3.5] Steady state Response of LTI system with FS Representation

$$x(t) \longrightarrow \boxed{LTI} \longrightarrow y(t)$$

$$x(t) = \sum_{n=-\infty}^{\infty} X_n \, e^{jn\omega_0 t} \quad , \quad H(\omega) = |H(\omega)| \, e^{j\theta_H(\omega)}$$

↳ Frequency response $[\omega = n\omega_0]$

↳ input (complex form of FS)

By applying the concept of of superposition, we get :-

$$\boxed{y(t) = \sum_{n=-\infty}^{\infty} X_n \, |H(n\omega_0)| \, e^{j(n\omega_0 t + \theta_H(n\omega_0))}}$$

$$x(t) = \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

$\hookrightarrow$ Sinusoidal form of FS (input)

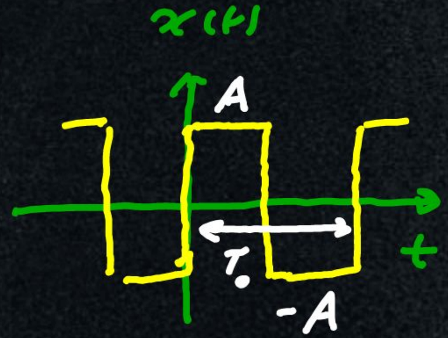
$$H(\omega) = |H(\omega)|\, e^{j\theta_H(\omega)}$$

where $\omega = n\omega_0$

By applying the concept of superposition, we get :-

$$\boxed{y(t) = \sum_{n=1}^{\infty} C_n |H(n\omega_0)| \cos(n\omega_0 t + \theta_n + \theta_H(n\omega_0))}$$

$\underline{Ex}$:- $H(\omega) = \dfrac{10}{10+j\omega}$



Find the SS response of $x(t)$

$a_o = 0$, $a_n = 0$, $b_n = 0$ for n even

$b_n = \dfrac{4A}{\pi n}$ for n odd

$$x(t) = \sum_{n=1,3,\dots}^{\infty} b_n \sin(n\omega_o t)$$

$$y(t) = \sum_{n=1,3,\dots}^{\infty} |H(n\omega_o)|\, b_n \sin(n\omega_o t + \theta_H(n\omega_o))$$

$$H(\omega) = \frac{10}{\sqrt{100+\omega^2}}\, e^{-j\tan^{-1}\left(\frac{n\omega_o}{10}\right)}$$

$$y(t) = \sum_{n=1,3,\ldots}^{\infty} \left[\frac{40A}{\sqrt{100+(n\omega_0)^2}} \frac{1}{\pi n} \sin\left(n\omega_0 t - \tan^{-1}\left(\frac{n\omega_0}{10}\right)\right) \right]$$

Write the SS response for $\omega_0 = 10$

$$y(t) = \sum_{n=1,3,\ldots}^{\infty} \frac{(4A/\pi)}{\sqrt{1+n^2}} \frac{1}{n} \sin\left(10nt - \tan^{-1}(n)\right)$$

Exercise: Determine the response of the same system to the input

$$x(t) = 10\cos(10t + \pi/2) + 20\sin(30t - \pi/3) + 15\cos(50t + \pi/4)$$

3.6] RMS Calculation Using FS

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$$X_{RMS}^2 = \frac{1}{T_0}\int_{T_0} x^2(t)\,dt = \frac{1}{T_0}\int_{T_0} x(t)\,x(t)\,dt$$

$$X_{RMS}^2 = \frac{1}{T_0}\left[\int_{T_0} x(t)\left[a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)\right]dt\right]$$

$$X_{RMS}^2 = \frac{1}{T_0}\left[a_0 \overbrace{\int_{T_0} x(t)\,dt}^{a_0 T_0} + \sum_{n=1}^{\infty} a_n \overbrace{\int_{T_0} x(t)\cos(n\omega_0 t)\,dt}^{a_n T_0/2} + \sum_{n=1}^{\infty} b_n \underbrace{\int_{T_0} x(t)\sin(n\omega_0 t)\,dt}_{b_n T_0/2}\right]$$

