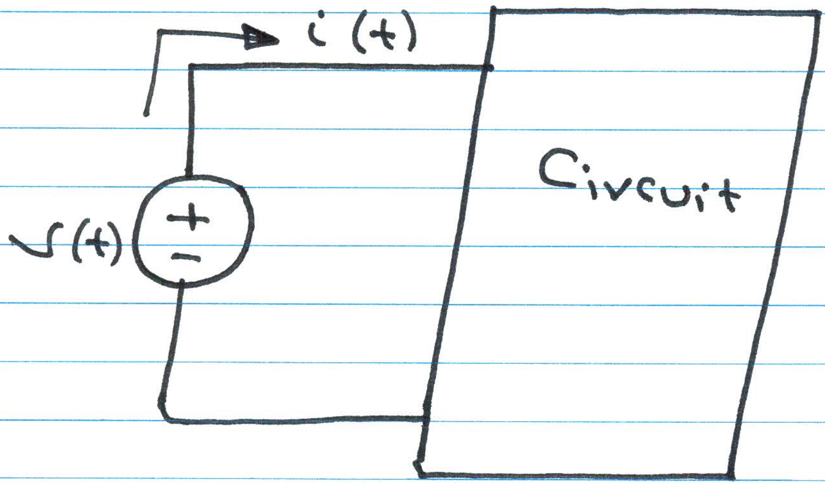
Sinusoidal Steady State
Power Calculation

Instantaneous Power : $p(t)$



$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \phi_i)$$

$$p(t) = v(t)\, i(t)$$

$$p(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \phi_i)$$

$$\cos\alpha \cos\beta = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$$

$$\therefore\ p(t) = \frac{V_m I_m}{2}\left[\cos(\theta_v - \phi_i) + \cos(2\omega t + \theta_v + \phi_i)\right]$$

Constant

Twice the excitation frequency

Example

$v(t) = 4 \cos(\omega t + 60°)$ V

$Z(j\omega) = 2\angle 30°$ Ω

Find $p(t)$.

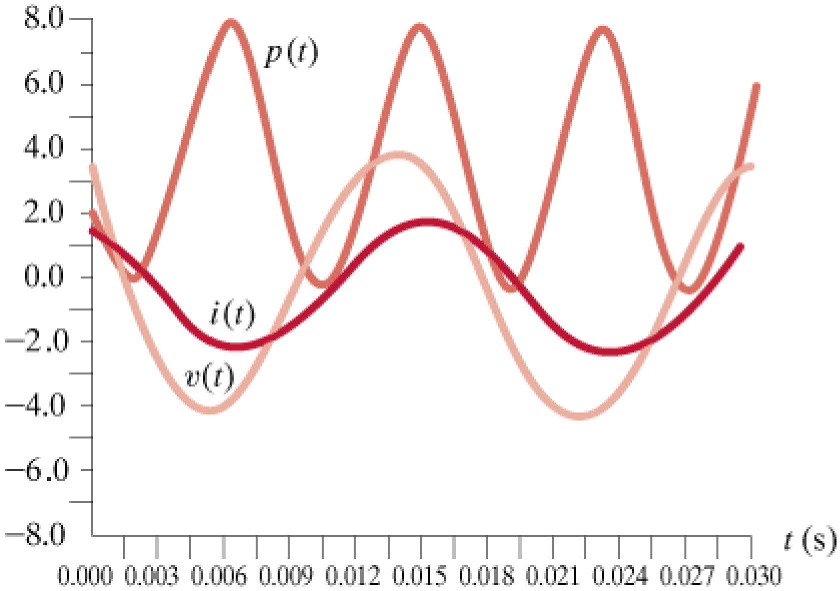
$$\vec{I} = \frac{\vec{V}}{Z} = \frac{4\angle 60°}{2\angle 30°} = 2\angle 30° \ A$$

$\therefore i(t) = 2 \cos(\omega t + 30°)$ A

$p(t) = v(t)\, i(t)$

$p(t) = 4 \cos 30° + 4 \cos(2\omega t + 90°)$

$p(t) = 3.46 + 4 \cos(2\omega t + 90°)$


8.0
6.0
4.0
2.0
0.0
−2.0
−4.0
−6.0
−8.0
$p(t)$
$i(t)$
$v(t)$
t (s)
0.000
0.003
0.006
0.009
0.012
0.015
0.018
0.021
0.024
0.027
0.030

Average Power : Real Power

$$P_{av} = \frac{1}{T}\int_0^T p(t)\,dt$$

$$P_{av} = \frac{1}{2} V_m I_m \cos(\theta_v - \phi_i)$$

$$\theta_v - \phi_i = \theta_z$$

$$\therefore \; P_{av} = \frac{1}{2} V_m I_m \cos\theta_z$$

1) For Resistor

$$\theta_v - \phi_i = 0 \longrightarrow \theta_z = 0$$

$$\therefore \; P_{av} = \frac{1}{2} V_m I_m = \frac{V_m^2}{2R} = \frac{I_m^2 R}{2}$$

2) For Inductor

$$\theta_v - \phi_i = 90^\circ$$

$$P_{av} = 0$$

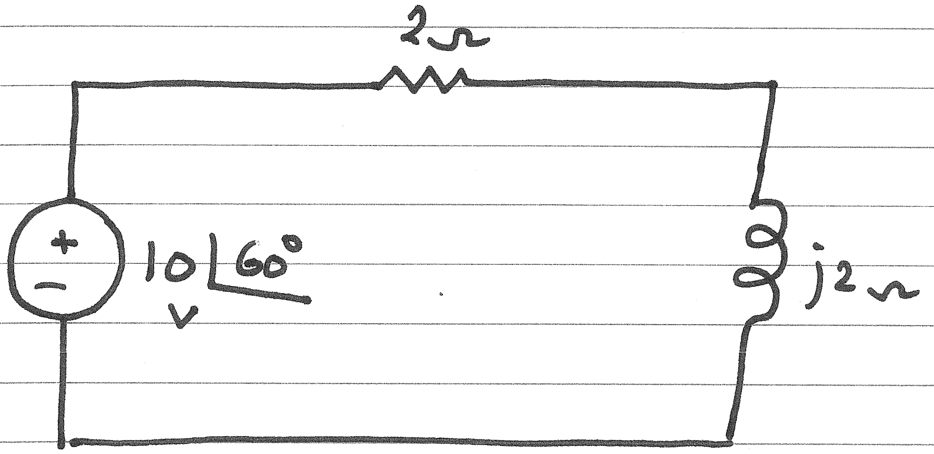
3) For Capacitor

$$\theta_v - \phi_i = -90^\circ$$

$\therefore \; P_{av} = 0$

$\therefore$ Reactive impedances absorb no average power

Example



Find the average power absorbed by each elemen.

$$\vec{I} = \frac{10\angle 60^\circ}{2+j2} = 3.53\angle 15^\circ \; A$$

$$P_{av} = \frac{V_m I_m}{2} \cos(\theta_v - \phi_i)$$

$$P_{av_{j2\Omega}} = 0$$

$$P_{av_{2\Omega}} = \frac{I_m^2 R}{2} = \frac{(3.53)^2 \cdot 2}{2} = 12.5\,W$$

To Calculate the average power supplied by the Source

$$P_{av_{Vs}} = \frac{V_m I_m}{2} \cos(\theta_v - \phi_i)$$

$I_m = 3.53\,A$

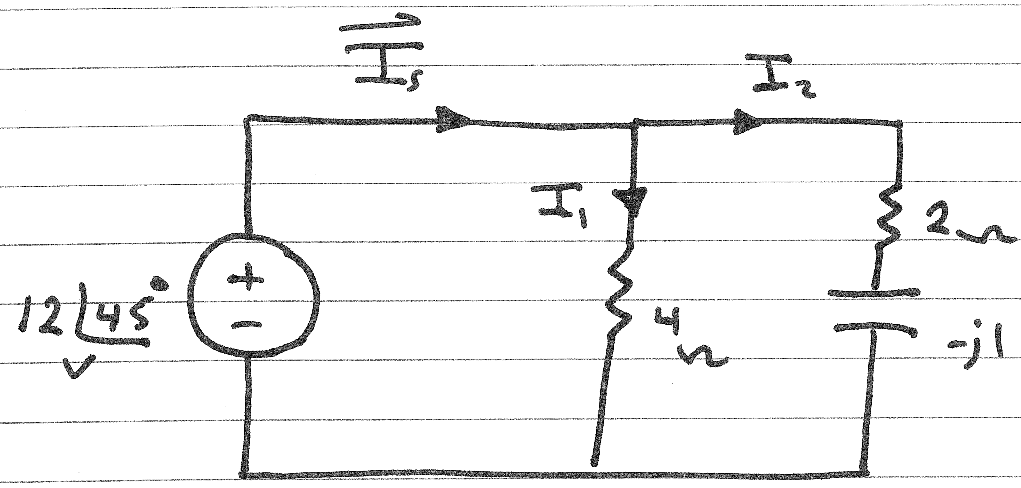
$V_m = 10\,V$

$\theta_v = 60^\circ$, $\phi_i = 15^\circ$

$$\therefore P_{av_{Vs}} = \frac{(10)(3.53)}{2} \cos(60 - 15^\circ)$$

$$= 12.5 \text{ Watt}$$

Example



Determine the average power absorbed by each resistor.

