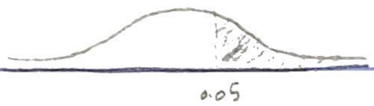
exercises :

Q1: Find the following chi-squared distribution values :

a. $\chi^2_{0.05}$ with df = 5 .



$\chi^2_{0.05} = 11.07$

b. $\chi^2_{0.025}$ with df = 15 = 27.488 .

c. $\chi^2_{0.975}$ with df = 20 : 9.591

d. $\chi^2_{0.01}$ with df = 10 : 23.209

e. $\chi^2_{0.95}$ with df = 18 : 9.390

Q2: a sample of 20 items provides a $\boxed{\text{sample standard deviation of } \overset{S}{5}}$.

a. compute a 0.90 CI estimate of the pop. variance σ^2 . df = 19 ($1-\alpha = 0.9 \rightarrow \frac{\alpha}{2} = 0.05$)

$$0.90 \text{ CI of } \sigma^2 = \left(\frac{(n-1)s^2}{\chi^2_{0.05}} , \frac{(n-1)s^2}{\chi^2_{0.95}} \right) = \left(\frac{19(25)}{30.144} , \frac{(19)(25)}{10.117} \right) = (15.76 , 46.95)$$

b. compute a 0.95 CI estimate of the pop. variance σ^2 : $1-\alpha = 0.95 \rightarrow \alpha = 0.05 \rightarrow \frac{\alpha}{2} = 0.0$

$$0.95 \text{ CI of } \sigma^2 = \left(\frac{475}{\chi_{0.025}} , \frac{475}{\chi_{0.975}} \right) = \left(\frac{475}{32.852} , \frac{475}{8.907} \right) = (14.46 , 53.33)$$

c. compute a 0.95 CI estimate of the pop. standard deviation σ :

$$0.95 \text{ CI of } \sigma = (\sqrt{14.46} , \sqrt{53.33}) = (3.80 , 7.30) .$$

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Q3: a sample of $\underset{n}{16}$ items provides a sample standard deviation of 9.5 $\overset{s}{}$.
Test the following hypothesis using $\alpha = 0.05$. What is your conclusion?
use both the p-value approach and the critical value approach:

$$H_0: \sigma^2 \le 50 \quad (\sigma_0^2) \qquad \text{upper tail test.}$$
$$H_1: \sigma^2 > 50.$$

→ test statistic: $\chi^2 = \frac{(n-1)S^2}{\sigma_0^2} = \frac{15(9.5)^2}{50} = \frac{1353.75}{50} = 27.08$

By critical value approach: Reject if $\chi^2 > \chi^2_\alpha$

$\chi^2_\alpha = \chi^2_{0.05}$ with $df = 15$ = 24.996.

$\Rightarrow$ 27.08 > 24.996 so we Reject H_0 ($\alpha = 0.05$).

By p-value approach: Reject if p-value $\le \alpha$.

find p-value with df = 15

χ^2_{test} وبشوف وين

df	0.05	0.025
15	24.996	27.488

p-value

p-value $\in$ (0.025, 0.05)

p-value $\le \alpha$

p-value $\le$ 0.05

So we Reject H_0 ($\alpha = 0.05$)

Q4: a sample of 18 units provided a sample variance of $S^2 = 0.36$.

a. construct a 0.90 CI estimate of the pop. variance for the weight ... .

$1-\alpha = 0.9 \rightarrow \alpha = 0.1 \rightarrow \frac{\alpha}{2} = 0.05 \rightarrow 1-\frac{\alpha}{2} = 0.95$, $df = 17$

$$0.90 \text{ CI for } \sigma^2 = \left(\frac{(18-1)0.36}{\chi^2_{0.05}} , \frac{(18-1)(0.36)}{\chi^2_{0.95}} \right)$$

$$= \left(\frac{6.12}{27.587} , \frac{6.12}{8.672} \right)$$

$$= (0.22 , 0.71).$$

b. construct a 0.90 CI estimate of pop. standard deviation.

$$0.90 \text{ CI for } \sigma = \left(\sqrt{0.22} , \sqrt{0.71} \right)$$

$$= (0.47 , 0.84)$$

Q5: $n=5$

Company	RoE %
–	25.83
–	25.35
–	22.60
–	28.72
–	17.10
–	15.40

s^2 s

a. Compute the sample variance and sample standard deviation for these data:

By calculater we find s, (shift $\xrightarrow{1} 2 \rightarrow 3$)

$s = 5.24$

$s^2 = 27.46$.

b. What is the 0.95 CI for the pop. variance?

$1-\alpha = 0.95 \rightarrow \alpha = 0.05 \rightarrow \frac{\alpha}{2} = 0.025 \rightarrow 1-\frac{\alpha}{2} = 0.975$.

$$0.95 \text{ CI for } \sigma^2 = \left(\frac{(5)(27.46)}{\chi^2_{0.025}}, \frac{(5)(27.46)}{\chi^2_{0.975}}\right) = \left(\frac{137.3}{12.832}, \frac{137.3}{0.831}\right)$$

$$= (10.7, 165.2)$$

c. What is the 0.95 CI for the pop. st. dev.?

$$0.95 \text{ CI for } \sigma = \left(\sqrt{10.7}, \sqrt{165.2}\right) = (3.27, 12.9)$$

s

Q6: $n = 20$, sample mean = 290, sample standard deviation = 30

a. What is the point estimate of pop. variance?

