

Birzeit University
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Engineering Probability and Statistics ENEE 2307
Quiz # 1

Instructors: Dr. Wael Hashlamoun,

Date: October 12, 2017

Problem:

A system is made up of two components A and B connected in parallel. The system works if at least one component works. The probability that A works is $P(A) = 1/4$ and the probability that B works is $P(B) = 1/3$. Given that $P(A \cap B) = 1/8$

- a. Are A and B independent?
- b. Find the probability that the system works.
- c. Find the probability that either A or B but not both work.

a. $P(A \cap B) = \frac{1}{8} \neq P(A) \cdot P(B) = \left(\frac{1}{4}\right) \left(\frac{1}{3}\right) = \frac{1}{12}$
 \Rightarrow A and B are dependent 3/10

b. $P(\text{system works}) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $= \frac{1}{4} + \frac{1}{3} - \frac{1}{8}$
 $= \frac{44}{96} \quad \frac{4}{10}$

c. $P(A \text{ or } B \text{ but not both}) = P(A \cup B) - P(A \cap B)$
 $= \frac{44}{96} - \frac{1}{8} = \frac{256}{768} = \frac{1}{3}$
 $\frac{3}{10}$

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The discrete probability function for a sample space S is:

$$P(x) = kx(x+1); \quad 1 \leq x \leq 4.$$

Let $A = \{1, 2\}$, $B = \{2\}$ be two events defined over S.

1. Find k so that $P(x)$ is a valid discrete probability function.
2. Are A and B independent? Explain

$$1. \quad \sum_{x=1}^4 P(x) = 1$$

$$k[2 + 6 + 12 + 20] = 1 \quad \Rightarrow \quad k = \frac{1}{40} \quad \frac{4}{10}$$

$$2. \quad P(A) = P(1) + P(2) = \frac{2}{40} + \frac{6}{40} = \frac{8}{40}$$

$$P(B) = P(2) = \frac{6}{40}$$

$$\{A \cap B\} = \{2\} \quad \Rightarrow \quad P(A \cap B) = P(2) = \frac{6}{40}$$

Test $P(A \cap B) \stackrel{?}{=} P(A) P(B)$

$$\left(\frac{6}{40}\right) \stackrel{?}{=} \left(\frac{8}{40}\right) \left(\frac{6}{40}\right)$$

NO

\Rightarrow A and B are dependent

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 Quiz # 2

Instructors: Dr. Wael Hashlamoun,

Date: November 2, 2017

Mugs produced by a certain pottery factory have an 8 % defective rate. A quality assurance inspector randomly selects five mugs and inspects them.

- a. Find the probability that the inspector finds at least one defective mug.
- b. What is the probability that all five mugs are defective?

Binomial Distribution

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x} \quad \left. \begin{array}{l} \text{know that} \\ \Rightarrow \frac{4}{10} \end{array} \right\}$$

a. , $p = 0.08, n = 5$

$$\begin{aligned} P(X \geq 1) &= 1 - P(X=0) \\ &= 1 - \binom{5}{0} (0.08)^0 (0.92)^5 \\ &= 1 - 0.65908 \\ &= 0.3409 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \frac{4}{10}$$

b. $P(X=5) = \binom{5}{5} (0.08)^5 (1-0.08)^0$

$$\begin{aligned} &= (0.08)^5 \\ &= 3.2768 \times 10^{-6} \end{aligned} \quad \frac{2}{10}$$

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 Quiz # 2

Instructors: Dr. Wael Hashlamoun,

Date: November 2, 2017

Let X be a random variable with the following probability density function:

$$f_X(x) = \begin{cases} kx, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

- a. Find k so that $f_X(x)$ is a valid probability density function
- b. Find the variance of X

a.
$$\int_0^2 f_X(x) dx = 1 = \int_0^2 kx dx = k \left(\frac{x^2}{2} \right) \Big|_0^2 = 2k$$

$\Rightarrow k = 1/2$ 4/10

b.
$$\sigma_x^2 = E(X^2) - E(X)^2$$
 2/10

$E(X) = \int_0^2 x f_X(x) dx = \int_0^2 kx^2 dx = \frac{4}{3}$ 2/10

$E(X^2) = \int_0^2 x^2 f_X(x) dx = \int_0^2 kx^3 dx = 2$ 2/10

$\sigma_x^2 = 2 - \left(\frac{4}{3}\right)^2 = \frac{2}{9}$

same

Also,
$$\sigma_x^2 = \int_0^2 \left(x - \frac{4}{3}\right)^2 kx dx = \frac{2}{9}$$

