Malek Zeghari

1

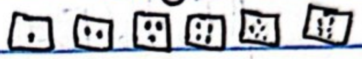
ENEE 2307:
STAT:-

Ch 1: Fundmental of probality:-

* Experement:-

① can be repeated (infinite times). like

- flipping a coin
- tossing a dice



(H) head (T) tail

$S = \{H, T\}$

sample outcome — sample outcome

الترتيب مش مهم

sample space كل الفضاء $S = \{1, 2, 3, 4, 5, 6\}$ ممنوع التكرار

sample outcome (عناصر المجموعة)

Exp: A: even num.

$A = \{2, 4, 6\}$

B: div by 3

$B = \{3, 6\}$

C: less than 1

$C = \varnothing$

D: less than 10

$D = S = \{1, 2, 3, 4, 5, 6\}$

② has a well-known outcomes:-

* Sample Space (S) الفضاء العيني
* Sample outcomes.
* Event

Exp1:- Consider the experiment of flipping a green coin and red coin.

1. Write the sample space?

$S = \{\underset{R,G}{HH}, \underset{R,G}{HT}, \underset{R,G}{TH}, \underset{R,G}{TT}\}$

2. A: a match

$A = \{HH, TT\}$

3. B: the red coin is Head.

$B = \{\underset{RG}{HH}, \underset{RG}{HT}\}$

Exp2:- Consider the experiment of flipping two similler coins at the same time in the same place.

[1] write the Sample Space:-

$$S = \{HH, HT, TH, TT\}.$$

Exp3:- Consider the experiment of flipping a coin and tossing a dice.

[1] write the Sample Space:

$$S = \{(H,1), (H,2), (H,3), (H,4), (H,5), (H,6), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)\}$$

[2] A: Head is observed in the coin and an even number is observed on the dice.

$$A = \{(H,2), (H,4), (H,6)\}$$

[3] B: odd numbers are observed on the dice.

$$B = \{(H,1), (H,3), (H,5), (T,1), (T,3), (T,5)\}$$

Exp 1:- let us Consider the experiment of tossing two dice.

[1] A: the Sum of the numbers observed is 6.

$$A = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}.$$

[2] B: the Sum of the numbers observed is less than 5.

$$B = \{(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)\}.$$

[3] C: a match is observed.

$$C = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$$

* Algebra of the sets :-

⇒ Intersection " $\cap$ " → "تقاطع" العناصر المشتركة بين مجموعتين

4. D: the sum of the two numbers is less than 5 **and** the two numbers match.

في $D = B \cap C = \{(1,1), (2,2)\}$

⇒ Union " $\cup$ " → "اتحاد" وجميع العناصر في المجموعتين ولكن بدون تكرار

5. E: the sum of the numbers observed is 6 or there is a match

في $E = A \cup C = \{(1,5), (2,4), (3,3), (4,2), (5,1), (1,1), (2,2), (4,4), (5,5), (6,6)\}$

⇒ Complement → المتمم (تكملة المجموعة) ، يعني بوخذ العناصر إلي مش موجودة في المجموعة

$\overline{C} = \{(1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,1), (2,3), (2,4), (2,5), (2,6),$
$(3,1), (3,2), (3,4), (3,5), (3,6),$
$(4,1), (4,2), (4,3), (4,5), (4,6),$
$(5,1), (5,2), (5,3), (5,4), (5,6),$
$(6,1), (6,2), (6,3), (6,4), (6,5)\}$

<- هات C في $C = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

فمتممها بتكون بإجي العناصر ماعدا العناصر الموجودة في C.

* Demorgan's Laws.

1. $\overline{A \cup B} = \overline{A} \cap \overline{B}$
2. $\overline{A \cap B} = \overline{A} \cup \overline{B}$

Exp2) Let the sample space $S = \{1,2,3,4,5,6\}$ and the event $A = \{1,3,5\}$, $B = \{2,3,4\}$

1. $\overline{A \cup B} = \overline{\{1,3,5\} \cup \{2,3,4\}} = \overline{\{1,2,3,4,5\}} \rightarrow = \{6\}$

or

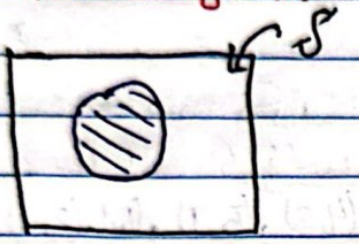
$\overline{A} \cap \overline{B} \rightarrow \overline{A} = \{2,4,6\}$, $\overline{B} = \{1,5,6\}$ $\rightarrow \overline{A} \cap \overline{B} = \{6\}$

$\boxed{2}$ $\overline{A\cap B} = \overline{\{1,3,5\}\cap\{2,3,4\}} = \overline{\{3\}} \longrightarrow = \{1,2,4,5,6\}$

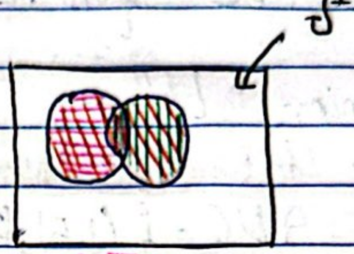
or

~~[scribble]~~ $\bar{A}\cup\bar{B} = \{2,4,6\}\cup\{1,5,6\} \longrightarrow \{1,2,4,5,6\}$

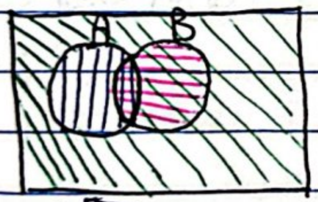
* Venn Diagrams:-



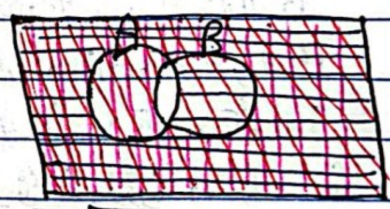
legend A



A
B
$A\cap B$
$A\cup B$



A
B
$A\cap B$
$\overline{A\cap B}$



$\bar{A}$
$\bar{B}$
$\bar{A}\cup\bar{B}$

* $S = \{1,2,3,4,5,6\}$

$A = \{1,2\}$

$B = \{2,4\}$

$C = A\cap B = \{2\}$, $\bar{C} = \overline{A\cap B} = \{1,3,4,5,6\}$

Exp 3: Let us consider the sample space. $S = \{4 \leq x \leq 10\}$

$A = \{5 \leq x \leq 10\}$, $B = \{4 < x \leq 7\}$

1. $\bar{A} = \{4 \leq x < 5\}$

2. $A \cap B = \{5 \leq x \leq 7\}$

3. $\bar{B} = \{4,\ 7 < x \leq 10\}$

---

* Probability :- ← Classical "a priopri" قبل التجربة

⇒ 1. $P(A) = \dfrac{\text{number of sample outcomes in } A}{\text{number of sample outcomes in } S}$ مهم

Exp 4) Let $S = \{1, 2, 3, 5, 6\}$, $A = \{1, 5, 6\}$, $B = \{2, 3, 5, 6\}$

1. $P(A) = \dfrac{3}{5}$ 2. $P(B) = \dfrac{4}{5}$

3. $P(A \cap B) = A \cap B = \{5, 6\} \rightarrow P(A \cap B) = \dfrac{2}{5}$

4. $P(\overline{A \cap B}) = \overline{A \cap B} = \bar{A} \cup \bar{B} = \{1, 2, 3\} \rightarrow P(\overline{A \cap B}) = \dfrac{3}{5}$

$\bar{A} = \{2, 3\}$

$\bar{B} = \{1\}$

بعد التجربة "a post propri"

⇒ 2. Relative frequency

$$P(A) = \lim_{n \to \infty} \frac{\text{number of times } A \text{ occurs}}{\text{number of trails } (n)}$$ مهم

Exp4) A coin is flipped for 10^{12} times, assume head is observed in 4×10^{11} times. what is the probability of observing head after flipping the coin?

$$P(H) = \frac{4\times10^{11}}{10^{12} = 10\times10^{11}} = \frac{4}{10} = 0.4$$

* Fair coin → $P(H) = 0.5$, $P(T) = 0.5$
* Biased coin → $P(H) \neq P(T)$.

⇒ 3 Subjective.

احتمال شخصي يعتمد على حدسك

Rev.

→ Definitions of probability:-

1 Classical → $P(A) = \frac{\text{num. of outcomes in } A}{\text{num. of outcomes in } S}$

2 Relative frequency → $P(A) = \lim_{n\to\infty} \frac{\text{numbers of time } A \text{ occurs}}{\text{number of trails } (n)}$

3 Subjective

→ 4 Axiomatic

1 $P(A) \geq 0$

2 $P(S) = 1$

} $0 \leq P(A) \leq 1$

3 a if events A and B are disjoint (mutually exclusive)

then $P(A \cup B) = P(A) + P(B)$

b if three events are disjoints (mutually exclusive)

then $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

* Notes:-

1 two events A and B are said to be disjoint (mutually exclusive) if $A \cap B = \emptyset$

2 three events A, B and C are said disjoint (" ") if

a $A \cap B = \emptyset$ c $B \cap C = \emptyset$

b $A \cap C = \emptyset$ d $A \cap B \cap C = \emptyset$

* لازم الاربعة يتحققوا، لو واحد خطأ بكون مش

\# disjoint

Exp 1) let $S = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 2\}$, $B = \{3, 4\}$, $C = \{5\}$, $D = \{1, 6\}$.

1) Are A and B disjoint?

$A \cap B \overset{??}{=} \emptyset$

$\{1, 2\} \cap \{3, 4\} = \emptyset \rightarrow$ yes, they are disjoint.

2) Are A and D disjoint?

$A \cap D = \{1, 2\} \cap \{1, 6\} = \{1\} \neq \emptyset \rightarrow$ No, they aren't disjoint

3) Are A, B and D disjoint?

$\rightarrow A \cap B = \emptyset$

$\times\ A \cap D \neq \emptyset \rightarrow$ So A, B and D are not disjoint.

4) Are A, B and C are disjoint?

$\rightarrow A \cap B = \{1, 2\} \cap \{3, 4\} = \emptyset$

$\rightarrow A \cap C = \{1, 2\} \cap \{5\} = \emptyset$

$\rightarrow B \cap C = \{3, 4\} \cap \{5\} = \emptyset$

$\rightarrow A \cap B \cap C = \{1, 2\} \cap \{3, 4\} \cap \{5\} = \emptyset$

تحققوا الاربع شروط

So, A, B and C are disjoint.

5) $P(A \cup B) = ?$

① $P(A \cup B) = P(\{1, 2, 3, 4\}) = 4/6$

or

② $P(\underset{\text{disjoint}}{A \cup B}) = P(A) + P(B) = 2/6 + 2/6 = 4/6$

6) $P(A \cap D) = P(\{1\}) = 1/6$

7) $P(A \cup B \cup C) = P(\{1, 2\} \cup \{3, 4\} \cup \{5, 6\})$

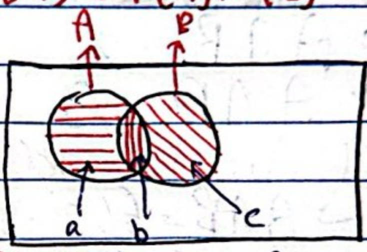
$= P(\{1, 2, 3, 4, 6\}) = 5/6$

8) $P(A \cap B \cap C) = P(\emptyset) = 0$

Notes:

1. $A \cap A = A$
2. $A \cup A = A$
3. $A \cap \bar{A} = \emptyset$
4. $A \cup \bar{A} = S$
5. $S^c = \emptyset$ (c complement)
6. $\emptyset^c = S$
7. $S \cap \emptyset = \emptyset$
8. $S \cup \emptyset = S$
9. $S \cap A = A$
10. $S \cup A = S$
11. $\emptyset \cap A = \emptyset$
12. $\emptyset \cup A = A$

Exp2) Use Venn diagram to prove $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



$P(A \cup B) = P(a \cup b \cup c)$ disjoint

$= P(a) + P(b) + P(c)$

$P(a) + P(b) + P(c) + P(b) - P(b)$

disjoint disjoint

$P(a \cup b) + P(c \cup b) - P(b)$

$\rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ///

a, b, c:

$a \cap b = \emptyset$

$a \cap c = \emptyset$

$b \cap c = \emptyset$

$a \cap b \cap c = \emptyset$

$\rightarrow$ a, b, c disjoint

Exp3) let $S = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 3\}$, $B = \{2, 4\}$, $C = \{3, 5\}$

1. $P(A \cup B) \overset{①}{=} P(\{1, 2, 3, 4\}) = 4/6$

   disjoint ② $= P(A) + P(B) = 2/6 + 2/6 = 4/6$

2. $P(A \cup C) \overset{①}{=} P(\{1, 3, 5\}) = 3/5$

   not disjoint ② $= P(A) + P(C) - P(A \cap C)$

   $= 2/6 + 2/6 - 1/6 = 3/6$

Exp 4) Let the event A, B and C be disjoint and defined over the sample space S, Assume $P(A) = 0.25$, $P(B) = 0.75$ and $A \cup B \cup C = S$ Determine $P(C)$?

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

disjoint

$P(S) = 0.25 + 0.75 + P(C)$

$1 = 1 + P(C) \longrightarrow P(C) = 0$

Exp 5) Let A and B be two events defined over the sample space S. Use the axioms of probability to prove the following:

① $P(\bar{A}) = 1 - P(A)$

$P(A \cup \bar{A}) = P(S)$

disjoint

$A \cap \bar{A} = \emptyset$

$P(A) + P(\bar{A}) = 1 \longrightarrow P(\bar{A}) = 1 - P(A)$

② $P(\emptyset) = 0$

$P(S \cup \emptyset) = P(S)$

disjoint

$\cancel{P(S)} + P(\emptyset) = \cancel{P(S)} \rightarrow P(\emptyset) = 0$

Exp 6) Let the event A and B be defined over S. $P(A) = 0.25$ and $P(B) = 0.4$. Assume A and B are disjoint.

ⓐ determine $P(\bar{A} \cap \bar{B})$

$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

disjoint

$= 1 - \{P(A) + P(B)\} = 1 - \{0.25 + 0.4\}$

$1 - 0.65 = \boxed{0.35}$

ⓑ $P(\bar{A} \cup \bar{B}) = ?$

$P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B})$

$= 1 - P(A \cap B)$

$= 1 - 0 = \boxed{1}$

* if A, B, and C are three events, then:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Exp (2.5): One integer is chosen at random from the numbers $\{1, 2, \ldots, 50\}$. What is the probability that the chosen number is divisible by 6? Assume all 50 outcomes are equally likely.

$\frac{1}{50}$ دايما بفرضها والعكس في هلق عندي 50 رقم، احتمال يطلع اي رقم منهم

$\Rightarrow S = \{1, 2, 3, 4, \ldots, 50\}$

$A = \{6, 12, 18, 24, 30, 36, 42, 48\}$

$$P(A) = \frac{\text{number of elements in } A}{\text{number of elements in } S} = \frac{8}{50}$$

Exp (2.6): if the probability of occurrence of an even number is twice as likely as that of an odd number in Exp (2.5). Find $P(A)$: A is defined above.

$\hookrightarrow P(\text{even}) = 2P(\text{odd})$, odd P, even 2P

$P(S) = P(\text{even}) + P(\text{odd}) = 1 \Rightarrow P(\{1\} \cup \{2\} \cup \{3\} \cdots \cup \{50\})$

$= P(\{1\}) + P(\{2\}) + P(\{3\}) + \cdots + P(\{50\})$

$= P + 2P + P + \cdots + 2P = 25P + 25(2P)$

$\rightarrow 1 = 75P \rightarrow P = 1/75$

$2 \times \frac{1}{75}$

$\checkmark P(A) = 8 \times \frac{2}{75} = \boxed{\frac{16}{75}}$

الاعداد إلي بتقسم على 6 هي الاعداد الزوجية وإلي عددها 8 وهي الي إلها 2P

حل الكتاب

Let (P) be the probability of occurrence of an odd number then (2P) will be the " " " of an even number

$P(S) = 25(P) + 25(2P) \Rightarrow 1 = 75P \rightarrow P = \frac{1}{75}$

odd, 25 even, 25

$P(A) = 8 \rightarrow 2P = 8 \times \frac{2}{75} = \boxed{\frac{16}{75}}$

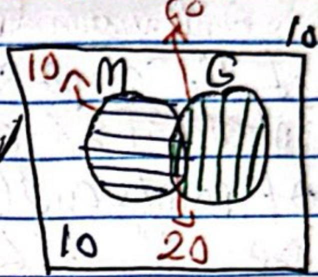
Exp (2.7):
Suppose that a company has 100 employees who are classified according to their ~~mat~~ marital status and according to whether they are college graduates or not, it is known that 30% of the employees are married, and the ~~power~~ percent of graduate is 80%.
More over, 10 employees are neither married nor graduates.
What proportion of married employees are graduates? $\rightarrow M \cap G$

$M \cup G = 90 \rightarrow 100 - 10 \rightarrow$ neither M nor G

$P(M \cup G) = 0.9$

$P(M) = 0.3$

$P(G) = 0.8$



$P(M \cup G) = P(M) + P(G) - P(M \cap G)$ = 30 (horizontal lines)

$0.9 = 0.3 + 0.8 - P(M \cap G)$ = 80 (vertical lines)

$\rightarrow P(M \cap G) = 0.2$ = ?? $\rightarrow M \cap G$

$\Rightarrow$ two third of the married employees in the company are graduates

من ٣٠٪ كم واحد خريج G

$\frac{M \cap G \rightarrow 20}{M \rightarrow 30} = \frac{2}{3}$

Exp (2.8): An experiment has two possible outcomes, the first occurs with probability (P), the second with probability (P^2)
find P?

$P(S) = 1 \rightarrow P(S) = P(A \cup B)$

$= P(A) + P(B)$

$\longrightarrow P + P^2 = 1$

$P^2 + P - 1 = 0 \rightarrow \boxed{P = \frac{-1 + \sqrt{5}}{2}}$

القانون العام

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1,\ b = 1,\ c = -1 \rightarrow \frac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}$ ← السالب بنهمل
فش نسبة بالسالب

Exp (2.9), Exp (2.10)
شوفها من السلايدز

* Discrete probability ~~fun~~ function :-

if the Sample Space generated by an experiment contains either a finite or a countable infinite number of outcomes then its is called a discrete Sample space.

Ex: "S" = {1, 2, 3, 4, 5, 6} is countable finite معدود

Ex: "S" = {1, 2, 3, 4, ...} is countable infinite غير معدود

* Continuous Probability function :-

if the Sample Space associated (مرتبط) with an experiment is an interval of real number then (S) an uncountable infinite number of points and (S) is said to be continuous.

Ex: "S" = {$1 \leq x \leq 2$} is uncountable infinite ما بقدر نعدهم (لأعداد عشرية)

يحتوي على اعداد عشرية (حقيقية)

← هاي مقدمة عن موضوع راح نوخذه بشكل موسع اكثر في الصفحات الجاية

* Conditional Probabilities and Statistical Independence :-

~~Def~~ Def :-

Given two events A and B with P(A) and P(B) > 0, we define the conditional probability of A given B has occurred as

$$\rightarrow P(A/B) = \frac{P(A \cap B)}{P(B)}$$

given تقرأ

and the probability of B given A has occurred as :-

$$\rightarrow P(B/A) = \frac{P(A \cap B)}{P(A)}$$

given تقرأ

Exp: $S = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 4, 6\}$, $B = \{3, 6\}$, $C = \{1, 2, 3, 4\}$

a) $P(A), P(B), P(C)$?

$P(A) = 3/6$, $P(B) = 2/6$, $P(C) = 4/6$.

b) What is the probability of ~~[illegible]~~ observing an even number (A) divisible by 3 (B)?

$P(A \cap B) = P(\{6\}) = 1/6$.

هلأ الـ S راح تكون فيها even بس، S new هي

c) Assume an even number (A) is observed, what is the probability that is divisible by 3 (B)?

حل ① $S_{new} = \{2, 4, 6\}$.

$B_{new} = \{6\} \rightarrow P(B_{new}) = 1/3$.

حل ② Conditional probability :-

$P(B/A) \longrightarrow$ probability of B given A (A occurred)

given: الحدث المعروف — A وقع

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{1/6}{3/6} = 1/3$$

d) Assume a number divisible by 3 (B, يعني وقع) is observed, what is the probability that is an even number (A)?

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/6}{2/6} = 1/2$$

e) Assume the number is less than 5 (C, يعني وقع), what is the probability that is an even number (A)?

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{2/6}{4/6} = 2/4 = 1/2$$

Exp1: Let us consider an experiment with the sample space $S = \{1,2,3,4,5,6\}$. Let us assume that the probability of observing a sample is proportional to its value.

