Engineering statistics "ENEE2307"

Chapter 1

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Notes, questions and forms

Chapter 1

FUNDAMENTAL CONCEPTS OF PROBABILITY

Experiment:

By an experiment, we mean any procedure that:

1- Can be repeated, theoretically, an infinite number of times.

2- Has a well-defined set of possible outcomes.

① يمكن اعادتها عدد مرات لا نهائي "للأبد"

② لديها عدد معين من المخرجات

Sample Outcome:

Each of the potential eventualities of an experiment is referred to as a sample outcome(s).

هو أحد المخرجات من التجربة

Sample Space:

The totality of sample outcomes is called the sample space (S).

جميع الاحتمالات الممكنة

Event:

Any designated collection of sample outcomes, including individual outcomes, the entire sample space and the null space, constitute an event.

هو اي مجموعة معينة من الاحداث

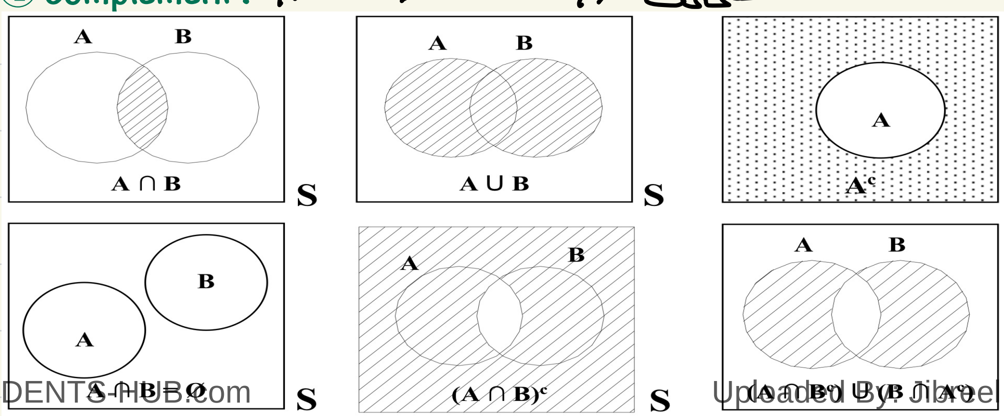
Algebra of events:

① intersection : $A \cap B$ تقاطع

② Union : $A \cup B$ اتحاد

③ disjoint : mutualy Execlusive $\Rightarrow A \cap B = \emptyset$

④ complement : $A^c \Rightarrow$ A متممة



De Morgan's Laws:

1- $(A \cap B)^c = A^c \cup B^c$

2- $(A \cup B)^c = A^c \cap B^c$

Example 1:

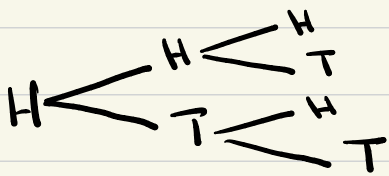
Consider the experiment of flipping a coin three times.

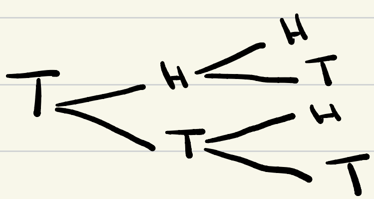
a- What is the sample space?

b- Which sample outcomes make up the event:

A : Majority of coins show heads.

a:- $S = \{(H,H,H), (H,H,T), (H,T,T), (H,T,H), (T,H,H), (T,T,H), (T,T,T), (T,H,T)\}$





b:- $A = \{(H,H,H), (H,H,T), (H,T,H), (T,H,H)\}$

Example 2 :

. An experiment has its sample space specified as:

S = {1, 2, 3, ..., 48, 49, 50}. Define the events
A : set of numbers divisible by 6
B : set of elements divisible by 8
C : set of numbers which satisfy the relation 2n , n = 1, 2, 3,...

Find: 1- A, B, C
2- A U B U C
3- A ∩ B ∩ C

$A = \{6, 12, 18, 24, 30, 36, 42, 48\}$

$B = \{8, 16, 24, 32, 40, 48\}$

$C = \{2, 4, 6 ----, 50\}$ even numbers

$A \cup B \cup C = \{2, 4, 6 --- 50\}$

$A \cap B \cap C = \{24, 48\}$

Definitions of Probability:

① Classical :

$$P(A) = \frac{\text{Number of outcomes } A}{\text{Number of sample space}}$$

*مهم :
يستخدم عندما تكون نواتج التجربة عدد محدود + الأحتمالات متساوية .

$$P(A) = \frac{\text{عدد الأحتمال الذي أريده}}{\text{العدد الكلي للأحتمالات}}$$

② Relative frequency :

$$P(A) = \lim_{n\to\infty} \frac{\text{Number of times } A \text{ occurs}}{\text{Number of trials}} \Rightarrow P(A) = \lim_{n\to\infty} \frac{f(A)}{n}$$