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 Quiz # 3

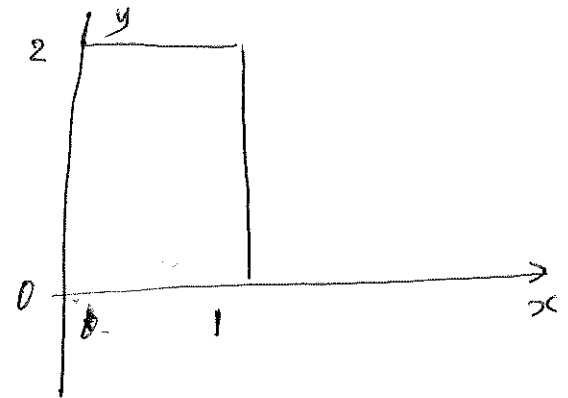
Instructors: Dr. Wael Hashlamoun,

Date: December 7, 2017

The two random variables X and Y are related by the following joint probability density function (pdf)

$$f_{X,Y}(x,y) = \begin{cases} kx, & 0 \leq x \leq 1, \quad 0 \leq y \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

- a) Find k so that $f_{X,Y}(x,y)$ is a valid joint pdf.
 b) Compute the marginal pdfs $f_X(x)$ and $f_Y(y)$



$$a. \quad k \int_0^2 \int_0^1 kx \, dx \, dy = 1$$

$$k \int_0^2 \left. \frac{x^2}{2} \right|_0^1 dy = 1$$

$$\frac{k}{2} \int_0^2 dy = 1 \Rightarrow \frac{k}{2} y \Big|_0^2 = 1 \Rightarrow \boxed{k=1} \quad \frac{4}{10}$$

$$b. \quad f_X(x) = \int_0^2 x \, dy = \begin{cases} 2x & 0 < x \leq 1 \\ 0 & \text{o.w} \end{cases} \quad \frac{3}{10}$$

$$f_Y(y) = \int_0^1 x \, dx = \begin{cases} \frac{1}{2} & 0 < y \leq 2 \\ 0 & \text{o.w} \end{cases} \quad \frac{3}{10}$$

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 Quiz # 3

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Date: November 7, 2017

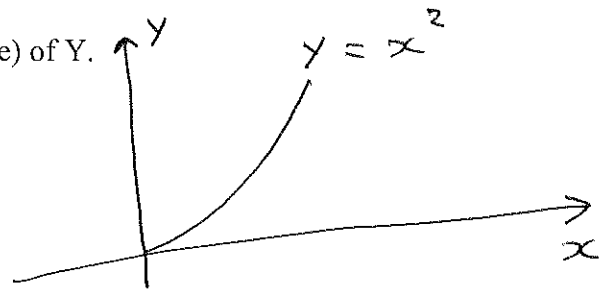
Let X be a random variable with the following probability density function:

$$f_X(x) = \begin{cases} \frac{1}{2}x, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

If Y is another random variable, related to X by $Y = X^2$,

- a. Find the probability density function of Y and the interval over which it is define.
- b. Find the mean value (i.e., the expected value) of Y.

$$f_X(x) = \begin{cases} \frac{1}{2}x & 0 < x \leq 2 \\ 0 & \text{o.w.} \end{cases}$$



a. $f_Y(y) = \frac{f_X(x)}{|dy/dx|}$

$$= \frac{\frac{1}{2}x}{2x} = \frac{1}{4} \quad 4/10$$

part a: $\frac{6}{10}$

when $x=0, y=0$
 $x=2, y=4$

$0 < y \leq 4 \quad 2/10$

$$\Rightarrow f_Y(y) = \begin{cases} \frac{1}{4} \\ 0 \end{cases}$$

b. can be solved in two ways (both are correct)

1. $E(Y) = E(X^2) = \int_0^2 x^2 f_X(x) dx = \int_0^2 x^2 \cdot \frac{1}{2}x dx$
 $= \frac{1}{2} \left. \frac{x^4}{4} \right|_0^2 = \frac{16}{8} = \boxed{2} \quad 4/10$

2. $E(Y) = \int_0^4 y f_Y(y) dy = \int_0^4 y \cdot \frac{1}{4} dy = \frac{1}{4} \left. \frac{y^2}{2} \right|_0^4 = \frac{16}{8}$
 $= \boxed{2}$

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 Quiz # 4

Instructors: Dr. Wael Hashlamoun,

Date: January 9, 2018

The speed of a vehicle, measured at different times, is given in the table

Time(s) x	1	1.3	2.3	3	3.5	4
Speed (m/s) y	5.7	6.3	7.4	8.4	11.9	13.7

We suspect that the data follow a linear function of the form $y = \alpha x + \beta$.