نسبة الرقم نفس قيمته

1 A: even numbers, Find P(A).

$A = \{2,4,6\}$, $P(A) = P(\{2,4,6\}) = ??$

$\overset{2k + 4k + 6k}{\rightarrow \frac{2}{21} + \frac{4}{21} + \frac{6}{21}} = \boxed{12/21}$

$\rightarrow P(S) = 1$

$P(\{1,2,3,4,5,6\}) = 1$

disjoint

$P(\{1\} \cup \{2\} \cup \{3\} \cup \{4\} \cup \{5\} \cup \{6\}) = 1$

$P(\{1\}) + P\{2\} + P(\{3\}) + P(\{4\}) + P(\{5\}) + P(\{6\}) = 1$

$K + 2K + 3K + 4K + 5K + 6K = 1 \rightarrow \boxed{K = 1/21}$

2 B: number less than 5 is observed.

$P(B) = P(\{1,2,3,4\}) = 1k + 2k + 3k + 4k$

$= 1/21 + 2/21 + 3/21 + 4/21 = \boxed{10/21}$

3 C: an even number less than 5 is observed. (even: A, less than 5: B)

$C = A \cap B = \{2,4\} \rightarrow P(C) = (2+4)K = \boxed{6/21}$

4 D: number divisible by 3.

$D = \{3,6\} \rightarrow P(D) = (3+6)K = \boxed{9/21}$

5 $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{6/21}{10/21} = \boxed{6/10}$

Assume a number less than 5 is observed, what is the probability that it is even?

6 $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{6/21}{12/21} = \boxed{6/12}$

Note:

→ Two events A and B are said to be **statistically independent** if $P(A \cap B) = P(A)\,P(B)$.

→ Three events (A, B, and C) are said to be **statistically independent** if:
① $P(A \cap B) = P(A)\,P(B)$
② $P(B \cap C) = P(B) \cdot P(C)$
③ $P(A \cap C) = P(A)\,P(C)$
④ $P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$

* لازم الاربعة يتحققوا ، اذا واحد ما تحقق بكونش Stat. ind.

$\boxed{7}$ Are A and B statistically independent?

$P(A \cap B) \stackrel{??}{=} P(A)P(B)$

$6/21 \stackrel{??}{=} 12/21 \cdot 10/21 \rightarrow 6/21 \neq \frac{120}{441}$

→ So A and B are not Stat. ind.

Exp2: Consider the experiment of tossing a dice,
A: even number is observed, B: number less than 5 is observed

a) $P(A) = ?$

$S = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 4, 6\} \rightarrow P(A) = 3/6$

b) $P(B) = ?$

$B = \{1, 2, 3, 4\} \rightarrow P(B) = 4/6$

c) $P(A/B) = ?$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{2/6}{4/6} = 2/4 = 1/2$

d) $P(B/A) = ?$

$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{2/6}{3/6} = 2/3$

e) Are A and B statistically independent?

$P(A \cap B) \stackrel{??}{=} P(A) \cdot P(B)$

$2/6 \stackrel{?}{=} \underset{1/3}{\cancel{2/6}} \cdot \underset{2/3}{\cancel{4/6}} \rightarrow 2/6 = 2/6$

→ Yes, so A, B are stat. indep.

[f] C: number is divisible by 3, Are A and C Stat. indep?

$C = \{3, 6\}$

$P(A \cap C) \overset{??}{=} P(A) \cdot P(C)$

$\frac{1}{6} \overset{??}{=} \frac{3}{6} \cdot \frac{2}{6} \longrightarrow \frac{1}{6} = \frac{\cancel{6}^{1}}{\cancel{18}_{6}}$

Yes, they are Stat. indep.

[g] ~~Are~~ Are B and C stat. indep?

$P(B \cap C) \overset{?}{=} P(B) \cdot P(C)$

$\frac{1}{6} \overset{?}{=} \underset{2/3}{\cancel{\frac{4}{6}}} \cdot \underset{1/3}{\cancel{\frac{2}{6}}} \rightarrow \frac{1}{6} \neq \cancel{2/4}\ 2/9$

So B, C are not Stat. indep.

[h] Are B and C disjoint?

$B \cap C = \{3\} \neq \emptyset \rightarrow$ So they are not disjoint.

Exp 3: $S = \{1, 2, 3, 4\}$, $A = \{1, 2\}$, $B = \{1, 3\}$, $C = \{1, 4\}$, are A, B, and C stat. indep?

$P(A) = \frac{1}{2}$
$P(B) = \frac{1}{2}$
$P(C) = \frac{1}{2}$

① $P(A \cap B) \overset{??}{=} P(A) P(B)$

$\frac{1}{4} \overset{??}{=} \frac{1}{2} \cdot \frac{1}{2}$

$\frac{1}{4} = \frac{1}{4}$

② $P(A \cap C) \overset{??}{=} P(A) P(C)$

$\frac{1}{4} \overset{?}{=} \frac{1}{2} \cdot \frac{1}{2} \rightarrow \frac{1}{4} = \frac{1}{4}$

③ $P(B \cap C) \overset{??}{=} P(B)\, P(C)$

$\frac{1}{4} \overset{?}{=} \frac{1}{2} \cdot \frac{1}{2} \longrightarrow \frac{1}{4} = \frac{1}{4}$

④ $P(A \cap B \cap C) \overset{?}{=} P(A) \cdot P(B) \cdot P(C)$

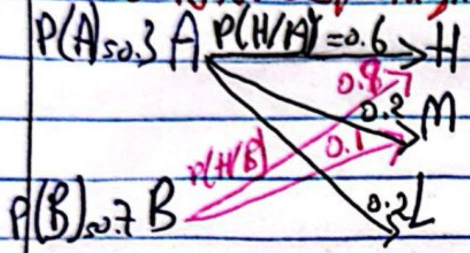
$\frac{1}{4} \overset{??}{=} \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

$\frac{1}{4} \neq \frac{1}{8}$

So they are not stat. indep.

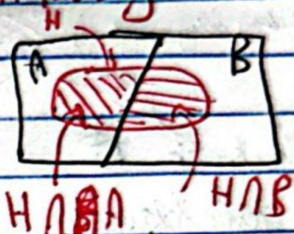
* Total probability theorem :

Ex P4: A factory consist of 2 production lines A and B, Production line A produces 30% of the products out of which 60% are of high quality, 20% of medium quality and 20% of low quality. Production line B produces the rest of the products and 90% are of high quality and 10% of medium quality.



$$P(A/B) = \frac{P(A\cap B)}{P(B)}$$

$$P(A\cap B) = P(B)\,P(A/B)$$



$$P(H) = P(\underline{H\cap A} \cup \underline{H\cap B})$$

disjoint

$$= P(H\cap A) + P(H\cap B)$$

$$= P(A)\,P(H/A) + P(B)\,P(H/B)$$

(a) An item/product is select randomly, what is the probability that it is of high quality?

$$P(H) = P(A)\,P(H/A) + P(B)\,P(H/B)$$

$$= 0.7 \cdot 0.6 + 0.7 \cdot 0.9 = \boxed{0.81}$$

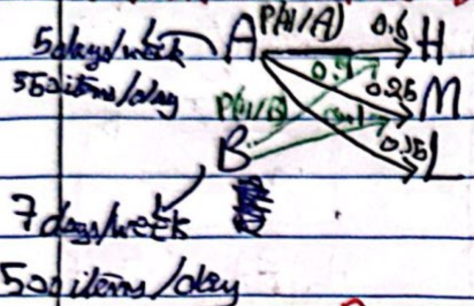
(b) An item is selected at random, what is the probability that it is of low quality?

$$P(L) = P(A)\,P(L/A) + P(B)\,P(L/B)$$

$$= 0.3 \cdot 0.2 + \cancel{0.7 \cdot 0}$$

$$= \boxed{0.06}$$

Exp:- A factory has two productions lines A and B. Production line A work 5 days a week and produces 350 items per day out of which 60% of high quality and 25% of medium quality and 15% of low quality, Production line B work 7 days a week and produces 500 items per day out which 90% of high quality and 10% of medium quality:-



→ [a] Determine the probability of selecting an item produced by production line A after 10 weeks.

$$\text{Total items} = \underbrace{\overset{weeks}{10} \times \overset{A\ days}{5} \times \overset{items}{350}} + \underbrace{\overset{week}{10} \times \overset{B\ days}{7} \times \overset{items}{500}}$$

$$P(A) = \frac{\cancel{10} \times \cancel{5} \times \cancel{350}^{50}}{\cancel{10} \times \cancel{5} \times \cancel{350}_{50} + \cancel{10} \times \cancel{7} \times \cancel{500}_{100}} = \frac{5}{15} = 1/3$$

[b] $P(B) = 1 - P(A) = 2/3$

$$\text{or } \frac{\cancel{10} \times \cancel{7} \times \cancel{500}^{2}}{\cancel{10} \times \cancel{5} \times \cancel{350}_{\cancel{50}\ 1} + \cancel{10} \times \cancel{7} \times \cancel{500}_{\cancel{100}\ 2}} = 2/3$$

[c] An item is selected at random, what is the probability that it is of high quality?

$$P(H) = P(A)P(H/A) + P(B)P(H/B)$$

$$= \cancel{1/3} * \overset{0.2}{\cancel{0.6}} + \cancel{2/3} * \overset{0.3}{\cancel{0.9}} = 0.8$$

d if a high quality item is selected (H given), what is the probability that it was produced by A?

$$P(A/H) = \frac{P(A \cap H)}{P(H)} = \frac{P(A)\,P(H/A)}{P(H)}$$

(given)

$P(A)\,P(A/H)$ ✗ (المقلوب / بالعكس) → $P(A/H)$ هادا اللي بدي قيمتها، هنا بنحط اوفر ماضت ذكنا

$$\to \frac{P(A)\,P(H/A)}{P(H)} = \frac{1/3 * 0.6}{0.8} = \frac{0.2}{0.8} = \boxed{\frac{1}{4}}$$

$$* \; P(A/H) = \frac{P(A \cap H)}{P(H)}$$

$$P(H/A) = \frac{P(A \cap H)}{P(A)}$$

* دايماً ال given (الحكي اللي) بيجي ~~بعد~~ بنحطه في الأعلى

e An item is selected and found to be ~~of~~ medium quality, What is the probability that it was ~~produced~~ produced by A?

* في الامتحان بيجي هيك السؤال لانه هاض شاكل الي قولكم قبل.

$$P(M) = P(A)\,P(M/A) + P(B)\,P(M/B)$$

$$= 1/3 * 0.25 + 2/3 * 0.1 = 0.15$$

$$\to P(A/M) = \frac{P(A \cap M)}{P(M)} = \frac{P(A)\,P(M/A)}{P(M)} = \frac{1/3 * 0.25}{0.15} = \boxed{\frac{5}{9}}$$

لو فش افرع قبل كان لازم احسب $P(A)$, $P(B)$ وبعدين احل هيك ؟
فهيك بيجي في الامتحان احسب حكي الدكتور (!).

f An item is selected at random and found to be of low quality What is the ~~probability~~ probability that it was produced by A?

$$P(L) = P(A)\,P(L/A) + P(B)\,P(L/B)$$

$$= 1/3 * 0.15 + \cancel{2/3 * 0} \to = \boxed{0.05}$$

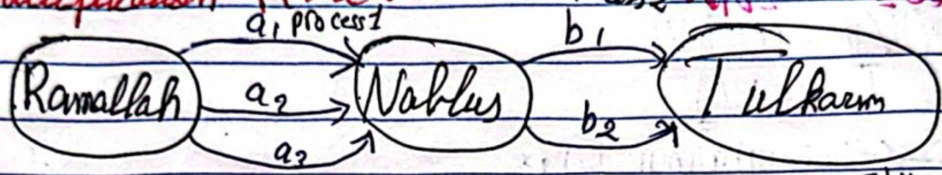
$$P(A/L) = \frac{P(A \cap L)}{P(L)} = \frac{P(A)\,P(L/A)}{P(L)} = \frac{1/3 * 0.15}{0.05} = \boxed{1}$$

9 what is the probability of selected a medium quality item produced by A?

$$p(A \cap M) = p(A)\, p(M/A)$$
$$= \tfrac{1}{3} * 0.25 = \frac{0.25}{3} = \boxed{1/12}$$

* Counting Techniques :- *عند ما نستخدم هاي الطريقة لما يكون في proc 1 و proc 2 او اكثر وما يكون في علاقة بينهم.

→ **Multiplication Rule:**



عدد الطرق من رام الله لنابلس = ٣ ← $3 \times 2 = 6$ → عدد الطرق من رام الله الطولكرم

عدد الطرق من نابلس الطولكرم = ٢

$S = \{ a_1b_1, a_1b_2, a_2b_1, a_2b_2, a_3b_1, a_3b_2 \}$

التكرار ممنوع

→ **Permutation** → without replacement (repetition is not allowed)

تباديل → order is important

الترتيب مهم

عدد الخيارات الممكنة

$$\boxed{P^n_k = \frac{n!}{(n-k)!}}$$

عدد الخيارات الي بختارها

S.P ① ② ③ ④ ⑤

$\underset{\text{عشرات}}{4} \times \underset{\text{احاد}}{5} = 20$

مثلا سحبت اول مرة ٣ ، وبعدها مرة ١

وسحبت اول مرة ١ ، وبعدها مرة ٣

هدول خيارين مختلفين يعني الترتيب مهم

5 كورات

P^5_2 كل مرة بسحب كرتين وراء بعض (الترتيب مهم)

$$= \frac{5!}{(5-2)!} = \frac{5 \times 4 \times 3!}{3!} = \boxed{20}$$

التكرار ممنوع

→ **Campination** → without replacement (repetition is not allowed)

توافيق

order is not important

الترتيب مش مهم

عدد الخيارات الممكنة

S.P ③ ④ ⑤ ① ②

$$\boxed{C^n_k = \frac{n!}{(n-k)!\,k!}}$$

عدد الخيارات الي بختارها

اذا سحبت كرتين بنفس الوقت (الترتيب مش مهم)

$$C^5_2 = \frac{5!}{(5-2)!\,2!} = \frac{5 \times 4 \times 3!}{3! \times 2!} = \boxed{10}$$

Exp: A person type a 5 letter computer password, what is the ~~pron~~ probability that no letter is repeated? (# of letters is 26)

$\underbrace{\text{no letter is repeated}}_{A}$

# of Sample outcomes is $S = 26 * 26 * 26 * 26 * 26 = 26^5$

# of Sample outcomes in $A = 26 * 25 * 24 * 23 * 22$

[illegible]

$$\rightarrow P^{n=26}_{k=5} = \frac{26!}{(26-5)!} = \frac{26 * 25 * 24 * 23 * 22 * \cancel{21!}}{\cancel{21!}}$$

$$P(A) = \frac{26 * 25 * 24 * 23 * 22}{26^5} = 0.66436749$$

→ Sample space:

	1	2	3	4	5
1	1,1	1,2	1,3	1,4	1,5
2	2,1	2,2	2,3	2,4	2,5
3	3,1	3,2	3,3	3,4	3,5
4	4,1	4,2	4,3	4,4	4,5
5	5,1	5,2	5,3	5,4	5,5

ـ اذا كان التكرار ممنوع (تباديل) ← 20

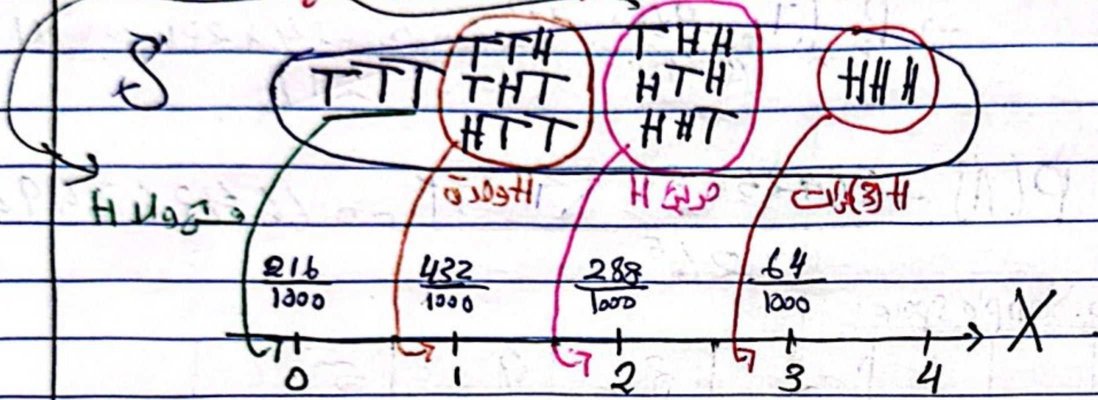
ـ اذا كان الترتيب مش مهم (وكمان التكرار ممنوع) (توافيق) ← 15

يعني مثلاً (1,2) و (2,1) ⇐

يعتبرهم خيار واحد

Chapter 2: Single Random Variables and Probability Distribution

Exp 1: let us consider the experiment of flipping three coins. Assume the $P(H) = 4/10$ and $P(T) = 6/10$ for each of the coins.
→ let us define the Random Variable X such that it represent the number of heads observed.



(a) Determine the probability of observing exactly one head.

$P(TTH, THT, HTT) = P(TTH) + P(THT) + P(HTT)$

$\rightarrow P(TTH) = P(T)\,P(T)\,P(H)$

$= \frac{6}{10} * \frac{6}{10} * \frac{4}{10} = \frac{144}{1000}$ +

$P(THT) = P(T)\,P(H)\,P(T)$

$= \frac{6}{10} * \frac{4}{10} * \frac{6}{10} = \frac{144}{1000}$ +

$P(TTH) = \text{\_\_\_\_\_\_} = \frac{144}{1000}$

$\rightarrow P(HTT, THT, TTH) = \boxed{\frac{432}{1000}}$

$= 3 * \frac{4}{10} * \left(\frac{6}{10}\right)^2 = \boxed{\frac{432}{1000}}$

(b) Determine the probability of observing ~~the~~ exactly two heads.

$P(THH, HTH, HHT) = 3 * P(T)\left(P(H)^2\right)$

$= 3 * \frac{6}{10} * \left(\frac{4}{10}\right)^2 = \boxed{\frac{288}{1000}}$

c) Determine the probability of observing three heads?

$P(HHH) = \left(\frac{4}{10}\right)^3 = \boxed{\frac{64}{1000}}$

d) Determine $P(X=1)$?

$P(X=1) = P(\{HTT, THT, TTH\}) = \boxed{\frac{432}{1000}}$

e) $P(X=0) = P(\{TTT\}) = \left(\frac{6}{10}\right)^3 = \boxed{\frac{216}{1000}}$

f) Determine the Probability Mass function of X?

$P(X=2) = \frac{288}{1000}$, $P(X=4) = 0$

$P(X=3) = \frac{64}{1000}$, $P(X=-1) = 0$

$$\rightarrow P(X=x) = \begin{cases} \frac{216}{1000} & x=0 \\ \frac{432}{1000} & x=1 \\ \frac{288}{1000} & x=2 \\ \frac{64}{1000} & x=3 \\ 0 & \text{otherwise} \end{cases}$$

Properties of P.M.F:-

* $P(X=x) \geq 0$
* $\sum_{-\infty}^{\infty} P(X=x) = 1$

* Comulative Distribution Function :-

$$F_X(x) = P(X \leq x)$$

g) $F_X(0.5) = P(X \leq 0.5) = P(X=0) = \frac{216}{1000}$

h) $F_X(1.5) = P(X \leq 1.5)$
$= P(X=0) + P(X=1) = \frac{216}{1000} + \frac{432}{1000} = \frac{648}{1000}$

i) $F_X(2.7) = P(X \leq 2.7)$
$= P(X=0) + P(X=1) + P(X=2) = \frac{936}{1000}$

j) $F_X(2) = P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = \frac{936}{1000}$

معناها يعني $X \leq 2$

k) $F_X(2^-) = P(X < 2) = P(X=0) + P(X=1) = \frac{648}{1000}$

معناها يعني $X \geq 2$ ← [illegible]

l) $F_X(2^+) = P(X \leq 2) = \frac{936}{1000}$
$P(X > 2) +$
$P(X \geq 2) +$

Notes

Properties of CDF :-

1) $F_X(-\infty) = P(X \leq -\infty) = 0$

2) $F_X(\infty) = P(X \leq \infty) = 1$

3) $P(X = x) = F_X(x) - F_X(x^-)$

Ex 2 :- Let X be a R.V with the following PMF :-

$$P_X(X=x) = \begin{cases} 0.2 & x=-1 \\ 0.3 & x=1 \\ \cancel{G}\ 0.1 & x=2 \\ 0.4 & x=3 \\ 0 & \text{otherwise} \end{cases}$$

① Determine the value of constant G ?

$$\sum_{-\infty}^{\infty} P(X=x) = 1 \rightarrow 0.2 + 0.3 + G + 0.4 = 1 \rightarrow \boxed{G=0.1}$$

② $F_X(1) = P(X \leq 1) = P(1) + P(-1) = 0.3 + 0.2 = \boxed{0.5}$

③ $F_X(-\infty) = ??$

$= P(X \leq -\infty) = 0$

④ $F_X(\infty) = ??$

$P(X \leq \infty) = P(X=-1) + P(X=1) + P(X=2) + P(X=3) = 1$

⑤ $F_X(2^-) = P(X < 2) = P(-1) + P(1) = 0.5$

⑥ $P(X > 2)$? $= P(X=3) = 0.4$

or

$1 - P(X \leq 2)$

$1 - F_X(2) = 1 - 0.6 = 0.4$

Exp 3): Let X be a R.V. with the following CDF.

$$F_X(x) = \begin{cases} \cancel{A}\ 0 & x < -2 \\ 0.2 & -2 \le x < 0 \\ 0.5 & 0 \le x < 3 \\ \cancel{K}\ 1 & 3 \le x \end{cases}$$

1) Determine the value of k, A?