*** Note :-**

① $\frac{f(A)}{n} = 0 \Rightarrow$ A doesn't occur any time

② $\frac{f(A)}{n} = 1 \Rightarrow$ A has Occured all times

Example -3 :

Let $s = (1, 2, 3, 4, 5, 6)$; $A = (1, 5, 6)$, $B = (2, 3, 5, 6)$

P(A): $P(A) = \frac{\text{num of } A}{\text{num of } S} \Rightarrow P(A) = \frac{3}{6} \Rightarrow P(A) = \frac{1}{2}$

P(B): $P(B) = \frac{\text{num of } B}{\text{num of } S} \Rightarrow P(B) = \frac{4}{6} \Rightarrow P(B) = \frac{2}{3}$

P(A∩B): $= \frac{(1, 5, 6) \cap (2, 3, 5, 6)}{S} = \frac{(5, 6)}{S} \Rightarrow P(A \cap B) = \frac{2}{6}$

P($\overline{A \cap B}$): $A \cap B = (5, 6) \Rightarrow \overline{A \cap B} = (1, 2, 3, 4)$

$P(\overline{A \cap B}) = \frac{4}{6} \Rightarrow P(\overline{A \cap B}) = \frac{2}{3}$

To Check: –

$$P(A \cap B) + P(\overline{A \cap B}) = 1$$

$$\frac{2}{6} + \frac{4}{6} = 1$$

③ subjective :

Probability is defined as a person's measure of belief that some given event will occur.

Example:
What is the probability of establishing an independent Palestinian state in the next 2 years?
Any number we might come up with would be our own personal

هو عبارة عن
توقع شخصيا ليس
مبني على أسس
منطقية

④ Axiomatic :

$P(A) > 0$: Probability is non-negative

$P(S) = 1$: Probability of the sample space is a certain

$P(A \cup B) = P(A) + P(B)$; $(A \cap B = \varnothing)$ $\Longrightarrow$ iff mutually Disjoint

$P(A \cup B) = P(A) + P(B) - p(A \cap B)$; $(A \cap B \neq \varnothing)$ $\Longrightarrow$ Not Disjoint

Note

اذا لم يكونوا "Disjoint" :-
الجمع تجمع - تقاطعهم

Basic Theorems for Probability

5- **$P(A^c) = 1 - P(A)$**

<u>*Proof:*</u> $S = A \cup A^c$

$P(S) = P(A) + P(A^c)$

$1 = P(A) + P(A^c) \quad \rightarrow \quad P(A^c) = 1 - P(A)$

6- **$P(\emptyset) = 0$**

<u>*Proof:*</u>

$S = S \cup S^c$

$S = S \cup \emptyset \ ; S^c = \emptyset$

$P(S) = P(S) + P(\emptyset) \quad \rightarrow \quad P(\emptyset) = 0$

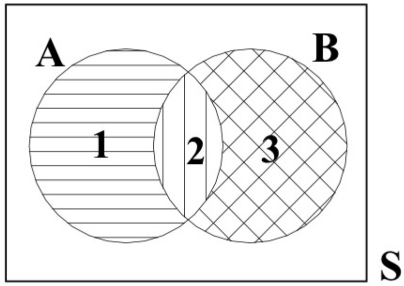
7- **$P(A \cup B) = P(A) + P(B) - P(A \cap B)$**

<u>*Proof:*</u>

For events (A) and (B) in a sample space:

$\{A \cup B\} = \{A \cap B^c\} \cup \{A \cap B\} \cup \{B \cap A^c\}$

$\{A \cup B\} = \quad \mathbf{1} \quad \cup \quad \mathbf{2} \quad \cup \quad \mathbf{3}$



Where events (1) and (2) and (3) are mutually exclusive

$P(A \cup B) = P(1) + P(2) + P(3)$

$P(A) = P(1) + P(2)$

$P(B) = P(2) + P(3)$

$\rightarrow \ P(A \cup B) = \{P(1) + P(2)\} + \{P(2) + P(3)\} - \{P(2)\}$

$\rightarrow \ P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- ***Theorem:***

If A, B, and C are three events, then:

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Theorem:

If A, B, and C are three events, then:

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

مجموعهم ناقص (تقاطع كل واحده مع الأخرى وتقاطعهم جميعاً)

Example 4:-

If the probability of occurrence of an even number is twice as likely as that of an odd number, A is the numbers divisible by 6, and the sample space is 1-50
Find P(A)

$S = \{1, 2, 3 ---- , 50\}$

$A = \{6, 12, 18, 24, 30, 36, 42, 48\}$

$P(S) = P(odd) + P(even) = 1$

$P(S) = P(odd) + 2P(odd) = 1$

$25 * P(odd) + 25 * 2P(odd)$

$75P = 1 \Rightarrow P = \frac{1}{75}$

$P(A) = 8 * 2P \Rightarrow P(A) = \frac{16}{75}$ #

عدد عناصر A

احتمالية even

Example 5:-

Suppose that a company has 100 employees who are classified according to their marital status and according to whether they are college graduates or not. It is known that 30% of the employees are married, and the percent of graduate employees is 80%. Moreover, 10 employees are neither married nor graduates. What proportion of married employees are graduates?