Determine the total average power absorbed and the average power supplied by the source.

$$\vec{I}_1 = \frac{12\angle 45^\circ}{4\Omega} = 3\angle 45^\circ \text{ A}$$

$$\vec{I}_2 = \frac{12\angle 45^\circ}{2-j1} = 5.36\angle 71.57^\circ \text{ A}$$

$$\vec{I}_s = \vec{I}_1 + \vec{I}_2 = 8.15\angle 62.1^\circ \text{ A}$$

1) $P_{4\Omega} = \dfrac{I_{1m}^2 \cdot 4}{2} = 18\text{W}$

2) $P_{2\Omega} = \dfrac{I_{2m}^2 \cdot 2}{2} = 28.7\text{ W}$

$\therefore$ Total Average power absorbed = 46.7 W

$$P_{V_s} = \frac{V_m I_m}{2} \cos(\theta_v - \phi_i)$$

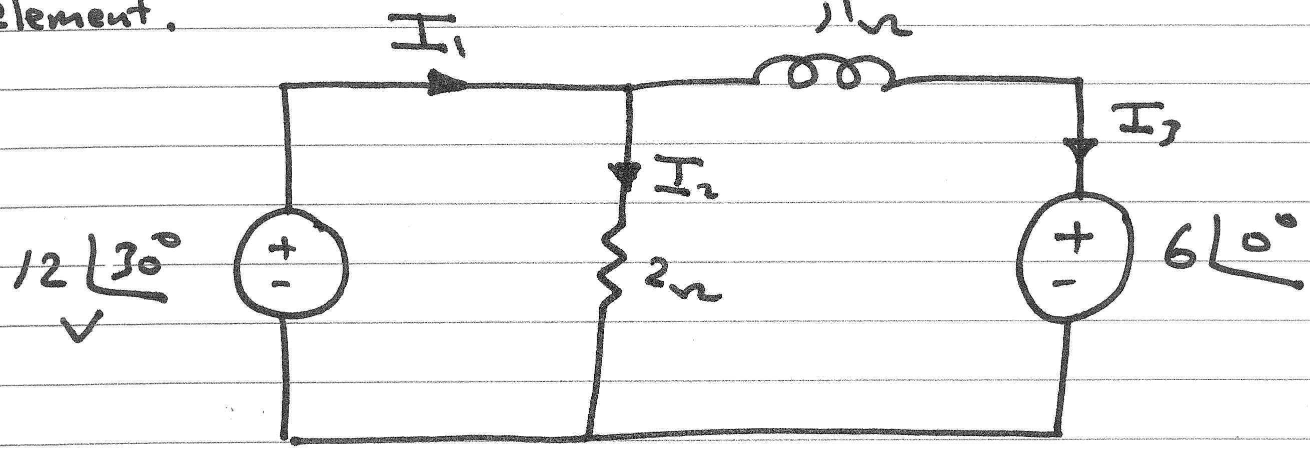
$$P_{V_s} = \frac{(12)(8.16)}{2} \cos(45 - 62.1)$$

$$P_{V_s} = 46.7\ W$$

$$\therefore \quad P_{V_s} = P_{4\Omega} + P_{2\Omega} + P_{-j1}$$

Example

Determine average power absorbed or supplied by each element.



$$\vec{I_2} = \frac{12\angle 30^\circ}{2} = 6\angle 30^\circ \text{ A}$$

$$\vec{I_3} = \frac{12\angle 30^\circ - 6\angle 30^\circ}{j1} = 7.43\angle -36.19^\circ \text{ A}$$

$$\vec{I_1} = \vec{I_2} + \vec{I_3} = 11.29\angle -7.07^\circ \text{ A}$$

$$P_{2\Omega} = \frac{I_{2m}^2 \cdot 2}{2} = 36 \text{ W}$$

$$P_{12\angle 30^\circ} = \frac{V_m I_m}{2} \cos(\theta_v - \phi_i)$$

$$P_{12\angle 30^\circ} = \frac{(12)(11.29)}{2} \cos\left(30 - (-7.07)\right)$$

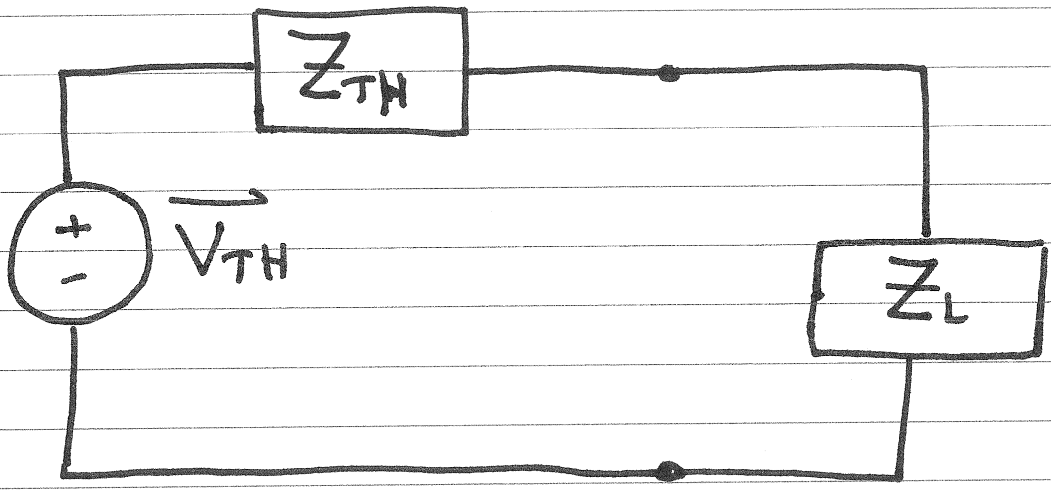
$$P_{12\angle 30^\circ} = 54 \text{ W} \quad \text{Supply}$$

$$P_{6\angle 0^\circ} = \frac{V_m I_m}{2} \cos(\theta_v - \phi_i)$$

$$P_{6\angle 0^\circ} = \frac{(6)(7.43)}{2} \cos(0 - (-36.19^\circ))$$

$P_{6\angle 0^\circ} = 18 \text{ W}$ absorbed

Maximum Average power Transfer



$$Z_{TH} = R_{TH} + j\,X_{TH}$$

$$Z_L = R_L + j\,X_L$$

$$P_L = \frac{I_{Lm}^2 \cdot R_L}{2}$$

$$\vec{I} = \frac{\vec{V}_{TH}}{Z_{TH} + Z_L}$$

$$\vec{I} = \frac{\vec{V}_{TH}}{(R_{TH} + R_L) + j\,(X_{TH} + X_L)}$$

$$P_L = \frac{I_m^2\,R_L}{2}$$

$$P_L = \frac{1}{2} \frac{V_{TH}^2 \cdot R_L}{(R_{TH}+R_L)^2 + (X_{TH}+X_L)^2}$$

$$\frac{\partial P_L}{\partial R_L} = 0 \quad ; \quad \frac{\partial P_L}{\partial X_L} = 0$$

$$\frac{\partial P_L}{\partial X_L} = \frac{-2 V_{TH}^2 R_L (X_L + X_{TH})}{2\left[(R_L+R_{TH})^2 + (X_L+X_{TH})^2\right]^2}$$

For $\dfrac{\partial P_L}{\partial X_L} = 0 \longrightarrow X_L = -X_{TH}$

$$\frac{\partial P_L}{\partial R_L} = \frac{V_{TH}^2 \left[(R_L+R_{TH})^2 + (X_L+X_{TH})^2 - 2R_L(R_L+R_{TH})\right]}{2\left[(R_L+R_{TH})^2 + (X_L+X_{TH})^2\right]^2}$$

For $\dfrac{\partial P_L}{\partial R_L} = 0 \longrightarrow R_L = \sqrt{R_{TH}^2 + (X_L+X_{TH})^2}$

$$X_L = -X_{TH}$$

$$\therefore R_L = R_{TH}$$

$$\therefore Z_L = Z_{TH}^*$$

$$P_{L,max} = \frac{1}{8} \frac{V_{TH}^2}{R_L}$$

$\therefore$ For maximum average power Transfer

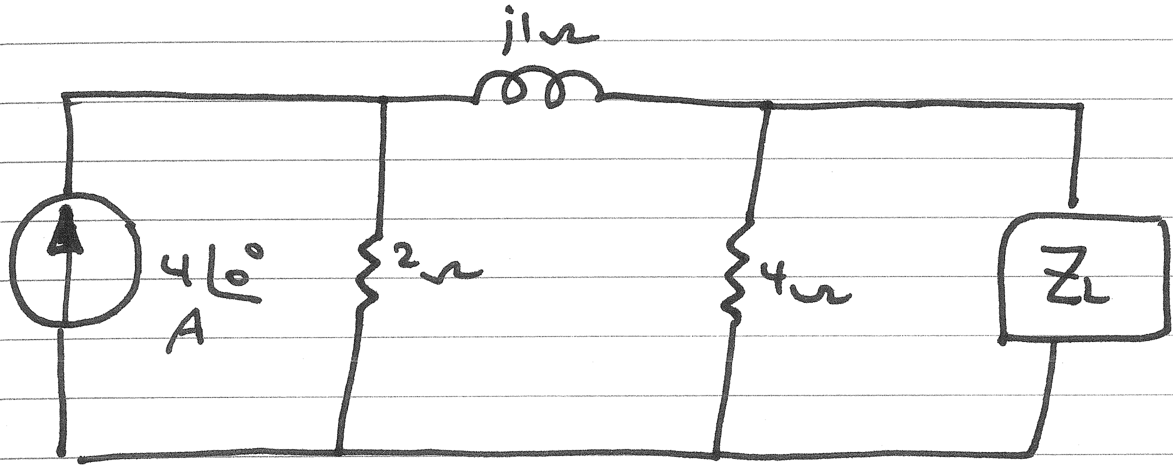
$$Z_L = Z_{TH}^*$$

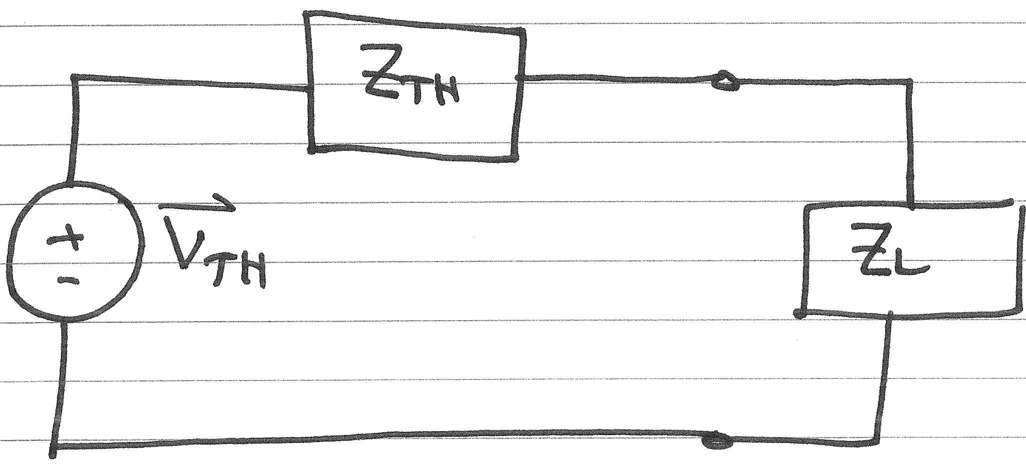
$$P_{L,max} = \frac{1}{8}\frac{V_{TH}^2}{R_{TH}}$$

Example

Find Z_L for maximum average power transfer.