$s^2 = 900$

$df = 19$

b. provide a 90 % CI estimate of the pop. variance? $1-\alpha = 0.9 \rightarrow \alpha = 0.1 \rightarrow$ $\frac{\alpha}{2} = 0.05$, $1-\frac{\alpha}{2} = 0.95$.

$$0.90 \text{ CI for } \sigma^2 = \left(\frac{19(900)}{30.144}, \frac{19(900)}{10.117}\right) = (567.3, 1690.2)$$

c. provide a 90% CI estimate of the pop. st. dev.?

$$0.90 \text{ CI for } \sigma = \left(\sqrt{567.3}, \sqrt{1690.2}\right) = (23.8, 41.1)$$

Q7:

a. compute the mean, variance, standard deviation for the quarterly returns.

	1st	2nd	3rd	4th
2001	-6.91	5.80	-9.64	6.45
2002	0.83	-6.48	-14.03	5.58
2003	-2.27	6.43	0.85	9.33
2004	1.34	1.11	-0.77	8.03
2005	-2.46	0.89	2.55	1.78

Mean = 										لشو بالزبط ؟؟

St. dev. = 										$n = 5$???

Variance =

b. construct 95% CI for the pop. standard deviation .. ?

Q8: $n = 20$.

a. compute the sample mean = 260.16.

b. compute the sample standard deviation s = 70.69 $\rightsquigarrow s^2 = 4996.8$

c. Compute a 0.95 CI for the pop. standard deviation σ.

$1-\alpha = 0.95 \rightarrow \alpha = 0.05 \rightarrow \boxed{\frac{\alpha}{2} = 0.025 \rightarrow 1-\frac{\alpha}{2} = 0.975}$, $\boxed{df = 19}$

$$0.95 \text{ CI for } \sigma^2 = \left(\frac{19(4996.8)}{\chi^2_{0.025}} , \frac{19(4996.8)}{\chi^2_{0.975}} \right) = (2889.9 , 10658.9)$$

$$0.95 \text{ CI for } \sigma = \left(\sqrt{2889.9} , \sqrt{10658.9} \right) = (53.76 , 103.24).$$

Q9: $n = 12$

a. compute the sample variance s^2 and sample st. dev. s ..? By calcolater

$S = 7.6$

$S^2 = 57.8$

b. construct a 0.95 CI for the population variance σ^2: $\frac{\alpha}{2} = 0.025$, $1-\frac{\alpha}{2} = 0.975$, $df = 11$

$$0.95 \text{ CI for } \sigma^2 = \left(\frac{11(57.8)}{21.920} , \frac{11(57.8)}{3.816} \right) = (29 , 166.6)$$

c. construct a 0.95 CI for the population standard deviation:

$$0.95 \text{ CI for } \sigma = \left(\sqrt{29} , \sqrt{166.6} \right) = (5.4 , 12.9)$$

Q6: $n=15$, $\sigma_0^2 = 0.0025$, $S=0.066$.

a. use $\alpha=0.1$ to determine whether the sample indicates that the maximum acceptable variance is being exceeded.

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$$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2} = 24.39 .$$

P-value

df	0.05	0.025
14	23.685	26.119

P-Value $\in (0.025, 0.05)$

$\alpha = 0.1 \geq$ P-value $\in (0.025, 0.05)$

$\Rightarrow$ P-Value $\leq \alpha$ so we reject H_0

b. compute a 0.9 CI estimate for the variance of the ball bearings in the pop. ?

$1-\alpha=0.9 \rightarrow \alpha=0.1 \rightarrow \boxed{\frac{\alpha}{2}=0.05 \ , 1-\frac{\alpha}{2}=0.95, df=14}$

0.9 CI for $\sigma^2 = \left(\frac{14(0.066)^2}{\chi^2_{0.05}}, \frac{14(0.066)^2}{\chi^2_{0.95}}\right) = \left(\frac{0.060984}{23.685}, \frac{0.060984}{6.571}\right) = (0.00257, 0.00928)$.

Q11: $n=36$ $S=22.2$

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lower tail test.

Q12: $S=8$, 0.95 CI $= (5.86, 12.62)$ for σ .

a. was a sample size of 10 or 15 used in this statistical analysis?

Try $n=15 \rightarrow$ $\alpha = 0.05$, $\frac{\alpha}{2} = 0.025$, $1-\frac{\alpha}{2} = 0.975$, $df = 14$

× $n=10$
✓ $n=15$

$$0.95 \text{ CI for } \sigma^2 = \left(\frac{14(64)}{\chi^2_{0.025}} , \frac{14(64)}{\chi^2_{0.975}} \right) = \left(\frac{896}{26.119} , \frac{896}{5.629} \right) = (34.3 , 159.2)$$

$$0.95 \text{ CI for } \sigma = \left(\sqrt{34.3} , \sqrt{159.2} \right) = (5.86 , 12.62)$$

So $n = 15$

b. $S=8$, $n=25$, compute a 0.95 CI estimate of σ .

$\frac{\alpha}{2} = 0.025$, $1-\frac{\alpha}{2} = 0.975$, $df = 24$.

$$0.95 \text{ CI for } \sigma^2 = \left(\frac{24(64)}{39.364} , \frac{24(64)}{12.401} \right) = (39.02 , 123.86)$$

$$0.95 \text{ CI for } \sigma = \left(\sqrt{39.02} , \sqrt{123.86} \right) = (6.25 , 11.13)$$