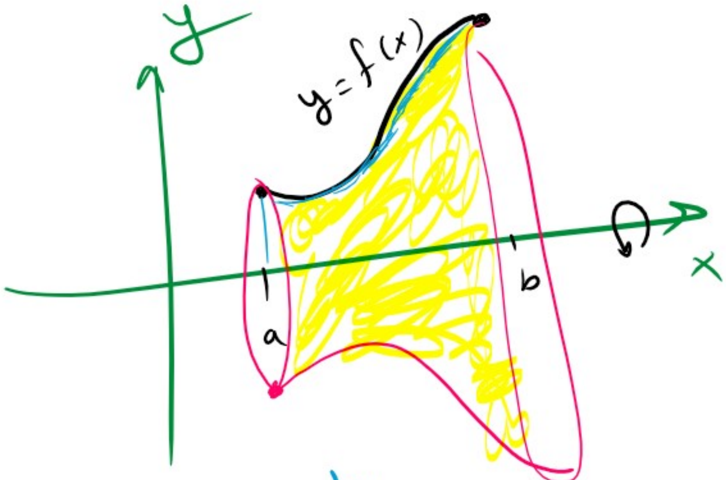
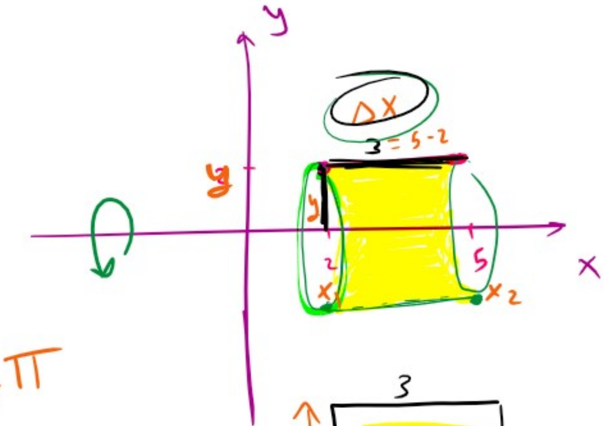
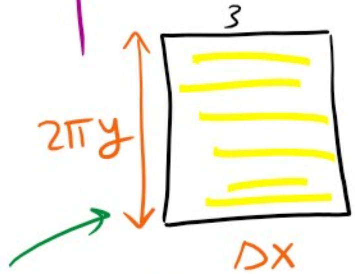
حجم الاسطوانة = مساحة القاعدة × الارتفاع