Compute the maximum average power supplied to the load.





$$\vec{V}_{TH} = 4\angle 0^\circ \; \frac{2}{2 + j1 + 4} \cdot 4 = 5.28 \angle -9.46^\circ \; V$$

$$Z_{TH} = 4\,\Omega \parallel (2 + j1)\,\Omega$$

$$Z_{TH} = (1.4 + j\,0.43)\,\Omega$$

$$\therefore Z_L = (1.4 - j\,0.43)\,\Omega$$

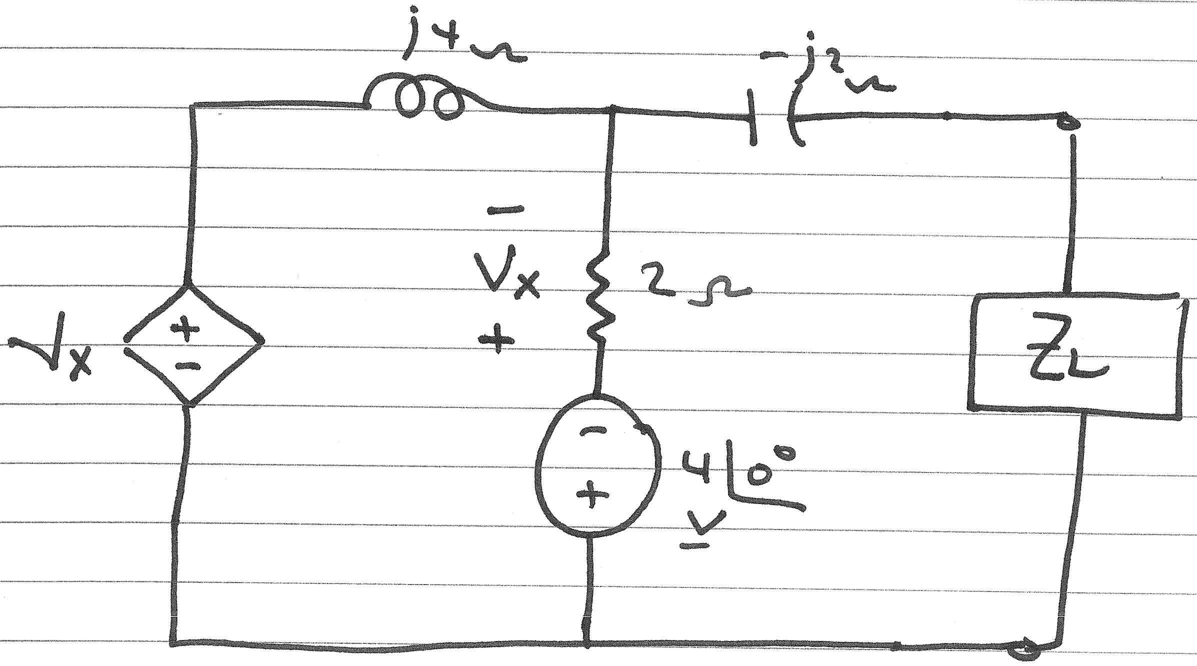
$$P_{,max} = \frac{1}{8}\,\frac{V_{TH}^2}{R_{TH}}$$

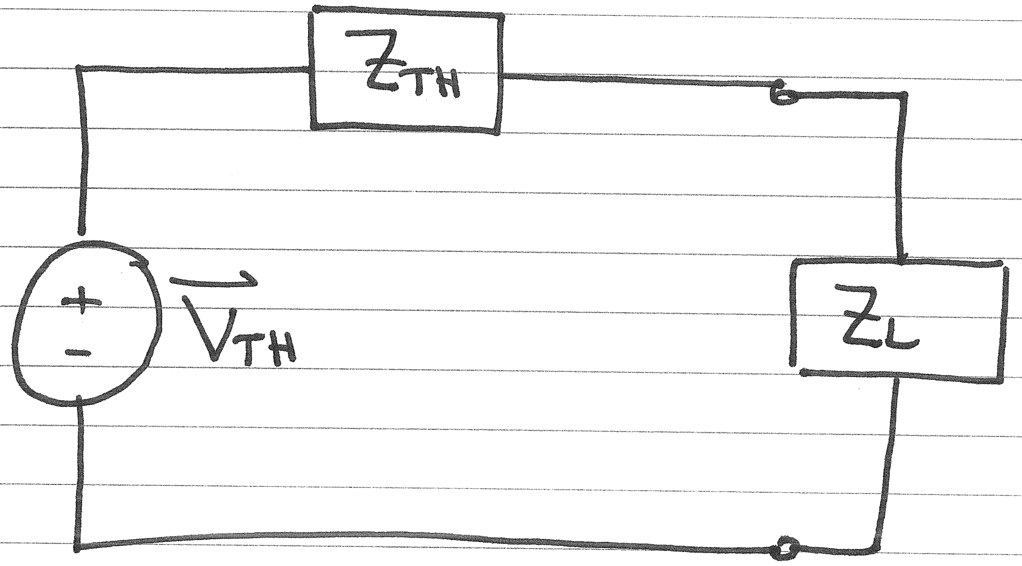
$$P_{,max} = 2.489\ W$$

Example

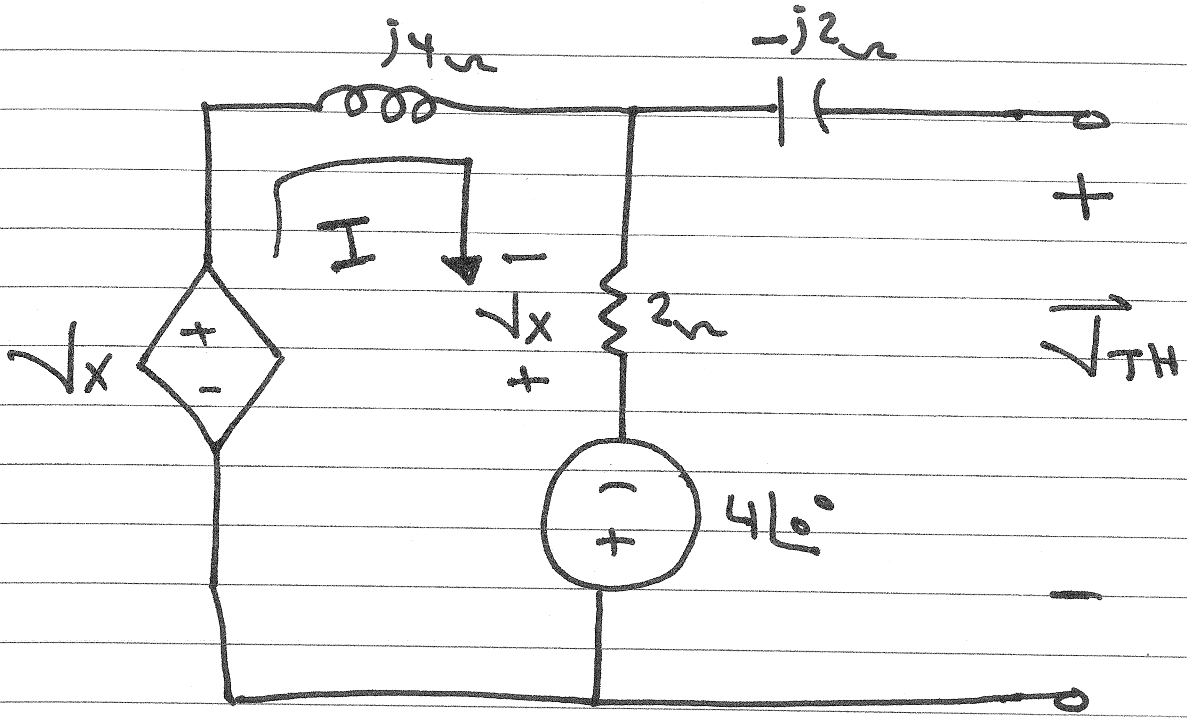
Find Z_L for maximum average power transfer

Compute the maximum average power supplied to Z_L





$$Z_L = Z_{TH}^*$$



$$\vec{V}_{TH} = 2\vec{I} - 4\angle 0^\circ$$

$$\vec{I} = \frac{V_x + 4\angle 0^\circ}{2 + j4}$$

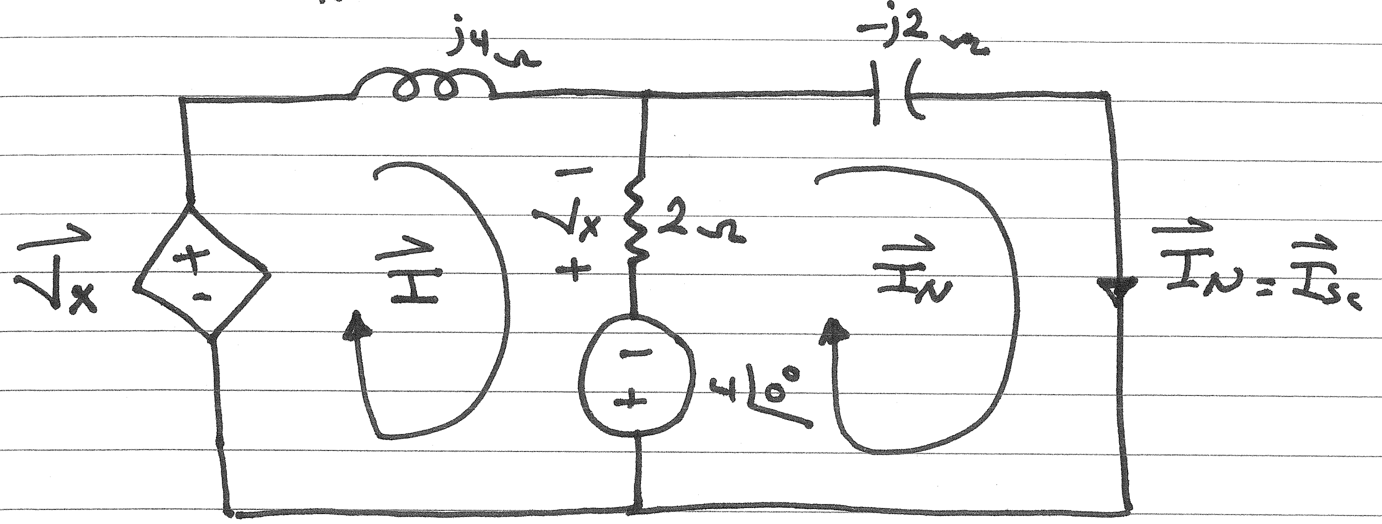
$$\vec{V}_x = -2\vec{I}$$

$$\vec{I} = 0.707\angle -45^\circ \text{ A}$$

$$\therefore \vec{V}_{TH} = (-3 - j1)\text{ V}$$

$$\vec{V}_{TH} = 3.16\angle 198.43^\circ \text{ V}$$

$$Z_{TH} = \frac{\vec{V}_{TH}}{\vec{I}_N}$$



KVL for mesh 1 :

