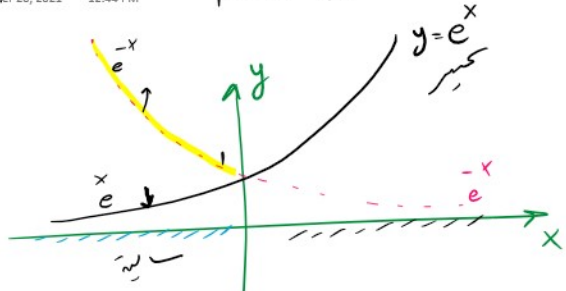
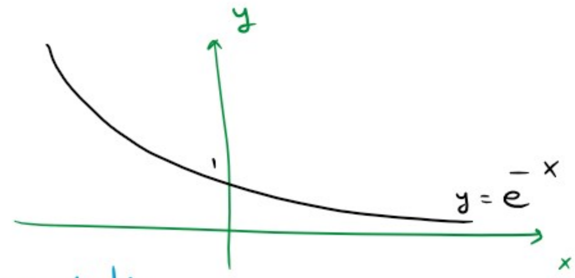
7.7 Hyperbolic functions

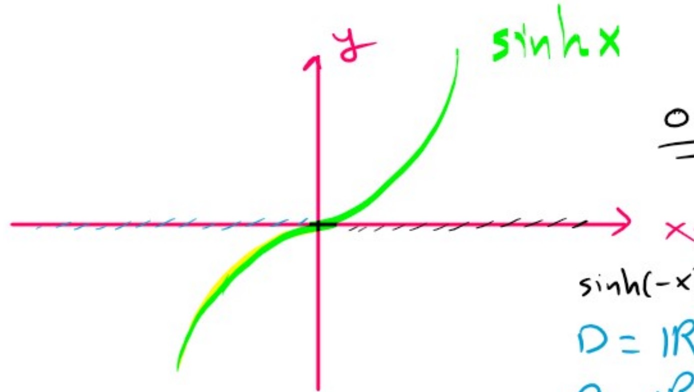
Monday, December 20, 2021 12:44 PM





h: hyperbolic

① $\sinh x = \dfrac{e^x - e^{-x}}{2}$



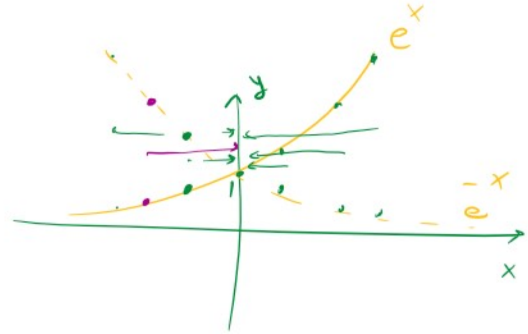
odd

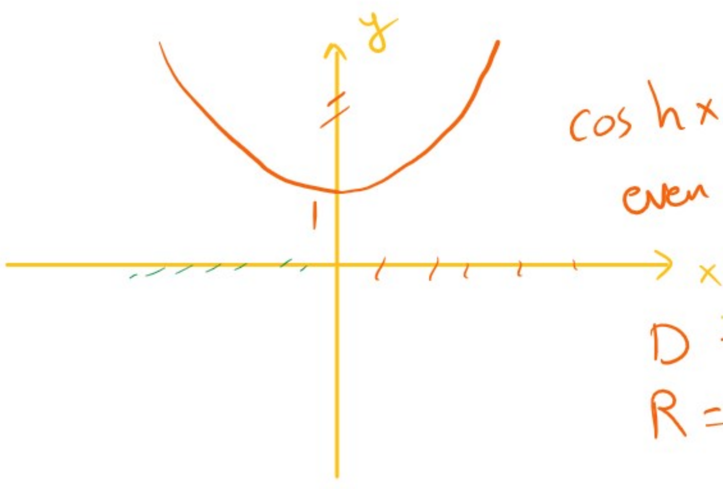
$\sinh(-x) = -\sinh x$

$D = \mathbb{R}$

$R = \mathbb{R}$

② $\cosh x = \dfrac{e^x + e^{-x}}{2}$





cosh x

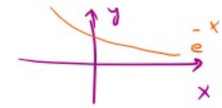
even $\Rightarrow \cosh(-x) = \cosh(x)$

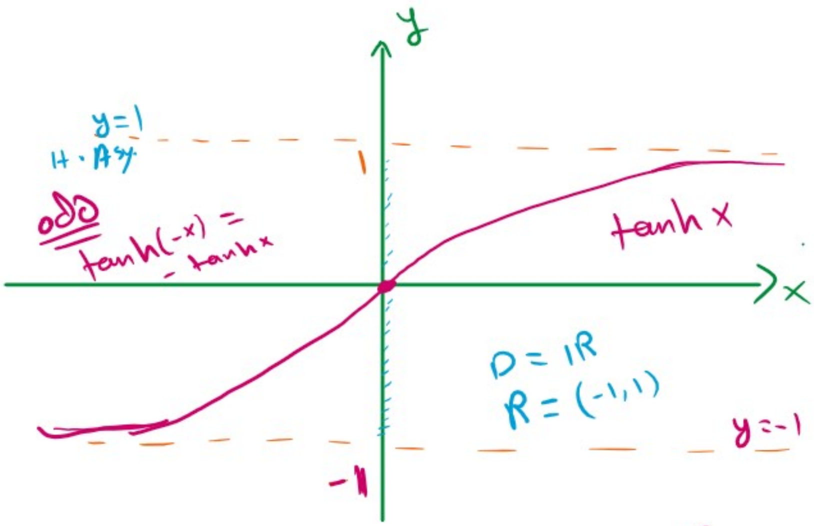
$D = \mathbb{R}$

$R = [1, \infty)$

③ $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$e + e$