M: set of married employees

G: set of gratuated employees

K: Set of people that are not married gratuatand

A: set of all employees

$N(M) = 0.3 \times 100 = 30$ Persons are married

$N(G) = 0.8 \times 100 = 80$ Persons are graduated

$N(K) = 0.1 * 100 = 10$ Persons are none

$N(M \cup G)^c = 10$

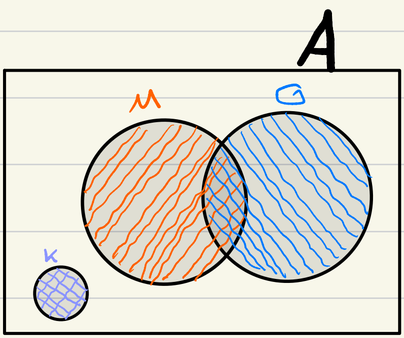
$N(M \cup G) = 100 - 10 \Rightarrow N(M \cup G) = 90$

$N(M \cup G) = N(M) + N(G) - N(M \cap G)$

$90 = 30 + 80 - N(M \cap G)$

$90 = 110 - N(M \cap G)$

$\Rightarrow N(M \cap G) = 20$ # Done



Example 6 :-

An experiment has two possible outcomes; the first occurs with probability (P), the second with probability (P^2), find (P).

$P(S) = 1$

$P(S_1) + P(S_2) = 1$

$P^2 + P - 1 = 0$

Answer

$$\boxed{P = \frac{-1+\sqrt{5}}{2}}$$

$P = \dfrac{-1 \pm \sqrt{1 - -4}}{2}$ القانون العام

$P = \dfrac{-1+\sqrt{5}}{2}$ OR $\boxed{P = \dfrac{-1-\sqrt{5}}{2}}$

يشطب
لأنه سالب

Example 7:-

A sample space "S" consists of the integers 1 to 6 inclusive. Each outcome has an associated probability proportional to its magnitude. If one number is chosen at random, what is the probability that an even number appears?

كل رقم احتماليته حسب حجمه ⇐ 2 احتماليتها 2P وهكذا

$S = \{1, 2, 3, 4, 5, 6\}$

$P(S) = \dfrac{1+2+3+4+5+6}{1+2+3+4+5+6} \Rightarrow P(S) = \dfrac{21}{21} \Rightarrow \text{Any } P = \dfrac{1}{21}$

$A = \{2, 4, 6\}$

$P(A) = \dfrac{P(2)+P(4)+P(6)}{P(S)} = \dfrac{2+4+6}{21} \Rightarrow \boxed{P(A) = \dfrac{12}{21}}$

Example 8 :-

Let (A) and (B) be any two events defined on (S). Suppose that P(A) = 0.4, P(B) = 0.5, and P(A ∩ B) = 0.1.
Find the probability that:

1- (A) or (B) but not both occur.
2- None of the events (A) or (B) will occur.
3- At least one event will occur.
4- Both events occur.

① $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.5 - 0.1 \Rightarrow P(A \cup B) = 0.8$

$P(A \text{ only}) = P(A) - P(A \cap B) \Rightarrow P(A \text{ only}) = 0.4 - 0.1 = 0.3$

$P(B \text{ only}) = P(B) - P(A \cap B) \Rightarrow P(B \text{ only}) = 0.5 - 0.1 = 0.4$

$P(A) \text{ or } P(B) = P(A \cup B) = 0.3 + 0.4 \Rightarrow \boxed{P(A \cup B) = 0.7}$

② $P(\text{None}) = P(A \cup B)^c \Rightarrow 1 - 0.8 \Rightarrow \boxed{P(\text{None}) = 0.2}$

③ $P(\text{At least one}) = \boxed{P(A \cup B) = 0.8}$

④ $P(\text{Both}) = \boxed{P(A \cap B) = 0.1}$

Conditional Probabilities and Statistical Independence

Rules :-

① $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

② $P(B/A) = \dfrac{P(A \cap B)}{P(A)}$

* نستخدم عادة عندما تكون التجربة قد حصلت
ونريد فقط نسب النتائج (Ocured, observed)

Notes :-

① A and B are statically independent if

1. $P(A \cap B) = P(A) * P(B)$

② A, B and C are statically independent if

1. $P(A \cap B) = P(A) * P(B)$
2. $P(A \cap C) = P(A) * P(C)$
3. $P(B \cap C) = P(B) * P(C)$
4. $P(A \cap B \cap C) = P(A) * P(B) * P(C)$

