10.2 Part 2

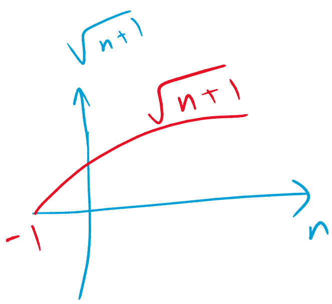
$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(1 - \frac{1}{\sqrt{n+1}}\right) = 1 - \left(\lim_{n\to\infty}\frac{1}{\sqrt{n+1}}\right) = 1 = L$$

Hence, $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$ converges to 1 by n^{th} partial sum Test

② Use n^{th} partial sum Test to check conv/Div

(37) $\sum_{n=1}^{\infty}\left(\ln\sqrt{n+1} - \ln\sqrt{n}\right)$ $\infty - \infty$

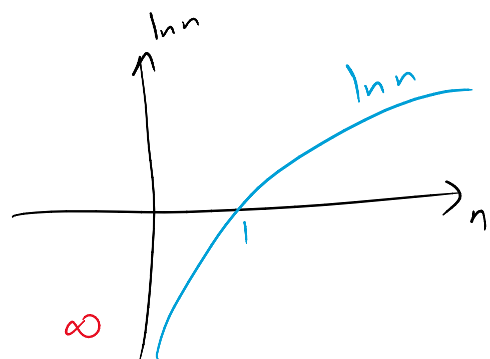
$$S_n = (\cancel{\ln\sqrt{2}} - \ln\sqrt{1}) + (\cancel{\ln\sqrt{3}} - \cancel{\ln\sqrt{2}}) + \dots + (\ln\sqrt{n+1} - \ln\sqrt{n})$$



$= -\ln\sqrt{1} + \ln\sqrt{n+1}$

$= -\ln 1 + \ln\sqrt{n+1}$

$= 0 + \ln\sqrt{n+1}$



$$S_n = \ln\sqrt{n+1}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\ln\sqrt{n+1} = \ln\left(\lim_{n\to\infty}\sqrt{n+1}\right) = \infty$$

Hence, $\sum_{n=1}^{\infty}\left(\ln\sqrt{n+1} - \ln\sqrt{n}\right)$ div by n^{th} partial Sum Test.

$\lim_{n\to\infty} a_n = 0 \Rightarrow$ we can not use Test 2

③ $\sum^{\infty} \frac{1}{n(n+1)}$ a_n

$S_n = \frac{1}{2} + \frac{1}{2(3)} + \dots$

Math 141 / 8.4 partial fraction

$$\sum_{n=1} \underbrace{\frac{1}{n(n+1)}}$$

partial fraction

$$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

$n=0$ $\qquad$ $n=-1$

$$A = \frac{1}{\boxed{0}+1} = 1$$

$$B = \frac{1}{\boxed{-1}} = -1$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

Telscop

$$S_n = \left(1 - \cancel{\tfrac{1}{2}}\right) + \left(\cancel{\tfrac{1}{2}} - \cancel{\tfrac{1}{3}}\right) + \left(\cancel{\tfrac{1}{3}} - \cancel{\tfrac{1}{4}}\right) + \cdots + \left(\cancel{\tfrac{1}{n}} - \frac{1}{n+1}\right)$$

$$= 1 - \frac{1}{n+1}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right) = 1 - 0 = 1$$

Hence, $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges to 1 by the n^{th} partial Sum Test

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Test 2 : n^{th} term test for div

$\sum_{n=1}^{\infty} a_n$?? conv/div

If $\lim_{n\to\infty} a_n \neq 0$ then $\sum_{n=1}^{\infty} a_n$ div.

