CH 1: Fundamental of signals

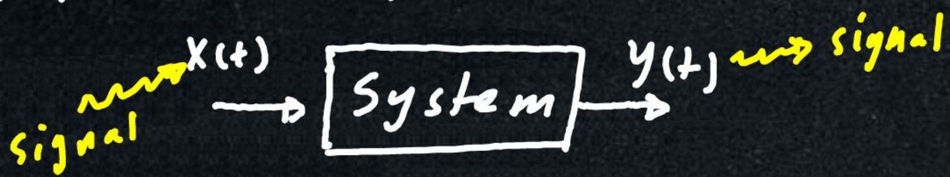
1.1) Introduction

- A signal is a quantitive description of a physical phenomenon, event or process.

- Some common examples
  - Electrical current or voltage in circuit
  - Audio signal
  - Heart rate
  - Blood pressure

- The signal is usually a function of one variable in time.

- The signal can be functions of more than one variable, e.g image

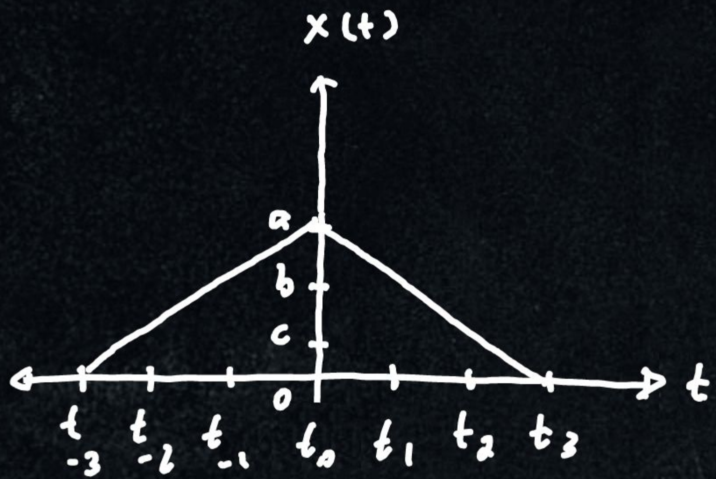


Types of signals:

Contineous time signals

→ specified for every value of time (t)

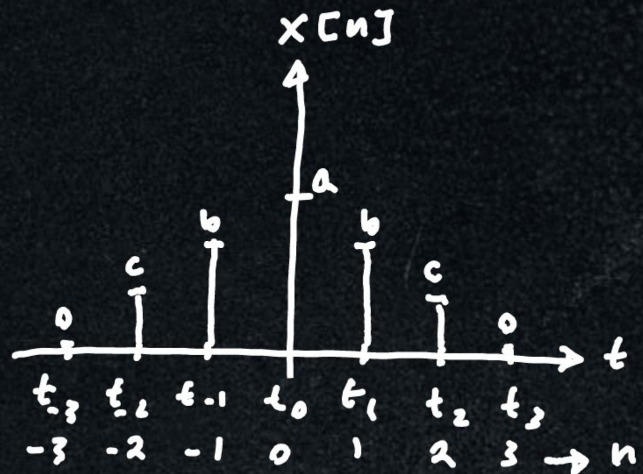
t: time



Discrete time signals

→ specified at discrete time intervals

n: integer



$$\Delta t = t_1 - t_0 = t_2 - t_1 = t_3 - t_2 = t_n - t_{n-1}$$

↳ sampling time

1.2) periodic and non-periodic signals

A signal $x(t)$ is periodic if $x(t+T_o) = x(t)$

where T_o is the fundamental period in [sec]

Note: $f_o = \frac{1}{T_o}$ where f_o is the fundamental frequency of the periodic signal in [Hz]

$\omega_o = \frac{2\pi}{T_o} = 2\pi f_o$ where ω_o is the fundamental electric radian frequency of the periodic signal in [rad/sec]