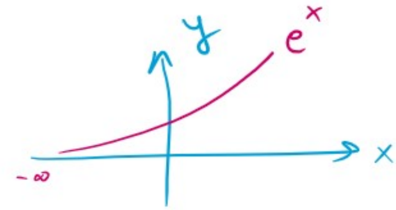
odd
$\tanh(-x) = -\tanh x$

$D = \mathbb{R}$
$R = (-1,1)$

$$\lim_{x\to\infty} \tanh x =$$

$$\lim_{x\to\infty} \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

$$\lim_{x\to\infty} \frac{e^{x}}{e^{x}} = \lim 1 = 1$$



$$\lim_{x\to-\infty} \tanh x = \lim_{x\to-\infty} \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

$$= \lim_{x\to-\infty} \frac{-e^{-x}}{e^{-x}} = -\lim_{x\to-\infty} 1 = -1$$

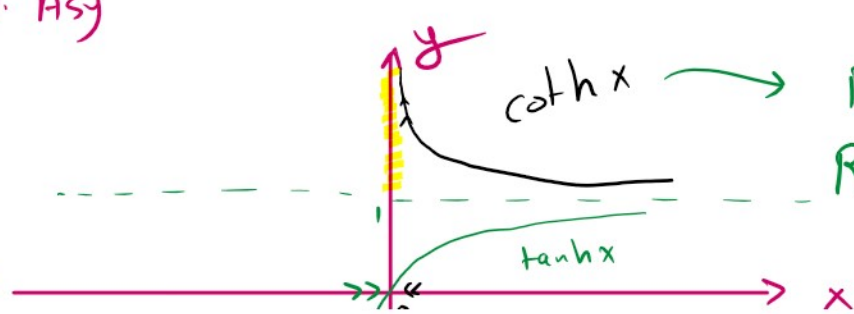
$$\tanh 0 = \frac{e^{0} - e^{0}}{e^{0} + e^{0}} = \frac{1-1}{1+1} = \frac{0}{2} = 0$$

④ $$\coth x = \frac{1}{\tanh x} = \frac{\cosh x}{\sinh x} = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

