6\,\Omega$

Use the closest value from Appendix H, which is $100\,\Omega$ to give

$$Q = 100(21{,}320)(2.2\times10^{-6}) = 4.69$$

[b] $\%$ error in $\omega_c = \dfrac{21{,}320 - 20{,}000}{20{,}000}(100) = 6.6\%$

$\%$ error in $Q = \dfrac{4.69-5}{5}(100) = -6.2\%$

P 14.27 **[a]** $\omega_o^2 = \dfrac{1}{LC} \quad$ so $\quad L = \dfrac{1}{[8000(2\pi)]^2(5\times10^{-9})} = 79.16\,\text{mH}$

$$R = \frac{\omega_o L}{Q} = \frac{8000(2\pi)(79.16\times10^{-3})}{2} = 1.99\,\text{k}\Omega$$

[b] $f_{c1} = 8000\left[-\dfrac{1}{4} + \sqrt{1+\dfrac{1}{16}}\right] = 6.25\,\text{kHz}$

[c] $f_{c2} = 8000\left[\dfrac{1}{4} + \sqrt{1+\dfrac{1}{16}}\right] = 10.25\,\text{kHz}$

[d] $\beta = f_{c2} - f_{c1} = 4\,\text{kHz}$

or

$$\beta = \frac{f_o}{Q} = \frac{8000}{2} = 4\,\text{kHz}$$

P 14.28 **[a]** We need ω_c close to $2\pi(8000) = 50{,}265.48$ rad/s. There are several possible approaches – this one starts by choosing $L = 10$ mH. Then,

$$C = \frac{1}{[2\pi(8000)]^2(0.01)} = 39.58\,\text{nF}$$

Use the closest value from Appendix H, which is $0.047\,\mu$F to give

$$\omega_c = \sqrt{\frac{1}{(0.01)(47\times10^{-9})}} = 46{,}126.56 \text{ rad/s} \quad \text{or} \quad f_c = 7341.27\,\text{Hz}$$

Then, $R = \frac{\omega_o L}{Q} = \frac{(46{,}126.56)(0.01)}{2} = 230\,\Omega$

Use the closest value from Appendix H, which is $220\,\Omega$ to give

$$Q = \frac{(46{,}126.56)(0.01)}{220} = 2.1$$

[b] % error in $f_c = \frac{7341.27 - 8000}{8000}(100) = -8.23\%$

% error in $Q = \frac{2.1 - 2}{2}(100) = 5\%$

P 14.29 **[a]** $\omega_o^2 = \frac{1}{LC} = \frac{1}{(40 \times 10^{-3})(40 \times 10^{-9})} = 625 \times 10^6$

$\omega_o = 25 \times 10^3$ rad/s $= 25$ krad/s

$$f_o = \frac{25{,}000}{2\pi} = 3978.87\,\text{Hz}$$

[b] $Q = \frac{\omega_o L}{R + R_i} = \frac{(25 \times 10^3)(40 \times 10^{-3})}{200} = 5$

[c] $\omega_{c1} = \omega_o \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 25{,}000\left[-\frac{1}{10} + \sqrt{1 + \frac{1}{100}}\right]$

$= 22.625$ krad/s or 3.60 kHz

[d] $\omega_{c2} = \omega_o \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 25{,}000\left[\frac{1}{10} + \sqrt{1 + \frac{1}{100}}\right]$

$= 27.625$ krad/s or 4.4 kHz

[e] $\beta = \omega_{c2} - \omega_{c1} = 27.62 - 22.62 = 5$ krad/s
or

$\beta = \frac{\omega_o}{Q} = \frac{25{,}000}{5} = 5$ krad/s or 795.77 Hz

P 14.30 **[a]** $H(s) = \frac{(R/L)s}{s^2 + \frac{(R+R_i)}{L}s + \frac{1}{LC}}$

For the numerical values in Problem 14.29 we have

$$H(s) = \frac{4500s}{s^2 + 5000s + 625 \times 10^6}$$

$$\therefore \quad H(j\omega) = \frac{4500j\omega}{(625 \times 10^6 - \omega^2) + j5000\omega}$$

$$H(j\omega_o) = \frac{j4500(25 \times 10^3)}{j5000(25 \times 10^3)} = 0.9\underline{/0^\circ}$$

$$\therefore \quad v_o(t) = 10(0.9)\cos 25{,}000 = 9\cos 25{,}000t\,\text{V}$$

[b] From the solution to Problem 14.29,

$$\omega_{c1} = 22.625\,\text{krad/s}$$

$$H(j22.625\,\text{k}) = \frac{j4500(22.625 \times 10^3)}{(113.12 + j113.12) \times 10^6} = 0.6364\underline{/45^\circ}$$

$$\therefore \quad v_o(t) = 10(0.6364)\cos(22{,}625t + 45^\circ) = 6.364\cos(22{,}625t + 45^\circ)\,\text{V}$$

[c] From the solution to Problem 14.29,

$$\omega_{c2} = 27.625 \text{ krad/s}$$

$$H(j27.625\,\text{k}) = \frac{j4500(27.625 \times 10^3)}{(-138.12 + j138.12) \times 10^6} = 0.6364\underline{/-45^\circ}$$

$$\therefore \quad v_o(t) = 10(0.6364)\cos(27{,}625t - 45^\circ) = 6.364\cos(27{,}625t - 45^\circ)\,\text{V}$$

P 14.31 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(5 \times 10^{-3})(200 \times 10^{-12})} = 10^{12}$

$$\omega_o = 1\,\text{Mrad/s}$$

[b] $\beta = \dfrac{R + R_L}{R_L} \cdot \dfrac{1}{RC} = \left(\dfrac{500 \times 10^3}{400 \times 10^3}\right)\left(\dfrac{1}{(100 \times 10^3)(200 \times 10^{-12})}\right) = 62.5\,\text{krad/s}$