Case I :-

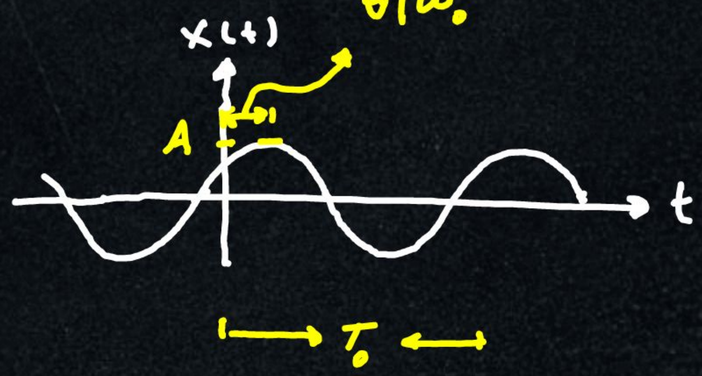
Sinusoidal signal "periodic signal"

$$x(t) = A\cos(\omega_0 t + \theta) = A\cos\left(\frac{2\pi}{T_0}t + \theta\right)$$

θ → phase

A → peak

ω_0 → Fundamental radian frequency



→ $x(t)$ is always periodic

Case II :-

sum of sinusoidal signals

$$x(t) = A_1\cos(\omega_1 t + \theta_1) + A_2\cos(\omega_2 t + \theta_2) + \cdots + A_n\cos(\omega_n t + \theta_n)$$

$x(t)$ is periodic if $\dfrac{\omega_i}{\omega_j} = \dfrac{N_i}{N_j}$ $\quad i \neq j$ → rational number

where N_i + N_j are integers

Ex:- $x(t) = \sin\left(\frac{5\pi}{6}t\right) + \cos\left(\frac{3\pi}{4}t\right) + \sin\left(\frac{\pi}{3}t\right)$

$$\omega_1 = \frac{5\pi}{6}, \quad \omega_2 = \frac{3\pi}{4}, \quad \omega_3 = \frac{\pi}{3}$$

$\frac{\omega_1}{\omega_2} = \frac{10}{9} \rightarrow$ rational

$\frac{\omega_1}{\omega_3} = \frac{5}{2} \rightarrow$ rational

$\longrightarrow \therefore x(t)$ is periodic

Ex:- $x(t) = \cos\left(\frac{10}{3}t\right) + \sin\left(\frac{5\pi}{4}t\right)$

$$\omega_1 = \frac{10}{3}, \quad \omega_2 = \frac{5\pi}{4}$$

$\frac{\omega_1}{\omega_2} = \frac{40}{15\pi} \rightarrow$ irrational $\{ \therefore x(t)$ is non-periodic

Calculation of ω_0 for the periodic signal

$$x(t) = A_1 \cos(\omega_1 t + \theta_1) + A_2 \cos(\omega_2 t + \theta_2) + \cdots + A_n \cos(\omega_n t + \theta_n)$$

$$\omega_0 = \underline{GCD}(\omega_1, \omega_2, \ldots, \omega_n)$$

↳ Greatest Common Divisor

or $$f_0 = \frac{\omega_0}{2\pi} = GCD(f_1, f_2, \ldots, f_n)$$

Calculation of T_0 for the periodic signal

$$x(t) = A_1 \cos\left(\frac{2\pi}{T_1} t + \theta_1\right) + A_2 \cos\left(\frac{2\pi}{T_2} t + \theta_2\right) + \cdots + A_n \cos\left(\frac{2\pi}{T_n} t + \theta_n\right)$$

$$T_0 = \frac{2\pi}{\omega_0} = LCM(T_1, T_2, \ldots, T_n)$$

↳ Least Common Multiplier

EX:- Calculate ω_0, f_0, and T_0 for the following periodic signal:

$$x(t) = \sin\left(\frac{5\pi}{6}t\right) + \cos\left(\frac{3\pi}{4}t\right) + \sin\left(\frac{\pi}{3}t\right)$$

$$\omega_0 = GCD\left(\frac{5\pi}{6}, \frac{3\pi}{4}, \frac{\pi}{3}\right) = \pi\, GCD\left(\frac{5}{6}, \frac{3}{4}, \frac{1}{3}\right)$$

$$\omega_0 = \frac{\pi}{12}\cancelto{1}{GCD(10, 9, 4)} = \frac{\pi}{12} \text{ rad/sec}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{24} \text{ Hz}$$