If $\lim a_n$ fails to exist then $\sum a_n$ div

If $\lim_{n\to\infty} a_n$ fails to exist ($\infty, -\infty$, DNE) then $\sum_{n=1} a_n$ div

$\lim_{n\to\infty} \frac{n^2-7}{n+1} = \infty$

$\lim_{n\to\infty} \frac{n}{n^3+1} = 0$

$\lim_{n\to\infty} \frac{2n^2+1}{1-4n^2}$ $\frac{2}{-4} = \frac{-1}{2}$

Exp check for Conv./Div

① $\sum_{n=1}^{\infty} \frac{n+1}{n}$ $\Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{D\,n+1}{D\,n'} = \frac{1}{1} = 1 \neq 0$

$\Rightarrow \sum_{n=1}^{\infty} \frac{n+1}{n}$ div by n^{th} term test.

② $\sum_{n=1}^{\infty} \sqrt{n}$ $\Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt{n} = \infty$ so

$\sum \sqrt{n}$ div by n^{th} term test

(64)

③ $\sum_{n=1}^{\infty} \frac{2^n + 4^n}{3^n + 4^n}$ $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{2^n + 4^n}{3^n + 4^n}$

$$\lim_{n\to\infty} \frac{\frac{2^n}{4^n} + 1}{\frac{3^n}{4^n} + 1} = \lim_{n\to\infty} \frac{\left(\frac{2}{4}\right)^n + 1}{\left(\frac{3}{4}\right)^n + 1} = \frac{1 + \lim_{n\to\infty} \left(\frac{1}{2}\right)^n}{1 + \lim_{n\to\infty} \left(\frac{3}{4}\right)^n} = 1$$

So our infinite series div by n^{th} term test

③ $\sum_{n=1}^{\infty} (-1)^{n+1}$ $S_n = 1 + (-1) + 1 + (-1) + 1 + \dots + (-1)^{n+1}$

$n+1$ WE $\lim S_n$ DNE

$n=1$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} (-1)^{n+1}$ DNE

$\lim_{n\to\infty} S_n$ DNE

so $\sum_{n=1}^{\infty} (-1)^{n+1}$ div

$$\sum_{n=1}^{\infty} (-1)^{n+1} = 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 - \cdots$$
$$= (1-1) + (1-1) + (1-1) + \cdots$$
$$= 0 + 0 + 0 + \cdots$$
$$= 0$$

$$\sum_{n=1}^{\infty} (-1)^{n+1} = 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + \cdots$$
$$= 1 + (-1+1) + (-1+1) + (-1+1) + \cdots$$
$$= 1 + 0 + 0 + 0 + \cdots$$
$$= 1$$

Div

Harmonic Series div $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n}$

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) + \left(\frac{1}{9} + \cdots + \frac{1}{16}\right) + \cdots$$
$$\boxed{\geq} 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right) + \left(\frac{1}{16} + \frac{1}{16} + \cdots + \frac{1}{16}\right) + \cdots$$
$$1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots$$

$1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\cdots$

$$\sum_{n=1}^{\infty} \frac{1}{n} \geq \infty$$

div

$$\sum_{n=1}^{\infty} a_n \ ??$$

Th. If $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty} a_n$ may div, may conv.

• If $\sum_{n=1}^{\infty} a_n$ conv. then $\lim_{n\to\infty} a_n = 0$

Converse is not true $\Rightarrow$ means

if $\lim_{n\to\infty} a_n = 0$ this does not mean $\sum_{n=1}^{\infty} a_n$ conv

??

Exp Harmonic Series

$\lim a_n = \lim \frac{1}{n} = 0$

Exp Harmonic series

$\sum_{n=1}^{\infty} \frac{1}{n}$ (a_n) div

but $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} = 0$

$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$ converg

$\lim_{n\to\infty} \frac{1}{n(n+1)} = 0$

$\sum_{n=1}^{\infty} \sqrt{n+1} - \sqrt{n}$ → div

$\lim_{n\to\infty} a_n \neq 0$

Sequence a_n	Series $\sum_{n=1}^{\infty} a_n$
$a_n = \frac{1}{n}$	$\sum \frac{1}{n}$ div
$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} = 0$	
a_n converges to zero	