Note :-
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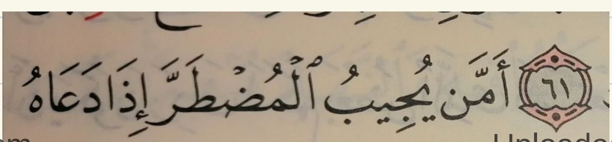
بلا أي استثناء



Example 1:-

A certain computer becomes inoperable if two components A and B both fail. The probability that A fails is 0.001 and the probability that B fails is 0.005. However, the probability that B fails increases by a factor of 4 if A has failed. Calculate the probability that:

a- The computer becomes inoperable.

b- A will fail if B has failed. happened = Conditional Probability

$P(A) = 0.001$, $P(B) = 0.005 \Rightarrow P(B) = 5P(A)$

if A fails $\Rightarrow P(B/A) = 0.02$

a- Both of them fail

$$P(B/A) = \frac{P(A) \cap P(B)}{P(A)} \Rightarrow P(A \cap B) = P(A) \cdot P(B/A)$$

$$P(A \cap B) = 0.02 * 0.001 \Rightarrow \boxed{P(A \cap B) = 0.00002}$$

b- A will fail if B has failed

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \Rightarrow \frac{0.00002}{0.005}$$

$$\boxed{P(A/B) = 0.004}$$

Example 2:-

A box contains 20 non-defective (N) items and 5 defective (D) items. Three items are drawn without replacement.
a. Find the probability that the sequence of objects obtained is (NND) in the given order.
b. Find the probability that exactly one defective item is obtained.

Sample space has 25 items

a- N N D

$P(N) = \frac{20}{25}$, $P(D) = \frac{5}{25}$

$P(NND) = P(N) * P(N) * P(D)$

$\frac{20}{25} * \frac{19}{24} * \frac{5}{23} = 0.137$

b- NND or NDN or DNN

$= P(NND) \cup P(NDN) \cup P(DNN)$

$= 0.137 + 0.137 + 0.137$

$= 3(0.137)$

$= 0.413$

Example 3:-

Let $S = \{1, 2, 3, 4\}$; $P_i = \frac{1}{4}$. $A = \{1, 2\}$ and $B = \{2, 3\}$. Are (A) and (B) independent?

SOLUTION: $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$ ➔ $(A \cap B) = \{2\}$, $P(A \cap B) = \frac{1}{4}$

➔ $P(A \cap B) = P(A)\,P(B)$ ➔ ***Events are independent***

Example 4 :-

Consider an experiment in which the sample space contains four outcomes $\{S_1, S_2, S_3, S_4\}$ such that $P(S_i) = \frac{1}{4}$. Let events (A), (B) and (C) be defined as:

$A = \{S_1, S_2\}$, $B = \{S_1, S_3\}$, $C = \{S_1, S_4\}$

Are these events independent?

SOLUTION: $P(A) = P(B) = P(C) = \frac{1}{2}$

$(A \cap B) = \{S_1\}$; $(A \cap C) = \{S_1\}$; $(B \cap C) = \{S_1\}$; $(A \cap B \cap C) = \{S_1\}$

$P(A \cap B) = \frac{1}{4}$; $P(A \cap C) = \frac{1}{4}$; $P(B \cap C) = \frac{1}{4}$; $P(A \cap B \cap C) = \frac{1}{4}$

Check the conditions:

$P(A \cap B) = \frac{1}{4} = P(A)\,P(B) = \frac{1}{2} \times \frac{1}{2}$ ✓ ; $P(A \cap C) = \frac{1}{4} = P(A)\,P(C) = \frac{1}{2} \times \frac{1}{2}$ ✓

$P(B \cap C) = \frac{1}{4} = P(B)\,P(C) = \frac{1}{2} \times \frac{1}{2}$ ✓

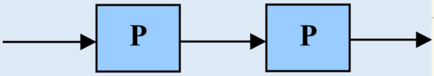
$P(A \cap B \cap C) = \frac{1}{4} \neq P(A)\,P(B)\,P(C) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ Fail in test 4

➔ **Events are not independent (even though they are pair-wise independent)**

Example 5 :-

على التوالي

Suppose that a system is made up of two components connected in series, each component has a probability (P) of working "**Reliability**". What is the probability that the system works assuming that components work independently?



SOLUTION:

