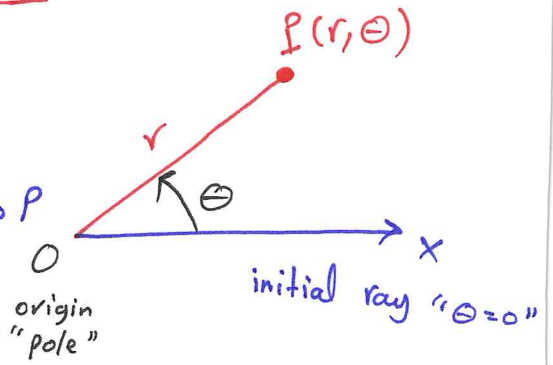


11.3 Polar Coordinates

①

$P(r, \theta)$ is Polar coordinate

- r is the directed distance from O to P
"r can be negative"



- θ is the directed angle from the initial ray to OP .

Polar coordinates are not unique:

$$P(r, \theta) = P(r, \theta + 2\pi m), \quad m = 0, \pm 1, \pm 2, \dots$$

$$= P(-r, (\theta \pm \pi) + 2\pi m) = P(-r, \theta \pm (2m+1)\pi)$$

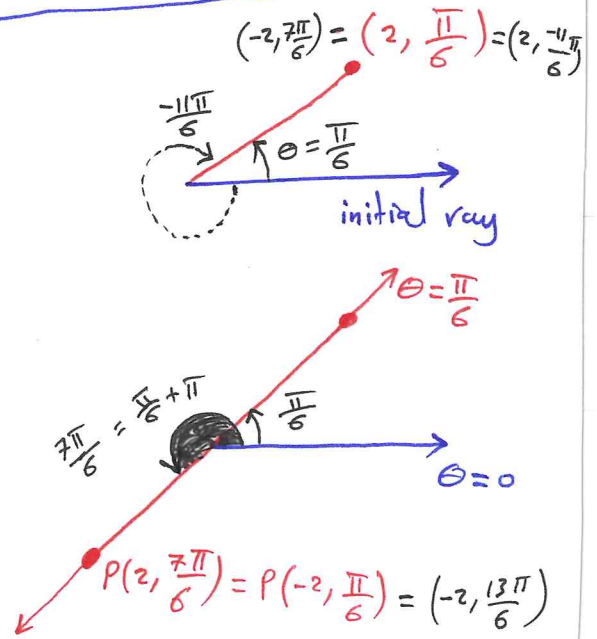
Exp Polar coordinate $P(2, \frac{\pi}{6})$

$$P(2, \frac{\pi}{6}) = (2, \frac{\pi}{6} - 2\pi) = (2, -\frac{11\pi}{6})$$

$$= (-2, \frac{\pi}{6} + \pi) = (-2, \frac{7\pi}{6})$$

$$P(-2, \frac{\pi}{6}) = (-2, \frac{\pi}{6} + 2\pi) = (-2, \frac{13\pi}{6})$$

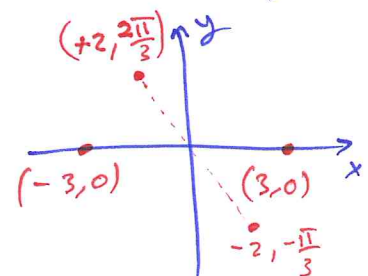
$$= (2, \frac{\pi}{6} + \pi) = (2, \frac{7\pi}{6})$$



Exp The following polar coordinates pair are the same:

$$P(r, \theta) = P(3, 0) = P(-3, \pi) \quad / \quad (-3, 0) = (-3, 2\pi)$$

$$(2, \frac{2\pi}{3}) = (-2, -\frac{\pi}{3}) \quad \dots$$



Polar Equations and Graphs:

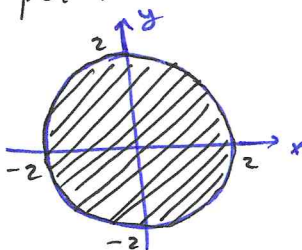
(2)

$r = a$ circle of radius $|a|$ centered at 0

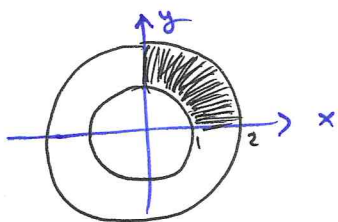
$\theta = \theta_0$ Line through 0 making an angle θ_0 with the initial ray.