$$\Rightarrow \quad x(t) = \underbrace{\sin(10\omega_0 t) + \cos(9\omega_0 t) + \sin(4\omega_0 t)}_{\text{Harmonics}}$$

$$T_0 = \frac{1}{f_0} = \boxed{24} \text{ sec}$$

or $T_0 = LCM(T_1, T_2, T_3)$

$$T_1 = \frac{2\pi}{\omega_1} = \frac{12}{5}, \quad T_2 = \frac{2\pi}{\omega_2} = \frac{8}{3}, \quad T_3 = \frac{2\pi}{\omega_3} = 6$$

$$T_0 = LCM\left(\frac{12}{5}, \frac{8}{3}, \frac{6}{1}\right) = \frac{1}{15} LCM(36, 40, 90)$$

	36	40	90
2	18	20	45
2	9	10	45
2	9	5	45
3	3	5	15
3	1	5	5
5	1	1	1

$$T_0 = \frac{1}{15}(2 \times 2 \times 2 \times 3 \times 3 \times 5) = \boxed{24} \text{ sec}$$

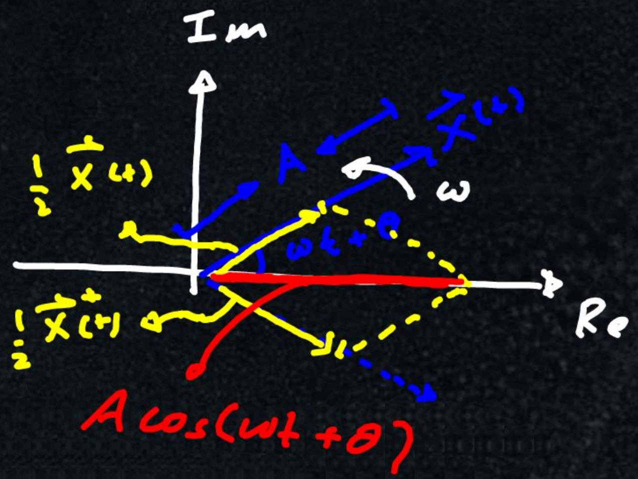
1.3) phasor signals and spectra

Complex sinusoidal function :-

$$\vec{X}(t) = A\, e^{j(\omega t+\theta)} \qquad \text{where } j=\sqrt{-1}$$

↓ Euler's formula

$$\vec{X}(t) = A\cos(\omega t+\theta) + jA\sin(\omega t+\theta)$$



$x(t) = A\cos(\omega t+\theta)$ can be represented using $\vec{X}(t)$ in 2 ways:

1) $x(t) = \mathrm{Re}[\vec{X}(t)]$

2) $x(t) = \frac{1}{2}[\vec{X}(t) + \vec{X}^*(t)]$

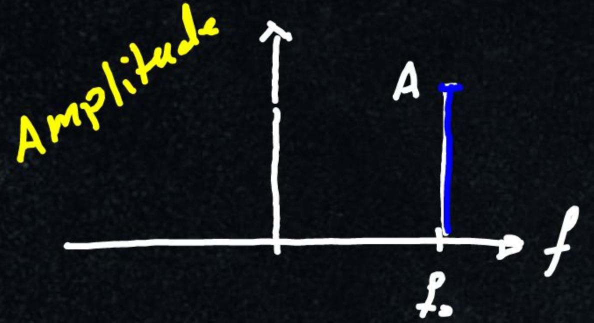
where $\vec{X}^*(t) = A\, e^{-j(\omega t+\theta)}$ ⟶ complex conjugate