$$P(\text{work}) = P \cap P$$
$$= P * P$$
$$= P^2$$

*Remark :-

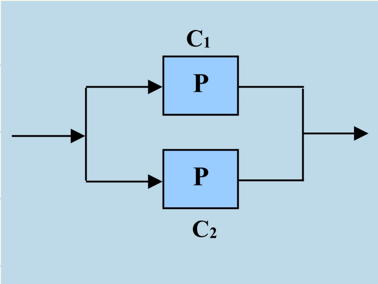
A and B : $A \cap B = A * B$

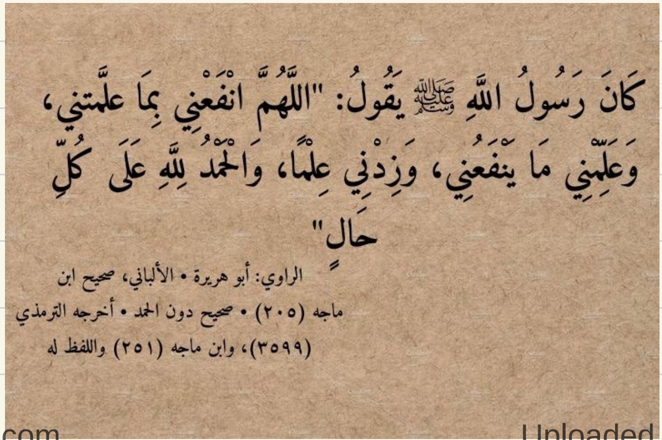
A or B : $A \cup B = A + B$

Example 6 :-

Suppose that a system is made up of two components connected in parallel. The system works if at least one component works properly. If each component has a probability (P) of working "**Reliability**" and components work independently, find the probability that the system works.

$$P(\text{work}) = P(C_1) \text{ or } P(C_2)$$
$$= (P(C_1) \cup P(C_2)) - P(C_1 \cap C_2)$$
$$= 2P - P^2$$





Total probability theroum

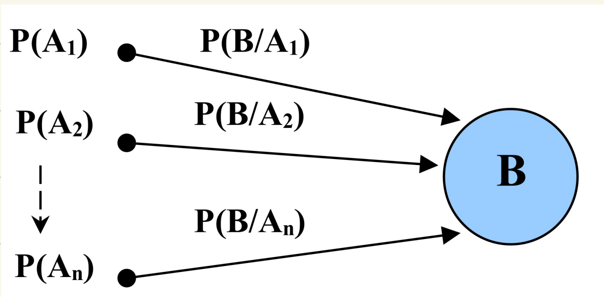
Let $A_1, A_2, \ldots, A_n$ be a set of events defined over (S) such that:

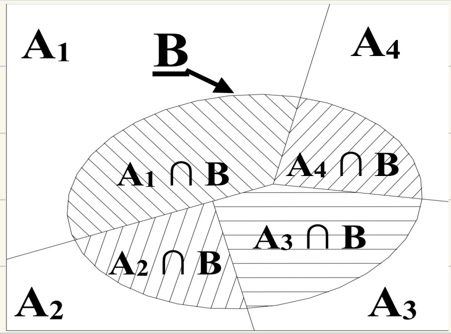
$S = A_1 \cup A_2 \cup \ldots \cup A_n \; ; \; A_i \cap A_j = \emptyset$ (They are disjoint)

هو عبارة عن S مقسم إلى مجموعات حسب تصنيف معين حيث يوجد event

B من تصنيف آخر مثال <= صف مقسم حسب التخصصات (A_1 , A_2 ---)

حيث يمثل الحدث B عدد الذكور في الصف





Goal	Given
To find $P(B)$	$P(A_1), P(A_2), \ldots, \ldots$ and $P(B/A_n) \ldots, \ldots$

Rule :-

$$P(B) = P(A_1 \cap B) \cup P(A_2 \cap B) \cup \ldots, \ldots \cup P(A_n \cap B)$$

$$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + \ldots, \ldots + P(A_n \cap B)$$

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$$\boxed{P(B) = P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2) + \ldots + P(A_n)\,P(B/A_n)}$$

Example 1 :-

If female students constitute 30% of the student body in the Faculty of Engineering and 40% of them have A GPA > 80, while 25 % of the male students have their GPA > 80. What is the probability that a person selected at random will have a GPA > 80?

M : male students , $P(M) = 70\%$, $P(G/M) = 25\%$

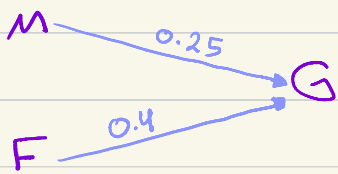
F : female students , $P(F) = 30\%$, $P(G/F) = 40\%$

G : GPA > 80 Find $P(G)$

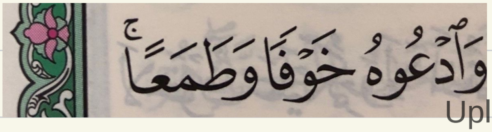
$$P(G) = P(M)\,P(G/M) + P(F) \times P(G/F)$$

$$(0.7 * 0.25) + (0.3 * 0.4)$$

$$0.175 + 0.12$$



$$\boxed{\therefore P(G) = 0.295}$$



Example 2 :-

Suppose that when a machine is adjusted properly, 50% of the items produced by it are of high quality and the other 50% are of medium quality. Suppose, however, that the machine is improperly adjusted during 10% of the time and that under these conditions 25% of the items produced by it are of high quality and 75% are of medium quality.

a- Suppose that one item produced by the machine is selected at random, find the probability that it is of medium quality.