$$\vec{V}_x + 4\angle 0° = (2+j4)\vec{I} - 2\vec{I}_N$$

$$\vec{V}_x = 2(\vec{I}_N - \vec{I})$$

KVL for mesh 2 :

$$-4\angle 0° = -2\vec{I} + (2-j2)\vec{I}_N$$

Solving for $\vec{I}_N$

$$\vec{I}_N = (-1-j2)\ A$$

$$\vec{I}_N = 2.24\angle 243.43°\ A$$

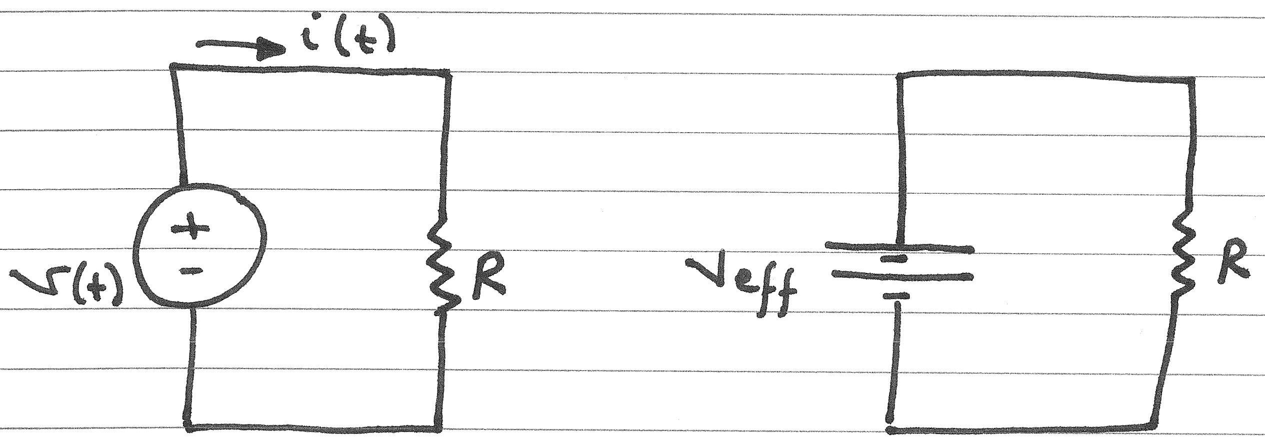
$$\therefore Z_{TH} = \frac{\vec{V}_{TH}}{\vec{I}_N} = 1.41\angle -45°\ \Omega = (1-j1)\ \Omega$$

$$\therefore Z_L = Z_{TH}^* = 1.41\angle +45°\ \Omega = (1+j1)\ \Omega$$

$$\therefore P_{L,max} = \frac{V_{TH}^2}{8R_{TH}} = 1.25\ W$$

Effective or RMS value

The effective value of a periodic voltage (current) is the dc voltage (current) that delivers the same average power to a resistor as the periodic voltage (current).



let $v(t) = V_m \cos(\omega t + \theta_v)$

$$\therefore \quad P_1 = \frac{V_m^2}{2R}$$

$$P_2 = \frac{V_{eff}^2}{R}$$

$$P_1 = P_2$$

$$\therefore \quad \frac{V_m^2}{2R} = \frac{V_{eff}^2}{R}$$

$$\therefore V_{eff} = \frac{V_m}{\sqrt{2}}$$

RMS : Root Mean Square

let $v(t) = V_m \cos(\omega t + \theta_v)$

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2(\omega t + \theta_v)\,dt}$$

$$V_{RMS} = V_m \sqrt{\frac{1}{T}\int_0^T \cos^2(\omega t + \theta_v)\,dt}$$

$$V_{RMS} = V_m \sqrt{\frac{1}{T}\int_0^T \frac{1}{2}\left(1 + \cos 2(\omega t + \theta_v)\right)dt}$$

$$V_{RMS} = V_m \frac{1}{\sqrt{2}}$$

$$P_{av} = \frac{V_m I_m}{2} \cos(\theta_v - \phi_i)$$

$$P_{av} = V_{rms} I_{rms} \cos(\theta_v - \phi_i)$$

For a resistor

$$P_{av} = V_{rms} I_{rms} \cos(\theta_v - \phi_i)$$

$$V_{rms} = R I_{rms} \quad ; \quad \theta_v - \phi_i = 0$$

$$\therefore \quad P_{av} = \frac{V_{rms}^2}{R}$$

$$\therefore \quad P_{av} = I_{rms}^2 R$$

Apparent power and power factor

$$P_{av} = V_{rms} I_{rms} \cos(\theta_v - \phi_i)$$

$$P_{apparent} = V_{rms} I_{rms}$$

$P_{apparent}$ measured in VA

PF = power factor

$$PF = \cos(\theta_v - \phi_i)$$

$$\therefore \quad P_{av} = P_a \cdot PF$$

1) For Resistor

$\theta_v - \phi_i = 0$

$\therefore PF = 1$

2) For inductor

$\theta_v - \phi_i = +90^\circ$

$\therefore PF = 0$

3) For Capacitor

$\theta_v - \phi_i = -90^\circ$

$\therefore PF = 0$

4) For Inductive Load

$90^\circ > \theta_v - \phi_i > 0$

$1 > PF > 0$ Lagging Power factor

5) For Capacitive Load

$-90^\circ < \theta_v - \phi_i < 0$

$1 > PF > 0$ Leading power factor

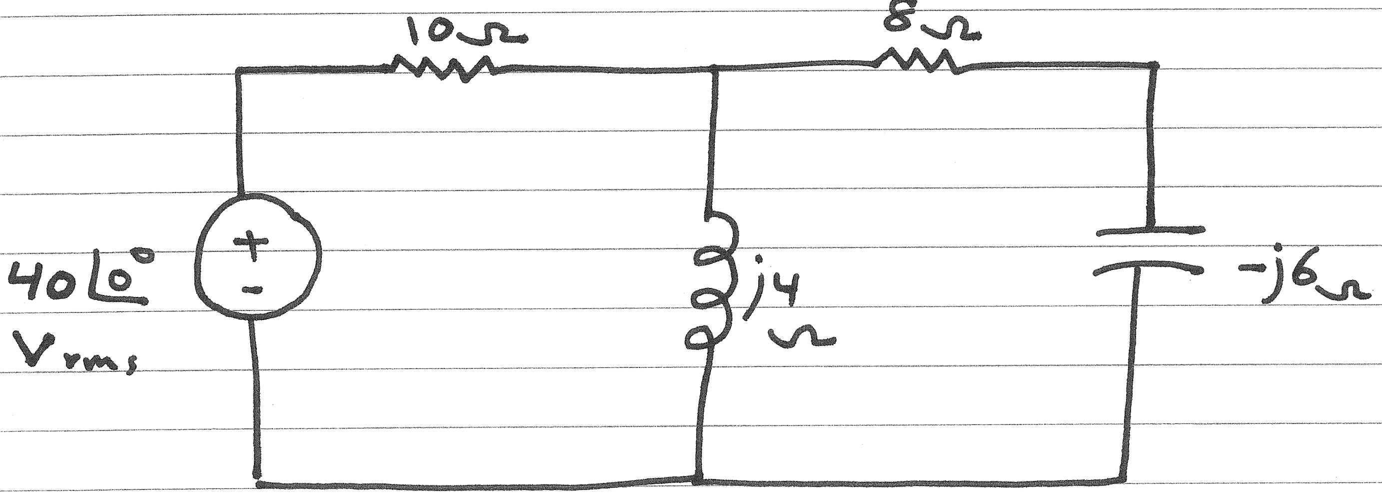
$$PF = \cos(\theta_v - \phi_i)$$

$$\cos(\alpha) = \cos(-\alpha)$$

Power factor is either Leading or Lagging reffering to the phase of the Current with respect to the voltage.

Example

Calculate the power factor seen by the source and the average power supplied by the source.



$$Z = 10 + j4 \parallel (8 - j6)$$

$$Z = 12.69 \angle 20.62^\circ \ \Omega$$

$$\vec{I}_s = \frac{40\angle 0^\circ}{Z} = 3.152 \angle -20.62^\circ \ A_{rms}$$

$$\theta_v = 0^\circ \ , \quad \phi_i = -20.62^\circ$$

$$PF = \cos(\theta_v - \phi_i)$$

$$PF = \cos(20.62^\circ)$$

$$PF = 0.936 \ \text{Lagging}$$

The average power supply by the source is equal to the average power absorbed by the circuit

