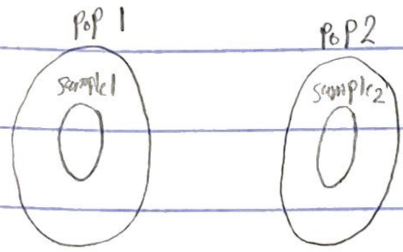
10.2 : Inferences about $\mu_1 - \mu_2$, σ_1 and σ_2 unknown.



μ_1 : Pop. 1 mean

μ_2 : Pop 2 mean

σ_1 : Pop 1 . st. dev.

σ_2 : Pop 2 . st. dev.

$\bar{X}_1$: sample 1 mean

$\bar{X}_2$: sample 2 mean

S_1 : sample 1 st. dev.

S_2 : sample 2 st. dev.

n_1 : size of sample 1

n_2 : size of sample 2.

Note :
- 95% : confidence level
- 0.95 : confidence coeffecient.

→ point estimater for $\mu_1 - \mu_2 = \bar{X}_1 - \bar{X}_2$

→ standard error $= \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$

→ margen of error $= t_{\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$

→ $$df = \left\lfloor \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\left(\frac{1}{n_1 - 1}\right)\left(\frac{S_1^2}{n_1}\right)^2 + \left(\frac{1}{n_2 - 1}\right)\left(\frac{S_2^2}{n_2}\right)^2} \right\rfloor$$

Note : $\lfloor \; \rfloor$ means Round down

exp. $\lfloor 9.4 \rfloor = 9$.

$$\rightarrow (1-\alpha)\,CI = (\bar{X}_1 - \bar{X}_2) \mp t_{\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}.$$

★ Assumptions : الشروط

1. sample 1 random sample from pop. 1.
2. sample 2 random sample from pop. 2.
3. samples 1 and 2 are independent.
4. Pop. 1 and pop. 2 have Normal distribution OR sample 1 and sample 2 are large enough.

↳ Large enough : $n_1 + n_2 \geq 20$ — ما بزبط $n_1 = 1$, $n_2 = 19$

$n_1 \simeq n_2$ — لازم تقريبا يكونو قد بعض

★ Hypothesis testing for $\mu_1 - \mu_2$.

① Lower Tailed test

$H_0 : \mu_1 - \mu_2 \geq D_0$

$H_1 : \mu_1 - \mu_2 < D_0$

② upper-Tailed test

$H_0 : \mu_1 - \mu_2 \leq D_0$

$H_1 : \mu_1 - \mu_2 > D_0$

③ Two Tailed test

$H_0 : \mu_1 - \mu_2 = D_0$

$H_1 : \mu_1 - \mu_2 \neq D_0$.

★ Test statistic :

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - D_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

★ under the same assumptions using

$$df = \left[\frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\left(\frac{1}{n_1-1}\right)\left(\frac{S_1^2}{n_1}\right)^2 + \left(\frac{1}{n_2-1}\right)\left(\frac{S_2^2}{n_2}\right)^2} \right]$$

→ partical advice :

If we additionally have $\sigma_1 = \sigma_2 \implies$

- Test statistic : $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - D_0}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$.

إذا كانت $\sigma_1 = \sigma_2$

- $df = n_1 + n_2 - 2$

- S^2 : Pooled estimate of the variance (pooled sample variance).

$$S^2 = \frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{n_1 + n_2 - 2}.$$