$x = 0$ is V. Asy

$$\lim_{x\to0^+} \coth x = \infty$$

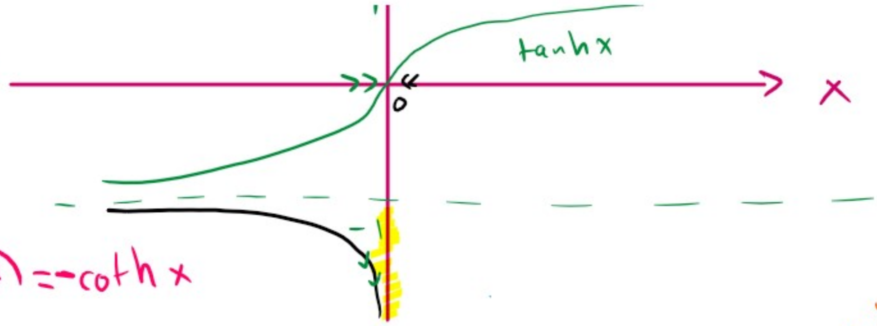
$$\lim \coth x = -\infty$$



$D = \mathbb{R} \setminus \{0\}$
$R = (-\infty,-1) \cup (1,\infty)$

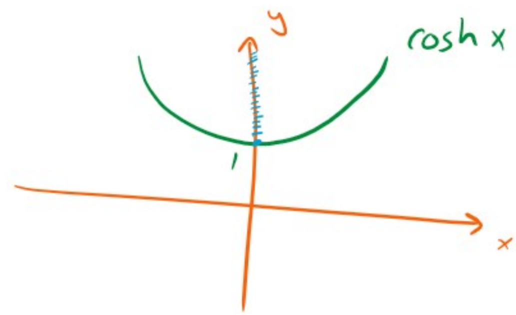
$x \to 0^+$

$$\lim_{x \to 0^-} \coth x = -\infty$$



$\coth(-x) = -\coth x$

$$⑤ \quad \operatorname{sech} x = \frac{1}{\cosh x}$$

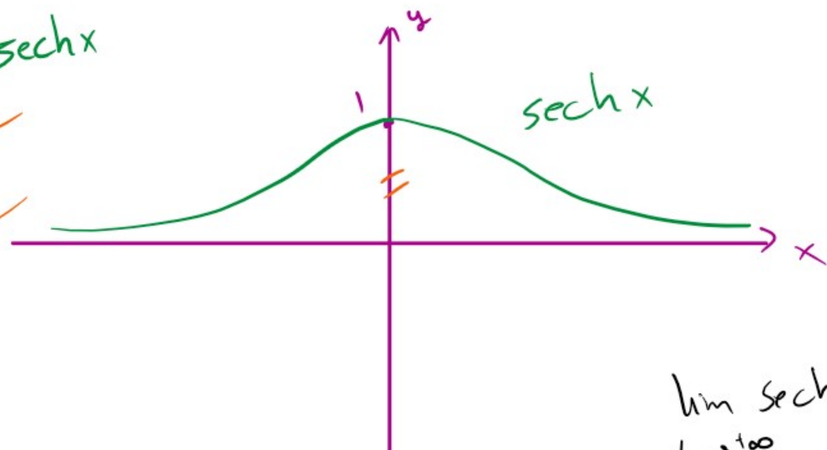


even

$\operatorname{sech}(-x) = \operatorname{sech} x$

$D = \mathbb{R}$ ✓

$R = (0, 1]$ ✓



$y = 0$ is H. Asy

$$\lim_{x \to \pm\infty} \operatorname{sech} x = 0$$

$$\lim_{x \to \infty} \frac{1}{\cosh x} = \lim_{x \to \infty} \frac{2}{e^{x} + e^{-x}}$$

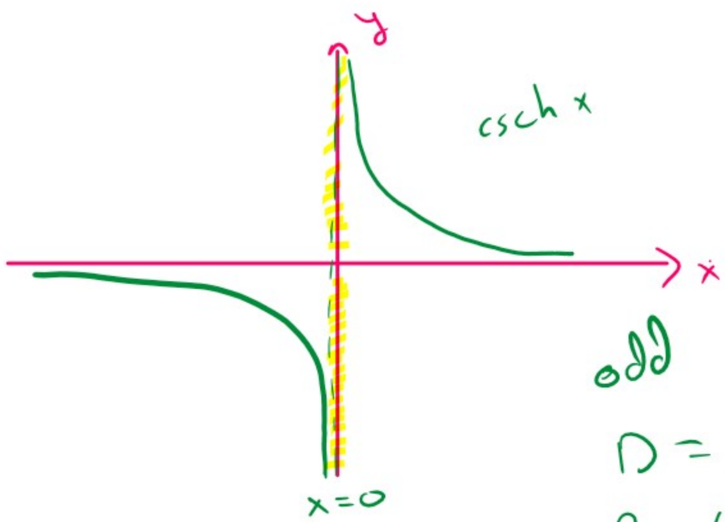
$$= \lim_{x \to \infty} \frac{2}{e^{x}} = 0$$