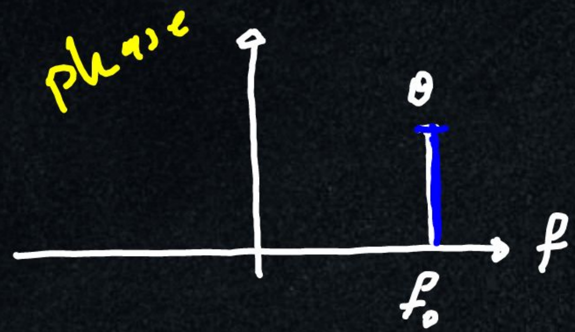
Frequency spectra

$x(t) = A\cos(\omega_0 t + \theta)$

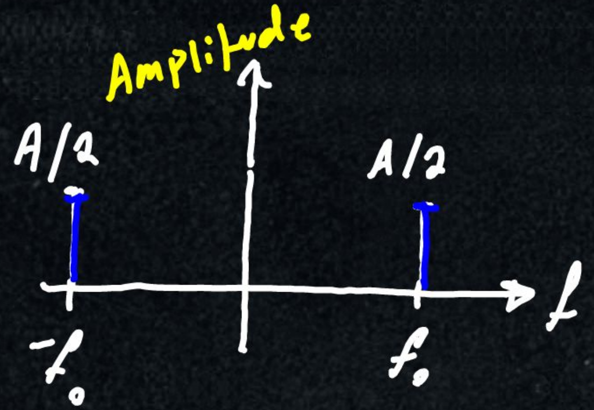
$x(t)$ can be plotted in frequency domain using 2 forms:

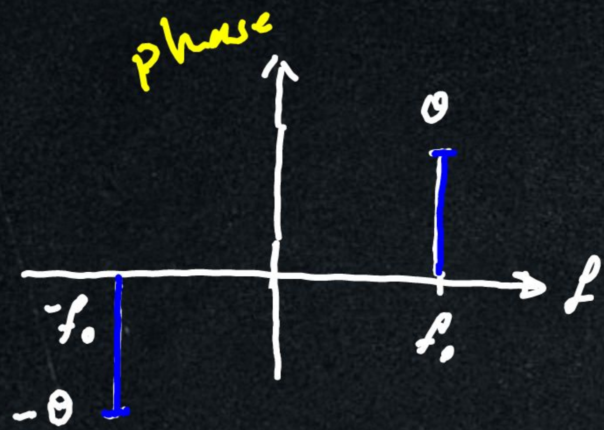
1) single-sided line spectra (Amplitude and phase)





2) Double-sided line spectra (Amplitude and phase)





EX : Given the signal

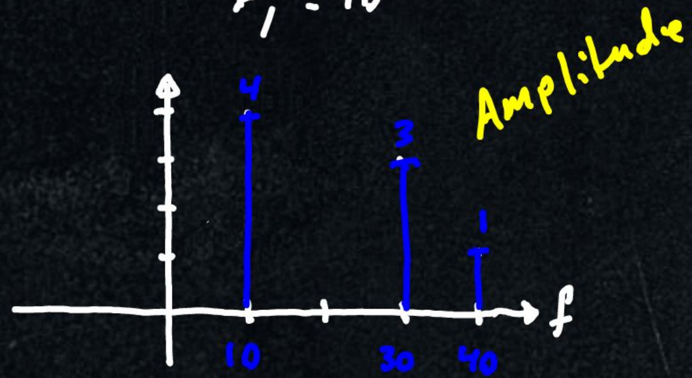
$$x(t) = 4\cos\left(20\pi t + \frac{\pi}{4}\right) + 3\cos\left(60\pi t - \frac{\pi}{6}\right) + \sin\left(80\pi t + \frac{\pi}{6}\right)$$