[c] $Q = \dfrac{\omega_o}{\beta} = \dfrac{10^6}{62.5 \times 10^3} = 16$

[d] $H(j\omega_o) = \dfrac{R_L}{R + R_L} = 0.8\underline{/0^\circ}$

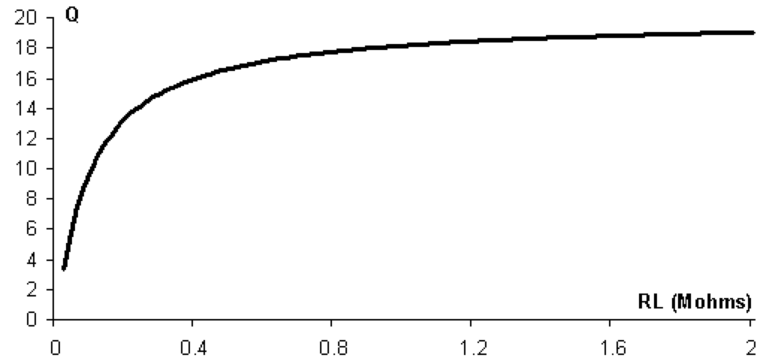
$$\therefore \quad v_o(t) = 250(0.8)\cos(10^6 t) = 200\cos 10^6 t\,\text{mV}$$

[e] $\beta = \left(1 + \dfrac{R}{R_L}\right)\dfrac{1}{RC} = \left(1 + \dfrac{100}{R_L}\right)(50 \times 10^3) \text{ rad/s}$

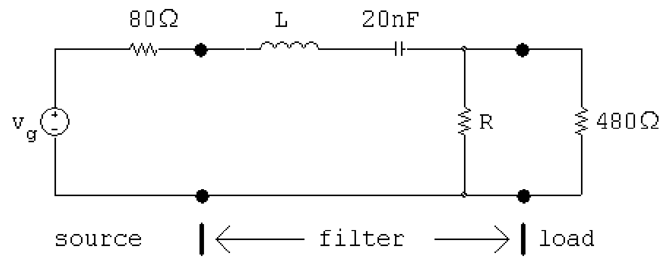
$$\omega_o = 10^6 \text{ rad/s}$$

$$Q = \frac{\omega_o}{\beta} = \frac{20}{1 + (100/R_L)} \qquad \text{where } R_L \text{ is in kilohms}$$

[**f**]



P 14.32 [**a**]



[**b**] $L = \dfrac{1}{\omega_o^2 C} = \dfrac{1}{(50 \times 10^3)^2 (20 \times 10^{-4})} = 20\,\text{mH}$

$$R = \frac{\omega_o L}{Q} = \frac{(50 \times 10^3)(20 \times 10^{-3})}{6.25} = 160\,\Omega$$

[**c**] $R_e = 160 \| 480 = 120\,\Omega$

$$R_e + R_i = 120 + 80 = 200\,\Omega$$

$$Q_{\text{system}} = \frac{\omega_o L}{R_e + R_i} = \frac{(50 \times 10^3)(20 \times 10^{-3})}{200} = 5$$

[**d**] $\beta_{\text{system}} = \dfrac{\omega_o}{Q_{\text{system}}} = \dfrac{50 \times 10^3}{5} = 10\,\text{krad/s}$

$$\beta_{\text{system}}(\text{Hz}) = \frac{10{,}000}{2\pi} = 1591.55\,\text{Hz}$$

P 14.33 [**a**] $\dfrac{V_o}{V_i} = \dfrac{Z}{Z + R}$ where $Z = \dfrac{1}{Y}$

and $Y = sC + \dfrac{1}{sL} + \dfrac{1}{R_L} = \dfrac{LCR_L s^2 + sL + R_L}{R_L L s}$

$$H(s) = \frac{R_L Ls}{R_L RLCs^2 + (R + R_L)Ls + RR_L}$$

$$= \frac{(1/RC)s}{s^2 + \left[\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)\right]s + \frac{1}{LC}}$$

$$= \frac{\left(\frac{R_L}{R+R_L}\right)\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)s}{s^2 + \left[\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)\right]s + \frac{1}{LC}}$$

$$= \frac{K\beta s}{s^2 + \beta s + \omega_o^2}, \qquad K = \frac{R_L}{R + R_L}, \qquad \beta = \frac{1}{(R\|R_L)C}$$

[b] $\beta = \left(\frac{R + R_L}{R_L}\right)\frac{1}{RC}$

[c] $\beta_U = \frac{1}{RC}$

$$\therefore \quad \beta_L = \left(\frac{R + R_L}{R_L}\right)\beta_U = \left(1 + \frac{R}{R_L}\right)\beta_U$$

[d] $Q = \frac{\omega_o}{\beta} = \frac{\omega_o RC}{\left(\frac{R+R_L}{R_L}\right)}$

[e] $Q_U = \omega_o RC$

$$\therefore \quad Q_L = \left(\frac{R_L}{R + R_L}\right)Q_U = \frac{1}{[1 + (R/R_L)]}Q_U$$

[f] $H(j\omega) = \frac{Kj\omega\beta}{\omega_o^2 - \omega^2 + j\omega\beta}$

$$H(j\omega_o) = K$$

Let ω_c represent a corner frequency. Then

$$|H(j\omega_c)| = \frac{K}{\sqrt{2}} = \frac{K\omega_c\beta}{\sqrt{(\omega_o^2 - \omega_c^2)^2 + \omega_c^2\beta^2}}$$

$$\therefore \quad \frac{1}{\sqrt{2}} = \frac{\omega_c\beta}{\sqrt{(\omega_o^2 - \omega_c^2)^2 + \omega_c^2\beta^2}}$$