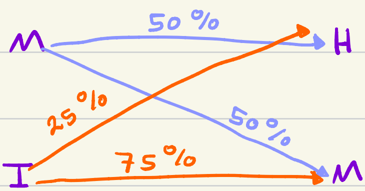
b- If one item is selected at random, and found to be of medium quality, what is the probability that the machine was adjusted properly.

A : adjusted Properly , $P(A) = 0.9$, $P(H/A) = 50\%$, $P(M/A) = 50\%$

I : improperly adjusted , $P(I) = 0.1$, $P(H/I) = 25\%$, $P(M/I) = 75\%$

H : High quality

M : medium quality



a- Medium quality :-

$$P(M) = P(A)\,P(M/A) + P(I)\,P(M/I)$$

$$(0.9 * 0.5) + (0.1 * 0.75)$$

$$\boxed{\therefore P(M) = 0.525}$$

b- adjusted Properely :-

$$P(A/M) = \frac{P(A \cap M)}{P(M)} = \frac{P(A)\,P(M/A)}{P(M)} = \frac{0.9 * 0.5}{0.525}$$

$$\boxed{\therefore P(A/M) = 0.856}$$

وَلَسَوْفَ يُعْطِيكَ رَبُّكَ فَتَرْضَىٰ

Example 3 :-

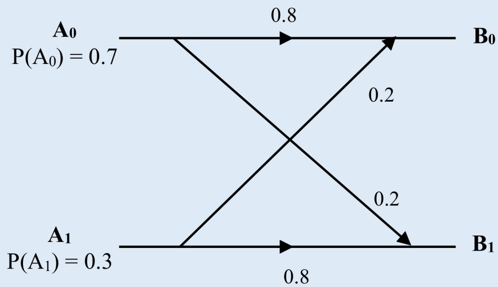
Consider the problem of transmitting binary data over a noisy communication channel. Due to the presence of noise, a certain amount of transmission error is introduced. Suppose that the probability of transmitting a binary 0 is 0.7 (70% of transmitted digits are zeros) and there is a 0.8 probability that a given 0 or 1 being received properly.

a- What is the probability of receiving a binary 1.
b- If a 1 is received, what is the probability that a 0 was sent.

a- $P(B_1) = P(A_0)\,P(B_1/A_0) + P(A_1)\,P(B_1/A_1)$
$\boldsymbol{P(B_1) = (0.7)(0.2) + (0.3)(0.8) = 0.38}$
$\boldsymbol{P(B_0) = 1 - P(B_1) = 0.62}$

b- $$P(A_0/B_1) = \frac{P(A_0 \cap B_1)}{P(B_1)} = \frac{P(A_0)\,P(B_1/A_0)}{P(B_1)}$$

$$\boldsymbol{P(A_0/B_1) = \frac{(0.7)(0.2)}{(0.38)} = 0.3684}$$



Example 4 :-

In a factory, four machines produce the same product. Machine A_1 produces 10% of the product, A_2 20%, A_3 30%, and A_4 40%. The proportion of defective items produced by the machines follows:

A_1: 0.001 ; A_2: 0.005 ; A_3: 0.005 ; A_4: 0.002

An item selected at random is found to be defective, what is the probability that the item was produced by machine A_1?

SOLUTION:

Let D be the event: Selected item is defective

$P(D) = P(A_1)\,P(D/A_1) + P(A_2)\,P(D/A_2) + P(A_3)\,P(D/A_3) + P(A_4)\,P(D/A_4)$
$P(D) = (0.1 \times 0.001) + (0.2 \times 0.005) + (0.3 \times 0.005) + (0.4 \times 0.002)$
$\boldsymbol{P(D) = 0.0034}$

$$P(A_1/D) = \frac{P(A_1)\,P(D/A_1)}{P(D)} = \frac{(0.1)\,(0.001)}{(0.0034)} = \frac{0.0001}{0.0034} = \frac{1}{34}$$

Counting Techniques

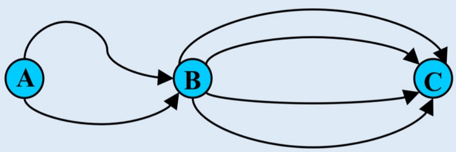
We use it when the outcomes are equally likely in the probability
Where $P(A) = \frac{1}{n}$

① multiplication rule :-

- Independent process — كل نتيجة غير مرتبطة باللي قبلها
- repetition is allowed — التكرار مسموح
- order is not important — الترتيب غير مهم

Example 1 :-

There are two roads between A and B and four roads between B and C. How many different routes can one travel between A and C.



SOLUTION:

$n = 2 \times 4 = 8$

Example 2 :-

How many different five-letter computer passwords can be formed:

a- If a letter can be used more than once.

b- If each word contains each letter no more than once.