Note ال مس

$F_X(-\infty) = \boxed{A = 0}$, $F_X(\infty) = \boxed{k = 1}$

2) $F_X(1) = ? = 0.5$

3) $P(X=0)$?

$P(X=0) = P(X \le 0) - P(X < 0)$

$= F_X(0) - F_X(0^-)$

$= 0.5 - 0.2 = 0.3$

4) $P(X=3)$?

$P(X=3) = P(X \le 3) - P(X < 3)$

$= F_X(3) - F_X(3^-)$

$1 - 0.5 = 0.5$

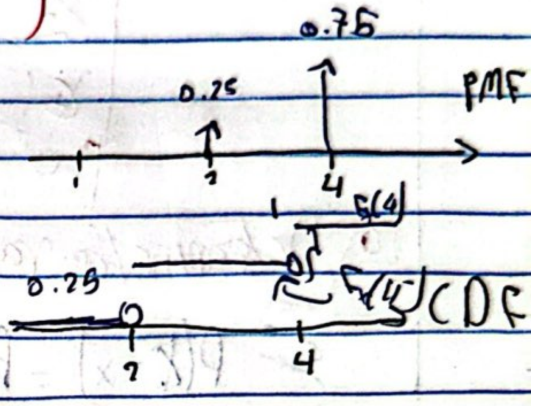
5) $F_X(7) = 1$

6) $P(X \ge 3) = ?$ → $P(X \ge 3) = 1 - P(X < 3)$

$= 1 - F_X(3^-) = 1 - 0.5 = 0.5$

Exp 4) let x be a R.V with the following PMF :-

$$P(X=x) = \begin{cases} 0.25 & x=2 \\ 0.75 & x=4 \\ 0 & \text{otherwise} \end{cases}$$



1) $F_X(1) = P(X \leq 1) = 0$

2) $F_X(5) = P(X \leq 5) = 1$

3) $F_X(7.5) = P(X \leq 7.5) = 1$

$$F_X(x) = \begin{cases} 0 & x < 2 \\ 0.25 & 2 \leq x < 4 \\ 1 & 4 \leq x \end{cases}$$

* Properties PMF :-

1. $P(X=x) \geq 0$
2. $\sum_{x=-\infty}^{\infty} P(X=x) = 1$

* Properties of CDF :-

1. $F_X(-\infty) = 0$
2. $F_X(\infty) = 1$
3. $F_X(x) = F_X(x^+)$

→ 4. $x_2 > x_1 \rightarrow F(x_2) \geq F(x_1)$

Exp 11) Let X be a R.V with the following PMF:-

$$P(X=x) = \begin{cases} 1/4 & x=-3 \\ 1/8 & x=-1 \\ 1/2 & x=1 \\ \cancel{G}\ 1/8 & x=2 \\ 0 & 0.w \end{cases}$$

a) Determine the value of the constant G.

$$\sum_{x=-\infty}^{\infty} P(X=x) = P(X=-3) + P(X=-1) + P(X=1) + P(X=2)$$
$$= 1/4 + 1/8 + 1/2 + G = 1$$
$$\rightarrow \boxed{G = 1/8}$$

b) $F_X(-\infty) = ??$

$$F_X(-\infty) = P(X \le -\infty) = 0$$

c) $F_X(\infty) = ??$

$$F_X(\infty) = P(X \le \infty) = 1$$

d) $F_X(1^-) = ??$

$$P(X \le 1^-) = P(X < 1) = 3/8$$

e) $F_X(1^+) = ??$

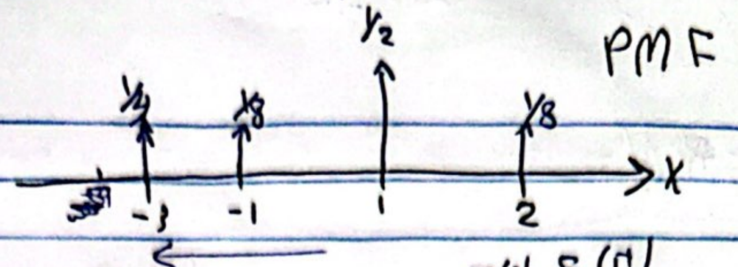
$$P(X \le 1^+) = P(X \le 1) = 7/8$$

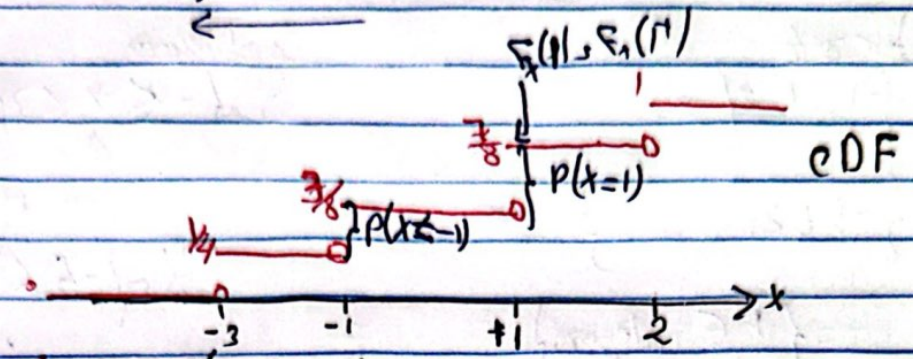
f) $F_X(1) = ?$

$$F_X(1) = F_X(1^+) = \frac{7}{8}$$

$P(X < 1)$

$P(X \le 1)$.





(g) $F_X(1.5) = P(X \leq 1.5) = 7/8$

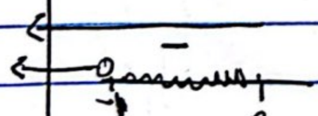
(h) $F_X(2.25) = P(X \leq 2.25) = 1$

(i) Write the CDF of X.

$$F_X(x) = \begin{cases} 0 & x < -3 \\ \frac{1}{4} & -3 \leq x < -1 \\ \frac{3}{8} & -1 \leq x < 1 \\ \frac{7}{8} & 1 \leq x < 2 \\ 1 & x \leq 2 \end{cases}$$

(j) $P(-1 \leq X \leq 2) = ? \rightarrow P(X=-1) + P(X=1) + P(X=2) = \frac{6}{8} = \frac{3}{4}$

or $\rightarrow P(X \leq 2) - P(X < -1)$



$= F_X(2) - F_X(-1^-) = 1 - 1/4 = \boxed{3/4}$

(k) $P(-1 < X \leq 2) \rightarrow P(1) + P(2) = 5/8$

or

$P(X \leq 2) - P(X \leq -1)$

$= F_X(2) - F_X(-1)$

$= 1 - \frac{3}{8} = \boxed{5/8}$

(l) $P(-1 \leq X < 2) = P(X < 2) - P(X < -1) = 7/8 - \frac{2 \times 1}{2 \times 4} = \boxed{5/8}$

(m) $P(-1 < X < 2) = P(X < 2) - P(X \leq -1) = 7/8 - \frac{3}{8} = 4/8 = \boxed{\frac{1}{2}}$

R.Vs

Discrete

$S = \{-1, 1, 2\}$

or $S\{-1, 1, 2, \ldots\}$

$= \{x \in \text{integers}\}$

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2, 4, 5\}$

$P(A) = 4/6$

⇐ بيكون فيها الأعمار بنوعه كامل راو عدد

Continuous

$S = \{-1 \le x < 3\}$

↑ real numbers

$S = \{-5 < x\}$

real

$S = \{\underbrace{-1 \le x < 3}_{4}\} \to 4$

$A = \{\underbrace{-1 \le x < 2}_{3}\} \to 3$

$P(A) = \frac{3}{4}$

$\cancel{P(x=0) = \frac{0}{\infty}} = 0$

عملنا قسمة لأنها بنعرفها

⊗ Probability density Function (PDF)

$$f_x(x) = \begin{cases} 1/4 & 1 \le x \le 5 \\ 0 & \text{o.w} \end{cases}$$

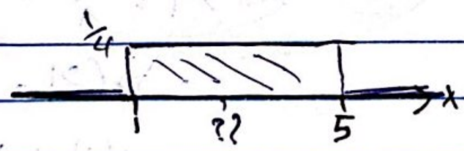
Valid PDF?

$\int_{-\infty}^{\infty} f_x(x)dx$

$= \int_{-\infty}^{1} 0\,dx + \int_{1}^{5} \frac{1}{4}dx + \int_{5}^{\infty} 0\,dx$

$= \frac{1}{4}x \Big|_1^5$

$\frac{1}{4}(5-1) = 1$



$$g(x) = \begin{cases} \frac{2x}{15} & 1 \le x \le 4 \\ 0 & \text{o.w} \end{cases}$$

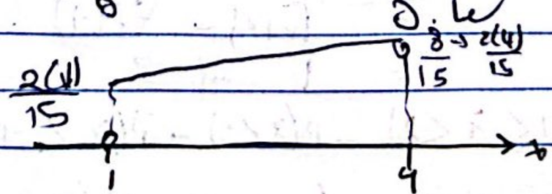
Valid PDF!

$\int_{-\infty}^{\infty} f_x\,dx$?? 1

$\int_1^4 \frac{2x}{15}dx = \frac{2x^2}{30}\Big|_1^4$

$\frac{2(16-1)}{30}$

$= 1$



* Properties of pdf?

① $f_x(x) \geq 0$

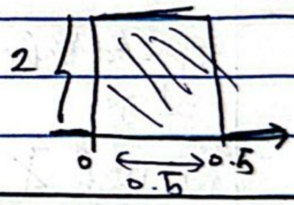
② $\int_{-\infty}^{\infty} f_x(x)\,dx = 1$

Exp 2):- Let x be a R.V with the following pdf?:-

$$f_x(x) = \begin{cases} \cancel{k}\ 2 & 0 < x < 0.5 \\ 0 & \text{o.w} \end{cases}$$

[a] Determine the value of the constant k?

$$\int_{-\infty}^{\infty} f_x(x)\,dx = 1 \to \int_0^{0.5} k\,dx = kx\Big|_0^{.5} = k(0.5) = 1 \to \boxed{k = 2}$$



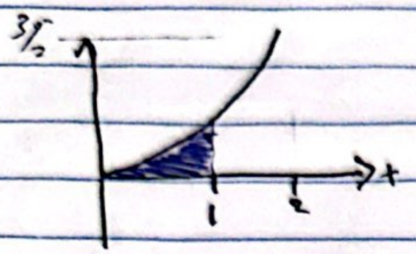
Exp 3): Let x be a R.V with the following pdf:-

$$f_x(x) = \begin{cases} kx^2 & 0 < x < 2 \\ 0 & \text{o.w} \end{cases}$$

① Determine the Value of the constant k:-

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^2 kx^2\,dx = \frac{kx^3}{3}\Big|_0^2 \to \frac{8k}{3} = 1 \to \boxed{k = \frac{3}{8}}$$

② $F_X(1) = ?$ $= P(X \le 1)$



$$\int_{-\infty}^{1} f(x)\,dx = \int_{0}^{1} \frac{3}{8} x^2\,dx$$

$$= \frac{x^3}{8}\Big|_0^1 = \boxed{\frac{1}{8}}$$

③ $P(0.5 \le X \le 1.5) = ?$

$$= \int_{0.5}^{1.5} \frac{3}{8} x^2\,dx = \frac{x^3}{8}\Big|_{0.5}^{1.5}$$

$$\frac{3x^3}{(3)8}\left[(1.5 - 0.5)^3\right] = \boxed{??}$$

Mean or Expected Value:-

- $E\{X\} = \mu_X \overset{D}{=} \sum_{x=-\infty}^{\infty} x\,P(X = x)$

expected Value of X or mean of X

$$\overset{C}{=} \int_{-\infty}^{\infty} x\,f_X(x)\,dx$$

- $E\{g(x)\} \overset{D}{=} \sum g(x)\,P(X = x)$

$$\overset{C}{=} \int_{-\infty}^{\infty} g(x)\,f_X(x)\,dx$$

Variance of X

$$\sigma_X^2 = Var\{X\} = E\{(X - \mu_X)^2\}$$

$$\overset{D}{=} \sum_{x=-\infty}^{\infty} (x - \mu_X)^2\,P(X = x)$$

$$\overset{C}{=} \int_{-\infty}^{\infty} (x - \mu_X)^2\,f_X(x)\,dx$$

* Standard deviation of X :-

$\sigma_X = \sqrt{\sigma_X^2}$

Exp1. Let X be a R.V with the following :-

$$P(X=x) = \begin{cases} 1/4 & x=-2 \\ 1/4 & x=-1 \\ 1/4 & x=1 \\ 1/4 & x=2 \\ 0 & o.w \end{cases}$$

Determine the :-

1. $\mu_X = \sum_{x=-\infty}^{\infty} x P(X=x) = (-2)(1/4) + (-1)(1/4) + 1(1/4) + 2(\frac{1}{4}) = \boxed{0}$

2. $\sigma_X^2 = E\{(X-\mu_X)^2\} = E\{(X-0)^2\} = E\{X^2\} = \sum_{x=-\infty}^{\infty} x^2 P(X=x)$

$= (-2)^2 \cdot (\frac{1}{4}) + (-1)^2 \cdot (\frac{1}{4}) + (1)^2 (\frac{1}{4}) + (2)^2 (\frac{1}{4}) = \frac{10}{4} = \boxed{2.5}$

3. $\sigma_X = \sqrt{\sigma_X^2} = \sqrt{2.5} = 1.58114$

Exp2 :- $P(X=x) = \begin{cases} 1/4 & x=-4 \\ 1/4 & x=-3 \\ 1/4 & x=-1 \\ 1/4 & x=0 \\ 0 & o.w \end{cases}$

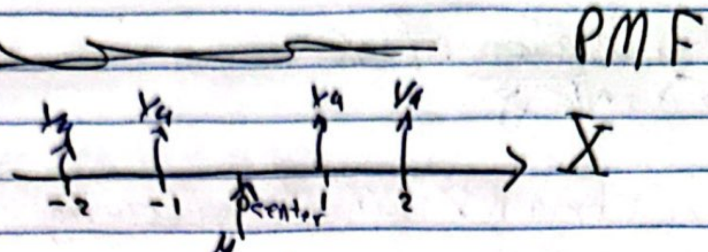
a. $\mu_X = \sum_{x=-\infty}^{\infty} x P(X=x) = -4 \cdot \frac{1}{4} + -3 \cdot \frac{1}{4} + -1 \cdot \frac{1}{4} + 0 \cdot \frac{1}{4} = \boxed{-2}$

b. $\sigma_X^2 = E\{(X-\mu_X)^2\} = E\{(X+2)^2\} = \sum_{x=-\infty}^{\infty} (x+2)^2 P(X=x)$

~~σ_X~~ $= \overset{-4+2}{(-2)^2} \cdot \frac{1}{4} + \overset{-3+2}{(-1)^2} \cdot \frac{1}{4} + \overset{-1+2}{(1)^2} \cdot \frac{1}{4} + \overset{0+2}{(2)^2} \cdot \frac{1}{4} = 2.5$

c. $\sigma_X = \sqrt{\sigma_X^2} = \sqrt{2.5} = 1.58114$

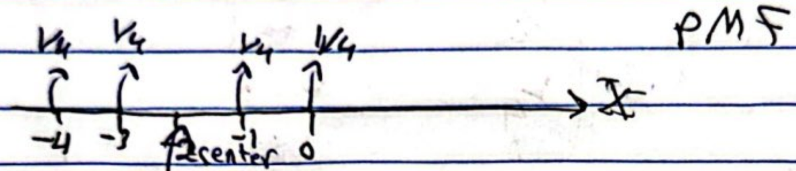
Exp1



$\mu_x = 0$

$\sigma_x^2 = 2.5$

Exp2



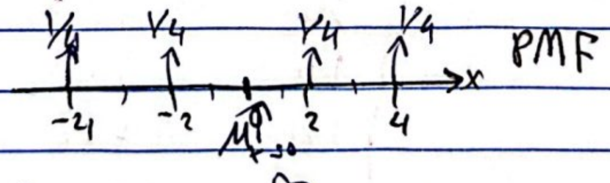
$\mu_x = -2$

$\sigma_x^2 = 2.5$

Exp3 $P(X=x) = \begin{cases} 1/4 & x=-4 \\ 1/4 & x=-2 \\ 1/4 & x=2 \\ 1/4 & x=4 \\ 0 & \text{o.w} \end{cases}$

a) $\mu_x = \sum_{x=-\infty}^{\infty} x P(X=x) = (-4)\frac{1}{4} + (-2)\frac{1}{4} + (2)\cdot\frac{1}{4} + (4)\frac{1}{4} = 0$

b) σ_x^2



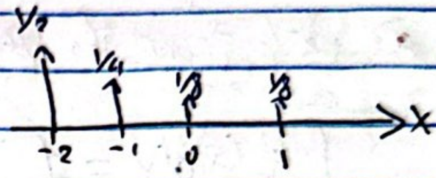
$\to E\{(X-\mu)^2\} = E\{x^2\} = \sum_{x=-\infty}^{\infty} x^2 P(X=x)$

$= (-4)^2\cdot\frac{1}{4} + (-2)^2\cdot\frac{1}{4} + (2)^2\cdot\frac{1}{4} + (4)^2\cdot\frac{1}{4} = \frac{40}{4} = \boxed{10}$

c) $\sigma_x = \sqrt{\sigma_x^2} = \sqrt{10} = 3.16228$

لما تكون σ_x^2 قليلة يعني النتائج قريبة على بعض
اما اذا كان عالي يعني النتائج بعيدة عن بعض (متشتتة) (متوزعة)
كلما زاد التشتت بزيد الـ Variance

Ex4: $P(X=x) = \begin{cases} 1/2 & x=-2 \\ 1/4 & x=-1 \\ 1/8 & x=0 \\ 1/8 & x=1 \\ 0 & \text{O.W} \end{cases}$



1) $\mu_x = \sum_{x=-\infty}^{\infty} x\,P(X=x)$

$= -2\cdot\frac{1}{2} + -1\cdot\frac{1}{4} + 0\cdot\frac{1}{8} + 1\cdot\frac{1}{8} = \boxed{-9/8}$

2) $\sigma_x^2 = E\{(x - -9/8)^2\} = E\{(9/8 + x)^2\} = \sum_{-\infty}^{\infty} (9/8 + x)^2 \cdot P(X=x)$

$= (\frac{9}{8} + -2)^2\cdot\frac{1}{2} + (\frac{9}{8} + -1)^2\cdot\frac{1}{4} + (\frac{9}{8}+0)^2\cdot\frac{1}{8} + (\frac{9}{8}+1)^2\cdot\frac{1}{8}$

$= (-\frac{7}{8})^2\cdot\frac{1}{2} + (\frac{1}{8})^2\cdot\frac{1}{4} + (\frac{9}{8})^2\cdot\frac{1}{8} + (\frac{17}{8})^2\cdot\frac{1}{8}$

$= \boxed{1.109375} \approx 1.10938$

3) $\sigma_x = \sqrt{\sigma_x^2} = \sqrt{1.10938} = 1.05327$

Exp 5: Let X be R.V with the following Pdf

$$f_x(x) = \begin{cases} k(1-x^2) & -1<x<1 \\ 0 & o.w \end{cases}$$

a) Determine the value of k?

$$\int_{-\infty}^{\infty} f_x(x)\,dx = 1 \rightarrow \int_{-1}^{1} k(1-x^2)\,dx = 1 = k\left\{x-\frac{x^3}{3}\right\}\Big|_{-1}^{1} = 1$$

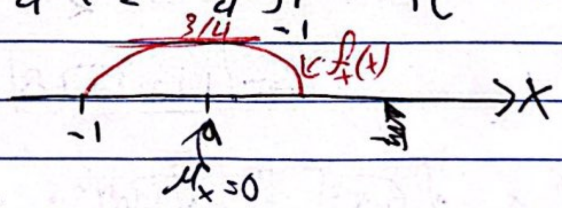
$$= k\left\{\left(1-\frac{1}{3}\right)-\left(-1+\frac{1}{3}\right)\right\} = 1$$

$$\frac{4}{3}k = 1 \rightarrow \boxed{k = 3/4}$$

b) Determine the expected value of X:

$$\mu_x = E\{x\} = \int_{-\infty}^{\infty} x\, f_x(x)\,dx = \int_{-1}^{1} x\cdot\frac{3}{4}(1-x^2)\,dx = \int_{-1}^{1}\frac{3}{4}(x-x^3)\,dx$$

$$= \frac{3}{4}\left\{\frac{x^2}{2}-\frac{x^4}{4}\right\}\Big|_{-1}^{1} = \frac{3}{4}\left\{\left(\frac{1}{2}-\frac{1}{4}\right)-\left(\frac{1}{2}-\frac{1}{4}\right)\right\} = \boxed{0}$$



b) $\sigma_x^2 = ?$

$$\sigma_x^2 = E\{x^2\} = \int_{-\infty}^{\infty} x^2 f_x(x)\,dx = \int_{-1}^{1} x^2\cdot 3/4(1-x^2)\,dx$$