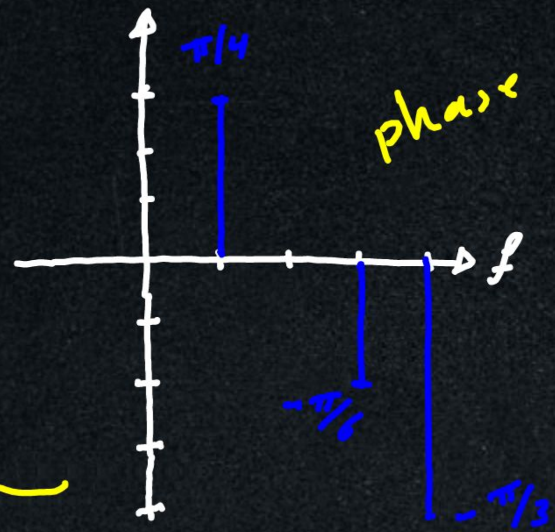
a) Sketch its single-sided spectra

b) Sketch its double-sided spectra

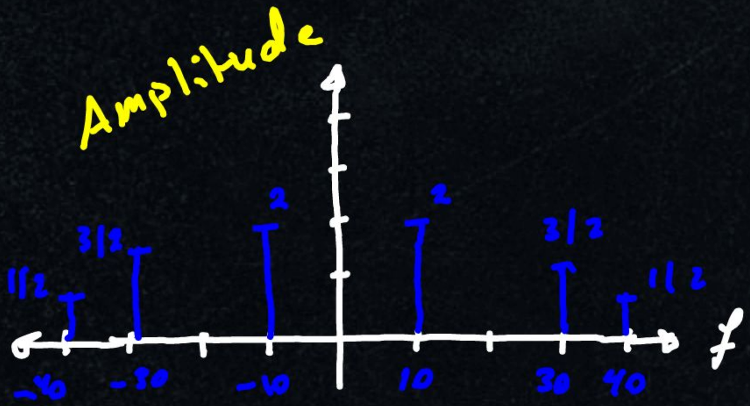
$$x(t) = 4\cos\left(20\pi t + \frac{\pi}{4}\right) + 3\cos\left(60\pi t - \frac{\pi}{6}\right) + \cos\left(80\pi t - \frac{\pi}{3}\right)$$

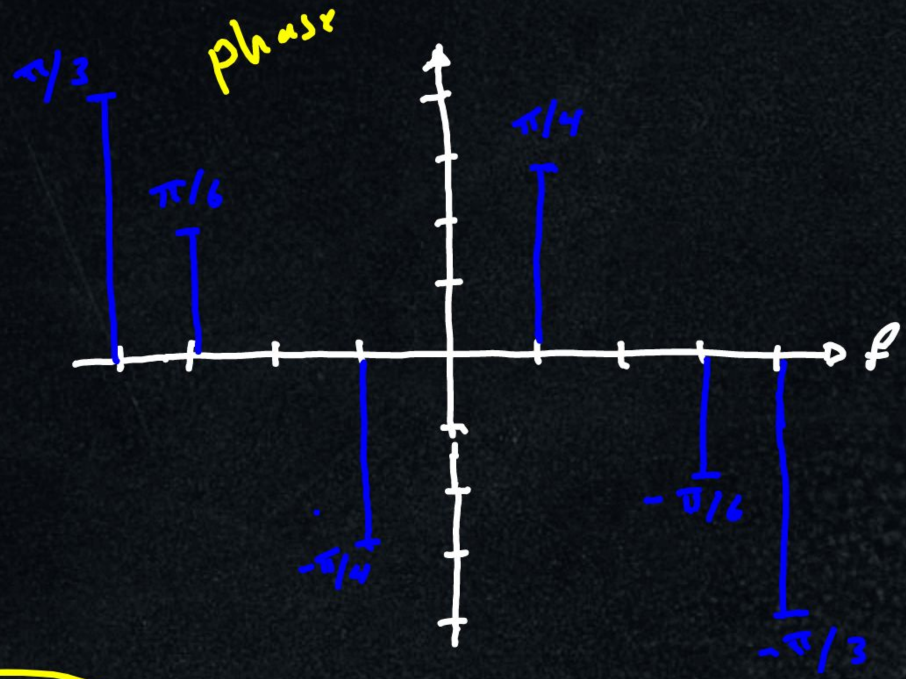
$f_1 = 10$ $\quad$ $f_2 = 30$ $\quad$ $f_3 = 40$





Single





Double-sided

$\underline{EX}$: Given the signal

$$x(t) = 6\cos\left(20\pi t - \frac{\pi}{3}\right) + 4\sin^2\left(30\pi t - \frac{\pi}{6}\right)$$

a) Sketch its single-sided spectra

b) Sketch its double-sided spectra

$$x(t) = 6\cos\left(20\pi t - \frac{\pi}{3}\right) + \frac{4}{2}\left(1 - \cos\left(60\pi t - \frac{\pi}{3}\right)\right)$$