SOLUTION:

a- $N = (26)^5$

b- $N = \frac{26!}{(26-5)!}$

② permutation - التباديل :-

- repetition is not allowed – التكرار ممنوع
- order is important – الترتيب مهم

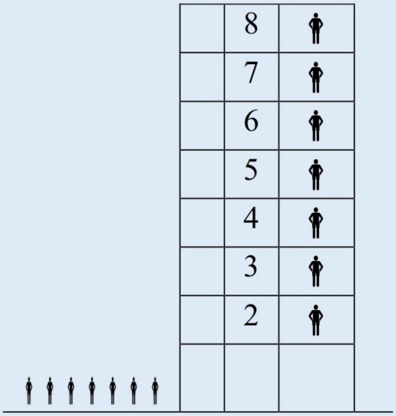
$$N = \frac{n!}{(n-k)!}$$

Since n is the sample space number
k is the times we did the experiment

Example 1 :-

An apartment building has eight floors (numbered 1 to 8). If seven people get on the elevator on the fist floor, what is the probability that:

a- All get off on different floors?
b- All get off on the same floor?



SOLUTION:

Number of points in the sample space:
First person can get off at any of the 7 floors.

Person (2) can get off at any of the 7 floors and so on.

➔ The number of ways people can get off:

(N) = $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^7$

a- Here the problem is to find the number of permutations of 7 objects taking 7 at a time.

$$P = \frac{7!}{7^7}$$

b- Here there are 7 ways whereby all seven persons get off on the same floor.

$$P = \frac{7}{7^7}$$

فإنك بأعيننا

③ combination – التوافيق :-

- repetition is not allowed
- order is not important

$$\binom{n}{k} = \frac{n!}{k(n-k)!}$$

* عدد الطرق التي يمكنني فيها ترتيب أو عمل شيء معين

Example 1 :-

From four persons (set of elements), how many committees (subsets) of two members (elements) may be chosen?

SOLUTION:

Let the persons be identified by the initials A, B, C and D

Subsets: (A , B) , (A , C) , (A , D) , (B , C) , (B , D) , (C , D)

$$N = \binom{4}{2} = \frac{4!}{2!\,(4-2)!} = 6$$

Missing sequences: (A , A) , (B , B) , (C , C) , (D , D) ➔ *(repetition is not allowed)*

Missing sequences: (B , A) , (C , A) , (D , A)

(C , B) , (D , B) , (D , C) ➔ *(order is not important)*

Example 2 :-

Consider the rolling of a die twice, how many pairs of numbers can be formed for each case?

SOLUTION:

$n = 6$ and $k = 2$

Case I: Permutation

a- With repetition

$N = n^k = 6^2 = 36$

b- Without repetition

$$N = \frac{n!}{(n-k)!} = \frac{6!}{(6-2)!} = 30$$

Case I: Combination

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{6!}{2!(6-2)!} = 15$$

D_1 \ D_2	1	2	3	4	5	6
1	*(1,1)*	**(1,2)**	**(1,3)**	**(1,4)**	**(1,5)**	**(1,6)**
2	**(2,1)**	*(2,2)*	**(2,3)**	**(2,4)**	**(2,5)**	**(2,6)**
3	**(3,1)**	**(3,2)**	*(3,3)*	**(3,4)**	**(3,5)**	**(3,6)**
4	**(4,1)**	**(4,2)**	**(4,3)**	*(4,4)*	**(4,5)**	**(4,6)**
5	**(5,1)**	**(5,2)**	**(5,3)**	**(5,4)**	*(5,5)*	**(5,6)**
6	**(6,1)**	**(6,2)**	**(6,3)**	**(6,4)**	**(6,5)**	*(6,6)*

Example 3 :-

In how many ways can we arrange 5 balls numbered 1 to 5 in 10 baskets each of which can accommodate one ball?

SOLUTION:

The number of ways $(N) = \frac{n!}{(n-k)!} = \frac{10!}{(10-5)!} = \frac{10!}{5!}$

NOTE:

If we remove the numbers of the balls so that the balls are no longer distinguishable, then:

The number of ways $\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{10!}{5!(10-5)!} = \frac{10!}{5!\ 5!}$

This is because the permutation within the 5 balls is no longer needed.

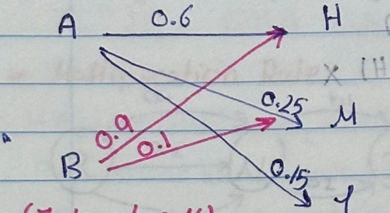
Forms

Question 1:-

Tuesday

Exp: A factory has two production lines A & B production line A works 5 day a week and produces 350 items perday out of which 60% of high quality & 25% of medium quality & 15% of low quality. production line B works 7 days a week and produces 500 items perday out of which 90% of high quality & 10% of meduim quality.

(5days/week) (350 item/day)



(7 days/week)

(500 item/day)

a. Determine the probability of selecting an item produced by production line A after 10 weeks.