$$X_{RMS}^2 = a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2) = a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} c_n^2$$

$$X_{RMS} = \sqrt{a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} C_n^2}$$

$$X_{RMS} = \sqrt{a_0^2 + \sum_{n=1}^{\infty} C_{RMS,n}^2}$$

where $C_{RMS,k} = \frac{C_k}{\sqrt{2}}$

$\rightarrow$ It is the RMS value of k^{th} sinusoidal harmonic $[C_k \cos(k\omega_0 t + \theta_k)]$

$$X_{RMS}^2 = a_0^2 + C_{RMS,1}^2 + C_{RMS,2}^2 + \cdots$$

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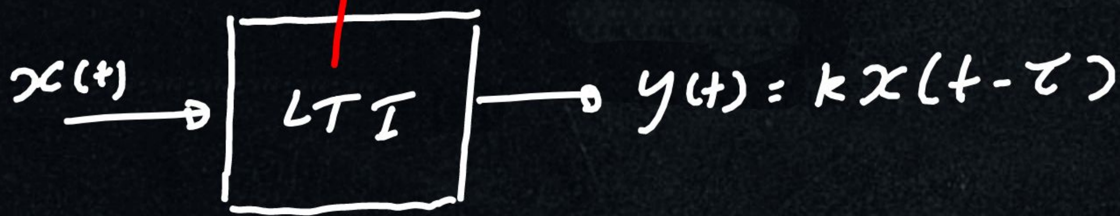
3.7] System and Signal Distortion

Distortionless system



$x(t)$ is undistroted if k & τ are constants

$\Rightarrow$ The frequency response of distortionless system is given by $H(\omega) = k e^{-j\omega\tau}$