- Set up the necessary equations needed to determine α and β .
- Solve the equations in Part a for α and β .

$$y = \alpha x + \beta$$

$$\begin{pmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \end{pmatrix} = \begin{pmatrix} \sum y_i \\ \sum x_i y_i \end{pmatrix}$$

$$\begin{pmatrix} 6 & 15.1 \\ 15.1 & 45.23 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \end{pmatrix} = \begin{pmatrix} 53.4 \\ 152.56 \end{pmatrix} \quad \frac{7}{10}$$

$$\begin{pmatrix} \beta \\ \alpha \end{pmatrix} = \begin{pmatrix} 2.5738 \\ 2.5137 \end{pmatrix} \quad \frac{3}{10}$$

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 Quiz # 2

Instructors: Dr. Wael Hashlamoun,

Date: October 25, 2018

The continuous probability function for a sample space S is:

$$f(x) = \frac{2}{15}x ; \quad 1 \leq x \leq 4.$$

- a. Find the mean (expected) value of X
- b. Find the variance of X

$$\begin{aligned} E(X) = \mu_x &= \int_1^4 x f_x(x) dx = \int_1^4 x \left(\frac{2}{15}\right) x dx \\ &= \frac{2}{15} \int_1^4 x^2 dx = \frac{2}{15} \left(\frac{x^3}{3}\right) \Big|_1^4 \\ &= \frac{126}{45} = 2.8 \end{aligned}$$

$$\sigma_x^2 = E(X^2) - \mu_x^2 \quad \text{Formula 2}$$

$$\begin{aligned} E(X^2) &= \int_1^4 x^2 f_x(x) dx = \int_1^4 x^2 \cdot \left(\frac{2}{15}\right) x dx \\ &= \frac{2}{15} \int_1^4 x^3 dx = \frac{2}{15} \left(\frac{x^4}{4}\right) \Big|_1^4 \\ &= \frac{2}{60} (256 - 1) = \frac{510}{60} = 8.5 \end{aligned}$$

$$\begin{aligned} \sigma_x^2 &= 8.5 - (2.8)^2 \\ &= 0.66 \end{aligned}$$

1 answer

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 Quiz # 4

Instructors: Dr. Wael Hashlamoun

Date: August 9, 2018

Let X and Y be two independent random variables with the marginal pdf's

$$f_X(x) = \begin{cases} 1/2, & 0 \leq x \leq 2, \\ 0, & \text{otherwise} \end{cases} \quad f_Y(y) = \begin{cases} 1/3, & 0 \leq y \leq 3, \\ 0, & \text{otherwise} \end{cases}$$

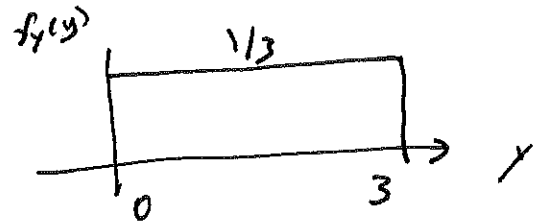
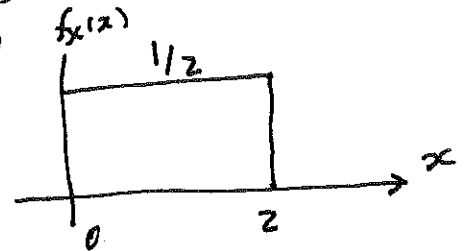
Define $Z = X + 2Y$

- Find the joint pdf $f_{X,Y}(x,y)$ of the two variables and the region over which it is defined
- Find the mean and variance of Z.

a. since X and Y are independent, then 4/10

$$f_{X,Y}(x,y) = f_X(x) f_Y(y) = \begin{cases} \frac{1}{6} & 0 \leq x \leq 2, 0 \leq y \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

$0 \leq x \leq 2, 0 \leq y \leq 3$
 o.w $f_X(x)$



b. For X

$$\mu_X = \frac{a+b}{2} = 1 \quad 1.5$$

$$\sigma_X^2 = \frac{(b-a)^2}{12} = \frac{4}{12} = \frac{1}{3} \quad 1.5$$

For Y

$$\mu_Y = \frac{a+b}{2} = \frac{3}{2} \quad 1.5$$

$$\sigma_Y^2 = \frac{(b-a)^2}{12} = \frac{9}{12}$$

$$Z = X + 2Y$$

$$1.5 \quad \mu_Z = \mu_X + 2\mu_Y = (1) + 2\left(\frac{3}{2}\right) = 4$$

$$1.5 \quad \sigma_Z^2 = \sigma_X^2 + 4\sigma_Y^2 = \frac{1}{3} + 4 \cdot \frac{9}{12} = \frac{1+9}{3} = \frac{10}{3}$$