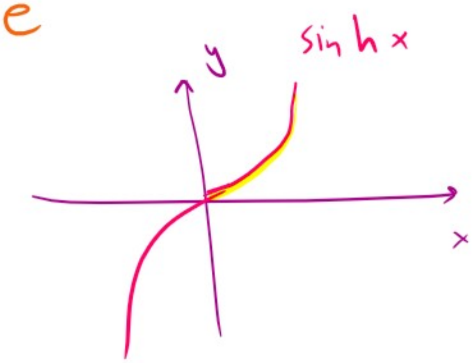
$$\lim_{x \to -\infty} \frac{2}{e^{x} + e^{-x}} = \frac{2}{\infty} = 0$$

$e^{x} \to 0$

$$⑥ \quad \operatorname{csch} x = \frac{1}{\ldots} = \frac{2}{\ldots}$$

⑥ $\operatorname{csch} x = \dfrac{1}{\sinh x} = \dfrac{2}{e^{x} - e^{-x}}$





odd $\Rightarrow \operatorname{csch}(-x) = -\operatorname{csh} x$

$D = \mathbb{R} \setminus \{0\}$

$R = (-\infty, 0) \cup (0, \infty)$

- $\cos^2 x + \sin^2 x = 1$
- $\sin 2x = 2 \sin x \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x$

  $= 2\cos^2 x - 1$

  $= 1 - 2\sin^2 x$
- $1 + \tan^2 x = \sec^2 x$
- $\cot^2 x + 1 = \csc^2 x$

$\boxed{1} \Rightarrow \cosh^2 x - \sinh^2 x = 1$

$\boxed{2}$ $\sinh 2x = 2 \sinh x \cosh x$

$\boxed{3}$ $\cosh 2x = \cosh^2 x + \sinh^2 x$

$= 2\cosh^2 x - 1$

$= 1 + 2\sinh^2 x$

$\boxed{4}$ $1 - \tanh^2 x = \operatorname{sech}^2 x$

$\boxed{5}$ $-\coth^2 x + 1 = -\operatorname{csch}^2 x$

$1 = \coth^2 x - \operatorname{csch}^2 x$

$1 = \coth x - \operatorname{csch} x$

Proof ① $\cosh^2 x - \sinh^2 x = 1$

$$\left(\frac{e^{x}+e^{-x}}{2}\right)^2 - \left(\frac{e^{x}-e^{-x}}{2}\right)^2 \overset{?}{=} 1$$

$$\frac{e^{2x}+2e^{x}e^{-x}+e^{-2x}}{4} - \frac{e^{2x}-2e^{x}e^{-x}+e^{-2x}}{4} \overset{?}{=} 1$$

$$\frac{e^{2x}+2+e^{-2x}}{4} - \frac{e^{2x}-2+e^{-2x}}{4} \overset{?}{=} 1$$

$$\frac{\cancel{e^{2x}}+2+\cancel{e^{-2x}}-\cancel{e^{2x}}+2-\cancel{e^{-2x}}}{4} \overset{?}{=} 1$$

$$\frac{4}{4} = 1$$

① $(\sinh x)' = \cosh x \Rightarrow \int \cosh x \, dx = \sinh x + c$

$(\quad)'$

② $(\cosh x)' = \sinh x$ $\Rightarrow \int \sinh x \, dx = \cosh x + c$

③ $(\tanh x)' = \operatorname{sech}^2 x$ $\Rightarrow \int \operatorname{sech}^2 x \, dx = \tanh x + c$

④ $(\coth x)' = -\operatorname{csch}^2 x$ $\Rightarrow \int \operatorname{csch}^2 x \, dx = -\coth x + c$

⑤ $(\operatorname{sech} x)' = -\operatorname{sech} x \tanh x$ $\Rightarrow \int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + c$

⑥ $(\operatorname{csch} x)' = -\operatorname{csch} x \coth x$ $\Rightarrow \int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x + c$

