

Fourier Transform of Useful Functions

$x(t)$	$X(f)$
$\delta(t)$	1
$u(t)$	$\frac{1}{j2\pi f} + \frac{1}{2} \delta(f)$
$\text{sgn}(t)$	$\frac{1}{j\pi f}$
$e^{-at} u(t)$	$\frac{1}{a + j2\pi f}$
$\cos(2\pi f_0 t)$	$\frac{1}{2} \delta(f - f_0) + \frac{1}{2} \delta(f + f_0)$

Fourier Transform Theorems:

① Superposition	$F[\alpha_1 x_1(t) + \alpha_2 x_2(t)] = \alpha_1 X_1(f) + \alpha_2 X_2(f)$
② Delay	$F[x(t - \tau)] = e^{-j2\pi f\tau} X(f)$
③ Scaling	$F[x(\alpha t)] = \frac{1}{ \alpha } X(f/\alpha)$
④ Inversion	$F[x(-t)] = X(-f)$
⑤ Duality	$F[X(t)] = x(-f) \quad \text{if} \quad F[x(t)] = X(f)$
⑥ Frequency shift	$F[x(t) e^{j2\pi f_0 t}] = X(f - f_0)$
⑦ Convolution	$F[x_1(t) * x_2(t)] = X_1(f) X_2(f)$
⑧ Multiplication	$F[x_1(t) x_2(t)] = X_1(f) * X_2(f)$

Fourier Transform Theorems:

⑨ Differentiation	$F \left[\frac{d^n x(t)}{dt^n} \right] = (j2\pi f)^n X(f)$
⑩ Integration	$F \left[\int_{-\infty}^t x(\lambda) d\lambda \right] = \frac{1}{j2\pi f} X(f) + \frac{1}{2} \delta(f) X(0)$
⑪ Modulation	$F [x(t) * c(t)] = \frac{A_c}{2} X(f - f_c) + \frac{A_c}{2} X(f + f_c)$ <p style="text-align: center;">where $c(t) = A_c \cos(2\pi f_c t)$</p>