SA : Surface Area



$2y\pi$





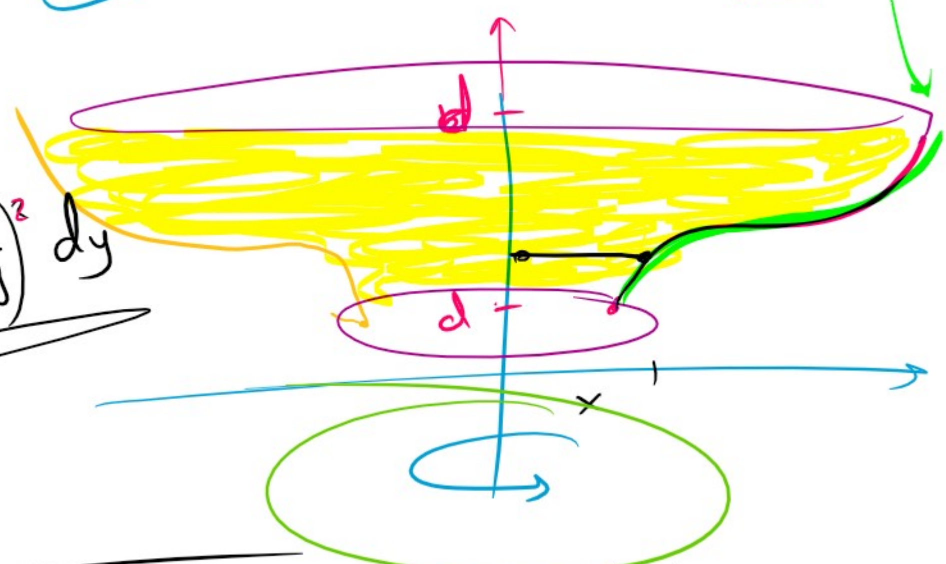
$$SA = 2\pi\, y\, \Delta x$$

$$SA = 2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$y > 0$

$x \geq 0$

$$SA = \int_c^d 2\pi\, x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$



Exp Find Surface Area generated by revolving the curve

Ex: Find the area of the surface generated by revolving the curve

(1) $y = 2\sqrt{x}$, $1 \le x \le 2$, about x-axis

$y \ge 0$ on $[1,2]$

$$SA = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = 2 \cdot \frac{1}{2\sqrt{x}}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{x}$$

$$= \int_1^2 2\pi (2\sqrt{x}) \sqrt{1 + \frac{1}{x}}\, dx$$

$$= 4\pi \int_1^2 \sqrt{x} \sqrt{\frac{x+1}{x}}\, dx$$

$$= 4\pi \int_1^2 \sqrt{x} \frac{\sqrt{x+1}}{\sqrt{x}}\, dx$$

$$= 4\pi \int_1^2 \sqrt{x+1}\, dx$$

$u = x+1$

$du = dx$

$x = 1 \Rightarrow u = 2$

$x = 2 \Rightarrow u = 3$

$$= 4\pi \int_2^3 \sqrt{u}\, du$$

$$= 4\pi \left. \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right|_2^3 = \cdots = \frac{8\pi}{3}\left(3\sqrt{3} - 2\sqrt{2}\right)$$

Q18

Find area of surface generated by revolving $x = \frac{1}{3}y^{\frac{3}{2}} - \sqrt{y}$, $1 \le y \le 3$ about y-axis

$\frac{1}{3} - 1$

$-x = -\left(\frac{1}{3}y^{\frac{3}{2}} - \sqrt{y}\right)$

$x = \left(\frac{1}{3}\right)\sqrt{y^3} - \sqrt{y} < 0$

$y^{\frac{3}{2}}$

$SA = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$\Rightarrow$ take $x = \sqrt{y} - \frac{1}{3}\sqrt{y^3}$

$x > 0$ on $[1,3]$

$= \int_1^3 2\pi\left(\sqrt{y} - \frac{1}{3}\sqrt{y^3}\right)\sqrt{1 + \frac{1}{4}\left(\frac{1}{\sqrt{y}} - \sqrt{y}\right)^2}\, dy$

$\sqrt{1 + \frac{1}{4}\left(\frac{1}{y} - 2 + y\right)}$

$\frac{dx}{dy} = \frac{1}{2\sqrt{y}} - \frac{1}{3}\cdot\frac{3}{2}y^{\frac{1}{2}}$

$= \frac{1}{2\sqrt{y}} - \frac{1}{2}\sqrt{y}$

$= \frac{1}{2}\left(\frac{1}{\sqrt{y}} - \sqrt{y}\right)$

$\sqrt{1 + \frac{1}{4y}\left(-\frac{1}{2}\right) + \frac{1}{4}y}$

$\sqrt{\frac{1}{4y} + \frac{1}{2} + \frac{1}{4}y}$

$\sqrt{\frac{1}{4}\left(\frac{1}{y} + 2 + y\right)}$

$\sqrt{\left(\frac{1}{2} + \sqrt{y}\right)^2}$

$$\frac{1}{2}\sqrt{\left(\frac{1}{\sqrt{y}}+\sqrt{y}\right)^2}$$

$$SA=\int_1^3 2\pi\left(\sqrt{y}-\frac{1}{3}y^{\frac{3}{2}}\right)\frac{1}{2}\left(\frac{1}{\sqrt{y}}+\sqrt{y}\right)dy$$

$$\frac{16\pi}{9}$$

$$SA=\int_a^b 2\pi\, x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

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$x>0$

$x<0$

$-x>0$

Q20 Find the area for the surface generated by revolving

surface generated by revolving the curve

$$x = \sqrt{2y-1}, \quad \frac{5}{8} \le y \le 1$$

about y-axis

$x = \sqrt{2\left(\frac{5}{8}\right)-1} = \sqrt{\frac{10}{8}-1} > 0$

$x = \sqrt{2(1)-1} = \sqrt{1} > 0$

$x \ge 0$

$$L = \int_c^d 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$\frac{dx}{dy} = \frac{1 \times 2}{2\sqrt{2y-1}}$$