---

$$P_{av} = V_{rms} I_{rms} \cos(\theta_v - \phi_i)$$

$V_{rms} = 40 \, V_{rms}$

$I_{rms} = 3.152 \, A_{rms}$

$\theta_v = 0°$

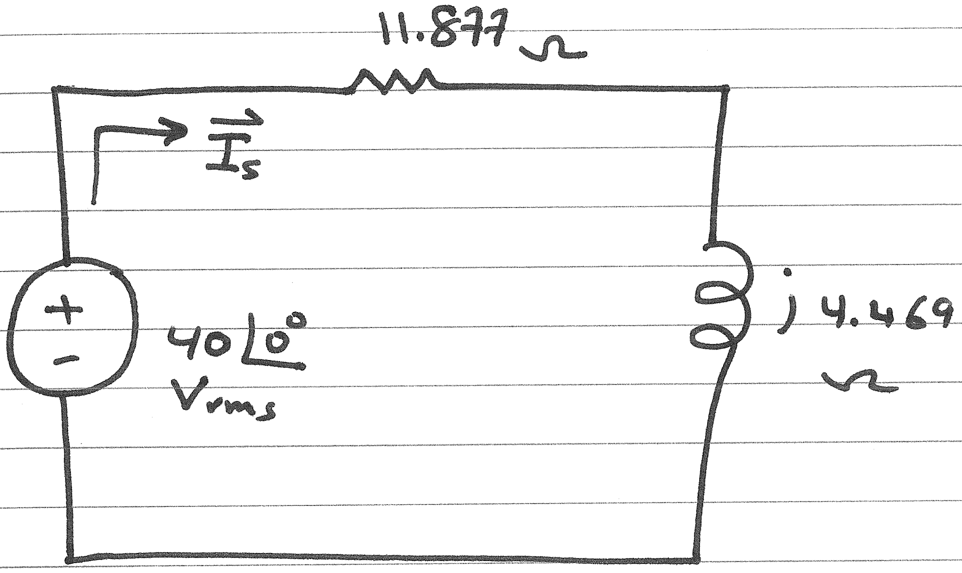
$\phi_i = -20.62°$

$\therefore P_{av} = (40)(3.152) \cos(0 - (-20.62°))$

$\therefore P_{av} = 118$ Watt

$$Z = 12.69 \angle 20.62^\circ \ \Omega$$

$$Z = 11.877 + j\,4.469 \ \Omega$$



$$\therefore \ P_{av} = I_{rms}^2 R$$

$$P_{av} = (3.152)^2 (11.877)$$

$$P_{av} = 118 \ W$$

also $P_{av} = P_{av_{10\Omega}} + P_{av_{8\Omega}} + P_{av_{-j6}} + P_{av_{j4}}$

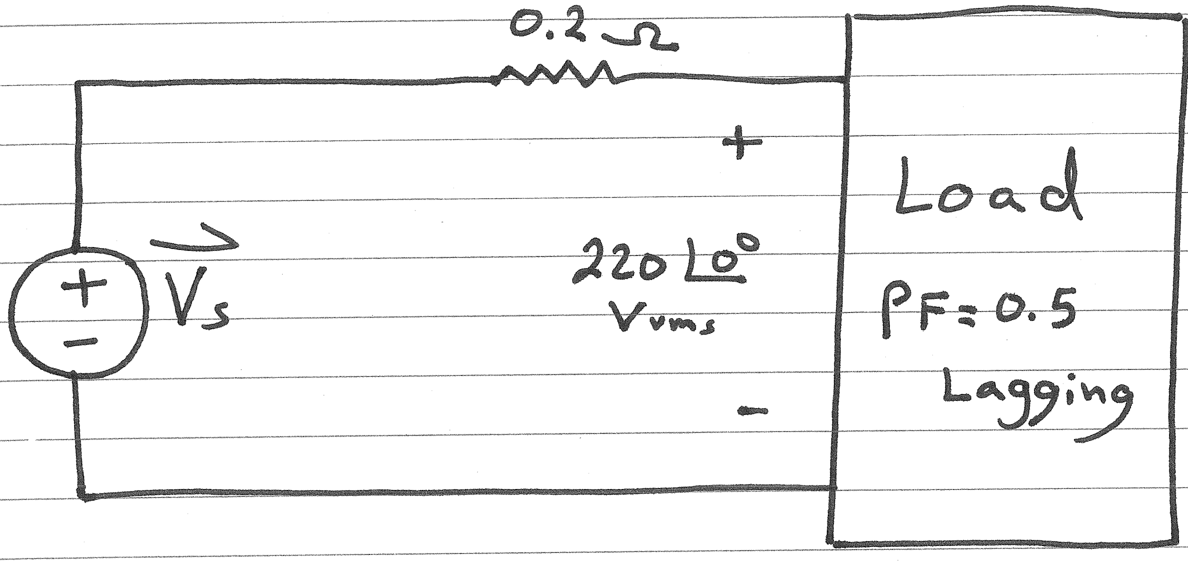
$$P_{av} = P_{av_{10\Omega}} + P_{av_{8\Omega}}$$

Example

An industrial Load Consumes 11KW at 0.5 PF Lagging from a 220V rms Line. The transmission Line resistance from the power Company to the plant is 0.2 Ω.

1) Determine the average power that must be Supplied by the power Company

2) Repeat 1 if the Power factor is changed to unity.



$$P_{av_L} = V_{rms} \cdot I_{rms} \cdot PF$$

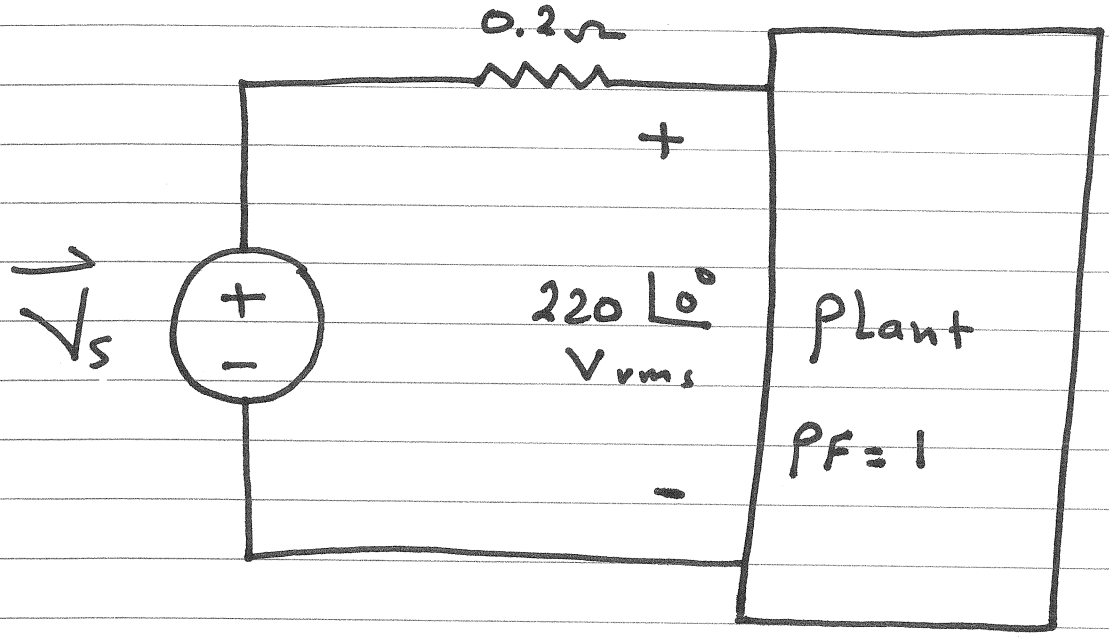
$$\therefore \; I_{rms} = \frac{P_{av_L}}{V_{rms} \cdot PF}$$

$$I_{rms} = \frac{11\,KW}{(220)(0.5)} = 100 \; A_{rms}$$

$$P_{Loss} = (I_{rms}^2) \cdot (0.2) = 2\,KW$$

$$\therefore \; P_{av_{sup}} = P_{av_L} + P_{av_{Loss}}$$

$$P_{av_{sup}} = 13\,KW$$



$$P_{av_{Load}} = V_{rms} \cdot I_{rms} \cdot PF$$

$$\therefore \ I_{rms} = \frac{P_{av}}{V_{rms} \cdot PF} = 50 \ A_{rms}$$

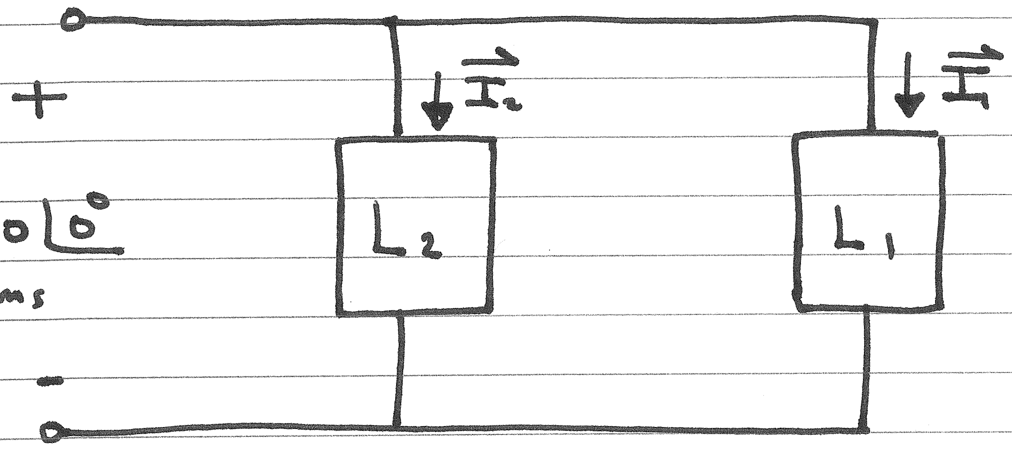
$$P_{Loss} = I_{rms}^2 \cdot R = (50)^2 \cdot (0.2) = 0.5 \, KW$$

$$\therefore \ P_{av_{sup}} = 0.5 \, KW + 11 \, KW$$

$$P_{av_{sup}} = 11.5 \, KW$$

Example

Find the power factor of the two loads



Load 1 : 10 KW , 0.9 Lagging PF

Load 2 : 5 KW , 0.95 Leading PF

$$\vec{I}_1 = \frac{10,000}{(2300)(0.9)} \angle -\cos^{-1} 0.9$$

$$\vec{I}_1 = 4.83 \angle -25.84^\circ \text{ A rms}$$

$$\vec{I}_2 = \frac{5000}{(2300)(0.95)} \angle +\cos^{-1} 0.95$$

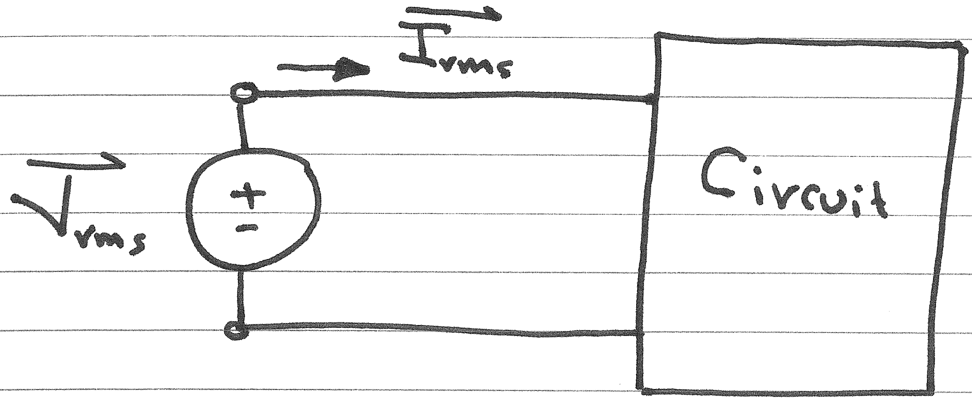
$$\vec{I}_2 = 2.288 \angle 18.195^\circ \text{ A rms}$$