Total items = A + B

$= 10.5.350 + 10.7.500 =$

items produced by A $= 10.5.350$

$p(A) = \dfrac{10.5.350}{10.5.350 + 10.7.500} = \dfrac{5}{15} = \dfrac{1}{3}$

b. $p(B) = 1 - p(A) = \dfrac{2}{3}$

or $\dfrac{10.7.500}{10.5.350 + 10.7.500} = \dfrac{2}{3}$

c. An item is selected at random What is the probability that it is of high quality?

$$p(H) = p(A)p(H/A) + p(B)p(H/B)$$
$$= \frac{1}{3} \times 0.6 + \frac{2}{3} \times 0.9 = 0.8$$

d. If a high quality item is selected, what is the probability that it was produced by A?

$$p(A/H) = \frac{p(A \cap H)}{p(H)}$$

given

$p(A \cap H) \to p(A)\,p(H/A)$; $p(H)\,p(A/H)$ ✗

$$= \frac{\frac{1}{3} * 0.6}{0.8} = \frac{0.2}{0.8} = \frac{1}{4}$$

e. An item is selected and found to be of medium quality what is the probability that it was produced by A.

$$p(M) = p(A)\,p(M/A) + p(B)\,p(M/B)$$
$$= \frac{1}{3} * 0.25 + \frac{2}{3} * 0.1 = 0.15$$
$$p(A/M) = \frac{p(A \cap M)}{p(M)} = \frac{p(A)\,p(M/A)}{p(M)} = \frac{\frac{1}{3} * 0.25}{0.15} = \frac{5}{9}$$

f. An item is selected at random and found to be of low quality, what is the probability that it was produced by A.

$$p(L) = p(A)\,p(L/A) + p(B)\,p(L/B)$$
$$= \frac{1}{3} * 0.15 + \frac{2}{3} * 0$$
$$= 0.05$$

Quiz by Dr. Jubran

BIRZEIT UNIVERSITY
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Probability and Statistical Engineering, ENEE2307
Dr. Mohammad K. Jubran
Quiz#1 – Section 1

Date: Thursday, 7/4/2022 Time: 30 minutes
Name: Razan Abdelrahman. Student #: 1200531

Problem 1 (20pts):
In a game experiment a coin is flipped for two times and a dice is rolled for one time. The probability of observing a head in the coin is three times the probability of tail. Let A be the event of observing at least one head, and B is the event that two heads are observed and an even number is observed on the dice.

a) Compute $P(A)$.

+7

$A = \{(H,H,1),(H,T,1)(T,H,1),(H,H,2),(H,T,2),(T,H,2)$
$(H,H,3),(H,T,3),(T,H,3),(H,H,4),(H,T,4),(T,H,4)$
$(H,H,5),(H,T,5),(T,H,5),(H,H,6),(H,T,6),(T,H,6)\}$

$p(A) = \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6}$
$+ \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6}$
$+ \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6}$

$= \frac{18}{96} + \frac{36}{96} = \boxed{\frac{54}{96}}$

b) Compute $P(B)$.

$B = \{(H,H,2),(H,H,4),(H,H,6)\}$

$p(B) = p(\{HH2\} \cup \{HH4\} \cup \{HH6\})$

$= \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{6}$

$= \frac{9}{96} + \frac{9}{96} + \frac{9}{96}$

+4

$= 0,28125 = \boxed{\frac{27}{96}}$

c) Compute $P(A/B)$.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P\{\{H,H,2\} \cup \{H,H,4\}, \{H,H,6\}\}}{P(B)}$$

$$= \frac{P(B)}{P(B)} = \boxed{1}$$

4

d) Are A and B statistically independent?

$$P(A \cap B) \stackrel{?}{=} P(A)\,P(B).$$

$$P(B) \stackrel{?}{=} P(A)\;P(B)$$

$$\frac{54}{96} \stackrel{?}{=} \frac{54}{96} \cdot \frac{27}{96}$$

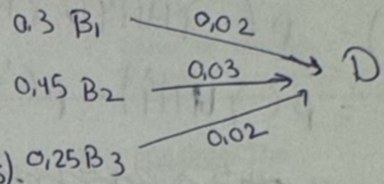
No, so they are not statically independent.

4

Problem 2 (10pts):

In a certain assembly plant, three machines, B1, B2, and B3, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected.

1) What is the probability that it is defective?



$$P(D) = P(D/B_1)P(B_1) + P(D/B_2)P(B_2) + P(D/B_3)P(B_3)$$

$$= (0{,}02)(0{,}3) + (0{,}03)(0{,}45) + (0{,}02)(0{,}25).$$

$$= \boxed{0{,}0245.}$$

2) If a defective item is selected, what is the probability that it was made by machine B2.

$$P(B_2/D) = \frac{P(B_2 \cap D)}{P(B_2)}$$

$$= \frac{P(D/B_2)P(B_2)}{P(D)}$$

$$= \frac{0{,}03(0{,}45)}{0{,}0245}$$

$$= \boxed{0{,}551}$$

10