Proof $(\sinh x)' \stackrel{?}{=} \cosh x$

$$\left(\frac{e^x - e^{-x}}{2} \right)' = \frac{e^x - e^{-x}(-1)}{2} = \frac{e^x + e^{-x}}{2} = \cosh x$$

Exp Find y' if ① $y = \ln(\sinh\sqrt{x})$

$$y' = \frac{(\cosh\sqrt{x})\,\frac{1}{2\sqrt{x}}}{\sinh\sqrt{x}}$$

$$= \frac{\cosh\sqrt{x}}{\sinh\sqrt{x}}\,\frac{1}{2\sqrt{x}}$$

$$= \frac{\coth\sqrt{x}}{2\sqrt{x}}$$

② $y = 4\cosh\frac{x^3}{3}$

$$y' = 4\sinh\frac{x^3}{3}\left(\frac{\cancel{3}}{\cancel{3}}x^2\right)$$

$$= 4x^2\sinh\frac{x^3}{3}$$

Exp ① $\int \sinh 2x\,dx = \int \sinh u\,\frac{du}{2}$

$u = 2x$

$du = 2\,dx$

$\frac{du}{2} = dx$

$\int \sinh u\,du$

$$= \frac{1}{2} \int \sinh u \, du$$

$$= \frac{1}{2} \cosh u + C$$

$$= \frac{1}{2} \cosh(2x) + C$$

2) $\int \coth x \, dx = \int \frac{\cosh x \, dx}{\sinh x}$

or

$u = \sinh x$

$du = \cosh x \, dx$

$$\int \frac{du}{u} = \ln|u| + C$$

$$= \ln|\sinh x| + C$$

Exp 15) Find y' if $y = 2\sqrt{t} \tanh \sqrt{t}$

$$y' = \cancel{2\sqrt{t}} \underbrace{\operatorname{sech}^2\sqrt{t} \left(\frac{1}{\cancel{2\sqrt{t}}}\right)}_{\text{مشتقة الثاني}} + \underbrace{\tanh \sqrt{t}}_{\text{ثاني}} \; \frac{\cancel{2}}{\cancel{2}\sqrt{t}}$$

الأول

$$= \operatorname{sech}^2\sqrt{t} + \frac{1}{\sqrt{t}} \tanh \sqrt{t}$$

$\int^{\ln 2}$

54

$$\int_0^{\ln 2} 4\, e^{-\theta} \sinh\theta \, d\theta$$

$$\int_0^{\ln 2} \cancel{4}^{2}\, e^{-\theta} \left(\frac{e^{\theta} - e^{-\theta}}{\cancel{2}_{1}} \right) d\theta$$

$$2 \int_0^{\ln 2} \left(1 - e^{-2\theta} \right) d\theta$$

$$2 \left(\theta - \frac{e^{-2\theta}}{-2} \right) \Big|_0^{\ln 2}$$

$$2 \left(\theta + \frac{1}{2} e^{-2\theta} \right) \Big|_0^{\ln 2}$$

$$2 \left[\left(\ln 2 + \frac{1}{2} e^{-2\ln 2} \right) - \left(0 + \frac{1}{2} e^{0} \right) \right]$$

$$2 \left[\ln 2 + \frac{1}{2} e^{\ln 2^{-2}} - \frac{1}{2} \right]$$

$$2\left[\ln 2 + \frac{1}{2}(e) - \frac{1}{2}\right]$$

$$2\left[\ln 2 + \frac{1}{2}\cdot 2^{-2} - \frac{1}{2}\right]$$

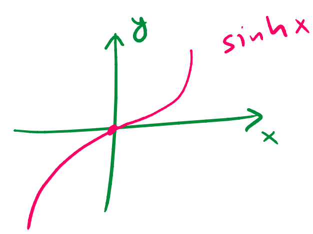
$$2\ln 2 + 2^{-2} - 1$$

$$\ln 4 + \frac{1}{4} - 1$$

$$\ln 4 - \frac{3}{4}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \qquad \frac{0}{0}$$

$$\lim_{x\to 0} \frac{\sinh x}{x} = 1 \qquad \frac{0}{0}$$



$$\lim_{x\to 0} \frac{\cosh x}{1} = \cosh 0$$

$$= 1$$