Squaring both sides leads to

$(\omega_o^2 - \omega_c^2)^2 = \omega_c^2\beta^2$ or $(\omega_o^2 - \omega_c^2) = \pm\omega_c\beta$

$$\therefore \quad \omega_c^2 \pm \omega_c\beta - \omega_o^2 = 0$$

or

$$\omega_c = \mp\frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

The two positive roots are

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2} \quad \text{and} \quad \omega_{c2} = \frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

where

$$\beta = \left(1 + \frac{R}{R_L}\right)\frac{1}{RC} \text{ and } \omega_o^2 = \frac{1}{LC}$$

P 14.34 $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(2 \times 10^{-6})(50 \times 10^{-12})} = 10^{16}$

$$\omega_o = 100\,\text{Mrad/s}$$

$$Q_U = \omega_o RC = (100 \times 10^6)(2.4 \times 10^3)(50 \times 10^{-12}) = 12$$

$$\therefore \quad \left(\frac{R_L}{R + R_L}\right) 12 = 7.5; \qquad \therefore \quad R_L = \frac{7.5}{4.5} R = 4\,\text{k}\Omega$$

P 14.35 $H(s) = \dfrac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$

[a] $\omega_o = \sqrt{\dfrac{1}{LC}} = \sqrt{\dfrac{1}{(0.15625)(100 \times 10^{-9})}} = 8000 \text{ rad/s}$

[b] $f_o = \dfrac{\omega_o}{2\pi} = 1273.24\,\text{Hz}$

[c] $Q = \sqrt{\dfrac{L}{R^2 C}} = \sqrt{\dfrac{0.15625}{(1875)^2(100 \times 10^{-9})}} = \dfrac{2}{3}$

[d] $\beta = \dfrac{R}{L} = \dfrac{1875}{0.15625} = 12{,}000 \text{ rad/s}$

$$\beta \text{ (Hertz)} = \frac{12{,}000}{2\pi} = 1909.86\,\text{Hz}$$

[e] $\omega_{c1} = \dfrac{-\beta}{2} + \sqrt{\left(\dfrac{\beta}{2}\right)^2 + \omega_o^2}$

$$= \frac{-12{,}000}{2} + \sqrt{\left(\frac{12{,}000}{2}\right)^2 + 8000^2} = 4000 \text{ rad/s}$$

[f] $f_{c1} = \dfrac{4000}{2\pi} = 636.62\,\text{Hz}$

[g] $\omega_{c2} = \dfrac{\beta}{2} + \sqrt{\left(\dfrac{\beta}{2}\right)^2 + \omega_o^2}$

$$= \frac{12{,}000}{2} + \sqrt{\left(\frac{12{,}000}{2}\right)^2 + 8000^2} = 16{,}000 \text{ rad/s}$$

[h] $f_{c2} = \dfrac{16{,}000}{2\pi} = 2546.48\,\text{Hz}$

P 14.36 **[a]** $H(j\omega) = \dfrac{\dfrac{1}{LC} - \omega^2}{\dfrac{1}{LC} - \omega^2 + j\dfrac{R}{L}\omega} = \dfrac{8000^2 - \omega^2}{8000^2 - \omega^2 + j12{,}000\omega}$

$\omega_o = 8000$ rad/s :

$$H(j\omega_o) = \frac{8000^2 - 8000^2}{8000^2 - 8000^2 + j12{,}000(8000)} = 0$$

$\omega_{c1} = 4000$ rad/s :

$$H(j\omega_{c1}) = \frac{8000^2 - 4000^2}{8000^2 - 4000^2 + j12{,}000(4000)} = 0.7071\underline{/-45^\circ}$$

$\omega_{c2} = 16{,}000$ rad/s :

$$H(j\omega_{c2}) = \frac{8000^2 - 16{,}000^2}{8000^2 - 16{,}000^2 + j12{,}000(16{,}000)} = 0.7071\underline{/45^\circ}$$

$0.1\omega_o = 800$ rad/s :

$$H(j0.1\omega_o) = \frac{8000^2 - 800^2}{8000^2 - 800^2 + j12{,}000(800)} = 0.9887\underline{/-8.62^\circ}$$

$10\omega_o = 80{,}000$ rad/s :

$$H(j10\omega_o) = \frac{8000^2 - 80{,}000^2}{8000^2 - 80{,}000^2 + j12{,}000(80{,}000)} = 0.9887\underline{/8.62^\circ}$$

[b] $\omega = \omega_o = 8000$ rad/s : $\qquad V_o = H(j8000)V_i = 0V_i$

$$v_o(t) = 0$$

$\omega = \omega_{c1} = 4000$ rad/s :

$$V_o = H(j4000)V_i = (0.7071\underline{/-45^\circ})(80) = 56.57\underline{/-45^\circ}$$

$$v_o(t) = 56.57\cos(4000t - 45^\circ)\,\text{V}$$

$\omega = \omega_{c2} = 16{,}000$ rad/s :

$$V_o = H(j16{,}000)V_i = (0.7071\underline{/45^\circ})(80) = 56.57\underline{/45^\circ}$$

$$v_o(t) = 56.57\cos(16{,}000t + 45^\circ)\,\text{V}$$

$\omega = 0.1\omega_o = 800$ rad/s :

$$V_o = H(j800)V_i = (0.9887\underline{/-8.62^\circ})(80) = 79.1\underline{/-8.62^\circ}$$

$$v_o(t) = 79.1\cos(800t - 8.62^\circ)\,\text{V}$$

$\omega = 10\omega_o = 80{,}000$ rad/s :

$$V_o = H(j80{,}000)V_i = (0.9887\underline{/8.62^\circ})(80) = 79.1\underline{/8.62^\circ}$$

$$v_o(t) = 79.1\cos(80{,}000t + 8.62^\circ)\,\text{V}$$

P 14.37 **[a]** In analyzing the circuit qualitatively we visualize v_i as a sinusoidal voltage and we seek the steady-state nature of the output voltage v_o.
At zero frequency the inductor provides a direct connection between the input and the output, hence $v_o = v_i$ when $\omega = 0$.
At infinite frequency the capacitor provides the direct connection, hence $v_o = v_i$ when $\omega = \infty$.
At the resonant frequency of the parallel combination of L and C the impedance of the combination is infinite and hence the output voltage will be zero when $\omega = \omega_o$.
At frequencies on either side of ω_o the amplitude of the output voltage will be nonzero but less than the amplitude of the input voltage.
Thus the circuit behaves like a band-reject filter.