$E\{(x-0)^2\}$

$$= \frac{3}{4}\int_{-1}^{1}(x^2-x^4)\,dx = \frac{3}{4}\left\{\frac{x^3}{3}-\frac{x^5}{5}\right\}\Big|_{-1}^{1}$$

$$= \frac{3}{4}\left\{\left(\frac{1}{3}-\frac{1}{5}\right)-\left(\frac{-1}{3}+\frac{1}{5}\right)\right\} = \frac{3}{4}\left\{\frac{2}{3}-\frac{2}{5}\right\} = \boxed{0.2}$$

c) $\sigma_x = ?$

$$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{0.2} = 0.44721$$

Exp 6) $f_x(x) = \begin{cases} \underset{3/32}{\cancel{k}}(4-x^2) & -2 \le x \le 2 \\ 0 & \text{O.W} \end{cases}$

$\mu_x ?,\ \sigma_x^2 ?,\ \sigma_x ?$

find $k \rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 1 \rightarrow k\int_{-2}^{2}(4-x^2)\,dx = k\left[\int_{-2}^{2}4\,dx - \int_{-2}^{2}x^2\,dx\right] = 1$

$k\left[4(2+2) - \left(\frac{x^3}{3}\Big|_{-2}^{2}\right)\right] = 1$

$k\left[16 - \left(\frac{8}{3} - \frac{-8}{3}\right)\right] = 1$

$\rightarrow k\left[16 - \frac{16}{3}\right] = 1 \rightarrow \frac{32}{3}k = 1 \rightarrow \boxed{k = \frac{3}{32}}$

$\rightarrow \mu_x = \int_{-\infty}^{\infty} x\, f_x(x)\,dx \rightarrow \int_{-2}^{2}\frac{3}{32}(4-x^2).x\ dx \rightarrow \frac{3}{32}\int_{-2}^{2}4x - x^3\ dx$

$= \frac{3}{32}\left[2x^2 - \frac{x^3}{3}\Big|_{-2}^{2}\right] \rightarrow \frac{2}{32}\left[\cancel{\left(8-\frac{8}{3}\right)} - \cancel{\left(8-\frac{8}{3}\right)}\right] = 0$

$\sigma_x^2 = E(x^2 - \mu_x) = E x^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

$= \int_{-2}^{2} x^2 \cdot \frac{3}{32}(4-x^2)\,dx \rightarrow \frac{3}{32}\int_{-2}^{2}(4x^2 - x^4)\,dx = \frac{3}{32}\left[\frac{4x^3}{3} - \frac{x^5}{5}\right]$

$= \frac{3}{32}\left[\left(\frac{32}{3} - \frac{32}{5}\right)\right] \rightarrow \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{32}}}\left[\frac{\overset{2}{\cancel{64}}}{\underset{5}{\cancel{15}}}\right] = 2/5 = \boxed{0.4}$

$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{0.4} = \boxed{0.63246}$

Exp7) $f_x(x) = \begin{cases} H & -2 < x < 2 \\ 0 & \text{o.w} \end{cases}$

μ_x ?
σ_x^2 ??
σ_x ?

$\int_{-\infty}^{\infty} f(x)\,dx = 1 \rightarrow \int_{-2}^{2} H\,dx = 1 \rightarrow H\{2 - -2\} = 1 \rightarrow 4H = 1$ $\boxed{H = \frac{1}{4}}$

$\mu_x = \int_{-2}^{2} x f_x(x)\,dx = \int_{-2}^{2} \frac{1}{4} x\,dx = \frac{1}{4}\frac{x^2}{2}\Big|_{-2}^{2}$

$= \frac{1}{8}\{4 - 4\} = \boxed{0}$

$\sigma_x^2 = E(x-\mu_x)^2 = Ex^2 = \int_{-\infty}^{\infty} x^2 f_x(x)\,dx$

$= \int_{-2}^{2} \frac{1}{4}x^2\,dx = \frac{1}{4}\frac{x^3}{3}\Big|_{-2}^{2}$

$= \frac{1}{4}\left[\frac{8}{3} - -\frac{8}{3}\right] = \frac{1}{4}\left(\frac{16}{3}\right) = \boxed{\frac{4}{3}}$

$= 1.33$

$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{1.33} = \boxed{1.154}$

Exp 1: Let X be a R.V with the following pdf:-

$$f_X(x) = \begin{cases} k(x^2+x) & 0 \le x \le 2 \\ 0 & O.W \end{cases}$$

$3/14$

1) Determine the value of the constant k?

$$\int_{-\infty}^{\infty} f_X(x)\,dx = \int_0^2 k(x^2+x)\,dx = 1 \to k\left[\frac{x^3}{3}+\frac{x^2}{2}\Big|_0^2\right] = 1$$

$$\to k\left\{\left(\frac{8}{3}+\frac{4}{2}\right)-0\right\} = 1 \to k\left(\frac{14}{3}\right) = 1$$

$$\boxed{k = 3/14}$$

2) $P(X \ge 1) = \int_1^{\infty} f_X(x)\,dx = \int_1^2 \frac{3}{14}(x^2+x)\,dx = \frac{3}{14}\left[\left(\frac{x^3}{3}+\frac{x^2}{2}\right)\Big|_1^2\right]$

$$\to \frac{3}{14}\left[\left(\frac{8}{3}+\frac{4}{2}\right)-\left(\frac{1}{3}+\frac{1}{2}\right)\right] \to \frac{3}{14}\left[\left(\frac{7}{3}+\frac{3}{2}\right)\right] \to \frac{1}{14}\cdot\frac{23}{2} = \boxed{\frac{23}{28}}$$

3) $P(\underset{A}{X \ge 1.5} / \underset{B}{X > 1})$ given $= \frac{P(X \ge 1.5 \cap X > 1)}{P(X > 1)}$

$$= \frac{P(X \ge 1.5)}{P(X>1)} = \frac{\int_{1.5}^{\infty} f_X(x)\,dx}{\int_1^{\infty} f_X(x)\,dx} = \frac{\int_{1.5}^{2}\frac{3}{14}(x^2+x)\,dx}{\int_1^2 \frac{3}{14}(x^2+x)\,dx} = \frac{\frac{3}{14}\left(\frac{x^3}{3}+\frac{x^2}{2}\right)\Big|_{1.5}^2}{\frac{3}{14}\left(\frac{x^3}{3}+\frac{x^2}{2}\right)\Big|_1^2}$$

$$= \frac{\frac{3}{14}\left(\frac{8}{3}+\frac{4}{2}\right)-\left(\frac{(1.5)^3}{3}+\frac{(1.5)^2}{2}\right)}{23/28} = \frac{\frac{3}{14}\left(\frac{14}{3}-\frac{9}{4}\right)}{23/28} = \frac{0.51785}{23/28} = \boxed{0.63043}$$

4) $P(X \le 1.4 / X > 1) = \frac{P(X \le 1.4 \cap X > 1)}{P(X>1)} = \frac{P(1 < X \le 1.4)}{P(X>1)}$

$$= \frac{\int_1^{1.4}\frac{3}{14}(x^2+x)\,dx}{\frac{23}{28}} = \frac{\frac{3}{14}\left(\frac{x^3}{3}+\frac{x^2}{2}\right)\Big|_1^{1.4}}{23/28} = \frac{\frac{3}{14}\left(\frac{(1.4)^3}{3}+\frac{(1.4)^2}{2}\right)-\left(\frac{1}{3}+\frac{1}{2}\right)}{23/28}$$

$$= \frac{0.22742}{23/28}$$

$$= \boxed{0.2769}$$

5 $P(x>1.5 \mid x<3) = \dfrac{P(x>1.5 \cap x<3)}{P(x<3)} = \dfrac{P(1.5<x<3)}{P(x<3)}$

$$= \frac{\int_{1.5}^{2} \frac{3}{14}(t^2+t)\,dx}{\int_{0}^{2} \frac{3}{14}(t^2+x)\,dx} = \frac{\frac{3}{14}\left(\frac{t^3}{3}+\frac{t^2}{2}\Big|_{1.5}^{2}\right)}{1} = \frac{0.5179}{1} \quad \boxed{0.5179}$$

6 Determine the ~~variance~~ mean of x:-

$$\mu_x = \int_{-\infty}^{\infty} x\, f_x(t)\,dx = \int_0^2 x \cdot \frac{3}{14}(t^2+x) = \frac{3}{14}\int_0^2 (x^3+t^2)\,dx$$

$$= \frac{3}{14}\left[\frac{x^4}{4}+\frac{t^3}{3}\Big|_0^2\right] = \frac{3}{14}\left\{\left(\frac{16}{4}+\frac{8}{3}\right)-0\right\} = \frac{3}{14}\left\{\frac{20}{3}\right\} = \frac{20}{14} = \boxed{\frac{10}{7}}$$

7 Determine the variance of x:-

$$\sigma_x = E(x-\mu_x)^2 = \int_{-\infty}^{\infty}\left(t-\frac{10}{7}\right)^2 f_x(t)\,dx = \int_0^2 \left(x-\frac{10}{7}\right)^2 \cdot \frac{3}{14}(t^2+x)\,dx$$

$$= \frac{3}{14}\int_0^2 \left(x-\frac{10}{7}\right)^2 (x^2+x)\,dx = \boxed{\frac{46}{245}}$$

Note :-

* Mode of the distribution

x_{mode} : $f_x(x_{mode}) = Max\,[f_x(t)]$

8 Determine the mode of $f_x(x)$:-

$$f'_x = 0 \rightarrow \frac{d f_x(t)}{dx} = 0 \rightarrow \frac{d\left[\frac{3}{14}(t^2+t)\right]}{dx} = 0 = \frac{3}{14}(2x+1) = 0$$

$$\rightarrow 2x+1 = 0 \rightarrow \frac{2x}{2} = -\frac{1}{2} \rightarrow \boxed{x = -\frac{1}{2}} \quad \alpha$$

check of the ends of the interval $[0,2]$

$f_x(0) = 0$, $f_x(2) = \frac{3}{14}(2^2+2) = \boxed{\frac{18}{14}}$, $\frac{18}{14} > 0$

So, the mode of $f_x(t)$ is at $x=2$

Ex 23) let x be a R.V with the following pdf:-

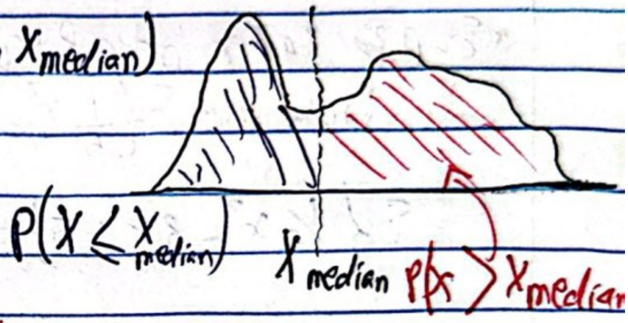
$$f_X(x) = \begin{cases} \frac{2}{9}x & 0 < x \le 3 \\ 0 & o.w \end{cases}$$

[1] Determine the mode of $f_X(x)$?

$$\frac{df_X}{dx} = 0 \to \frac{2}{9} \neq 0, \quad \begin{matrix} f_X(0) = 0 \\ f_X(3) = 2/3 \end{matrix} \Bigg] \to \text{so, the mode of } f_X(x) \text{ is } 3$$

*** Note:** • Median of distribution

$$X_{median}: P(X \le X_{median}) = P(X > X_{median})$$



[2] Determine the median of $f_X(x)$:-

$$P(X \le \underbrace{x_0}_{median}) = \int_0^{x_0} \frac{2}{9}x\,dx = \frac{x^2}{9}\Big|_0^{x_0} = \left(\frac{x_0^2}{9} - 0\right)$$

$$= \frac{x_0^2}{9} = \frac{1}{2} \to x^2 = \frac{9}{2} \to \cancel{x_0 = \pm} \quad \boxed{x_0 = \pm\frac{3}{\sqrt{2}}} \quad \cancel{-\frac{3}{\sqrt{2}}}$$

$$\boxed{x_0 = \frac{3}{\sqrt{2}}}$$

*** Notes:-**

[1] $E\{\underset{\text{constant}}{a}x\} = \int_{-\infty}^{\infty} a\,x\,f_X(x)\,dx = a\int_{-\infty}^{\infty} f_X(x)\,dx = aE(x)$

[2] $E\{\underset{\text{constant}}{b}\} = \int_{-\infty}^{\infty} b\,f_X(x)\,dx = b\int_{-\infty}^{\infty} \underbrace{f_X(x)\,dx}_{1} = b$

[3] $E\{ax+b\} = ?? = \int_{-\infty}^{\infty} [ax+b]\,f_X(x)\,dx = a\int_{-\infty}^{\infty} f_X(x)\,dx + b\int f_X(x)\,dx$

$= aE(x) + b$

4. $\mathcal{E}\{x^2\} = \int_{-\infty}^{\infty} x^2 f_x(x)\,dx \neq (\mathcal{E}\{x\})^2$

5. $\sigma_x^2 = \mathcal{E}\{(x-\mu_x)^2\} = \mathcal{E}\{x^2 - 2\mu_x x + \mu_x^2\}$

$= \mathcal{E}\{x^2\} - \mathcal{E}\{2\mu_x x\} + \mathcal{E}\{\mu_x^2\}$

$= \mathcal{E}\{x^2\} - 2\mu_x \mathcal{E}\{x\} + \mu_x^2$

$= \mathcal{E}\{x^2\} - 2\mu_x^2 + \mu_x^2$

$\boxed{\sigma_x^2 = \mathcal{E}\{x^2\} - \mu_x^2}$

* Exp 3) let x be a R.V with $\mu_x = 3$ and $\sigma_x^2 = 6$
Determine $\mathcal{E}\{\frac{1}{5}x^2 - 3x + 5\}$?

$\mathcal{E}\{\frac{1}{5}x^2 - 3x + 5\} = \frac{1}{5}\mathcal{E}\{x^2\} - 3\,\cancel{\mathcal{E}\{x\}}^{\,3} + 5$

$\rightarrow \frac{1}{5} \cdot 15 - 9 + 5 = \boxed{-1}$

, $\sigma_x^2 = \mathcal{E}\{x^2\} - \mu_x^2$

$6 = \mathcal{E}\{x^2\} - 9$

$\boxed{\mathcal{E}\{x^2\} = 15}$

* Common Discrete Distribution:-

1. Binomial Distribution:

Experiment repeated for n times (Trails)

* Only two outcomes "S", "f" — معروف كم مرة بتكرر
* P(S) and P(f) do not change — عدد النجاح
* Trails are independent — المسألة كم مرة رح تنجح

X: number of success in the n trails.

$$P(X=x) = \begin{cases} \binom{n}{x} P(S)^x \left[1-P(S)\right]^{n-x} & x = 0, 1, 2, \ldots, n \\ 0 & \text{o.w} \end{cases}$$

$\mu_x = n\,P(S)$, $\sigma_x^2 = n\,P(S)\left[1-P(S)\right]$.

2 Geometric Distribution:

Experiment is repeated

* Only two outcomes "S" and "F". مش معروف كم مرة بتكرر.
* p(S) and P(F) do not change.
* trails are independent

X: number of trails to the first success. (هي بتتبع لكل مرة) (احتمالية النجاح لأول مرة)

$$P(X=x) = P(F)^{x-1}\, P(S)$$

$$\mu_x = \frac{1}{P(S)}, \quad \sigma_x^2 = \frac{1-P(S)}{P(S)^2}.$$

ExP4: let us consider the experiment of tossing the dice.

a what is the probability of getting a number divisible by 3 in 4 trails assuming the dice is tossed for 10 times.

معروف عدد المحاولات

$n=10$, S: divisible by 3, $p(\text{"S"}) = \frac{2}{6} = \frac{1}{3}$, $P(F) = \frac{2}{3}$

X: number of success $\xrightarrow{\{3,6\}}$ in the 10 trails.

$$P(X=x) = \begin{cases} \binom{10}{x}\left(\frac{1}{3}\right)^x\left(\frac{2}{3}\right)^{10-x} & x = 0,1,2,\dots,10 \\ 0 & \text{o.w} \end{cases}$$

$$P(X=4) = \binom{10}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^6 = \frac{10!}{(10-4)!\,4!}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^6 = \boxed{0.227601}.$$

b what is the probability of getting a number divisible by 3 the first time in the fourth trial?

Geo.

X: Success in the 4^{th} Trail

$$FFFS \rightarrow P(X=x) = \begin{cases} P(F)^{x-1}\,P(S) & , x = 0,1,2,\dots \\ 0 & \text{o.w} \end{cases}$$

$$P(X=4) = \left(\frac{2}{3}\right)^{4-1}\left(\frac{1}{3}\right) = \left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right) = 0.098765.$$

[C] Assume the dice is tossed for 8 trials, what is the average number of trials at which a number less than 5 is ~~also~~ observed

S : Success: number less than 5 = $\{1,2,3,4\}$

$$P(S) = 4/6 = 2/3$$

X : number of success in 8 trails

bion

$$\mu_x = n\,P(S) = 8 \cdot \frac{2}{3} = \boxed{16/3}$$

Ber Common Distribution :-

[1] Binomial Distribution.

[2] Geometric Distribution.

→ [3] Hyper-geometric Distribution

$$P(X=x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}, \quad x = 0,1,2,\ldots,\min(K,n)$$

0 o.w

لازم مجموع الرقمين إلي فوق يساووا إلي تحت

Type1 K / Type2 N-K

Type1 x / Type2 n-x

$n < K$

$$\rightarrow P = \frac{K}{N}, \quad \mu_x = nP, \quad \sigma_x^2 = nP(1-P)\left(\frac{N-n}{N-1}\right).$$

Exp :- let us consider an experiment of picking a ball from a box, The box contains 5 red balls and 3 blue balls.

[1] Assume three balls will be picked, what is the probability that only one of them is red?

$$P(X=x) = \frac{\binom{5}{x}\binom{3}{3-x}}{\binom{8}{3}}, \quad x = 0,1,2,\ldots,\min(3,5)$$

0 o.w

$8 = 3 + 5$

$3 = x + 3 - x$

K=5 / N-K=3 , N=8

Type I red / Type II blue

x / n-x

$n < N$

$$P(X=1) = \frac{\binom{5}{1}\binom{3}{2}}{\binom{8}{3}} = \frac{\frac{5!}{4!\,1!}\cdot\frac{3!}{1!\,2!}}{\frac{8!}{5!\,3!}} = \boxed{\frac{15}{56}}$$

X : number of red balls in the 3 selected balls.

2) Assume 5 balls to be selected, what is the probability that at least 2 balls are blue?

red / blue: $N-k=5$ / $k=3$, $N=8$

$5-x$ / x, $n=5$

X: number of blue balls.

$$P(X=x) = \begin{cases} \dfrac{\binom{3}{x}\binom{5}{5-x}}{\binom{8}{5}} & x=0,1,2,\dots,\min(5,3) \\ 0 & \text{o.w} \end{cases}$$

$$P(X \geq 2) = P(x=2) + P(x=3) + P(x=4) + \dots$$
$$= \frac{\binom{3}{2}\binom{5}{3}}{\binom{8}{5}} + \frac{\binom{3}{3}\binom{5}{2}}{\binom{8}{5}} + 0 + 0 \longrightarrow \binom{3}{4} = 0$$

4) Poison Distribution:

$$P(X=x) = \begin{cases} \dfrac{e^{-b}\, b^{x}}{x!} & x=0,1,2,\dots \\ 0 & \text{o.w} \end{cases} \quad , b>0$$

$$, \mu_x = \sigma_x^2 = b$$

Example:- messages arrive to a computer server according to a poisson distribution with a rate of 10 messages/hour

a) What is the probability that 3 messages arrive in 2 hours??

$$b = \underset{\text{rate}}{\lambda} * \underset{\text{period}}{t} = 10 \frac{\text{message}}{\cancel{\text{hour}}} * 2\ \cancel{\text{hours}} = 20 \text{ messages}$$

$$\rightarrow P(X=x) = \begin{cases} \dfrac{e^{-20}(20)^x}{x!} & x=0,1,2,\dots \\ 0 & \text{ow} \end{cases}$$

T_1, T_2, t, $P(x=3)$, $P(x=3)$

$$P(X=3) = \frac{e^{-20}(20)^3}{3!} = \boxed{2.74820 * 10^{-6}}$$

6,728+b3

b) What is the Probability that at least 2 messages arrive in 30 minutes??