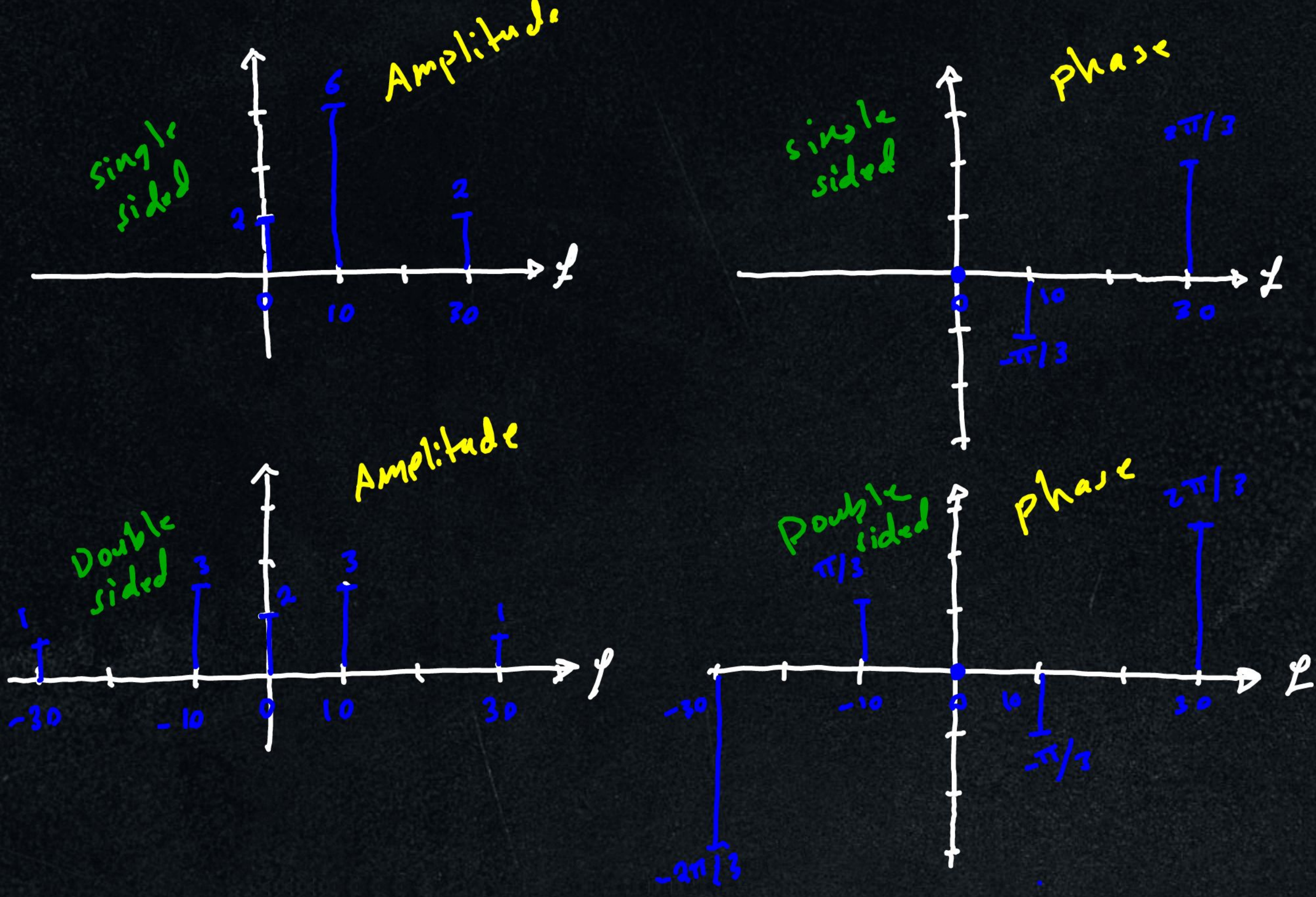
$$x(t) = 6\cos\left(20\pi t - \frac{\pi}{3}\right) + 2 - 2\cos\left(60\pi t - \frac{\pi}{3}\right)$$

$$x(t) = 2 + 6\cos\left(20\pi t - \frac{\pi}{3}\right) + 2\cos\left(60\pi t + \frac{2\pi}{3}\right)$$

$f = 0$ $\quad$ $\theta = 0$

$f_1 = 10$ $\quad$ $\theta = \pi/3$

$f_2 = 30$ $\quad$ $\theta = 2\pi/3$


Amplitude
single sided
Phase
single sided
Amplitude
Double sided
Phase
Double sided