[b] Let Z represent the impedance of the parallel branches L and C, thus

$$Z = \frac{sL(1/sC)}{sL + 1/sC} = \frac{sL}{s^2LC + 1}$$

Then

$$H(s) = \frac{V_o}{V_i} = \frac{R}{Z + R} = \frac{R(s^2LC + 1)}{sL + R(s^2LC + 1)}$$

$$= \frac{[s^2 + (1/LC)]}{s^2 + \left(\frac{1}{RC}\right)s + \left(\frac{1}{LC}\right)}$$

$$H(s) = \frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2}$$

[c] From part (b) we have

$$H(j\omega) = \frac{\omega_o^2 - \omega^2}{\omega_o^2 - \omega^2 + j\omega\beta}$$

It follows that $H(j\omega) = 0$ when $\omega = \omega_o$.

$$\therefore \quad \omega_o = \frac{1}{\sqrt{LC}}$$

[d] $$|H(j\omega)| = \frac{\omega_o^2 - \omega^2}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2\beta^2}}$$

$$|H(j\omega)| = \frac{1}{\sqrt{2}} \text{ when } \omega^2\beta^2 = (\omega_o^2 - \omega^2)^2$$

or $\pm\omega\beta = \omega_o^2 - \omega^2$, thus

$$\omega^2 \pm \beta\omega - \omega_o^2 = 0$$

The two positive roots of this quadratic are

$$\omega_{c_1} = \frac{-\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

$$\omega_{c_2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

Also note that since $\beta = \omega_o/Q$

$$\omega_{c_1} = \omega_o\left[\frac{-1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

$$\omega_{c_2} = \omega_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

[e] It follows from the equations derived in part (d) that

$$\beta = \omega_{c_2} - \omega_{c_1} = 1/RC$$

[f] By definition $Q = \omega_o/\beta = \omega_o RC$.

P 14.38 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(2.5\times10^{-3})(62.5\times10^{-9})} = 64\times10^8$

$$\therefore\quad \omega_o = 80 \text{ krad/s}$$

[b] $f_o = \dfrac{\omega_o}{2\pi} = 12.73\,\text{kHz}$

[c] $Q = \omega_o RC = (80{,}000)(3000)(62.5\times10^{-9}) = 15$

[d] $\omega_{c1} = \omega_o\left[-\dfrac{1}{2Q} + \sqrt{1 + \left(\dfrac{1}{2Q}\right)^2}\right] = 80{,}000\left[-\dfrac{1}{30} + \sqrt{1 + \dfrac{1}{900}}\right]$

$$= 77{,}377.77 \text{ rad/s}$$

[e] $f_{c1} = \dfrac{\omega_{c1}}{2\pi} = 12{,}315.05\,\text{Hz}$

[f] $\omega_{c2} = \omega_o\left[\dfrac{1}{2Q} + \sqrt{1 + \left(\dfrac{1}{2Q}\right)^2}\right] = 80{,}000\left[\dfrac{1}{30} + \sqrt{1 + \dfrac{1}{900}}\right]$

$$= 82{,}711.1 \text{ rad/s}$$

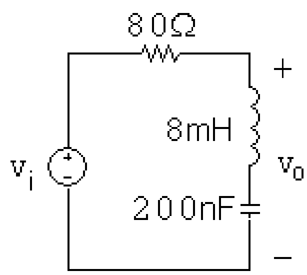
[g] $f_{c2} = \dfrac{\omega_{c1}}{2\pi} = 13{,}163.88\,\text{Hz}$

[h] $\beta = f_{c2} - f_{c1} = 848.8\,\text{Hz}$

P 14.39 **[a]** $\omega_o = \sqrt{1/LC} \quad \text{so} \quad L = \dfrac{1}{\omega_o^2 C} = \dfrac{1}{(25{,}000)^2(200\times10^{-9})} = 8\,\text{mH}$

$$Q = \frac{\omega_o}{\beta} \quad \text{so} \quad \beta = \frac{\omega_o}{Q} = \frac{25{,}000}{2.5} = 10{,}000 \text{ rad/s}$$

$$\beta = \frac{R}{L} \quad \text{so} \quad R = L\beta = (8\times10^{-3})(10{,}000) = 80\,\Omega$$



[b] From part (a), $\beta = 10{,}000$ rad/s.

$$\omega_{c1,2} = \pm\frac{\beta}{2} + \sqrt{\frac{\beta}{2} + \omega_o^2} = \pm\frac{10{,}000}{2} + \sqrt{\left(\frac{10{,}000}{2}\right)^2 + 25{,}000^2}$$

$$= \pm5000 + 25{,}495.1$$

$$\omega_{c1} = 20{,}495.1 \text{ rad/s} \qquad \omega_{c2} = 30{,}495.1 \text{ rad/s}$$

P 14.40 $H(j\omega) = \dfrac{\omega_o^2 - \omega^2}{\omega_o^2 - \omega^2 + j\omega\beta} = \dfrac{25{,}000^2 - \omega^2}{25{,}000^2 - \omega^2 + j\omega(10{,}000)}$