$b = \lambda t = 10 \frac{\text{messages}}{\text{houre}} \cdot \cancel{30}^{1} \text{min} \cdot \frac{1 \text{ houre}}{\cancel{60}_{2} \text{min}} = \frac{10}{2}$

$= \boxed{5 \text{ messages}}$

$$p(X=x) = \begin{cases} \dfrac{e^{-5}(5)^x}{x!} & x=0,1,\dots \\ 0 & \text{o.w} \end{cases}$$

$P(X \geq 2) = P(X=2) + P(X=3) + \dots$

$= 1 - p(X<2)$

$= 1 - p(X=0) - p(X=1) = 1 - \dfrac{e^{-5}(5)^0}{0!} - \dfrac{e^{-5}(5)^1}{1!}$

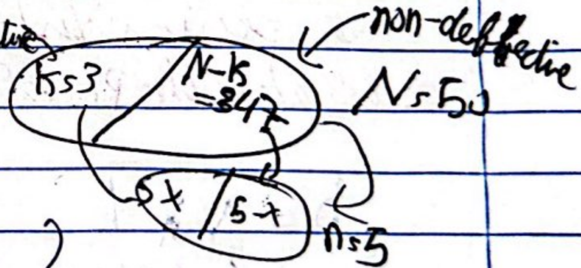
$= \boxed{0.933}$

Exp (3-19) P36:- ع التخيل :

50 electric Motors → ~~there~~ there are 3 defective motors.

5 motors are selected for inspection

X: number of defective motors in the 5 selected



$$P(X=x) = \begin{cases} \dfrac{\binom{3}{x}\binom{47}{5-x}}{\binom{50}{5}} & x=0,1,2,3 \\ 0 & \text{ow} \end{cases}$$

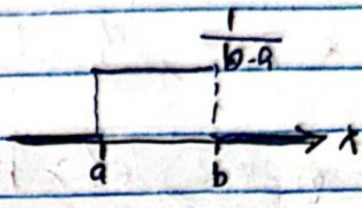
$P(X \geq 1) = 1 - p(X=0)$

$= 1 - \dfrac{\binom{3}{0}\binom{47}{5}}{\binom{50}{5}} = \boxed{0.28}$

Common Continuous Distributions :-

1 Uniform Distribution :-

$$f_X(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & o.w \end{cases}$$



$$\mu_X = \frac{a+b}{2} \qquad \sigma_X^2 = \frac{(b-a)^2}{12}$$

Exp: let x to be a R.V that follow the uniform distribution over the interval [-2, 4].

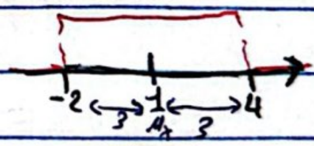
a Write the PdF of x :-

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0 & o.w \end{cases} = \begin{cases} \frac{1}{6}, & -2 \leq x \leq 4 \\ 0 & o.w \end{cases}$$

b Determine $E\{2x^2+3x-5\}$?

$$\rightarrow E\{2x^2+3x-5\} = 2E\{x^2\} + 3E\{x\} - 5$$

$$E\{x\} = \mu_X = \frac{a+b}{2} = \frac{-2+4}{2} = \frac{2}{2} = \boxed{1}$$



$$\sigma_X^2 = E\{x^2\} - \mu_X$$

$$\frac{(b-a)^2}{12} = E\{x^2\} - 1 \rightarrow E\{x^2\} = \frac{(4+2)^2}{12} + 1 = \frac{36}{12} + 1 = \boxed{4}$$

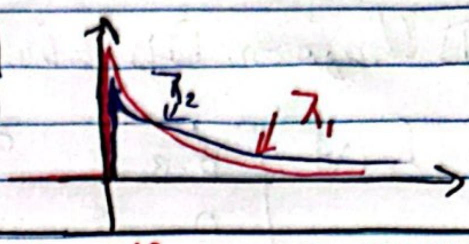
$$\Rightarrow 2E\{x^2\} + 3E\{x\} - 5 \rightarrow 2(4) + 3(1) - 5 = \boxed{6}$$

c $F_X(1.5) = ?$

$$F_X(1.5) = P(x \leq 1.5) = \int_{-2}^{1.5} \frac{1}{6} dx = \frac{x}{6}\Big|_{-2}^{1.5} = \frac{1.5+2}{6} = \frac{3.5}{6} = \ldots$$

2 Exponential Distribution :-

$$f_x(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & o.w \end{cases}$$



* $\lambda = 0$, $\mu_x = \frac{1}{\lambda}$, $\sigma_x^2 = \frac{1}{\lambda^2}$

Example 2) let x be a R.V that follows the exponential distribution with a mean of 0.5

a Write the pdf of x?

$\mu_x = \frac{1}{\lambda} \rightarrow 0.5 = \frac{1}{\lambda} \rightarrow \boxed{\lambda = 2}$

$$\rightarrow f_x(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & o.w. \end{cases}$$

b $p(x \geq 1)$?

$$= \int_1^\infty f_x(x)\,dx = \int_1^\infty 2e^{-2x}\,dx$$

$$= \left. \frac{2e^{-2x}}{-2} \right|_1^\infty = \left. -e^{-2x} \right|_1^\infty = -e^{-\infty} - (-e^{-2}) = \boxed{e^{-2}} = \boxed{\frac{1}{e^2}}$$

c $p(1 \leq x / x \leq 3) = ?$

$$= \frac{p(1 \leq x \cap x \leq 3)}{p(x \leq 3)} = \frac{p(1 \leq x \leq 3)}{p(x \leq 3)}$$

$$= \frac{\int_1^3 2e^{-2x}\,dx}{\int_0^3 2e^{-2x}\,dx} = \frac{\left. -e^{-2x} \right|_1^3}{\left. -e^{-2x} \right|_0^3} = \frac{-e^{-6} + e^{-2}}{-e^{-6} + e^{0}} = \frac{-e^{-6} + e^{-2}}{-e^{-6} + 1}$$

$$= \boxed{\frac{\frac{1}{e^2} - \frac{1}{e^6}}{1 - \frac{1}{e^6}}}$$

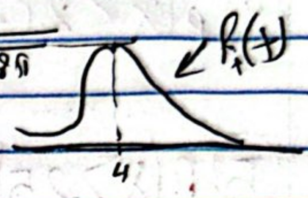
3) Normal (Gaussian) Distribution :-

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \cdot e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} \quad , -\infty < x < \infty$$

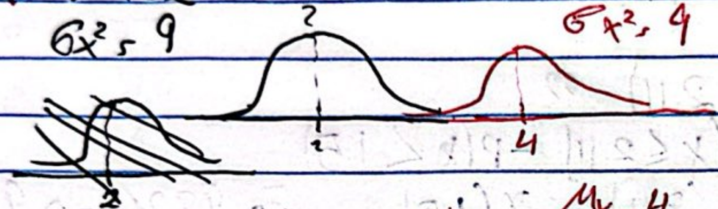
Exp3) : let x be a Gaussian with mean of 4 and standard deviation of 3. write the pdf of x. $(\sigma_x)^2 = (3)^2 \rightarrow \boxed{\sigma_x^2 = 9}$

$$f_x(x) = \frac{1}{\sqrt{2\pi \cdot 9}} e^{-\frac{(x-4)^2}{2 \cdot 9}} \quad -\infty < x < \infty$$

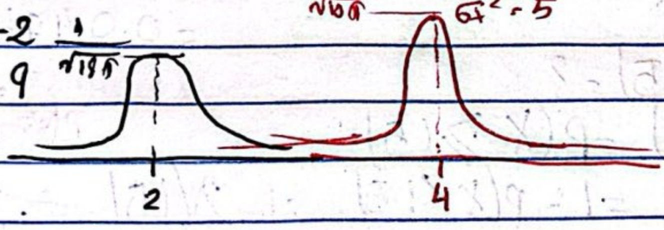
$\frac{1}{\sqrt{18\pi}}$ $f_x(x)$



* $\mu_x = 2$, $\sigma_x^2 = 9$ — $\mu_x = 4$, $\sigma_x^2 = 4$



* $\mu_x = 2$, $\sigma_x^2 = 9$, $\frac{1}{\sqrt{18\pi}}$ — $\frac{1}{\sqrt{10\pi}}$, $\mu_x = 4$, $\sigma_x^2 = 5$



Note!

* For Gaussian Distribution $\boxed{\text{mean} = \text{mode} = \text{median}}$

* Standard Gaussian (Normal) Distribution is a gaussian (normal) distribution ($\mu_x = 0$) and unity var. ($\sigma_x^2 = 1$)

Exp4) let x be a R.V that follows the Normal distribution with $\mu_x = 0$, and $\sigma_x^2 = 1$. Determain the following:-

1) $p(x < 0) = ? = \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = \Phi(0) = 0.5$

من الجدول

$\int_{-\infty}^{?}$ * عشان أستخدم الجدول لازم تكون standard و التكامل

② $p(x < 3.12) = ?$

$= \int_{-\infty}^{3.12} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = \emptyset(3.12) = 0.9991$

standard Gaussian

③ $p(x \leq 2.56) = ?$

$= \emptyset(2.56) = 0.9948$

④ $p(x < 2.12)$

$= p(x \leq 2.119999\ldots9) = \emptyset(2.12) = 0.9830$

⑤ $p(x > 2.5) = 1 - p(x \leq 2.5) = 1 - \emptyset(2.5) = 1 - 0.9938 = 0.0062$

⑥ $p(1.5 \leq x \leq 2.11) = ??$

$= p(x \leq 2.11) - p(x < 1.5)$

$= \emptyset(2.11) - \emptyset(1.5) \quad = 0.9826 - 0.9332$

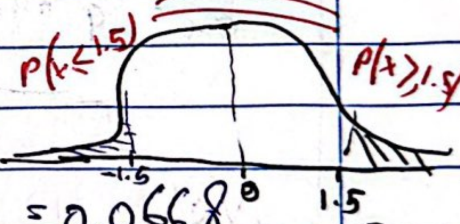
$= 0.0494$ مساحة

⑦ $p(x \leq -1.5) = ?$

$p(x \leq -1.5) = p(x \geq 1.5)$

$= 1 - p(x < 1.5) = 1 - \emptyset(1.5)$

$= 1 - 0.9332 = 0.0668$



*Note

$$\boxed{\emptyset(-a) = 1 - \emptyset(a)}$$

Exp 5 : let X be a Gaussian R.V with mean of 3 and Variance of 64 , Determine the following ?

① $P(X < 7)$?

$$f_X(x) = \frac{1}{\sqrt{2\pi * 64}} e^{\frac{-(x-3)^2}{2*64}} \qquad -\infty < x < \infty$$

not Standard.

$\mu_x = 3$, $\sigma_x^2 = 64$

$$P(X \le 7) = \int_{-\infty}^{7} \frac{1}{\sqrt{2\pi * 64}} e^{\frac{-(x-3)^2}{2*64}} dx$$

$\frac{-(7-3)^2}{2*64}$

$$Z = \frac{x-3}{8}$$

$$dz = \frac{dx}{8} \qquad \boxed{dx = 8dz}$$

$\rightarrow x = 7 \rightarrow Z = \frac{7-3}{8}$

$x = -\infty \rightarrow Z = -\infty$

$$\Rightarrow = \int_{\boxed{-\infty}}^{\frac{7-3}{8}} \frac{1}{\sqrt{2\pi * \cancel{64}}} e^{\frac{-z^2}{2}} \cancel{8} dz = \Phi\left(\frac{7-3}{8}\right)$$

μ_x, σ_x

$$= \Phi\left(\frac{4}{8}\right) = \Phi(0.5) = 0.6915.$$

* Z = Standard Gaussian

② $P(X \le 5) = ?$

$\mu_x = 3$

$\sigma_x^2 = 64$

$$= \Phi\left(\frac{5-3}{\sqrt{64}}\right) = \Phi\left(\frac{2}{8}\right) = \Phi(0.25)$$
$$= 0.5987$$

3 $P(X>11)=?$

$= 1-P(X\le 11)$

$= 1-\Phi\left(\frac{11-3}{8}\right) = 1-\Phi\left(\frac{8}{8}\right) = 1-\Phi(1)$

$= 1-0.8413$

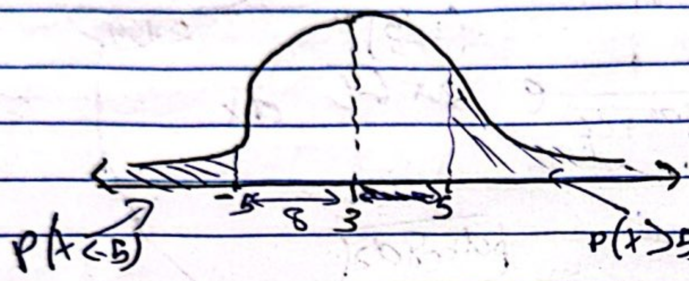
$= 0.1587$

4 $P(X<-5)$??

$P(X<-5) = \Phi\left(\frac{-5-3}{8}\right) = \Phi(-1) = 1-\Phi(1)$

$= 1-0.8413$

$= 0.1587$



5 $P(X<-5 / X<11) = ? \cdot \frac{P(X<-5 \cap X<11)}{P(X<11)}$

$= \frac{P(X<-5)}{P(X<11)} = \frac{\Phi\left(\frac{-5-3}{8}\right)}{\Phi\left(\frac{11-3}{8}\right)} = \frac{\Phi(-1)}{\Phi(1)} = \frac{1-\Phi(1)}{1-0.8413} = \frac{1-0.8413}{1-0.8413} = \boxed{1}$

~~Conti~~ Continuous — Discrete

$\underbrace{\text{PDF} \quad \text{CDF}}_{\text{Continuous}}$ $\underbrace{\text{CDF} \quad \text{PDF}}_{\text{Discrete}}$

PDF: $f_X(x) = \left\{ \quad \right\}$

CDF: $F_X(x) = \begin{cases} 0 & x<0 \\ 0.4 & 0\le x<2 \\ 0.6 & 2\le x<4 \\ 1 & 4\le x \end{cases}$

PDF: $P(X=x) = \begin{cases} \frac{1}{4} & x=1 \\ \frac{1}{4} & x=2 \\ \frac{1}{2} & x=5 \\ 0 & \text{o.w} \end{cases}$

$\sum P(X=x)$

$\sum P(X)$

$f_X(x) = \frac{dF_X(x)}{dx}$, $F_X(x) = \left\{ \right.$ $3+\ldots$

$F_X(x=5) = P(X\le 5) = \int_{-\infty}^{5} f_X(x)\,dx$

* Approximation:-

1 Hyper-geometric $\longrightarrow$ Binomial

$$P(X=x) = \begin{cases} \dfrac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} & , x=0,1,2,\dots,\min(n,K) \\ 0 & \text{o.w} \end{cases} \Rightarrow P(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & , x=0,1,\dots \\ 0 & \text{o.w} \end{cases}$$

$\boxed{p = \frac{K}{N}}$

$\boxed{\frac{n}{N} << 0.1}$

2 Binomial $\longrightarrow$ Poisson

$$P(X=x) = \lim_{n\to\infty} \binom{n}{x} p^x (1-p)^{n-x} \longrightarrow P(X=x) = \frac{e^{-b}\, b^x}{x!}$$

$b = n\,p(S)$

Exp (3-23) P.140: الحل

1000 bits $\qquad$ 10 in error $\Rightarrow X = 10$

$p(\text{error}) = \frac{1}{365}$

$$P(X=10) = \binom{1000}{10}\left(\frac{1}{365}\right)^{10}\left(\frac{364}{365}\right)^{1000-10}$$

$\rightarrow$ approximation using Poisson

$\Rightarrow b = np = \frac{1000}{365}$

$\mu_{\text{Poisson}} = \mu_{\text{Binomial}}$

$$P(X=10) \simeq \frac{e^{-\frac{1000}{365}} \left(\frac{1000}{365}\right)^{10}}{10!} = \boxed{\;\dots\;}$$

$= 6.566 \times 10^{-3}$.

[3] Binomial $\longrightarrow$ Gaussian

$$F_X(x) = P(X \le x) = \sum_{x=0}^{x} \binom{n}{x} p^x (1-p)^{n-x} \longrightarrow P(X \le x) = \emptyset\left(\frac{x-\mu_x}{\sqrt{\sigma_x^2}}\right)$$

$\mu_x = np(s)$

$\sigma_x^2 = np(s)(1-p(s))$

Exp (3-28) P 49

Consider a Binomial experiment with $n = 1000$ and $p(s) = 0.2$

[a] Determine the $P(X \le 240)$?

$$P(X = x) = \begin{cases} \binom{1000}{x} (0.2)^x (0.8)^{1000-x} & x = 0, 1, \ldots, 1000 \\ 0 & O.W \end{cases}$$

$$P(X \le 240) = P(X=0) + P(X=1) + \ldots\ldots + P(X=240) = \sum_{x=0}^{240} \binom{1000}{x} (0.2)^x (0.8)^{1000-x}$$

[b] Use the Gaussian approximation to determine $P(X \le 240)$?

$\mu_x = np(s) = 1000 \cdot 0.2 = 200$, $\sigma_x^2 = np(1-p) = 1000 \times 0.2 \times 0.8 = 160$

$$P(X \le 240) = \emptyset\left(\frac{240-\mu_x}{\sqrt{\sigma_x^2}}\right) = \emptyset\left(\frac{240-200}{\sqrt{160}}\right) = \emptyset(3.16) = 0.999216$$

[4] Poisson $\longrightarrow$ Gaussian

$(\mu_x = \sigma_x^2 = b)$

$$P(X \le x) = \sum_{x=0}^{x} \frac{e^{-b}\, b^x}{x!} \longrightarrow P(X \le x) = \emptyset\left(\frac{x-b}{\sqrt{b}}\right)$$

Exp (3-29) P 50) :-

Poisson , $\mu_x = 1000\ cm^3$

a) $P(X=950)$? $\mu_x = b = \lambda T$, $T = 1\ cm^3$ $= 1000 = \lambda(1) \rightarrow \boxed{\lambda = 1000}$ ~~= 1000(1) = 1000~~

$$P(X=950) = \frac{e^{-1000}(1000)^{950}}{950!}$$

b) $P(X=950$ in $3\ cm^3)$:

$\mu_x = b = \lambda T = 1000 = \lambda \cdot 1 \rightarrow \boxed{\lambda = 1000}$ (T: $1\ cm^3$) ~~1000 = λ·3~~

$\rightarrow b = \lambda T = 1000 \times 3 = 3000 = \mu_x = b$ (T: $3\ cm^3$)

$$P(X=950 \text{ in } 3\ cm^3) = \frac{e^{-3000}(3000)^{950}}{950!}$$

c) Use the Gaussian approximation to estimate the number of probability of having a most particle in $1\ cm^3$.

$P(X \le 950) = \Phi\left(\frac{950-1000}{\sqrt{1000}}\right) =$, $\mu_x = \sigma_x^2 = b = 1000$ $\boxed{=1000}$

Exp: Assume the number of viruses particles in a $1\ cm^3$ of water follow a poisson distribution with a mean of $\boxed{5000}$ viruses in $10\ cm^3$ of water if a $3\ cm^3$ of water are analyzed;

(μ_x = 5000, T = 10 cm³)

a) ~~wat~~ what is the probability that it contains 950 viruses?

