Homework 8 (chapter 9).

Abistract 1

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2. prove that A_n is normal in S_n.

every element of S_n is either odd or even.

The product of two odd or two even permutations is even.

The product of an odd permutations and an even permutation is odd.

→ A_n consists of all the even permutations.

If $g \in S_n \setminus A_n$ then $g^{-1} \in S_n \setminus A_n$

and so $a \in A_n \Rightarrow g^{-1} a g \in A_n$

Thus, A_n is a Normal subgroup of S_n.

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4. let $H = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}, a,b,d \in \mathbb{R}, ad \neq 0 \right\}$. Is H a Normal subgroup of $GL(2, \mathbb{R})$?

No.

let $X = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \in GL(2, R)$, Then $X^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

let $y = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \in H$

$$X^{-1} y X = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \notin H$$

Therefore, $X^{-1} y X \notin H$ so $H \ntriangleleft GL(2, \mathbb{R})$. ☐

7. Prove that if H has index 2 in G, then H is Normal in G.

Suppose H is a subgroup of G of index 2

Then there are only two cosets of G relative to H.

→ let $x \in G$, If $x \in H$ then $xH = H = Hx$.

If $x \notin H$ then xH is the set of element in G not in H.

But Hx is also the set of element in G not in H.

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12. Prove that a factor group of an Abelian group is Abelian.

let G be an Abelian group and consider its factor group $G \backslash H$

where H is Normal in G.

let aH and bH Then $aH\,bH = ab(H)$

$= ba(H)$

$= bHaH$ since G Abelian

Hence, the factor group is also Abelian.

14. What is the order of the element $14 + \langle 8 \rangle$ in the factor group $Z_{24} \setminus \langle 8 \rangle$?

$$14 + \langle 8 \rangle = 6 + \langle 8 \rangle$$

and $(6 + \langle 8 \rangle)^4 = 0 + \langle 8 \rangle$

$\Longrightarrow |6 + \langle 8 \rangle| = 4$.

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25. let $G = U(32)$ and $H = \{1, 31\}$. The group $G \setminus H$ is isomorphic to one of $\boxed{Z_8}$, $Z_4 \oplus Z_2$ or $Z_n \oplus Z_2 \oplus Z_2$, determine which one by elimination .

is Z_8 .

Take the element 3 and find its order in $G \setminus H$:

$3^1 = 3$

$3^2 = 9$

$3^3 = 27$

$3^4 = 17$.

The order can be only 1, 2, 3, 4 ,

since $3^1, 3^2$ and 3^4 do not lies in H , Then 3 has order equal to 8 in $G \setminus H$.

So the $G \setminus H$ is cyclic and $G \setminus H \cong Z_8$.