[a] $H(j25{,}000) = \dfrac{25{,}000^2 - 25{,}000^2}{25{,}000^2 - 25{,}000^2 + j(25{,}000)(10{,}000)} = 0$

$$V_o = (0)V_i \quad \therefore \quad v_o(t) = 0\,\text{mV}$$

[b] $H(j20{,}495.1) = \dfrac{25{,}000^2 - 20{,}495.1^2}{25{,}000^2 - 20{,}495.1^2 + j(20{,}495.1)(10{,}000)} = \dfrac{1}{\sqrt{2}}\angle -45^\circ$

$$V_o = \frac{1}{\sqrt{2}}\angle -45^\circ V_i \quad \therefore \quad v_o(t) = 7.071\cos(20{,}495.1t - 45^\circ)\,\text{V}$$

[c] $H(j30{,}495.1) = \dfrac{25{,}000^2 - 30{,}495.1^2}{25{,}000^2 - 30{,}495.1^2 + j(30{,}495.1)(10{,}000)} = \dfrac{1}{\sqrt{2}}\angle 45^\circ$

$$V_o = \frac{1}{\sqrt{2}}\angle 45^\circ V_i \quad \therefore \quad v_o(t) = 7.071\cos(30{,}495.1t + 45^\circ)\,\text{V}$$

[d] $H(j3125) = \dfrac{25{,}000^2 - 3125^2}{25{,}000^2 - 3125^2 + j(3125)(10{,}000)} = 0.9987\underline{/-2.91^\circ}$

$V_o = 0.9987\underline{/-2.91^\circ}V_i \quad \therefore \quad v_o(t) = 9.987\cos(3125t - 2.91^\circ)\,\text{mV}$

[e] $H(j200{,}000) = \dfrac{25{,}000^2 - 200{,}000^2}{25{,}000^2 - 200{,}000^2 + j(200{,}000)(10{,}000)} = 0.9987\underline{/2.91^\circ}$

$V_o = 0.9987\underline{/2.91^\circ}V_i \quad \therefore \quad v_o(t) = 9.987\cos(200{,}000t + 2.91^\circ)\,\text{mV}$

P 14.41 $H(j\omega) = \dfrac{j\omega\beta}{\omega_o^2 - \omega^2 + j\omega\beta} = \dfrac{j\omega(10{,}000)}{25{,}000^2 - \omega^2 + j\omega(10{,}000)}$

[a] $H(j25{,}000) = \dfrac{j(25{,}000)(10{,}000)}{25{,}000^2 - 25{,}000^2 + j(25{,}000)(10{,}000)} = 1$

$V_o = (1)V_i \quad \therefore \quad v_o(t) = 10\cos 25{,}000t\,\text{V}$

[b] $H(j20{,}495.1) = \dfrac{j(20{,}495.1)(10{,}000)}{25{,}000^2 - 20{,}495.1^2 + j(20{,}495.1)(10{,}000)} = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}V_i \quad \therefore \quad v_o(t) = 7.071\cos(20{,}495.1t + 45^\circ)\,\text{V}$

[c] $H(j30{,}495.1) = \dfrac{j(30{,}495.1)(10{,}000)}{25{,}000^2 - 30{,}495.1^2 + j(30{,}495.1)(10{,}000)} = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}V_i \quad \therefore \quad v_o(t) = 7.071\cos(30{,}495.1t - 45^\circ)\,\text{V}$

[d] $H(j3125) = \dfrac{j(3125)(10{,}000)}{25{,}000^2 - 3125^2 + j(3125)(10{,}000)} = 0.051\underline{/87.09^\circ}$

$V_o = 0.051\underline{/87.09^\circ}V_i \quad \therefore \quad v_o(t) = 0.51\cos(3125t + 87.09^\circ)\,\text{V}$

[e] $H(j200{,}000) = \dfrac{j(200{,}000)(10{,}000)}{25{,}000^2 - 200{,}000^2 + j(200{,}000)(10{,}000)} = 0.051\underline{/-87.09^\circ}$

$V_o = 0.051\underline{/-87.09^\circ}V_i \quad \therefore \quad v_o(t) = 0.51\cos(200{,}000t - 87.09^\circ)\,\text{V}$

P 14.42 **[a]** $\omega_o = 2\pi f_o = 8\pi\,\text{krad/s}$

$L = \dfrac{1}{\omega_o^2 C} = \dfrac{1}{(8000\pi)^2(0.5\times10^{-6})} = 3.17\,\text{mH}$

$R = \dfrac{Q}{\omega_o C} = \dfrac{5}{(8000\pi)(0.5\times10^{-6})} = 397.89\,\Omega$

[b] $f_{c2} = f_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 4000\left[\frac{1}{10} + \sqrt{1 + \frac{1}{100}}\right]$

$= 4.42\,\text{kHz}$

$f_{c1} = f_o\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 4000\left[-\frac{1}{10} + \sqrt{1 + \frac{1}{100}}\right]$

$= 3.62\,\text{kHz}$

[c] $\beta = f_{c2} - f_{c1} = 800\,\text{Hz}$

or

$\beta = \frac{f_o}{Q} = \frac{4000}{5} = 800\,\text{Hz}$

P 14.43 **[a]** $R_e = 397.89\|1000 = 284.63\,\Omega$

$Q = \omega_o R_e C = (8000\pi)(284.63)(0.5 \times 10^{-6}) = 3.58$

[b] $\beta = \frac{f_o}{Q} = \frac{4000}{3.58} = 1.12\,\text{kHz}$

[c] $f_{c2} = 4000\left[\frac{1}{7.16} + \sqrt{1 + \frac{1}{7.16^2}}\right] = 4.60\,\text{kHz}$