$b = \lambda T =$ (T: $3\ cm^3$) , from question $\mu_x = b = \lambda T$

$5000 = \lambda\, 10 \rightarrow \boxed{\lambda = 500 \text{ viruses}}$

$= 500 \frac{\text{viruses}}{cm^3} \cdot 3\ cm^3 = 1500$ viruses

$$P(X = 950) = \frac{e^{-1500}(1500)^{950}}{950!}$$

b) what is the probability that $2cm^3$ of water contains at least 3500 viruses, Use the gaussian to approximate the answer?

$$b = \lambda T = \frac{5000\,\text{vir}}{cm^3} \cdot 2cm^3 = 1000\ \text{vir}$$

$$\mu_T = \sigma_T^2 = b = 1000$$

$$P(X \geq 3500) = 1 - P(X < 3500)$$
$$= 1 - \Phi\left(\frac{3500-1000}{\sqrt{1000}}\right) = \Phi($$

* Transformation of R.V:-

1) Discrete :-

Exp1) let x be a R.V with the following P.M.F:-

$$P(X=x) = \begin{cases} \frac{1}{8} & x=-2 \\ \frac{1}{8} & x=-1 \\ \frac{1}{4} & x=1 \\ \frac{1}{2} & x=3 \\ 0 & \text{o.w} \end{cases}$$

a) let $y = 2x+5$, Determine the PMF of y.

x	P(X=x)	y = 2x+5	P(Y=y)
-2	$\frac{1}{8}$	$y=2(-2)+5 = \boxed{1}$	$\frac{1}{8}$
-1	$\frac{1}{8}$	$y=2(-1)+5 = \boxed{3}$	$\frac{1}{8}$
1	$\frac{1}{4}$	$y=2(1)+5 = \boxed{7}$	$\frac{1}{4}$
3	$\frac{1}{2}$	$y=2(3)+5 = \boxed{11}$	$\frac{1}{2}$

$$P(Y=y) = \begin{cases} 1/8 & , y=1 \\ 1/8 & , y=3 \\ 1/4 & , y=7 \\ 1/2 & , y=11 \\ 0 & , \text{o.w} \end{cases}$$

لما حكي $R=2$ إما $x=+1$ أو $x=-1$ والتنتين موجودات في السؤال وموجودين في R ماخد

فبنجمع $P(x=1) + P(x=-1)$

$= \frac{1}{8} + \frac{1}{4} = \boxed{\frac{3}{8}}$ ✓ صحيح

لما حكي $R=8$ إما $x=+2$ أو $x=-2$

بس $x=+2$ مش موجود في السؤال في $P(x=-2)$ و إما $P(x=2)=0$ فـ $P(x=2)+P(x=-2)$

$0 + \frac{1}{8} = \boxed{\frac{1}{8}}$

b) $R = 2x^2$

x	$P(X=x)$	R	$P(R=r)$
-2	$\frac{1}{8}$	$R=2(-2)^2 = \boxed{8}$	$\frac{1}{8}$
-1	$\frac{1}{8}$	$R=2(\pm1)^2 = \boxed{2}$	$\frac{3}{8}$
1	$\frac{1}{4}$		
3	$\frac{1}{2}$	$R=2(3)^2 = \boxed{18}$	

$$P(R=r) = \begin{cases} \frac{1}{8}, & r=8 \\ \frac{3}{8}, & r=2 \\ \frac{4}{4}\left(\frac{1}{2}\right) = \frac{4}{8}, & r=18 \\ 0, & o.w \end{cases}$$

Ex2) let x be a R.V that follows Binomial Distribution with $n=2$, $P(S)=1/4$. let a new R.V $y=2x+1$ Determine the PMF of y.

$$P(X=x) = \begin{cases} \binom{n}{x} P(s)^x (1-P(s))^{n-x} & x=0,1,2 \\ 0, & o.w \end{cases}$$

$$P(x=0) = \binom{2}{0}\cancel{\left(\frac{1}{4}\right)^0}\left(\frac{3}{4}\right)^2 = \frac{2!}{(2-0)!\,0!} \cdot \frac{9}{16} = \frac{9}{16}$$

$$P(x=1) = \binom{2}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^1 = \frac{2!}{(2-1)!\,1!} \cdot \frac{1}{4} \cdot \frac{3}{4} = \frac{6}{16}$$

$$P(x=2) = \binom{2}{2}\left(\frac{1}{4}\right)^2\cancel{\left(\frac{3}{4}\right)^0} = \frac{2!}{(2-2)!\,2!}\left(\frac{1}{4}\right)^2(1) = \frac{1}{16}$$

مجموعهم 1 = إذا صح

x	$P(x=x)$	$y=2x+1$	$P(y)$
0	$\frac{9}{16}$	$y=2(0)+1 = \boxed{1}$	$\frac{9}{16}$
1	$\frac{6}{16}$	$y=2(1)+1 = \boxed{3}$	$\frac{6}{16}$
2	$\frac{1}{16}$	$y=2(2)+1 = \boxed{5}$	$\frac{1}{16}$

$$P(Y=y) = \begin{cases} \frac{9}{16}, & x=1 \\ \frac{6}{16}, & x=3 \\ \frac{1}{16}, & x=5 \\ 0 & o.w \end{cases}$$

⇒ Continuous :-

Exp 3 ⇒ let x be a R.V that Follows uniform distribution over the interval $[-3, 7]$ ($a = -3$, $b = 7$)

(a) let $y = 2x - 5$. Determine the Pdf of Y.

(1) $f_x(x) = \begin{cases} \frac{1}{b-a} = \frac{1}{7-3} = \frac{1}{10} & , -3 \leq x \leq 7 \\ 0 & \text{o.w} \end{cases}$

(2) $y = 2x - 5$

$y + 5 = 2x$

$x = \frac{y+5}{2}$

(3) $\frac{dy}{dx} = 2$

(4) $f_y(y) = \left. \frac{f_x(x)}{\left|\frac{dy}{dx}\right|} \right|_{x = g(y)}$

(4a) $x < -3 \to y < -11$

$\times 2$: $2x < -6$

-5: $2x - 5 < -11$

$\Rightarrow f_y(y) = \frac{0}{2} = 0$

(4B) $-3 \leq x \leq 7 \longrightarrow -11 \leq y \leq 9$

$\times 2$: $-6 \leq 2x \leq 14$

-5: $-11 \leq 2x - 5 \leq 9$

(4C) $7 < x \to 9 < y$

$f_y(y) = 0$

$f_y(y) = \frac{\frac{1}{10}}{|2|} = \boxed{\frac{1}{20}}$

$f_y(y) = \begin{cases} 1/20 & , -11 \leq x \leq 9 \\ 0 & \text{o.w} \end{cases}$ uniform ($a = -11$, $b = 9$)

$\frac{1}{b-a} = \frac{1}{9-11} = \boxed{\frac{1}{20}}$

(b) let $U = X^2$. Determine the pdf of U?

① $f_X(x) = \begin{cases} \frac{1}{b-a} = \frac{1}{7+3} = \frac{1}{10}, & -3 \le x \le 7 \\ 0 & o.w \end{cases}$

② $U = X^2 \rightarrow x = \pm\sqrt{u}$

③ $\frac{du}{dx} = 2x$

④ $f_U(u) = \left.\frac{f_X(x)}{\left|\frac{du}{dx}\right|}\right|_{x=\sqrt{u}} + \left.\frac{f_X(x)}{\left|\frac{du}{dx}\right|}\right|_{x=-\sqrt{u}}$

4a $x < -7$ and $x > 7 \rightarrow u > 49$

$f_U(u) = 0 + 0 = 0$

4b $-7 < x < -3$ and $3 < x < 7 \rightarrow 9 < u < 49$

$f_U(u) = \left.\frac{\frac{1}{10}}{|2x|}\right|_{x=\sqrt{u}} + 0 = \frac{1}{20\sqrt{u}}$

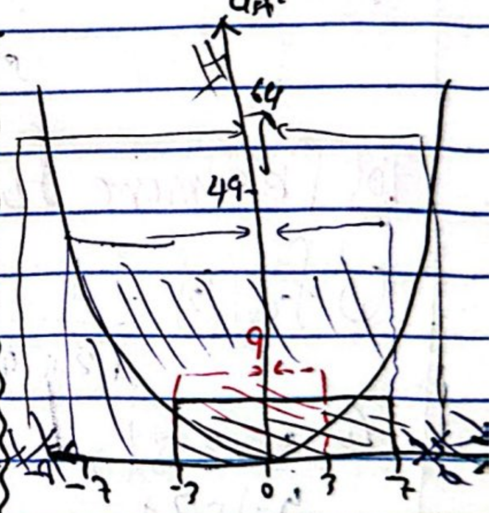
$u = 4 \rightarrow x = 2 = \sqrt{4}$, $x = -2 = -\sqrt{4}$

4c $-3 < x < 3$, $0 < u < 9$

$x \ne 0$

$f_U(u) = \left.\frac{\frac{1}{10}}{|2x|}\right|_{x=\sqrt{u}} + \left.\frac{\frac{1}{10}}{|2x|}\right|_{x=-\sqrt{u}}$

$= \frac{1}{20\sqrt{u}} + \frac{1}{20\sqrt{u}} = \frac{2}{20\sqrt{u}}$



$f_U(u) = \begin{cases} \frac{2}{20\sqrt{u}} & 0 < u < 9 \\ \frac{1}{20\sqrt{u}} & 9 < u < 49 \\ 0 & o.w \end{cases}$

Ex 4: let x be a Gaussian R.V with $\mu_x = 2$, $\sigma_x^2 = 9$
let $y = 2x + 1$ Determine

a) μ_y, σ_y^2?

$\mu_y = E\{y\} = E\{2x+1\} = 2\mu_x + 1 = 2(2)+1 = \boxed{5}$

$\sigma_y^2 = 4\sigma_x^2 = 4 \cdot 9 = 36$

Note: $y = ag + b$

$E\{y\} = a\,E\{g\} + b = a\mu_g + b$

$Var\{y\} = E\{(y-\mu_y)^2\} = E\{(ax+b-a\mu_x-b)^2\} = E\{(ax-a\mu_x)^2\}$

$= a^2 E\{(x-\mu_x)^2\} = \boxed{a\,\sigma_x^2}$

b) Determine $f_y(y)$?

① $f_y(y) = \frac{1}{\sqrt{2\pi \cdot 9}} e^{-\frac{(x-2)^2}{2*9}}$ $\quad -\infty < x < \infty$

② $y = 2x+1$
$y - 1 = 2x$
$\boxed{x = \frac{y-1}{2}}$

③ $\frac{dy}{dx} = 2$

④ $f_y(y) = f(y) = \left.\frac{f_x(x)}{\left|\frac{dy}{dx}\right|}\right|_{x=\frac{y-1}{2}}$

$= \frac{\frac{1}{\sqrt{2\pi*9}} e^{-\frac{\left(\frac{y-1}{2}-2\right)^2}{2*9}}}{2}$

$= \frac{1}{\sqrt{2\pi*9*4}} e^{-\frac{(y-5)^2}{2*9*4}}$ $\quad -\infty, y, \infty$

$= \frac{1}{\sqrt{2\pi*36}} e^{-\frac{(y-5)^2}{2*36}}$ $\quad -\infty, y < \infty$

Chapter 3 :- Probability Distribution for more than One Random variable.

$P(X=x, Y=y, R=r \dots)$

$P(X=x, Y=y)$

↳ joint - PMF

Exe: let X and Y be two R.V.s with the following Joint-PMF:

$$P(X=x) = \begin{cases} \frac{1}{8} & , x=-1, y=0 \\ \frac{1}{8} & , x=-1, y=1 \\ \frac{1}{4} & , x=0, y=0 \\ 1/8 & , x=1, y=0 \\ \cancel{K}\ 3/8 & , x=1, y=2 \\ 0 & , o.w \end{cases}$$

[a] Determine the value of constant K?

$$\sum_{y=-\infty}^{\infty} \sum_{x=-\infty}^{\infty} P(X=x, Y=y) = 1$$

$P(X=-1, Y=0) + P(X=-1, Y=1) + \text{---} \dots = 1$

$\frac{1}{8} + \frac{1}{8} + \frac{1}{4} + \frac{1}{8} + K = 1 \rightarrow \frac{5}{8} + K = 1 \rightarrow K = 1 - \frac{5}{8}$

$\boxed{K = \frac{3}{8}}$

~~[b] P(X = 1, y = 0~~

[b] $P(X \leq 0, Y \leq 0) = ?$

$= P(X=-1, Y=0) + P(X=0, Y=0)$

$= \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$

$y \leq 0$, $x \leq 0$

$x \backslash y$	0	1	2
-1	$\frac{1}{8}$	$\frac{1}{8}$	0
0	$\frac{1}{4}$	0	0
1	$\frac{1}{8}$	0	$\frac{3}{8}$

o.w = [illegible] صفر

$y \geq 1$ (مش موجودة فوقه)

[illegible]

[C] $P(X \leq 0, Y \geq 1) = ?$

$P((X=-1), (Y=1)) = \frac{1}{8}$

$x \leq 0$

$x \backslash y$	0	1	2
-1	$\frac{1}{8}$	$\frac{1}{8}$	0
0	$\frac{1}{4}$	0	0
1	$\frac{1}{8}$	0	$\frac{3}{8}$

* properties of Joint-PMF

1. $P(X=x, Y=y) \geq 0$
2. $\sum_{x=-\infty}^{\infty}\sum_{y=-\infty}^{\infty} P(X=x, Y=y) = 1$

- d
- e
- f
- g

x \ y	0	1	2
-1	$\frac{1}{8}$	$\frac{1}{8}$	0
0	$\frac{1}{4}$	0	0
1	$\frac{1}{8}$	0	$\frac{3}{8}$

$\frac{4\,2\,1}{}$ $\frac{1}{8}, \frac{3}{8}$

d ~~P(X ≤ 0, Y ≥ 1)~~ $P(X \leq 0 / Y \geq 1) = ?$

$$P(X \leq 0 / Y \geq 1) = \frac{P(X \leq 0, Y \geq 1)}{P(Y \geq 1)} = \frac{\frac{1}{8}}{\frac{4}{8}} = \boxed{\frac{1}{4}}$$

e $P(X > 0) = ?$

$$\frac{1}{8} + \frac{3}{8} = \boxed{\frac{4}{8}}$$

f $F_{XY}(0,1) = ?$

→ joint CDF of x, y

$$= P(X \leq 0, Y \leq 1) = \frac{1}{8} + \frac{1}{8} + \frac{1}{4} = \frac{4}{8} = \boxed{\frac{1}{2}}$$

g $F_{YX}(0,1)$

$$= P(Y \leq 0, X \leq 1) = \frac{1}{8} + \frac{1}{4} + \frac{1}{8} = \boxed{\frac{1}{2}}$$

H $F_{X,Y}(3,-2) = ?$

$$P(\underbrace{X \leq 3}_{\text{كل قيم x بالجدول}}, \underbrace{Y \leq -2}_{\text{فش قيم لـ y بالجدول}}) = \boxed{0}$$

I $F_{X,Y}(3,3) = ?$

$$P(\underbrace{X \leq 3}_{\text{كل قيم x}}, \underbrace{Y \leq 3}_{\text{كل قيم y}}) = 1$$

كل الجدول

J $F_{X,Y}(-3,-2)$

$$P(X \leq -3, Y \leq -2) = 0$$

K $F_{X,Y}(-\infty,-\infty)$

$= P(X \leq -\infty, Y \leq -\infty) = \boxed{0}$

L $F_{X,Y}(\infty,\infty)$

$= P(X \leq \infty, Y \leq \infty) = \boxed{1}$

M $F_{X,Y}(\infty,-\infty) = ?$

$F_{X,Y}(+\infty,-\infty) = P(X \leq \infty, Y \leq -\infty) = 0$

N Determine the marginal PMF of X ?

$\rightarrow P(X=-1) = P(\underline{X=-1}, \underbrace{-\infty < y < \infty}_{\text{كلهم}})$

$$= \sum_{y=-\infty}^{\infty} P(X=-1, Y=y)$$

$$= P(X=-1, Y=0) + P(X=-1, Y=1)$$

$$= \frac{1}{8} + \frac{1}{8} = \boxed{\frac{1}{4}}$$

$\rightarrow P(X=0)$

$$= \sum_{y=-\infty}^{\infty} P(X=0, Y=y) = \boxed{\frac{1}{4}}$$

$\rightarrow P(X=1)$

$$= \sum_{y=-\infty}^{\infty} P(X=1, Y=y) = \frac{1}{8} + \frac{3}{8} = \frac{4}{8} = \boxed{\frac{1}{2}}$$

So $P(X=x) = \begin{cases} \frac{1}{4} & x=-1 \\ \frac{1}{4} & x=0 \\ \frac{1}{2} & x=1 \\ 0 & \text{o.w} \end{cases}$

o) PMF of Y = ?

$p(Y=3) = \sum_{x=-\infty}^{\infty} p(Y=3, X=x) = 0$

~~$p(Y=2)$~~

$p(Y=2) = \sum_{x=-\infty}^{\infty} p(Y=2, X=x) = \boxed{\frac{3}{8}}$

$p(Y=1) = \sum_{x=-\infty}^{\infty} p(Y=1, X=x) = \boxed{\frac{1}{8}}$

$p(Y=0) = \sum_{x=-\infty}^{\infty} (p(Y=0, X=x) = \frac{1}{8} + \frac{1}{4} + \frac{1}{8} = \boxed{\frac{4}{8}}$

$$\text{So } p(Y=y) = \begin{cases} 4/8 & , y=0 \\ 1/8 & , y=1 \\ 3/8 & , y=2 \\ 0 & , y=3 \\ 0 & \text{o.w} \end{cases}$$

» Note: X and Y are said to be stat. Independent if: $p(X=x, Y=y) = p(X=x)\,p(Y=y)$.

p) Are x and y S. Independent ??

- $p(X=-1, Y=0) \overset{?}{=} p(X=-1)\,p(Y=0)$

$\frac{1}{8} \overset{?}{=} \frac{1}{4} * \frac{4}{8} = \frac{1}{8}$

- $p(X=-1, Y=1) \overset{?}{=} p(X=-1)\,p(Y=1)$

$\frac{1}{8} \overset{?}{=} \frac{1}{4} \cdot \frac{1}{8}$

$\frac{1}{8} \neq \frac{1}{32}$

So x, y are not Statistically Independent.

Q $p(X=-1 / Y=+1) = ??$

$$= \frac{p(X=-1, Y=1)}{p(Y=1)} = \frac{1/8}{1/8} = 1$$

R $p(\underbrace{X \geq 0}_{\text{ش1}} / \underbrace{Y \leq 0, X \leq 0}_{\text{ش2}}) = ??$

X=0 فقط عندي غيره

$$\rightarrow \frac{p(X \geq 0, X \leq 0, Y \leq 0)}{p(Y \leq 0, X \leq 0)} = \frac{P(X=0, Y<0)}{p(Y \leq 0, X<0)} = \frac{\frac{1}{4}}{\frac{1}{8}+\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{8}}$$

$$= \frac{\frac{2}{8}}{\frac{3}{8}} = \boxed{\frac{2}{3}}$$

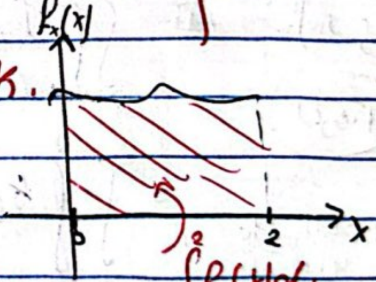
~~Continuous~~

Exp:- let X and Y be two R.Vs with the following Joint P.d

$$f_{x,y}(x,y) = \begin{cases} K & 0 \leq x \leq 1, 0 \leq y \leq 3 \\ 0 & o.w \end{cases}$$

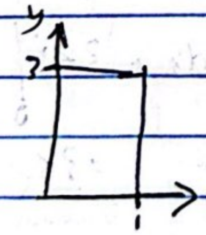
a Determine the value of the constant K.

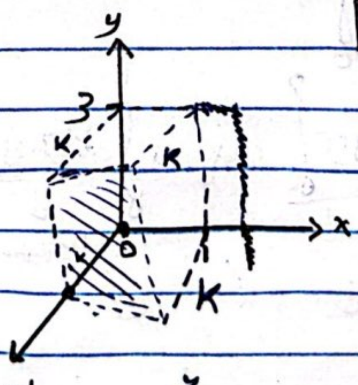
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{x,y}(x,y)\, dy\, dx = 1$$



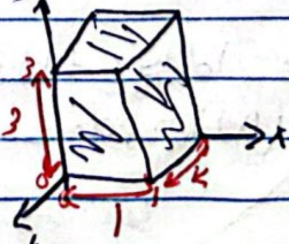
$\int_0^2 f_x(t)dt$

هاد الي افتكرناه قبل لما نكون 1R.V.





$f_{x,y}(x,y)$



$f_{x,y}(x,y)$

اذا شكل مستطيل

بقدر استخدم الارتفاع ثابت

$V = \text{الطول} \times \text{العرض} \times \text{الارتفاع}$

$= 1 \times 3 \times K$

$= 3K$

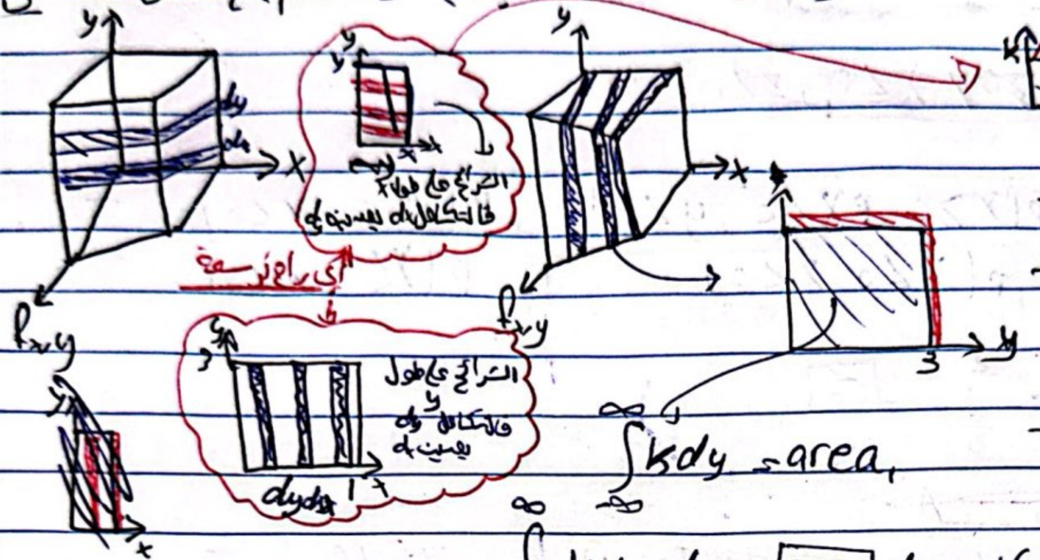
ثابتة $V = 1$

$1 = 3K \rightarrow \boxed{K = 1/3}$

$$\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f_{xy}(x,y)\,dy\right]dx = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f_{xy}(x,y)\,dx\right]dy$$

التكامل الاول — التكامل الاول

لو كاملت dy بعدين dx او dx بعدين dy رح يطلع نفس الشي .