$$\vec{I}_s = \vec{I}_1 + \vec{I}_2 = 6.78 \angle -12^\circ \text{ A rms}$$

$$PF = \cos(\theta_v - \theta_i) = \cos(0 - (-12^\circ))$$

$$PF = 0.978 \text{ Lagging}$$

Complex Power



$$\vec{V}_{rms} = V_{rms} \angle \theta_v$$

$$\vec{I}_{rms} = I_{rms} \angle \phi_i$$

$$\vec{S} \equiv \text{Complex power}$$

$$\vec{S} = \vec{V}_{rms} \cdot \vec{I}_{rms}^{\,*}$$

$$\vec{S} = V_{rms} I_{rms} \angle \theta_v - \phi_i$$

$$\vec{S} = V_{rms} I_{rms} \cos(\theta_v - \phi_i) + j V_{rms} I_{rms} \sin(\theta_v - \phi_i)$$

$$\vec{S} = P_{av} + j Q$$

$$\vec{S} = P_{av} + jQ$$

$P_{av} \equiv$ Average power in Watt

$Q \equiv$ Reactive Power in VAR

$\therefore \quad P_{av} = \text{Real}\left[\vec{S}\right]$

$Q = \text{Imajinary}\left[\vec{S}\right]$

$Q = V_{rms} I_{rms} \sin(\theta_v - \phi_i)$

1) For pure resistance

$$\theta_v - \phi_i = 0$$

$$\therefore \quad Q_R = 0$$

2) For pure inductance

$$\theta_v - \phi_i = +90°$$

$\therefore \quad Q_L = V_{rms} I_{rms}$

$V_{rms} = WL \, I_{rms}$

$$\therefore \quad Q_L = WL \, I_{rms}^2 = \frac{V_{rms}^2}{WL}$$

3) For pure Capacitance

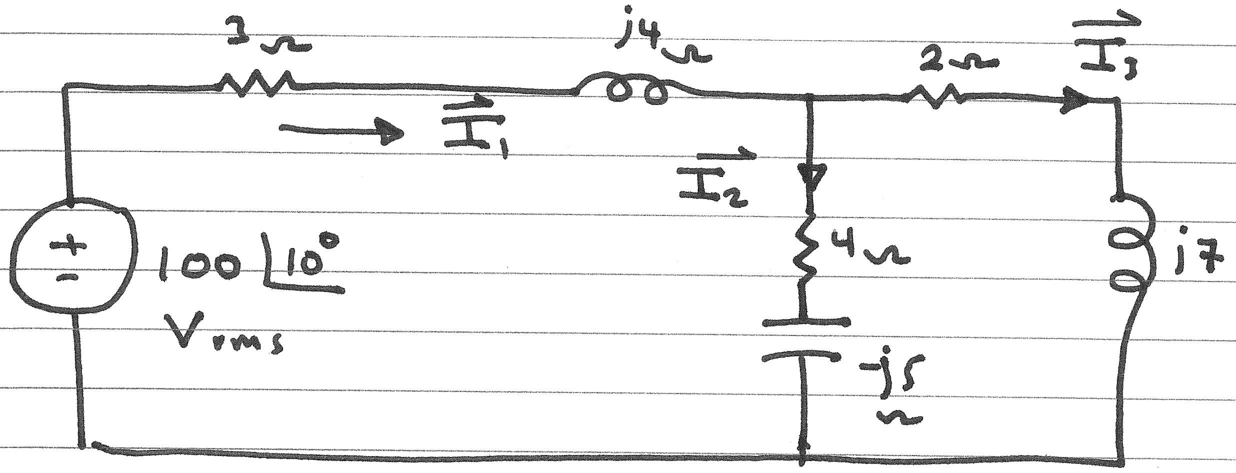
$$\theta_v - \Phi_i = -90^\circ$$

$$Q_c = -V_{rms} I_{rms}$$

$$I_{rms} = \omega C V_{rms}$$

$$\therefore Q_c = -\frac{I_{rms}^2}{\omega C} = -\omega C V_{rms}^2$$

What are the VARs Consumed by the Circuit



$$Q = V_{rms} I_{rms} \sin(\theta_v - \phi_i)$$

$$\vec{I}_1 = \frac{\vec{V}_s}{Z}$$

$$Z = (2+j7) \| (4-j5) + 3+j4$$

$$Z = 10.35 + j4.55 = 11.3 \angle 23.7^\circ \ \Omega$$

$$\therefore \vec{I}_1 = \frac{100 \angle 10^\circ}{11.3 \angle 23.7^\circ} = 8.84 \angle -13.7^\circ \ A_{rms}$$

$$\therefore Q = (100)(8.84) \sin\left(10 - (-13.7^\circ)\right)$$

$$\therefore Q = 355 \text{ VARS}$$

$$I_2 = 10.2 \ A_{rms}$$

$$I_3 = 8.95 \ A_{rms}$$

$$P_{av} = V_{rms} I_{rms} \cos(\theta_v - \phi_i)$$

$$Q = V_{rms} I_{rms} \sin(\theta_v - \phi_i)$$

$$\frac{Q}{P_{av}} = \tan(\theta_v - \phi_i)$$

$$\therefore Q = P_{av} \tan(\theta_v - \phi_i)$$

$$Q = P_{av} \tan\left[\cos^{-1}(PF)\right]$$

$$\vec{S} = P_{av} + jQ$$

$$\vec{S} = \sqrt{P_{av}^2 + Q^2} \angle \tan^{-1}\frac{Q}{P_{av}}$$

$$\vec{S} = V_{rms} I_{rms} \angle \theta_v - \phi_i$$

$$\therefore P_a = |S| = \sqrt{P_{av}^2 + Q^2}$$

$$\theta_v - \phi_i = \tan^{-1}\frac{Q}{P_{av}}$$

$$\theta_v - \phi_i = \tan^{-1} \frac{Q}{P_{av}}$$

To increase P.F, we need to decrease Q

$\therefore$ ~~For~~ inductive circuit, we add a Capacitor in paralleL to increase the Power factor

$$P_{av_T} = P_{av_1} + P_{av_2} + \cdots P_{av_n}$$

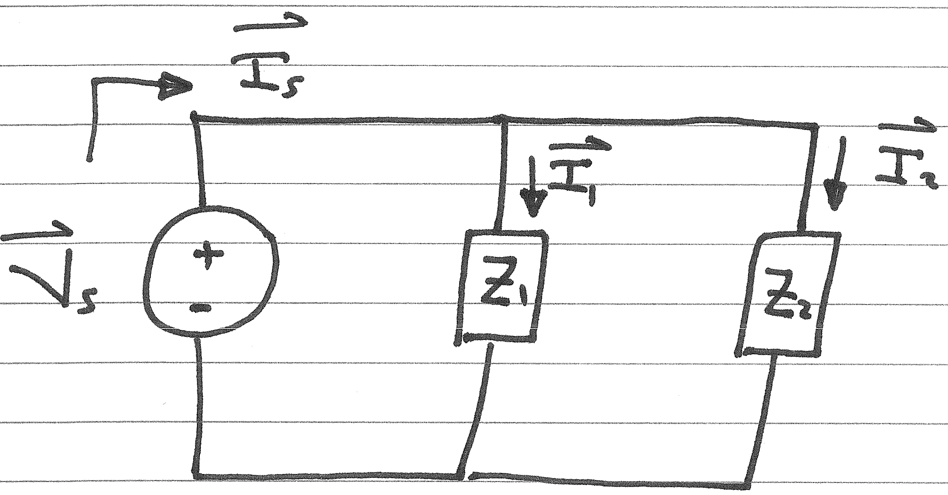
$$Q_T = Q_1 + Q_2 + \cdots Q_n$$

$$\vec{S}_T = P_{av_T} + j\,Q_T$$

$$\vec{S}_T = \vec{S}_1 + \vec{S}_2 + \cdots \vec{S}_n$$

Conservation of Ac Power

The Complex, real, and reactive powers of the Sources equal the respective Sum of the Complex, real, and reactive powers of the individual Loads.



$$\vec{S}_{source} = \vec{V}_s \cdot \vec{I}_s^{\,*}$$

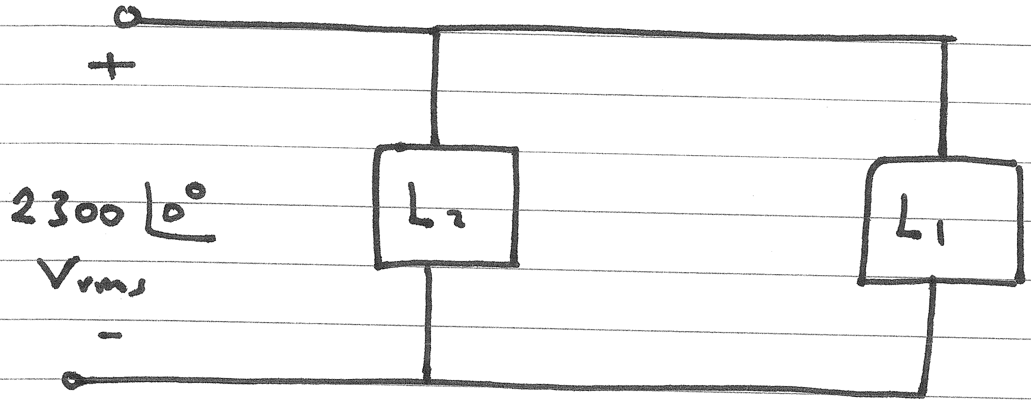
$$\vec{S}_{source} = \vec{V}_s \cdot \left(\vec{I}_1^{\,*} + \vec{I}_2^{\,*} \right)$$

$$\vec{S}_{source} = \vec{V}_s \cdot \vec{I}_1^{\,*} + \vec{V}_s \cdot \vec{I}_2^{\,*}$$

$$\vec{S}_{source} = \vec{S}_1 + \vec{S}_2$$

The Same results Can be obtained for a series Connection.