[d] $f_{c1} = 4000\left[-\frac{1}{7.16} + \sqrt{1 + \frac{1}{7.16^2}}\right] = 3.48\,\text{kHz}$

P 14.44 **[a]** We need $\omega_c = 2\pi(4000) = 25{,}132.74$ rad/s. There are several possible approaches – this one starts by choosing $L = 100\,\mu\text{H}$. Then,

$C = \frac{1}{[2\pi(4000)]^2(100 \times 10^{-6})} = 15.83\,\mu\text{F}$

Use the closest value from Appendix H, which is $22\,\mu$F, to give

$\omega_c = \sqrt{\frac{1}{100 \times 10^{-6})(22 \times 10^{-6})}} = 21{,}320.07$ rad/s so $f_c = 3393.19\,\text{Hz}$

Then, $R = \frac{Q}{\omega_o C} = \frac{5}{(21320.07)(22 \times 10^{-6})} = 10.66\,\Omega$

Use the closest value from Appendix H, which is $10\,\Omega$, to give

$Q = 10(21{,}320.07)(22 \times 10^{-6}) = 4.69$

[b] % error in $f_c = \frac{3393.19 - 4000}{4000}(100) = -15.2\%$

% error in $Q = \frac{4.69 - 5}{5}(100) = -6.2\%$

P 14.45 **[a]** Let $Z = \dfrac{R_L(sL + (1/sC))}{R_L + sL + (1/sC)}$

$$Z = \frac{R_L(s^2LC + 1)}{s^2LC + R_LCs + 1}$$

Then $H(s) = \dfrac{V_o}{V_i} = \dfrac{s^2R_LCL + R_L}{(R + R_L)LCs^2 + RR_LCs + R + R_L}$

Therefore

$$H(s) = \left(\frac{R_L}{R + R_L}\right) \cdot \frac{[s^2 + (1/LC)]}{\left[s^2 + \left(\frac{RR_L}{R+R_L}\right)\frac{s}{L} + \frac{1}{LC}\right]}$$

$$= \frac{K(s^2 + \omega_o^2)}{s^2 + \beta s + \omega_o^2}$$

where $\quad K = \dfrac{R_L}{R + R_L}; \quad \omega_o^2 = \dfrac{1}{LC}; \quad \beta = \left(\dfrac{RR_L}{R + R_L}\right)\dfrac{1}{L}$

[b] $\omega_o = \dfrac{1}{\sqrt{LC}}$

[c] $\beta = \left(\dfrac{RR_L}{R + R_L}\right)\dfrac{1}{L}$

[d] $Q = \dfrac{\omega_o}{\beta} = \dfrac{\omega_o L}{[RR_L/(R + R_L)]}$

[e] $H(j\omega) = \dfrac{K(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2) + j\beta\omega}$

$$H(j\omega_o) = 0$$

[f] $H(j0) = \dfrac{K\omega_o^2}{\omega_o^2} = K$

[g] $H(j\omega) = \dfrac{K\left[(\omega_o/\omega)^2 - 1\right]}{\left\{\left[(\omega_o/\omega)^2 - 1\right] + j\beta/\omega\right\}}$

$$H(j\infty) = \frac{-K}{-1} = K$$

[h] $H(j\omega) = \dfrac{K(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2) + j\beta\omega}$

$$H(j0) = H(j\infty) = K$$

Let ω_c represent a corner frequency. Then

$$|H(j\omega_c)| = \frac{K}{\sqrt{2}}$$

$$\therefore \quad \frac{K}{\sqrt{2}} = \frac{K(\omega_o^2 - \omega_c^2)}{\sqrt{(\omega_o^2 - \omega_c^2)^2 + \omega_c^2\beta^2}}$$

Squaring both sides leads to

$(\omega_o^2 - \omega_c^2)^2 = \omega_c^2\beta^2$ or $(\omega_o^2 - \omega_c^2) = \pm\omega_c\beta$

$$\therefore \quad \omega_c^2 \pm \omega_c\beta - \omega_o^2 = 0$$

or

$$\omega_c = \mp\frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

The two positive roots are

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2} \quad \text{and} \quad \omega_{c2} = \frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

where

$$\beta = \frac{RR_L}{R + R_L} \cdot \frac{1}{L} \text{ and } \omega_o^2 = \frac{1}{LC}$$

P 14.46 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(10^{-6})(4 \times 10^{-12})} = 0.25 \times 10^{18} = 25 \times 10^{16}$

$$\omega_o = 5 \times 10^8 = 500\,\text{Mrad/s}$$

$$\beta = \frac{RR_L}{R + R_L} \cdot \frac{1}{L} = \frac{(30)(150)}{180} \cdot \frac{1}{10^{-6}} = 25\,\text{Mrad/s} = 3.98\,\text{MHz}$$

$$Q = \frac{\omega_o}{\beta} = \frac{500\,\text{M}}{25\,\text{M}} = 20$$

[b] $H(j0) = \dfrac{R_L}{R + R_L} = \dfrac{150}{180} = 0.8333$

$$H(j\infty) = \frac{R_L}{R + R_L} = 0.8333$$

[c] $f_{c2} = \dfrac{250}{\pi}\left[\dfrac{1}{40} + \sqrt{1 + \dfrac{1}{1600}}\right] = 81.59\,\text{MHz}$

$$f_{c1} = \frac{250}{\pi}\left[-\frac{1}{40} + \sqrt{1 + \frac{1}{1600}}\right] = 77.61\,\text{MHz}$$