Exp Graph the set of points whose polar coordinates satisfy:

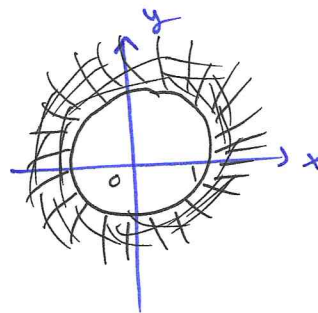
(a) $0 \leq r \leq 2$



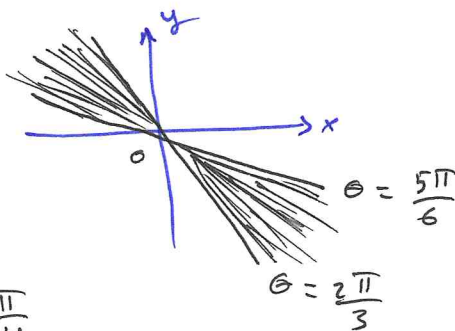
(b) $1 \leq r \leq 2$ and $0 \leq \theta \leq \frac{\pi}{2}$



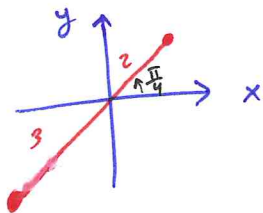
(c) $r \geq 1$



(d) $\frac{2\pi}{3} \leq \theta \leq \frac{5\pi}{6}$



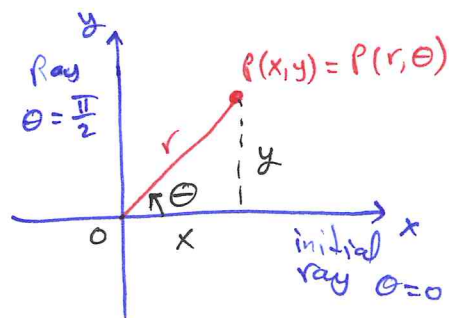
(e) $-3 \leq r \leq 2$ and $\theta = \frac{\pi}{4}$



Equations Relating Polar and Cartesian Coordinates:

$x = r \cos \theta$, $y = r \sin \theta$

$r^2 = x^2 + y^2$, $\tan \theta = \frac{y}{x}$



Exp Replace the polar equation with Cartesian equations: (3)

a) $r \cos \theta = 2 \Rightarrow x = 2$ vertical line passes (2,0)

b) $r = -3 \sec \theta \Rightarrow r = \frac{-3}{\cos \theta} \Rightarrow r \cos \theta = -3 \Leftrightarrow x = -3$

c) $r^2 = 1 \Rightarrow x^2 + y^2 = 1$ circle center (0,0) and $r=1$

d) $r = \csc \theta e^{r \cos \theta} \Rightarrow r \sin \theta = e^{r \cos \theta} \Rightarrow y = e^x \neq$

Exp Replace the Cartesian equation with equivalent polar eq.

a) $x = y \Rightarrow r \cos \theta = r \sin \theta \Rightarrow \cos \theta = \sin \theta \Rightarrow \theta = \frac{\pi}{4}$

b) $x^2 + y^2 = 4 \Rightarrow r^2 = 4 \Leftrightarrow r = \pm 2$

c) $(x-5)^2 + y^2 = 25$

$x^2 - 10x + 25 + y^2 = 25 \Leftrightarrow x^2 + y^2 = 10x$

$\Leftrightarrow r^2 = 10 r \cos \theta \Leftrightarrow r = 10 \cos \theta$