Find the power factor of the two Loads



Load 1 : 10 KW , 0.9 Lagging P.F

Load 2 : 5 KW , 0.95 Leading P.F

$$\vec{S_1} = P_{av_1} + j\,Q_1$$

$$Q_1 = P_{av_1} \tan\left[\cos^{-1}(P.F._1)\right] = 4843 \text{ VARS}$$

$$\therefore \vec{S_1} = 10000 + j\,4843$$

$$\vec{S_2} = P_{av_2} + j\,Q_2$$

$$Q_2 = -P_{av_2} \tan\left[\cos^{-1}(P.F._2)\right]$$

$$Q_2 = -1643 \text{ VARS}$$

$$\therefore \vec{S_2} = 5000 - j\,1643$$

$$\vec{S}_T = \vec{S}_1 + \vec{S}_2$$

$$\vec{S}_T = 15000 + j\,3200$$

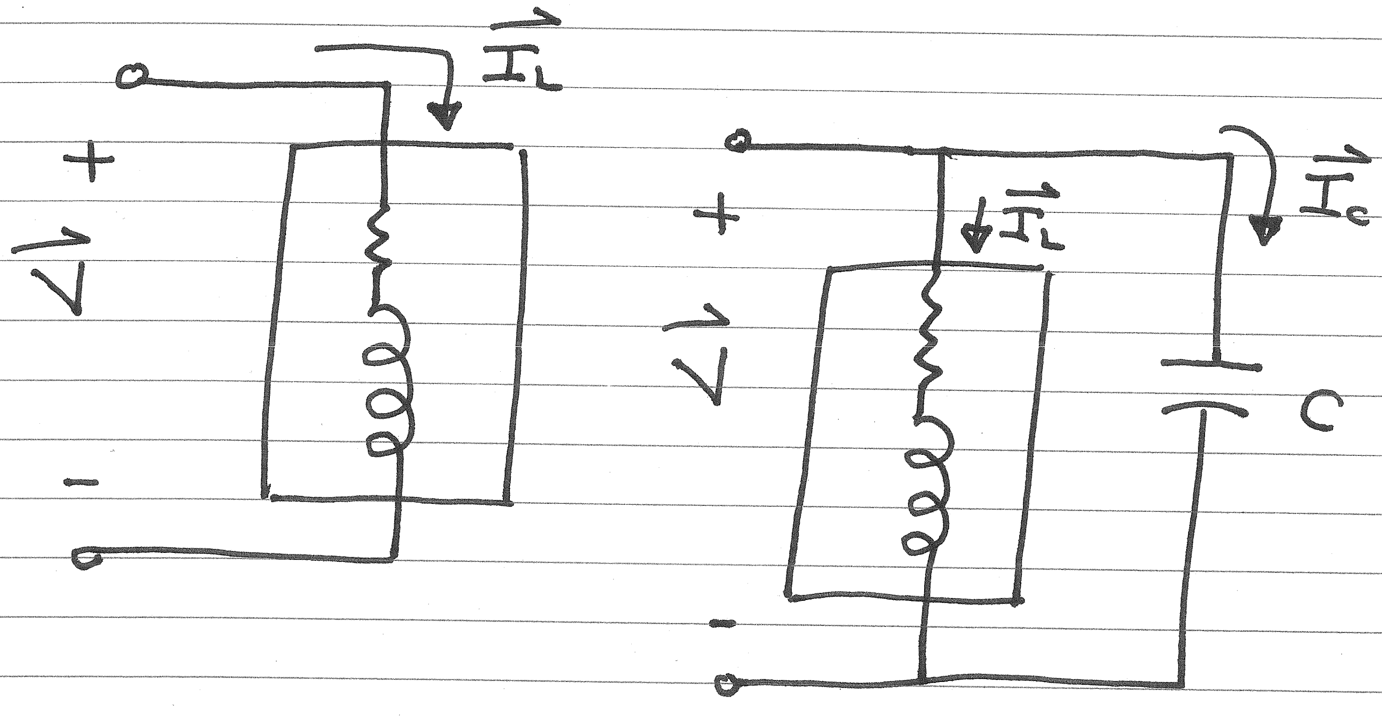
$$\vec{S}_T = 15337.5 \angle 12.02^\circ \quad VA$$

$$P.F = \cos(12.02)$$

$$P.F = 0.978 \text{ Lagging}$$

Power Factor Correction

Power factor Correction is the process of increasing the Power factor Without altering the voltage or Current to the original Load.



Power factor Correction is necessary for Economic Reason.

$$PF = \cos(\theta_v - \phi_i)$$

For R

$$P.F = 1$$

$$Q_R = 0$$

∴ To improve the power factor we must decrease the Reactive power

∴ For inductive Circuit, we add a Capacitor in parallel to the load.

$$Q_c = Q_{Final} - Q_{init}$$

$$C = -\frac{Q_c}{\omega V_{rms}^2}$$

Example

A certain industrial plant Consumes 1 MW at 0.7 Lagging power factor and a 2300 V rms.

What is the minimum Capacitor required to improve the power factor to 0.9 Lagging. $\omega = 377$ r/s