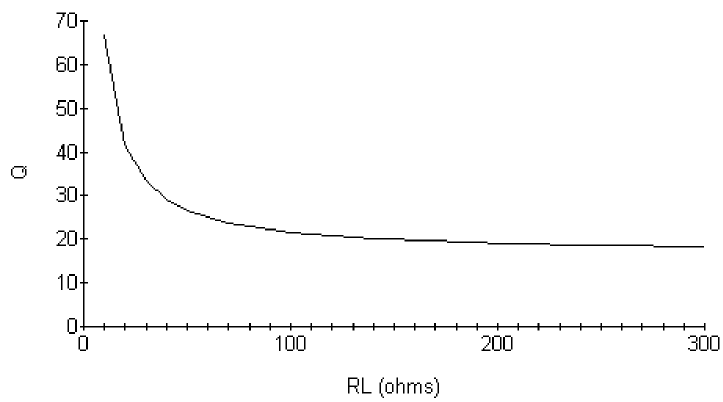
Check: $\beta = f_{c2} - f_{c1} = 3.98\,\text{MHz}.$

[d] $$Q = \frac{\omega_o}{\beta} = \frac{500\times 10^6}{\frac{RR_L}{R+R_L}\cdot\frac{1}{L}}$$

$$= \frac{500(R+R_L)}{RR_L} = \frac{50}{3}\left(1+\frac{30}{R_L}\right)$$

where R_L is in ohms.

[e]



P 14.47 **[a]** $\omega_o^2 = \dfrac{1}{LC} = 625\times 10^6$

$$\therefore \quad L = \frac{1}{(625\times 10^6)(25\times 10^{-9})} = 64\,\text{mH}$$

$$\frac{R_L}{R+R_L} = 0.9; \qquad \therefore \quad 0.1R_L = 0.9R$$

$$\therefore \qquad R_L = 9R \quad \therefore \quad R = \frac{500}{9} = 55.6\,\Omega$$

[b] $$\beta = \left(\frac{R_L}{R+R_L}\right)R\cdot\frac{1}{L} = 781.25 \text{ rad/s}$$

$$Q = \frac{\omega_o}{\beta} = \frac{25{,}000}{781.25} = 32$$

P 14.48 **[a]** $|H(j\omega)| = \dfrac{25\times 10^6}{\sqrt{(25\times 10^6-\omega^2)^2+(1000\omega)^2}} = 1$

$$\therefore \qquad 0 = 625\times 10^{12} = (25\times 10^6-\omega^2)^2+(1000\omega)^2$$

$$= -50\times 10^6\omega^2+\omega^4+10^6\omega^2$$

From the above equation it is obvious that $\omega = 0$ is one solution. The other is as follows:

$\omega^2 = 49\times 10^6 \quad$ so $\quad \omega = 7000\,\text{rad/s}$

[b] From the equation for $|H(j\omega)|$ in part (a), the frequency for which the magnitude is maximum is the frequency for which the denominator is minimum. This is the frequency for which the first derivative of the denominator with respect to ω is zero:

$$\frac{d}{d\omega}[(25\times10^6-\omega^2)^2+1000^2\omega^2]=4\omega^2+98\times10^6=0$$

Therefore, $\omega=\sqrt{98\times10^6/4}=4949.75\,\text{rad/s}$

[c] $|H(j4949.75)|=\dfrac{25\times10^6}{\sqrt{(25\times10^6-4949.75^2)^2+[1000(4949.75)]^2}}=5.025$

P 14.49 **[a]** $H(s)=\dfrac{sL}{R+sL+\frac{1}{sC}}=\dfrac{s^2LC}{RsC+s^2LC+1}=\dfrac{s^2}{s^2+\frac{R}{L}s+\frac{1}{LC}}$

[b] When $s=j\omega$ is very small (think of ω approaching 0),

$$H(s)\approx\frac{s^2}{\frac{1}{LC}}=0$$

[c] When $s=j\omega$ is very large (think of ω approaching ∞),

$$H(s)\approx\frac{s^2}{s^2}=1$$

[d] The magnitude of $H(s)$ approaches 0 as the frequency approaches 0, and approaches 1 as the frequency approaches ∞. Therefore, this circuit is behaving like a high pass filter when the output is the voltage across the inductor.

[e] $|H(j\omega_c)|=\dfrac{\omega_c^2}{\sqrt{(10^6-\omega_c^2)^2+(1500\omega_c)^2}}=\dfrac{1}{\sqrt{2}}$

$$2\omega_c^4=(10^6-\omega_c^2)^2+(1500\omega_c)^2=10^{12}-2\times10^6\omega_c^2+\omega_c^4+225\times10^4\omega_c^2$$

Simplifying, $\omega_c^4-25\times10^4\omega_c^2-10^{12}=0$

Solve for ω_c^2 and then ω_c:

$\omega_c^2=1{,}132{,}782.22$ so $\omega_c=1064.322$ rad/s

$f=169.4\,\text{Hz}$

P 14.50 **[a]** $H(s)=\dfrac{\frac{1}{sC}}{R+sL+\frac{1}{sC}}=\dfrac{1}{RsC+s^2LC+1}=\dfrac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$

[b] When $s=j\omega$ is very small (think of ω approaching 0),

$$H(s)\approx\frac{\frac{1}{LC}}{\frac{1}{LC}}=1$$

[c] When $s = j\omega$ is very large (think of ω approaching ∞),

$$H(s) \approx \frac{\frac{1}{LC}}{s^2} = 0$$

[d] The magnitude of $H(s)$ approaches 1 as the frequency approaches 0, and approaches 0 as the frequency approaches ∞. Therefore, this circuit is behaving like a low pass filter when the output is the voltage across the capacitor.