$\int_{-\infty}^{\infty} k\,dx = \text{area}_2$

$\int_{-\infty}^{\infty}\int k\,dx\,.dy = \text{area}_2\,.dy = \text{Volume slice}$

$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k\,dx\,dy = \text{Volume}$ كامل

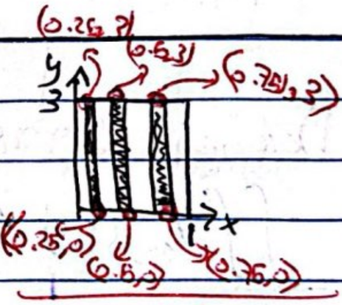
$\int_{-\infty}^{\infty} k\,dy = \text{area}_1$

$\int_{-\infty}^{\infty} k\,dy\,dx = \boxed{\text{area}}\,dx = \text{Volume of Slice}$

$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k\,dy\,dx = \text{Volume}$ كامل

حل السؤال ! 1 (الطريقة 1)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{xy}(x,y)\,dy\,dx$$



بنشتغل بالتكامل بالنسبة لل y بعدين x

$$= \int_0^1\int_0^3 k\,dy\,dx$$

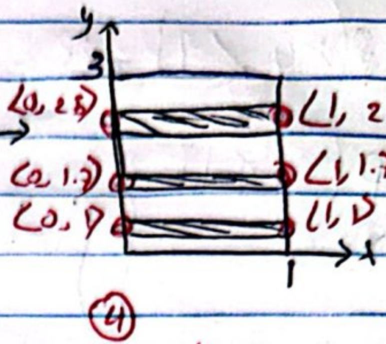
$$\int_0^1 \underbrace{ky\Big|_0^3}_{\text{تكامل بالنسبة لل y (1)}} dx \to \int_0^1 \underbrace{k\{3-0\}}_{\text{تعويض الحدود لل y (2)}} dx = \underbrace{\int_0^1 3k\,dx = 3kx\Big|_0^1}_{\text{عوضنا الحدود والحسبة لازم، تكاملنا بالنسبة لل x (3)}} = 3k\{1-0\}$$

$= 3k = 1 \to \boxed{k = \frac{1}{3}}$ بعد تعويض حدود تكامل x (5)

* ال Slice عمودي يعني اول اشي (تكامل y) dy بعدين (تكامل x) dx

(٢ مهم طريقة)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{x,y}(x,y)\, dx\, dy$$



بيكامل بالنسبة لـ x وبعدين لـ y

$$\int_0^3\int_0^1 k\,dx\,dy$$

$$\int_0^3 \underbrace{kx\Big|_0^1}_{\substack{\text{تكامل بالنسبة لـ } x \\ ①}} dy \rightarrow \int_0^3 \underbrace{k[1-0]}_{\substack{\text{عوضنا حدود } x \\ ②}} dy = \underbrace{\int_0^3 1k\,dy}_{\substack{\text{عوضنا الحدود والحين لازم تكامل بالنسبة لـ } y \\ ③}} = \overbrace{ky}^{\text{④}}\Big|_0^3 = \underbrace{k[3-0]}_{\substack{\text{نعوض حدود } y \\ ⑤}}$$

$$= \overbrace{3k}^{⑥} = 1$$

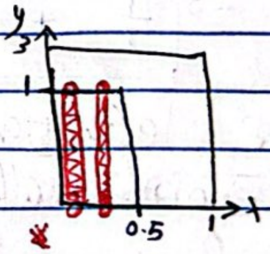
$$\boxed{k = \frac{1}{3}}$$

* الـ Slices افقي يعني اول اشي تكامل dx (x) ~~[illegible]~~ بعدين dy (التكامل).

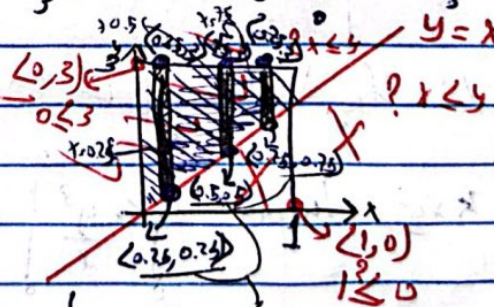
b $P(0 \le x \le 0.5,\ 0 \le y \le 1) = ?$

$$= \int_0^{0.5}\int_0^1 \frac{1}{3}\,dy\,dx = \int_0^{0.5} \frac{1}{3}y\Big|_0^1 dx$$

$$= \int_0^{0.5} \frac{1}{3}y\,dx = \frac{1}{3}x\Big|_0^{0.5} = \frac{1}{3}\cdot 0.5 = \boxed{1/6}$$



c $P(x \le y) = ?$



? x < y

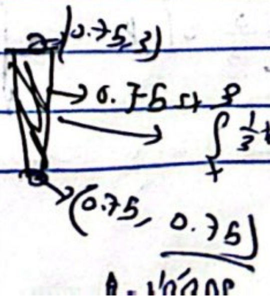
برسم قطعة وبحدد الجزء ابعرف اي قسم بحقق الشرط عن طريق عدة نقاط ويشوف متى بتحقق الشرط.

$$= \int_0^1\left[\int_x^3 \frac{1}{3}\,dy\right]dx$$

$$= \int_0^1 \frac{1}{3}y\Big|_x^3 dx = \int_0^1 \frac{1}{3}(3-x)\,dx$$

$$= \frac{1}{3}\left(3x - \frac{x^2}{2}\right)\Big|_0^1$$

$$= \frac{1}{3}\left[\left(3-\frac{1}{2}\right)-(0-0)\right] = \frac{1}{3}\cdot\frac{5}{2} = \boxed{\frac{5}{6}}$$



$0.75 < x < 3$

$$\int_x^3 \frac{1}{3}\,dy = \frac{1}{3}y\Big|_x^3$$

area $= \frac{1}{3}(3-x)$

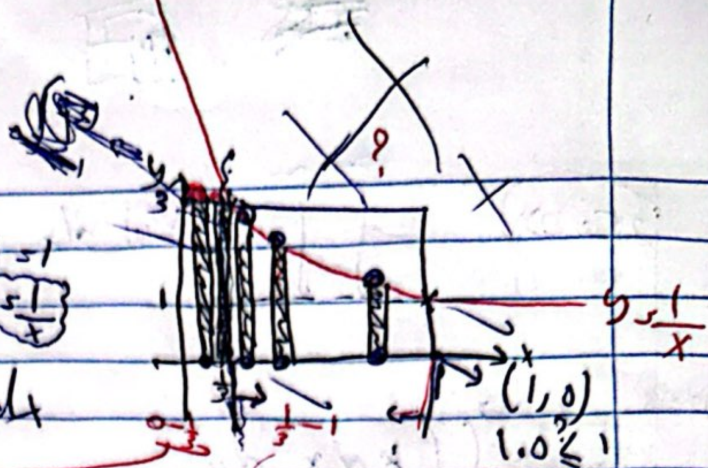
d $P(xy < 1) = ??$

$xy = 1$

$y = \frac{1}{x}$

$P(xy \leq 1)$

$= \iint \frac{1}{3}\, dy\, dx$



* نقطة التقاطع مع الـ $\frac{1}{3}$ بتحكي من $(\frac{1}{3} - 1)$

طبقة الحل ؟

$$\rightarrow \int_0^{\frac{1}{3}}\int_0^3 \frac{1}{3}\, dy\, dx + \int_{\frac{1}{3}}^{1}\int_0^{\frac{1}{x}} \frac{1}{3}\, dy\, dx$$

$$\int_0^{\frac{1}{3}} \frac{1}{3} y \Big|_0^3 \qquad\qquad \int_{\frac{1}{3}}^{1} \frac{1}{3} y \Big|_0^{\frac{1}{x}} dx$$

$$\int_0^{\frac{1}{3}} 1\, dx \qquad\qquad \int_{\frac{1}{3}}^{1} \frac{1}{3x}\, dx = \cancel{\frac{1}{3}\ln x \Big|_0} = \frac{1}{3}\ln x \Big|_{\frac{1}{3}}^{1}$$

$$= \frac{1}{3} \qquad\qquad = \frac{1}{3}\left[\ln x \Big|_{\frac{1}{3}}^{1}\right]$$

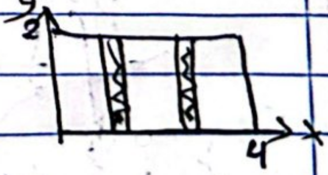
$$= \frac{1}{3}\ln 3$$

$$\rightarrow \boxed{\frac{1}{3} + \frac{1}{3}\ln 3}$$

Ex: let x and y be two R.Vs with the following Joint PDF:

$$f_{xy}(x,y) = \begin{cases} Kxy^2 & 0 < x < 4,\ 0 < y < 2 \\ 0 & \text{O.W} \end{cases}$$

a Determine the value of the constant K?



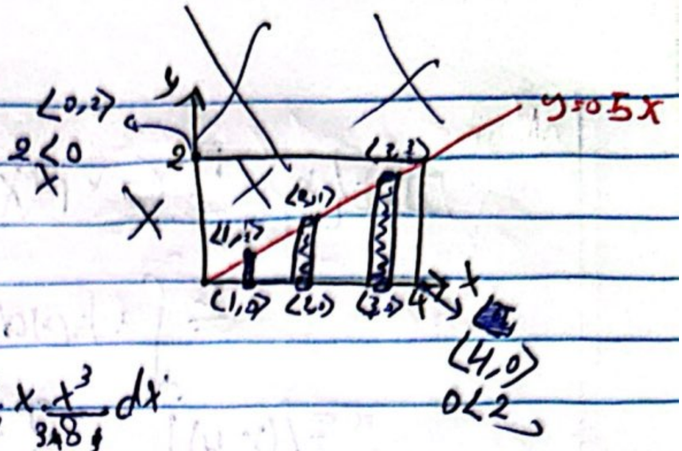
$$\int_0^4 \left[\int_0^2 Kxy^2\, dy\right] dx = \int_0^4 Kx \frac{y^3}{3}\Big|_0^2 dx$$

$$= \int_0^4 Kx\left[\frac{8}{3} - 0\right] dx = \int_0^4 \frac{8}{3} Kx\, dx = \left[\frac{8}{3} K \frac{x^2}{2}\right]_0^4$$

$$= \frac{8}{3} K\left[\frac{16}{2} - 0\right] \rightarrow \frac{8}{3} K \cdot 8 = 1 \rightarrow \frac{64}{3} K = 1$$

$$\boxed{K = 3/64}$$

b $P(Y < 0.5x) = ??$

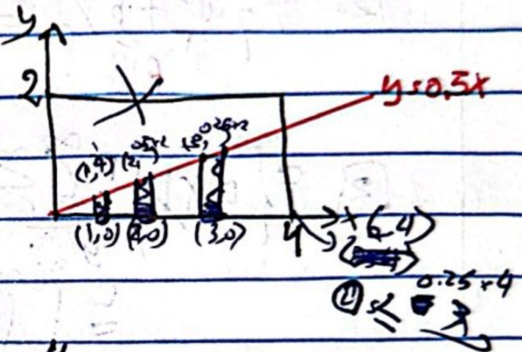


$$\int_0^4 \left[\int_0^{0.5x} \frac{3}{64} x y^2 \, dy\right] dx$$

$$= \int_0^4 \frac{3}{64} x \frac{y^3}{3} \Big|_0^{0.5x \to \frac{x}{2}} dx = \int_0^4 \frac{3}{64} x \frac{x^3}{3 \cdot 8} dx$$

$$= \int_0^4 \frac{3}{64} \frac{x^4}{8 \cdot 3} dx = \int_0^4 \frac{x^4}{64 \cdot 8} dx = \frac{x^5}{64 \cdot 8 \cdot 5} \Big|_0^4 = \frac{16 \cdot 16 \cdot 4}{64 \cdot 8 \cdot 5} = \boxed{\frac{2}{5}}$$

c $P(Y \leq 0.25x) = ??$

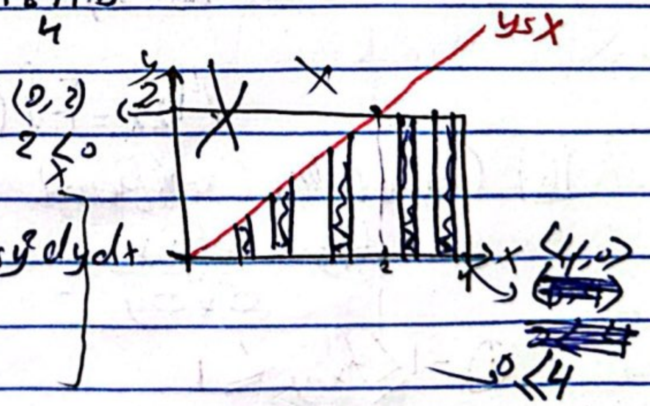


$$\int_0^4 \left[\int_0^{0.25x} \frac{3}{64} x y^2 \, dy\right] dx$$

$$\int_0^4 \frac{3}{64} x \frac{y^3}{3} \Big|_0^{0.25x} dx = \int_0^4 \frac{x \left(\frac{x}{4}\right)^3}{64} dx = \int_0^4 \frac{x^4}{64 \cdot 64} dx$$

$$= \frac{x^5}{64 \cdot 64 \cdot 5} \Big|_0^4 = \frac{4 \cdot 64 \cdot 16}{64 \cdot 64 \cdot 5} = \boxed{\frac{1}{20}}$$

d $P(Y \leq x) = ?$



$$\int_0^2 \left[\int_0^x \frac{3}{64} x y^2 \, dy\right] dx + \int_2^4 \left[\int_0^2 \frac{3}{64} x y^2 \, dy\right] dx$$

$$\frac{1}{2 \cdot 10} + \frac{5 \cdot 3}{3 \cdot 4} = \frac{2}{20} + \frac{15}{20} = \boxed{\frac{17}{20}}$$

Notes:-

1. ① $E\{X\} \overset{D}{=} \sum_{x=-\infty}^{\infty} x\, P(X=x)$ ② $E\{g(x)\} \overset{D}{=} \sum_{-\infty}^{\infty} g(x)\, P(X=x)$

$\overset{C}{=} \int_{-\infty}^{\infty} x\, f_X(x)\, dx$ $\overset{C}{=} \int_{-\infty}^{\infty} g(x)\, f_X(x)\, dx$

③ $E\{(X,Y)\} \overset{D}{=} \sum_{y=-\infty}^{\infty}\sum_{x=-\infty}^{\infty} g(x,y)\, P(X=x, Y=y).$

$\overset{C}{=} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\, f_{x,y}(x,y)\, dy\, dx$

2. $E\{aX+bY\} = aE\{X\} + b\{Y\}$ — R.V's

3. $E\{aXY\} \overset{D}{=} \sum_{y=-\infty}^{\infty}\sum_{x=-\infty}^{\infty} axy\, P(X=x, Y=y)$

$\overset{C}{=} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} axy\, f_{xy}(x,y)\, dy\, dx$

4. if X and Y are statistically independent $\rightarrow E\{aXY\} = aE\{X\}E\{Y\}$

5. Covariance μ_{xy}

$\rightarrow \mu_{xy} = E\{(X-\mu_x)(Y-\mu_y)\}$

$\mu_{xx} = E\{(X-\mu_x)(X-\mu_x)\} = E\{(X-\mu_x)^2\} = \sigma_x^2$

6. Correlation Coefficient

$\rho_{xy} = \dfrac{\mu_{xy}}{\sigma_x \sigma_y}$

① $-1 \leq \rho_{x,y} \leq 1$

② $\rho_{xy} = 0 \longleftarrow$ X and Y are uncorrelated

③ $\rho_{xy} = \pm 1 \longleftarrow$ X and Y are fully correlated

7) * if x and y are statistically independent
then x and y are uncorrelated

* if x and y are uncorrelated then they may or may not be S. independent.

8) $\mu_{xy} = E\{(x-\mu_x)(y-\mu_y)\} = E\{x\,y\} - \mu_x \mu_y$

9) * marginal pdf of x $\rightarrow f_x(x) = \int_{-\infty}^{\infty} f_{x,y}(x,y)\,dy$

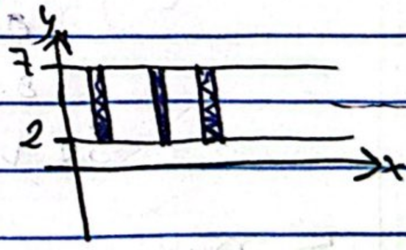
* marginal pdf of fy $\rightarrow f_y(y) = \int_{-\infty}^{\infty} f_{x,y}(x,y)\,dx$

10) x and y will be statistically independent if $f_{x,y}(x,y) = f_x(x)\,f_y(y)$

Exp: let x and y be two R.Vs with the following joint-pdf:

$$f_{x,y}(x,y) = \begin{cases} k e^{-0.2x} & 0 \le x;\ 2 \le y \le 7 \\ 0 & o.w \end{cases}$$

a) find k:



$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{x,y}(x,y)\,dy\,dx = 1$$

$$\rightarrow \int_0^{\infty}\left[\int_2^{7} k e^{-0.2x}\,dy\right]dx = \int_0^{\infty} k e^{-0.2x}\,y\Big|_2^7\,dx = 5k\int_0^{\infty} e^{-0.2x}\,dx$$

$$= \frac{5k\,e^{-0.2x}}{-0.2}\Bigg|_0^{\infty} = \frac{5k}{+0.2}\left[\overset{e^{-\infty}}{0} - \overset{e^{0}}{1}\right]_{+1} = 25k = 1 \rightarrow k = \frac{1}{25} = \boxed{0.04}$$

b) Are X and Y statistically independent??

$$f_{x,y}(x,y) = f_x(x)\, f_y(y)$$

$$\to f_x(x) = \int_{-\infty}^{\infty} f_{x,y}(x,y)\,dy = \int_2^7 0.04 e^{-0.2x}\,dy = 0.04 e^{-0.2x}\, y \Big|_2^7 = \boxed{0.2e^{-0.2x}}$$

exponential

$\lambda = 0.2$

↳ $\mu_x = \frac{1}{\lambda} = \frac{1}{0.2} = \boxed{5}$

$\sigma_x^2 = \frac{1}{\lambda^2} = \frac{1}{(0.2)^2} = \boxed{25}$

$$f_x(x) = \begin{cases} 0.2e^{-0.2x} & 0 < x \\ 0 & o.w \end{cases}$$

$$\to f_y(y) = \int_{-\infty}^{\infty} f_{x,y}(x,y)\,dx = \int_0^{\infty} 0.04 e^{-0.2x}\,dx = \frac{0.04e^{-0.2x}}{-0.2}\Big|_0^{\infty}$$

uniform $a = 2,\ b = 7$

$$= -0.2\,[e^{-\infty} - e^{0}] = -0.2\,[0 - 1] = \boxed{0.2}$$

$\to \frac{1}{b-a} = \frac{1}{5} = 0.2$

↳ $\mu_y = \frac{a+b}{2} = \frac{9}{2} = \boxed{4.5}$

$\sigma_y^2 = \frac{(b-a)^2}{12} = \boxed{\frac{25}{12}}$

$$f_y(y) = \begin{cases} 0.2 & 2 < y < 7 \\ 0 & o.w \end{cases}$$

$$\Longrightarrow f_{x,y}(x,y) \stackrel{?}{=} f_x(x)\, f_y(y)$$

$$0.4e^{-0.2} \stackrel{?}{=} 0.2e^{-0.2x} \cdot 0.2$$

$$0.4e^{-0.2x} = 0.4e^{-0.2x}$$

So yes, X and Y are S. independent.

c) Determine the correlation coefficient between X and Y.

$\rho_{xy} = \frac{\mu_{x,y}}{\sigma_x \sigma_y} = 0$ because X and Y are S. independent

الرجع إلي قبل

d) $E\{3xy\}$

$= 3E\{x\}E\{y\} = 3 * 5 * 4.5 = 67.5$

because X and Y are S. Indep.

Note: $R = ax + by$

$\rightarrow \mu_R = E\{R\} = a\mu_x + b\mu_y$

$\rightarrow \sigma_R^2 = Var\{R\} = a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab\sigma_x\sigma_y \rho_{x,y}$

$$= \boxed{a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab\,\mu_{x,y}}$$

e) $R = 2x - 3y$, Determine $Var\{R\}$??

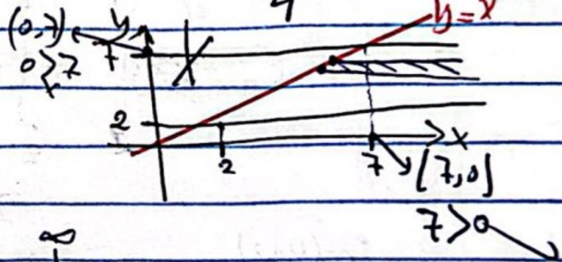
$\sigma_R^2 = (2)^2 * \sigma_x^2 + (-3)^2\sigma_y^2 + \cancel{2(+2)(-3)\sigma_x\sigma_y \underbrace{\rho_{x,y}}_{0}}$

0, a, bare S.I.