$\Rightarrow |H(\omega)| = k \quad , \quad \theta_H(\omega) = -\omega\tau$

Types of distortion

Amplitude distortion

↓

$|H(\omega)| \neq$ constant

phase distortion

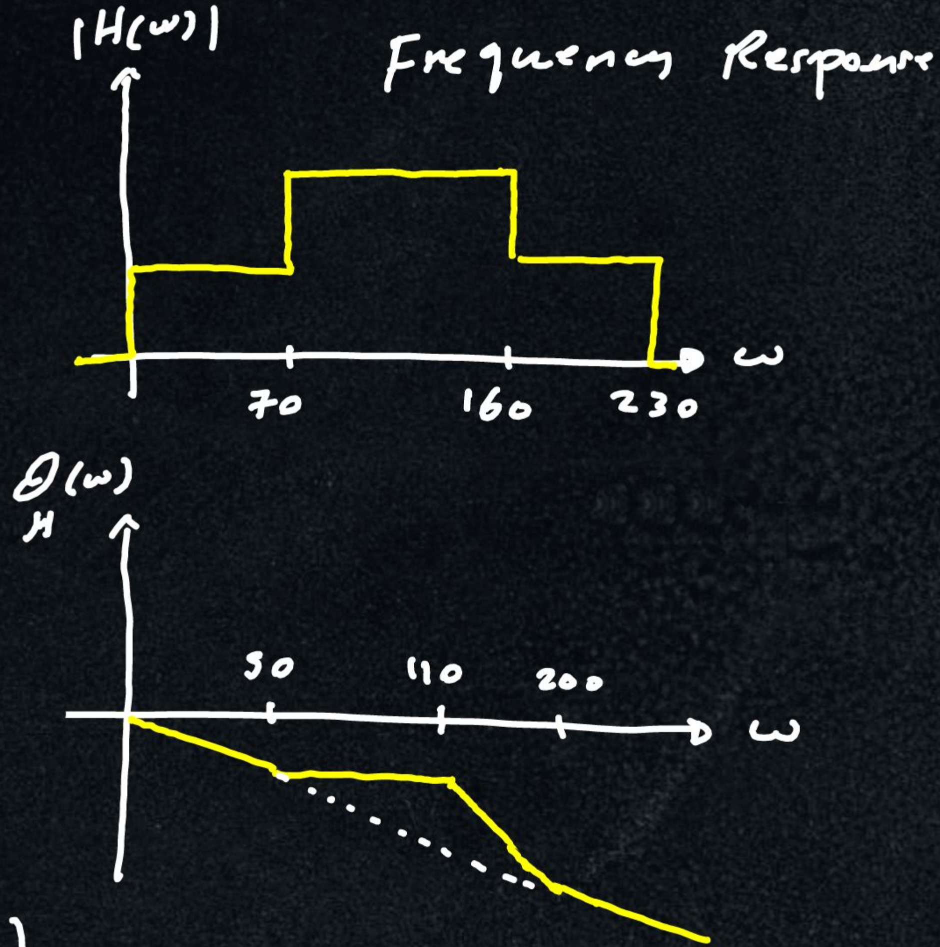
↓

$\theta_H(\omega) \neq -k\omega$

↓

It does not follow a constant slop linear characteristic

EX :-



1) $x(t) = 10\cos(10t + \pi/6) + 20\sin(30t - \pi/3)$

↳ Not distorted

2) $x(t) = 10\cos(10t + \pi/6) + 20\sin(60t - \pi/3)$

↳ distorted (phase distortion)

3) $x(t) = 10\cos(60t + \pi/6) + 20\sin(90t - \pi/3)$

↳ distorted (Amplitude + phase distortion)

4) $x(t) = 10\cos(40t + \pi/6) + 20\sin(215t - \pi/3)$

↳ Not distorted

3.8) Total Harmonic Distortion (THD) and Distortion Factor (DF)

$$THD = \frac{\sqrt{C_{RMS,2}^2 + C_{RMS,3}^2 + C_{RMS,3}^2 + \cdots}}{C_{RMS,1}} = \frac{\sqrt{X_{RMS}^2 - C_{RMS,1}^2}}{C_{RMS,1}}$$

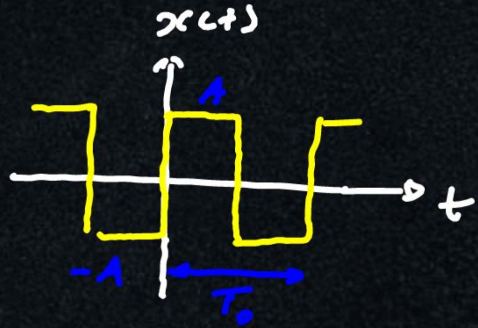
$$DF = \frac{C_{RMS,1}}{X_{RMS}}$$

$C_{RMS,1} \rightarrow$ RMS of the fundamental

$X_{RMS} \rightarrow$ RMS of the signal

Note: $THD = \sqrt{\left(\frac{1}{DF}\right)^2 - 1} \Rightarrow DF = \sqrt{\frac{1}{1 + (THD)^2}}$

EX :- Calculate the RMS, THD, and DF of the square wave signal



$$x(t) = \sum_{n=1,3,\dots}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

where $C_n = \frac{4A}{n\pi}$, $\theta_n = -\pi/2$

$$X_{RMS} = \sqrt{\frac{1}{T_0}\int_0^{T_0} x^2(t)\,dt} = \sqrt{\frac{1}{T_0}\int_0^{T_0} A^2\,dt} = \boxed{A}$$

$$C_{1,RMS} = \frac{4A}{\sqrt{2}\,\pi}$$

$$THD = \frac{\sqrt{X_{RMS}^2 - C_{1,RMS}^2}}{C_{1,RMS}} = \sqrt{\frac{\pi^2}{8} - 1} = \boxed{48.3}\ \%$$

$$DF = \frac{C_{1,RMS}}{X_{RMS}} = \frac{4}{\sqrt{2}\,\pi} = \boxed{0.9}$$