[e] $$|H(j\omega_c)| = \frac{10^6}{\sqrt{(10^6 - \omega_c^2)^2 + (1500\omega_c)^2}} = \frac{1}{\sqrt{2}}$$

$$2 \times 10^{12} = (10^6 - \omega_c^2)^2 + (1500\omega_c)^2 = 10^{12} - 2 \times 10^6\omega_c^2 + \omega_c^4 + 225 \times 10^4\omega_c^2$$

Simplifying, $\omega_c^4 + (1500^2 - 2 \times 10^6)\omega_c^2 - 10^{12} = 0$

Solve for ω_c^2 and then ω_c:

$\omega_c^2 = 882{,}782.22$ so $\omega_c = 939.565$ rad/s

$f = 149.54\,\text{Hz}$

P 14.51 **[a]** Use the cutoff frequencies to calculate the bandwidth:

$\omega_{c1} = 2\pi(697) = 4379.38$ rad/s $\qquad \omega_{c2} = 2\pi(941) = 5912.48$ rad/s

Thus $\beta = \omega_{c2} - \omega_{c1} = 1533.10$ rad/s

Calculate inductance using Eq. (14.32) and capacitance using Eq. (14.31):

$$L = \frac{R}{\beta} = \frac{600}{1533.10} = 0.39\,\text{H}$$

$$C = \frac{1}{L\omega_{c1}\omega_{c2}} = \frac{1}{(0.39)(4379.38)(5912.48)} = 0.10\,\mu\text{F}$$

[b] At the outermost two frequencies in the low-frequency group (687 Hz and 941 Hz) the amplitudes are

$$|\text{V}_{697\text{Hz}}| = |V_{941\text{Hz}}| = \frac{|V_{\text{peak}}|}{\sqrt{2}} = 0.707|V_{\text{peak}}|$$

because these are cutoff frequencies. We calculate the amplitudes at the other two low frequencies using Eq. (14.32):

$$|V| = (|V_{\text{peak}}|)(|H(j\omega)|) = |V_{\text{peak}}|\frac{\omega\beta}{\sqrt{(\omega_o^2 - \omega^2)^2 + (\omega\beta)^2}}$$

Therefore

$$|V_{770\text{Hz}}| = |V_{\text{peak}}| = \frac{(4838.05)(1533.10)}{\sqrt{(5088.52^2 - 4838.05^2)^2 + [(4838.05)(1533.10)]^2}}$$

$$= 0.948|V_{\text{peak}}|$$

and

$$|V_{852\text{Hz}}| = |V_{\text{peak}}| = \frac{(5353.27)(1533.10)}{\sqrt{(5088.52^2 - 5353.27^2)^2 + [(5353.27)(1533.10)]^2}}$$

$$= 0.948|V_{\text{peak}}|$$

It is not a coincidence that these two magnitudes are the same. The frequencies in both bands of the DTMF system were carefully chosen to produce this type of predictable behavior with linear filters. In other words, the frequencies were chosen to be equally far apart with respect to the response produced by a linear filter. Most musical scales consist of tones designed with this dame property – note intervals are selected to place the notes equally far apart. That is why the DTMF tones remind us of musical notes! Unlike musical scales, DTMF frequencies were selected to be harmonically unrelated, to lower the risk of misidentifying a tone's frequency if the circuit elements are not perfectly linear.

[**c**] The high-band frequency closest to the low-frequency band is 1209 Hz. The amplitude of a tone with this frequency is

$$|V_{1209\text{Hz}}| = |V_{\text{peak}}| = \frac{(7596.37)(1533.10)}{\sqrt{(5088.52^2 - 7596.37^2)^2 + [(7596.37)(1533.10)]^2}}$$

$$= 0.344|V_{\text{peak}}|$$

This is less than one half the amplitude of the signals with the low-band cutoff frequencies, ensuring adequate separation of the bands.

P 14.52 The cutoff frequencies and bandwidth are

$$\omega_{c_1} = 2\pi(1209) = 7596\,\text{rad/s}$$

$$\omega_{c_2} = 2\pi(1633) = 10.26\,\text{krad/s}$$

$$\beta = \omega_{c_2} - \omega_{c_1} = 2664\,\text{rad/s}$$

Telephone circuits always have $R = 600\,\Omega$. Therefore, the filters inductance and capacitance values are

$$L = \frac{R}{\beta} = \frac{600}{2664} = 0.225\,\text{H}$$

$$C = \frac{1}{\omega_{c_1}\omega_{c_2}L} = 0.057\,\mu\text{F}$$

At the highest of the low-band frequencies, 941 Hz, the amplitude is

$$|V_\omega| = |V_{\text{peak}}|\frac{\omega\beta}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2\beta^2}}$$

where $\omega_o = \sqrt{\omega_{c_1}\omega_{c_2}}$. Thus,

$$|V_\omega| = \frac{|V_{\text{peak}}|(5912)(2664)}{\sqrt{[(8828)^2 - (5912)^2]^2 + [(5912)(2664)]^2}}$$

$$= 0.344\ |V_{\text{peak}}|$$

Again it is not coincidental that this result is the same as the response of the low-band filter to the lowest of the high-band frequencies.

P 14.53 From Problem 14.51 the response to the largest of the DTMF low-band tones is $0.948|V_{\text{peak}}|$. The response to the 20 Hz tone is

$$|V_{20\text{Hz}}| = \frac{|V_{\text{peak}}|(125.6)(1533)}{[(5089^2 - 125.6^2)^2 + [(125.6)(1533)]^2]^{1/2}}$$

$$= 0.00744|V_{\text{peak}}|$$

$$\therefore\quad \frac{|V_{20\text{Hz}}|}{|V_{770\text{Hz}}|} = \frac{|V_{20\text{Hz}}|}{|V_{852\text{Hz}}|} = \frac{0.00744|V_{\text{peak}}|}{0.948|V_{\text{peak}}|} = 0.5$$

$$\therefore\quad |V_{20\text{Hz}}| = 63.7|V_{770\text{Hz}}|$$

Thus, the 20Hz signal can be 63.7 times as large as the DTMF tones.