$= 4 * 25 + 9 * \frac{25}{12}$

$\rightarrow = 100 + \frac{75}{4} = \boxed{118.75}$

f) $P(x > y)$



$$\int_2^7 \int_0^\infty 0.04 e^{-0.2x}\, dx\, dy$$

$$= \int_2^7 \left. \frac{0.04}{-0.2} e^{-0.2x} \right|_y^\infty dy$$

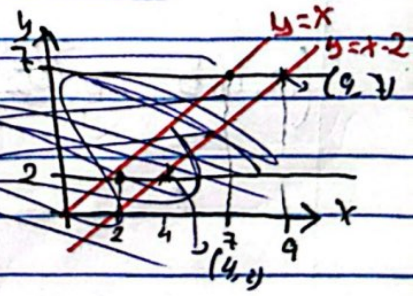
$$= \int_2^7 +0.2\left[0 + e^{-0.2y}\right] dy \rightarrow \int_2^7 0.2 e^{-0.2y}\, dy = \left. \frac{0.2 e^{-0.2y}}{-0.2} \right|_2^7$$

$$= -e^{-1.4} - (-e^{-0.4}) = e^{-0.4} - e^{-1.4} = \boxed{0.4237}$$

~~**g)** $P(y < x < y + 2)$??~~

~~$x = y + 2$~~
~~$y = x - 2$~~
~~$x = 9 \rightarrow y = 7$~~
~~$x = 4 \rightarrow y = 2$~~

~~$$\int_2^7 \int_y^{y+2} 0.4 e^{-0.2x}\, dx\, dy$$~~



$= \boxed{1.4}$

[9] $P(Y < X \leq Y+2) = ??$

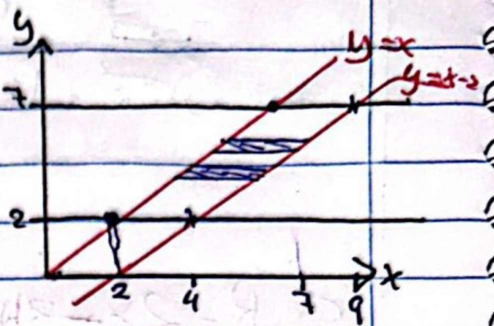
$x = y+2$

$y = x-2$

$x = 9 \rightarrow y = 7$

$x = 4 \rightarrow y = 2$

$x = 2 \rightarrow y = 0$



$$\int_2^7 \int_y^{y+2} 0.04 e^{-0.2x}\, dx\, dy$$

$$= \int_2^7 \frac{0.04}{-0.2} e^{-0.2x} \Big|_y^{y+2} dy = \int_2^7 -0.2 \left[e^{-0.2(y+2)} - e^{-0.2y} \right] dy$$

$$= \int_2^7 -0.2 e^{-0.2(y+2)} - 0.2 e^{-0.2y}\, dy$$

$$= \frac{-0.2 e^{-0.2(y+2)}}{-0.2} - \frac{0.2 e^{-0.2y}}{-0.2} \Big|_2^7$$

$$= e^{-2.5} + e^{-2.5}$$

$$\left(\frac{-0.2 e^{-0.2(9)}}{-0.2} + e^{-0.2(7)} \right) - \left(e^{-0.2(4)} + e^{-0.2(2)} \right)$$

$$(0.165 + 0.247) - (0.450 + 0.670)$$

$0.$

Transformation :-

Ex:- let X and Y be two R.Vs with the following joint PMF

x \ y	−1	1	2
−1	1/4	1/8	0
1	0	1/2	1/8

→ let $R = X + 2y^2$ - determine the PMF of R ?

x	y	$P(X=x, Y=y)$	$r = x + 2y^2$	$P(R=r)$
−1	−1	$\frac{1}{4}$	$r = (-1) + 2(-1)^2 = \boxed{1}$	$\frac{1}{4} + \frac{1}{8} = \boxed{\frac{3}{8}}$
−1	1	$\frac{1}{8}$	$r = (-1) + 2(1)^2 = \boxed{1}$	
1	1	1/2	$r = 1 + 2(1) = \boxed{3}$	$\boxed{1/2}$
1	2	1/8	$r = 1 + 2(2) = \boxed{5}$	$\boxed{1/8}$

مجموعهم = 1

انا

$$
\text{So } P(R=r) = \begin{cases} 3/8 & r=1 \\ 1/2 = 4/8 & r=3 \\ 1/8 & r=5 \\ 0 & \text{O.W} \end{cases}
$$

لما عوضنا X=-1

مع Y=-1 و عوضنا X=-1

مع Y=1

طلع جواب R

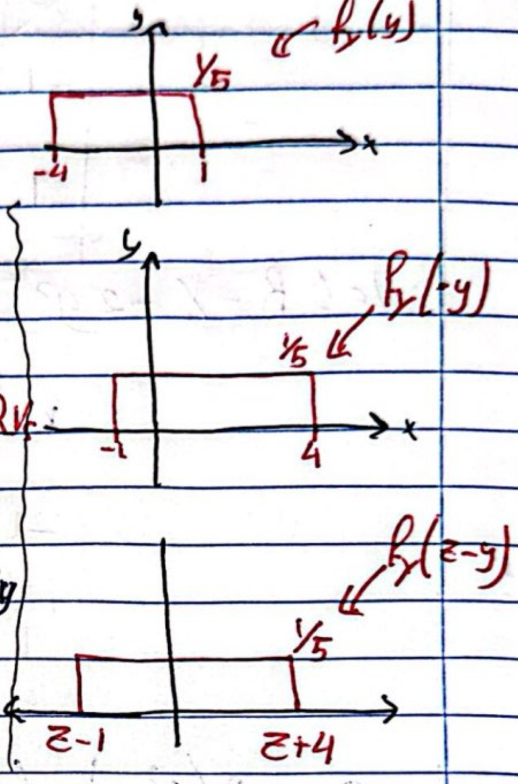
نفس الاشي (1)

فبنجمع الاحتماليات

Ex:- let X and Y be two statistically independent R.V.
X has a uniform distribution over the interval [-2, 6]
Y has also a uniform distribution over the interval [-4, 1].
let $\boxed{Z = X+Y}$, determine the pdf of Z?

$$f_X(x) = \begin{cases} \frac{1}{8} & -2 \le x \le 6 \\ 0 & o.w \end{cases}$$

$$f_Y(y) = \begin{cases} \frac{1}{5} & -4 \le x \le 1 \\ 0 & o.w \end{cases}$$



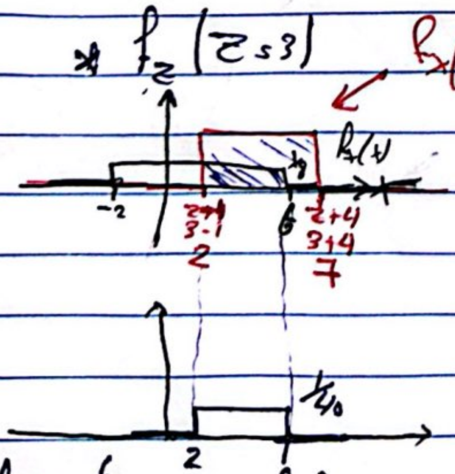
Note: if x and y are two S.I continuous R.V. and $Z = X+Y$, then the pdf of Z can be determined using the convolution Integral

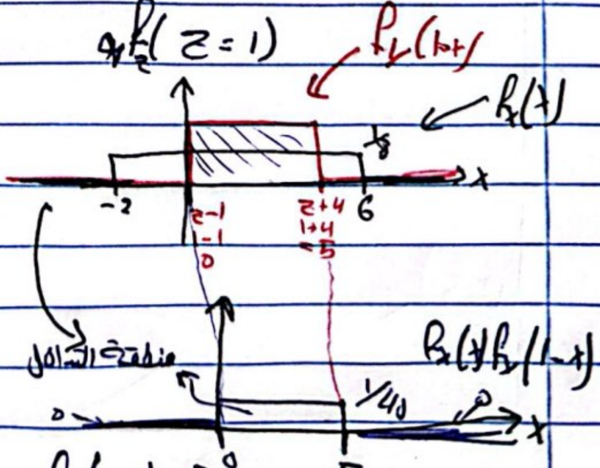
$$\rightarrow f_Z(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z-x)\,dx = \int_{-\infty}^{\infty} f_Y(y) f_X(z-y)\,dy$$

$$f_Y(z-x) = \begin{cases} \frac{1}{5} & -4 \le z-x \le -1 \\ 0 & o.w \end{cases}$$

$$= \begin{cases} \frac{1}{5} & (-z-4 \le -x \le -z+1) \\ 0 & o.w \end{cases} \Longrightarrow \begin{cases} \frac{1}{5} & z-1 \le x \le z+4 \\ 0 & o.w \end{cases}$$

$* f_Z(z=3)$ $* f_Z(z=1)$





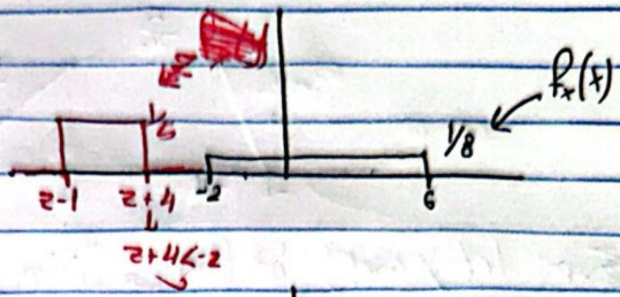
$$f_Z(z) = \int_2^6 \frac{1}{40}\,dx = \frac{x}{40}\Big|_2^6 = \frac{6-2}{40} = \boxed{\frac{4}{40}}$$

$$f_Z(z=1) = \int_0^5 \frac{1}{40}\,dt = \frac{x}{40}\Big|_0^5 = \boxed{\frac{5}{40}}$$

[1] $z+4<-2$

$z<-6$

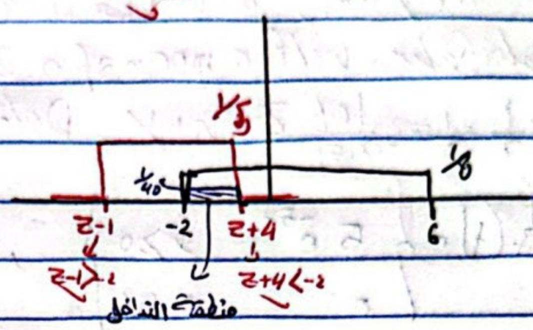
$f_z(z)=0$ فش تداخل



[2] $z+4>-2,\ z-1<-2$

$z>-6,\ z<-1$

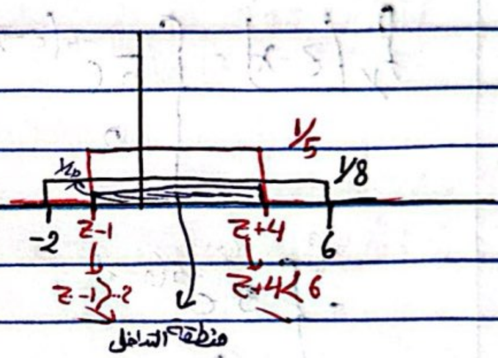
$$f_z(z)=\int_{-2}^{z+4}\frac{1}{40}\,dx=\frac{x}{40}\Big|_{-2}^{z+4}=\frac{z+4+2}{40}=\boxed{\frac{z+6}{40}}$$



[3] $z-1>-2,\ z+4<6$

$z>-1\ ,\ z<2$

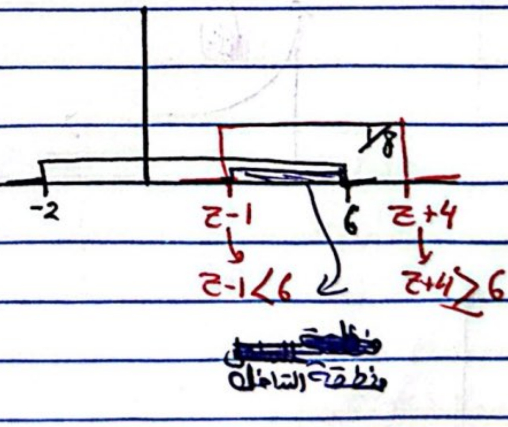
$$f_z(z)=\int_{z-1}^{z+4}\frac{1}{40}\,dx=\frac{x}{40}\Big|_{z-1}^{z+4}=\frac{z+4-z+1}{40}=\boxed{\frac{5}{40}}$$



[4] $z-1<6,\ z+4>6$

$z<7 \quad z>2$

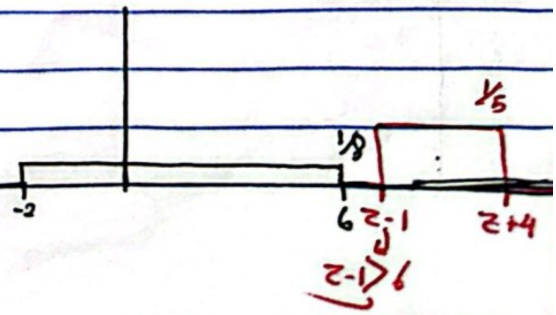
$$f_z(z)=\int_{z-1}^{6}\frac{1}{40}\,dx=\frac{x}{40}\Big|_{z-1}^{6}=\frac{6-z+1}{40}\ \boxed{\frac{7-z}{40}}$$

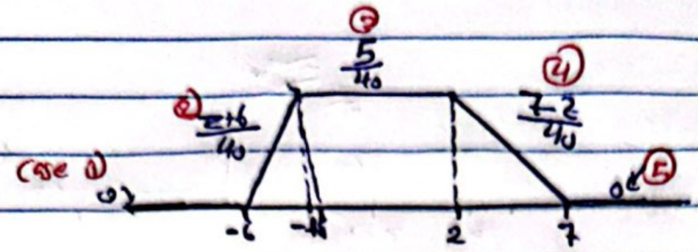


[5] $z-1>6$

$z>7$

$f_z(z)=0$ فش تداخل





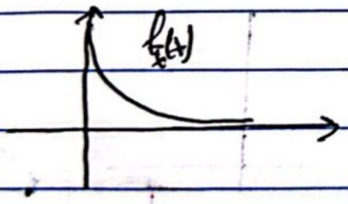
Exp:- let X and Y be two S. Indep. R.Vs with an exponential distribution with a mean of 0.2.

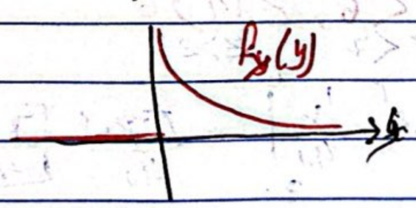
$\mu = \frac{1}{\lambda} \rightarrow \frac{1}{\lambda} = 0.2 \rightarrow \boxed{\lambda = 5}$ let $Z = X + Y$. Determine the pdf of Z

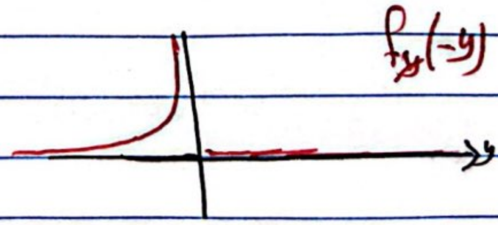
$$f_X(x) = \begin{cases} 5e^{-5x} & x \geq 0 \\ 0 & o.w \end{cases}, \quad f_Y(y) = \begin{cases} 5e^{-5y} & y \geq 0 \\ 0 & o.w \end{cases}$$

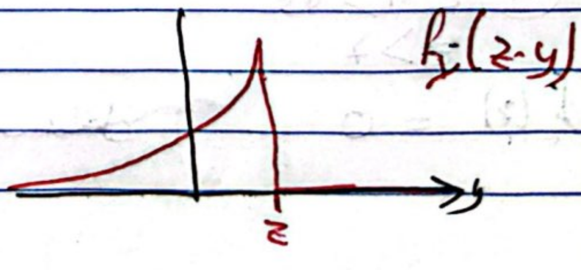
$$f_Y(z-x) = \begin{cases} 5e^{-5(z-x)} & z-x > 0 \\ 0 & o.w \end{cases}$$

$$= \begin{cases} 5e^{-5(z-x)} & z > x \\ 0 & o.w \end{cases}$$



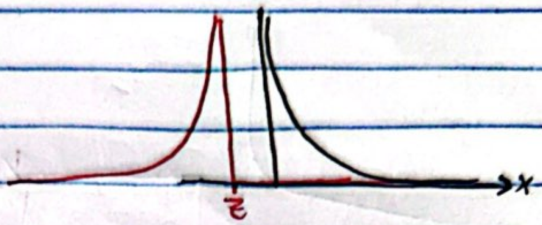




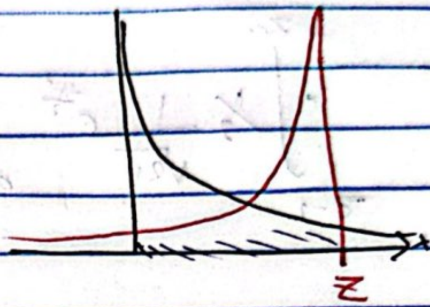


① $z<0$

$\therefore f_z(z) = 0$ فش تداخل



② $z>0$



$$f_z(z) = \int_0^z 5e^{-5x} * 5e^{-5(z-x)}\,dx$$

$$= \int_0^z 25e^{\cancel{-5x}-5z\cancel{+5x}}\,dx = \int_0^z 25e^{-5z}\,dx$$

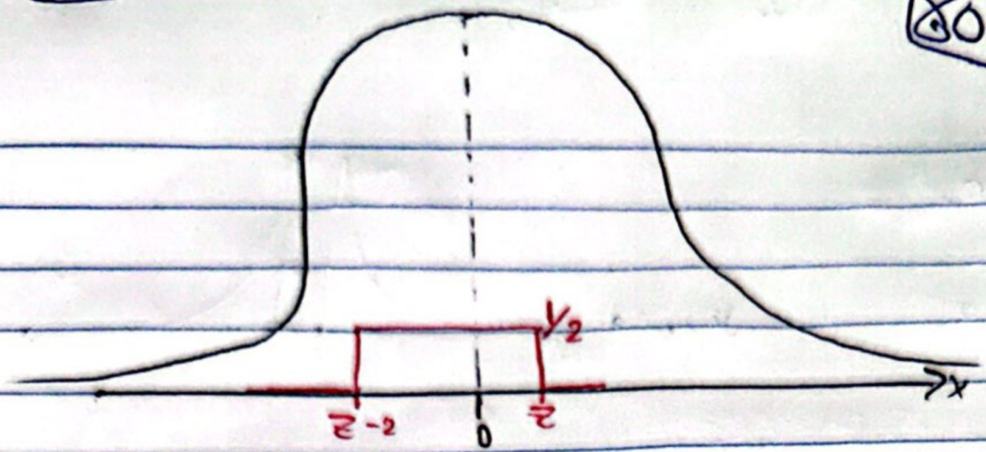
$$= 25e^{-5z}\cdot x\Big|_0^z = \boxed{25ze^{-5z}}$$

Exp: let x be a Gaussian R.V with zero mean and unity variance. y is another R.V with uniform distribution over the interval $[0,2]$. Assume x and y are S.I and $z = x+y$. Determine $f_z(0)$?

$$f_x(x) = \begin{cases} \dfrac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}} & -\infty<x<\infty \end{cases}$$

$$f_y(y) = \begin{cases} \frac{1}{2} & 0\le y\le 2 \\ 0 & o.w \end{cases} \rightarrow f_y(z-x) = \begin{cases} \frac{1}{2} & 0\le z-x\le 2 \\ 0 & o.w \end{cases}$$

$$= \begin{cases} \frac{1}{2} & -z\le -x\le 2-z \\ 0 & o.w \end{cases} \rightarrow = \begin{cases} \frac{1}{2} & z-2\le x\le z \\ 0 & o.w \end{cases}$$



$$= \int_{z-2}^{z} \frac{1}{2} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = \frac{1}{2} \int_{z-2}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$$

$$= \frac{1}{2} \left[\Phi(z) - \Phi(z-2) \right]$$

$$f_z(0) = \frac{1}{2} \left[\Phi(0) - \Phi(-2) \right] =$$

$$\Phi\left(\frac{0-0}{\sqrt{1}}\right) - \Phi\left(\frac{-2-0}{\sqrt{1}}\right)$$

$\mu_t = 0$, $\sigma_t^2 = 1$

* $t \rightarrow$ standard gaussian

$$P(1 \leq t \leq 5) = \int_{1}^{5} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = \Phi\left(\frac{5-0}{\sqrt{1}}\right) - \Phi\left(\frac{1-0}{\sqrt{1}}\right)$$

μ_t, σ_t^2