$$Q_{ini} = P_{av} \tan\left[\cos^{-1} PF_1\right]$$

$$Q_{ini} = 1M \tan\left[\cos^{-1} 0.7\right]$$

$$Q_{ini} = 1.02 \text{ MVARS}$$

$$Q_{Fin} = P_{av} \tan\left[\cos^{-1} PF_2\right]$$

$$Q_{Fin} = P_{av} \tan\left[\cos^{-1} 0.9\right]$$

$$Q_{Fin} = 0.484 \text{ MVARS}$$

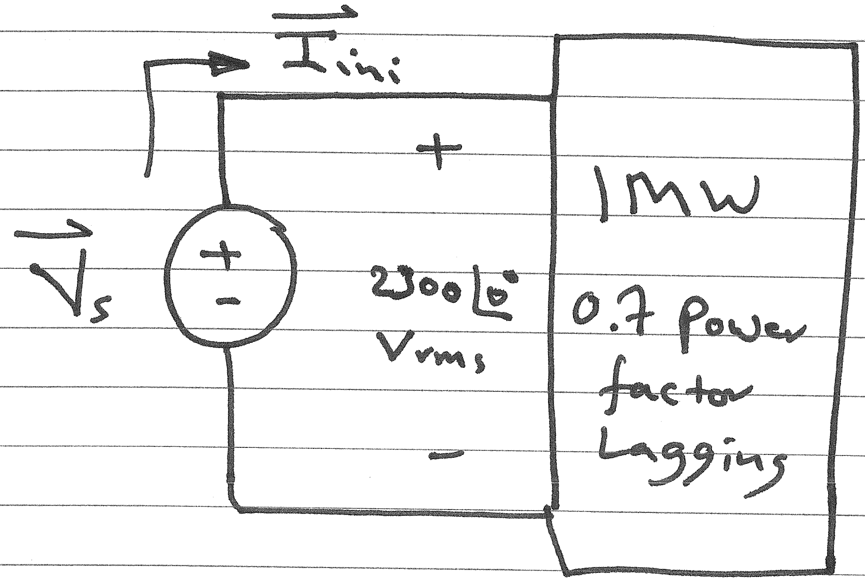
$$Q_c = Q_{Fin} - Q_{ini}$$

$$Q_c = -0.536 \text{ MVARS}$$

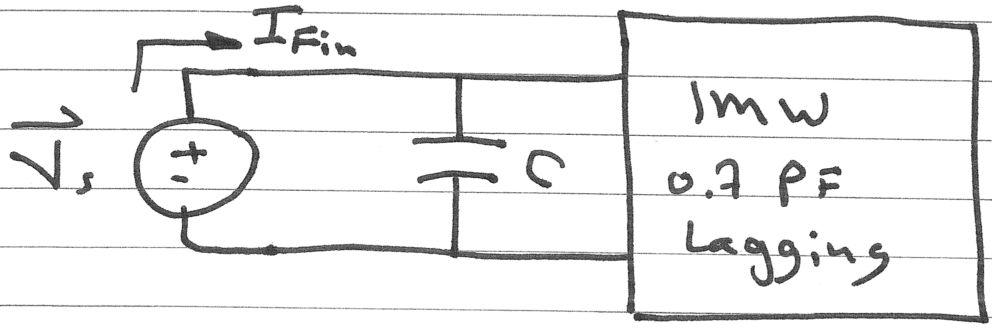
$$Q_c = -\frac{V_{rms}^2}{X_c}$$

$$Q_c = -\omega C V_{rms}^2$$

$$\therefore C = -\frac{Q_c}{\omega V_{rms}^2} = 269\ \mu F$$

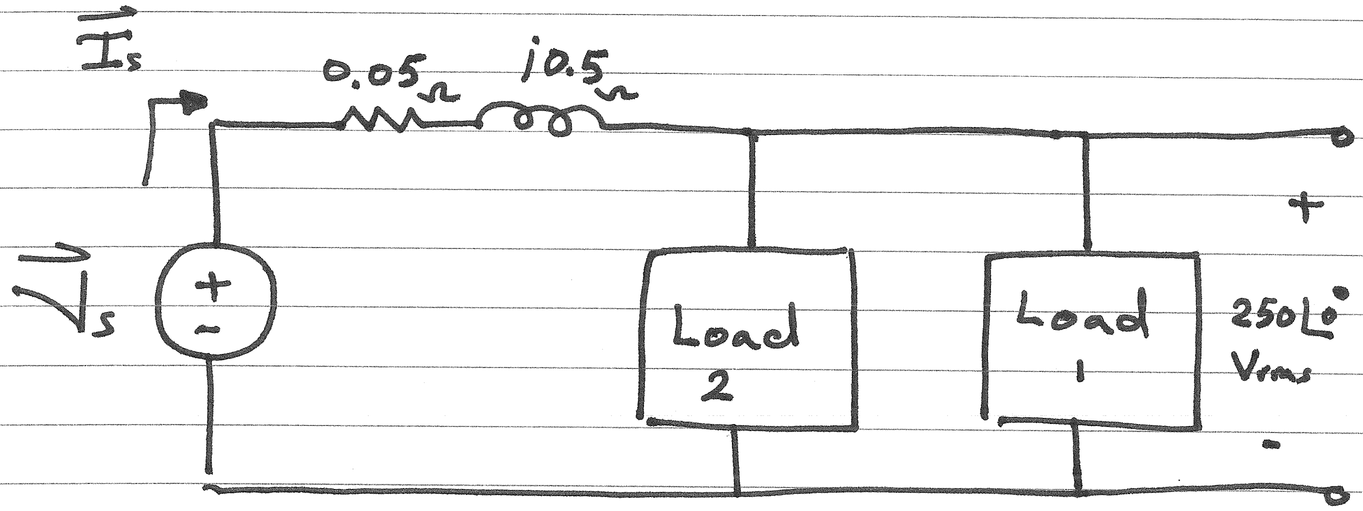


$$I_{ini} = \frac{P_{av}}{(V_{rms})(PF_1)} = 621\ A_{rms}$$



$$I_{Fin} = \frac{P_{av}}{(V_{rms})(PF_2)} = 483\ A_{rms}$$

Example



Load 1 : 8 KW ; 0.8 PF Leading

Load 2 : 20 KVA ; 0.6 PF Lagging

1) Determine the power factor of the two Loads in parallel

2) Determine the apparent power required to supply the loads; the magnitude of the Current I_s ; the average power loss in the transmission Line

3) Compute the value of the Capacitor that would Correct the power factor to 1 if

placed in Parallel with the two loads

$\omega = 377 \, r/s$

4) Repeat step 2

Load 1 : 8 KW ; 0.8 PF_1 leading

Load 2 : 20 KVA ; 0.6 PF_2 Lagging

$P_{av_1} = 8000\ W$

$\therefore\ Q_1 = -P_{av_1} \tan\left[\cos^{-1}(PF_1)\right] = -6000\ VARS$

$\therefore\ \overrightarrow{S_1} = P_{av_1} + j\,Q_1$

$\overrightarrow{S_1} = 8000 - j\,6000 \quad VA$

$P_{a_2} = 20000\ VA$; $PF_2 = 0.6$ Lagging

$\therefore\ P_{av_2} = P_a \cdot PF_2 = 12000\ W$

$Q_2 = P_{av_2} \tan\left[\cos^{-1}(PF_2)\right] = +16000\ VARS$

$\therefore\ \overrightarrow{S_2} = P_{av_2} + j\,Q_2$

$\overrightarrow{S_2} = 12000 + j\,16000 \quad VA$

$\overrightarrow{S_{T_L}} = \overrightarrow{S_1} + \overrightarrow{S_2}$

$\overrightarrow{S_{T_L}} = 20000 + j\,10000 \quad VA$

$\overrightarrow{S_{T_L}} = 22360 \angle 26.565^\circ \quad VA$

$\therefore\ PF = \cos(26.565) = 0.8944$ Lagging

Loads

$$\vec{S}_{L_T} = \vec{V}_{rms} \, \vec{I}_s^*$$

$$\therefore \vec{I}_s^* = \frac{\vec{S}_{L_T}}{\vec{V}_{rms}} = \frac{22360}{250} \angle 26.565^\circ$$

$$\therefore \vec{I}_s = 89.44 \angle -26.565^\circ \text{ A } rms$$

Since $\vec{S}_{L_T} = 22360 \angle 26.565^\circ$

$$\therefore P_a = |S_{L_T}| = 22360 \text{ VA}$$

$$P_{av_{Loss}} = |\vec{I}_s|^2 \cdot (0.05)$$

$$= 400 \text{ W}$$

3) Since $\vec{S}_{L_T} = 20000 + j\,10000$

$$\therefore Q_{ini} = 10000 \text{ VARS}$$

$$Q_{Fin} = 0$$

$$\therefore Q_c = Q_{Fin} - Q_{ini} = -10000 \text{ VARS}$$

$$\therefore C = \frac{-Q_c}{wV_{rms}^2} = 424.4 \, \mu F$$

4) Since $Q_{Fin} = 0$

$\therefore \vec{S}_F = P_{av} = 20000 \text{ VA}$

$\therefore P_a = P_{av} = 20000 \text{ VA}$

$\therefore \vec{S}_F = 20000 \angle 0° \text{ VA}$

$\vec{S}_F = \vec{V}_{rms} \cdot \vec{I}_s^*$

$\therefore \vec{I}_s^* = \dfrac{20000 \angle 0°}{250 \angle 0°} = 80 \angle 0° \text{ A}_{rms}$

$\therefore \vec{I}_s = 80 \angle 0° \text{ A}_{rms}$

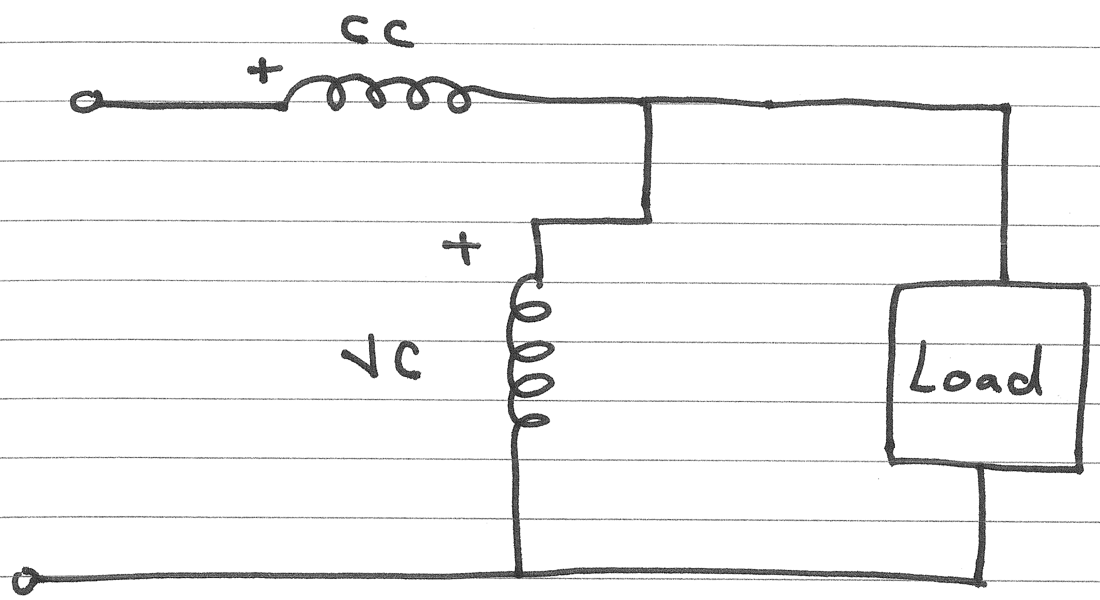
$P_{Loss} = |I_s|^2 \cdot (0.05)$

$P_{Loss} = 320 \text{ W}$

Power Measurement

Wattmeter is the instrument for measuring the average power

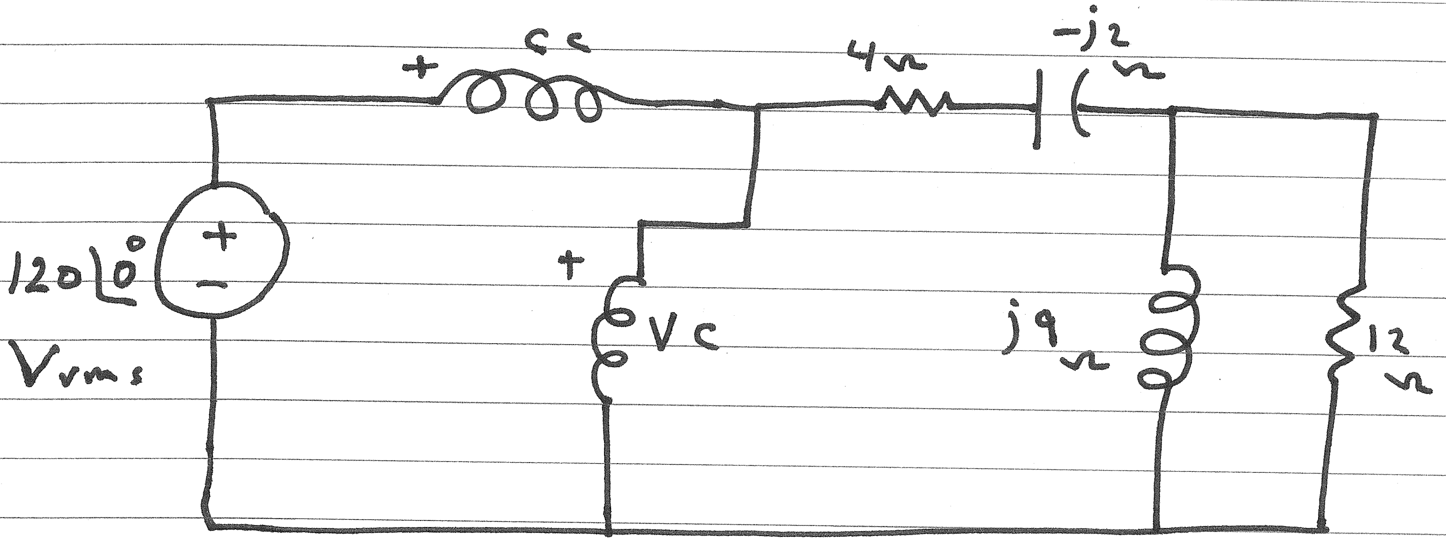
Two Coils are used, the high impedance Voltage Coil and the Low impedance Current Coil.



$$P = V_{rms} I_{rms} \cos(\theta_v - \phi_i)$$

Example

Find the Wattmeter reading



$$Z = 4 - j2 + (j9 \| 12)$$

$$Z = 9.13 \angle 24.32^\circ$$

$$\vec{I} = \frac{120 \angle 0^\circ}{9.13 \angle 24.32^\circ} = 13.14 \angle -24.32^\circ \ A_{rms}$$

$$P = (120)(13.14) \cos(0 + 24.32^\circ)$$

$$P = 1436.9 \ W$$