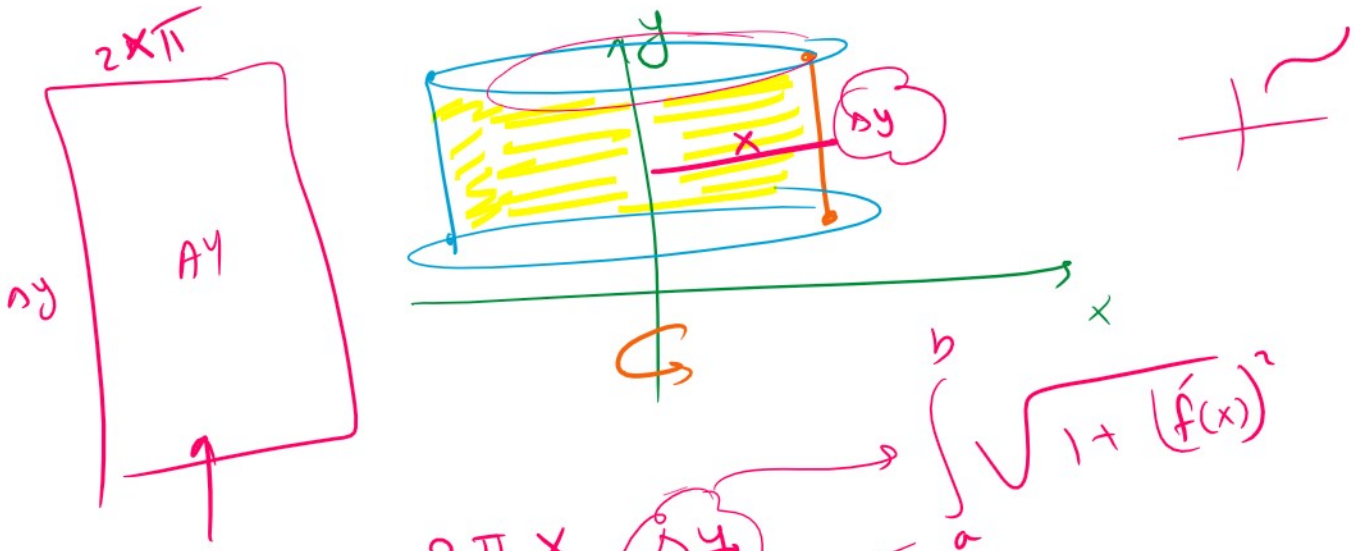
$$= 2\pi \int_{5/8}^{1} \sqrt{2y-1}\, \sqrt{1+\frac{1}{2y-1}}\, dy$$

$$= 2\pi \int_{5/8}^{1} \sqrt{2y-1}\, \sqrt{\frac{2y-1+1}{2y-1}}\, dy$$

$$= 2\pi \int_{5/8}^{1} \sqrt{2y-1}\, \frac{\sqrt{2y}}{\sqrt{2y-1}}\, dy$$

$$= 2\pi \int_{5/8}^{1} \sqrt{2}\, \sqrt{y}\, dy$$

$$= 2\sqrt{2}\pi \left. \frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right|_{\frac{5}{8}}^{1} = \cdots = \frac{11}{12}\left(16\sqrt{2} - 5\sqrt{2}\right)$$



$$SA = 2\pi x \, \Delta y \rightarrow \int_a^b \sqrt{1 + \left(f'(x)\right)^2}$$