CH.1 fundamental signals

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Fundamental of signals

Problem Solution

input output

Signals
- → Continuoes Signals :- $X(t)$ → any Real number
- → discrete Signals :- $x[n]$ → only integers , $x[1.5]$ is not define.

وصف سلوك معين سواء عملي أو نظري أو رياضي.

Periodic or A periodic

[1] Periodic :-

- the Signals is Periodis $\iff X(t+T_0) = X(t)$, $-\infty < t < \infty$

* يكرر نفسه كل فترة معينة T_0

* $T_0 \rightarrow$ fundemantal period [sec]

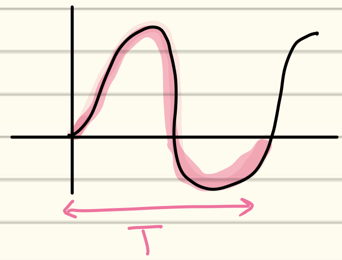
* $f_0 = \frac{1}{T_0}$, fundemantal frequency [Hz].

* $w_0 = 2\pi f_0 = \frac{2\pi}{T_0}$, angular frequency [rad/sec].

* $X(t) = A\cos(w_0 t + \theta)$

* $x(t) = A\sin(w_0 t + \theta)$

→ Phase shift.



Check if the following is periodic or not.

[1] $X(t) = A\cos(w_0 t + \theta)$

$x(t+T_0) = A\cos(w_0 t + w_0 T_0 + \theta)$ → $\frac{2\pi}{T_0} \times T_0 = 2\pi$

$= A\cos(w_0 t + \theta + 2\pi)$

$= A\cos(w_0 t + \theta)$

$= x(t)$

* its periodic.

2) $x(t) = A\sin(\omega_0 t + \theta)$

$$
\begin{aligned}
x(t+T_0) &= A\sin(\omega_0 t + \omega_0 T_0 + \theta) \\
&= A\sin(\omega_0 t + \theta + 2\pi) \\
&= A\sin(\omega_0 t + \theta) \\
&= x(t)
\end{aligned}
$$

* Periodice.

NOTE :-

عكس الإشارة

① $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

② $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$.

نفس الإشارة

3) $x(t) = A + B\cos(2\pi f_0 t)$.

$$
\begin{aligned}
x(t+T_0) &= A + B\cos(2\pi f_0 t + 2\pi f_0 T_0) \\
&= A + B\cos(2\pi f_0 t + 2\pi) \\
&= A + B\cos(2\pi f_0 t)
\end{aligned}
$$

* Periodic

4) $x(t) = 3\sin(\overset{\omega_0}{15} t)$

$$
\begin{aligned}
x(t+T_0) &= 3\sin(15t + 15T_0) \\
&= 3\sin(15t + 2\pi) \\
&= 3\sin(15t)
\end{aligned}
$$

$15 = \omega_0$

$15 = \frac{2\pi}{T_0} \rightarrow T_0 = \frac{2\pi}{15}$

substute

* Periodic.

Remark :-

إذا كانت sin أو cos لحالهم، فالعدد المضروب ب (t) هو ω_0.

Fundamental frequency

* To find fundemantal frequency, use [GCD].

* To find fundemantal period use [LCM].

for continuoes signal

Ex. find fundemantal frequency :-

1) $x(t) = 3\cos(400\pi t)$

$$
\begin{aligned}
\omega_0 &= 2\pi f_0 \\
400\pi &= 2\pi f_0 \\
f_0 &= 200 \text{ Hz}
\end{aligned}
$$

$\boxed{2}$ $x(t) = 2\cos(200\pi t) + 3\cos(400\pi t)$

$\omega_1 = 200\pi = 2\pi f_1$	$\omega_2 = 400\pi = 2\pi f_2$
$f_1 = 100$	$f_2 = 200$

$\frac{200}{100} = \frac{int}{int}$, لما كانت قسمتهم $\frac{int}{int}$ وتكون aperiodic وعندها يوجد f_0

$$f_0 = GCD(100, 200) = 100 \text{ Hz}$$

$\boxed{3}$ $x(t) = 2\sin(15t) + 3\cos(200\pi t)$

$\omega_1 = 15 = 2\pi f_1$	$\omega_2 = 200\pi = 2\pi f_2$	$\frac{f_1}{f_2} = \frac{100}{\frac{15}{2\pi}} = \frac{int}{rational}$
$f_1 = \frac{15}{2\pi}$	$f_2 = 100$	

doesn't have fundemantal frequency. so it's aperiodic

$\boxed{4}$ $x(t) = \cos\left(\frac{10\pi}{3} t\right) + \sin\left(\frac{5\pi}{4} t\right)$

$\omega_1 = 2\pi f_1 = \frac{10\pi}{3}$	$\omega_2 = 2\pi f_2 = \frac{5\pi}{4}$	$\frac{f_1}{f_2} = \frac{\cancel{5}}{3} \cdot \frac{8}{\cancel{8}} = \frac{8}{3} = \frac{int}{int}$ ✓
$f_1 = \frac{5}{3}$	$f_2 = \frac{5}{8}$	

$$f_0 = GCD\left(\frac{5}{3}, \frac{5}{8}\right) = \frac{GCD(5,5)}{LCM(3,8)} = \frac{5}{24} \text{ Hz}$$

* GCD : أكبر رقم مشترك بيقدر يقسموا عليه بدون باقي. [greatest common divisor]

* LCM : أقل عدد من مضاعفاتهم بينقسم عليهم بدون باقي. [least common Multiplier]

$\boxed{5}$ $x(t) = \sin\left(\frac{5\pi t}{6}\right) + \cos\left(\frac{3\pi t}{4}\right) + \sin\left(\frac{\pi}{3}t\right)$

$w_1 = \frac{5\pi}{6} = 2\pi f_1$ $f_1 = \frac{5}{12}$	$w_2 = \frac{3\pi}{4} = 2\pi f_2$ $f_2 = \frac{3}{8}$	$w_3 = \frac{\pi}{3} = 2\pi f_3$ $f_3 = \frac{1}{6}$

$$f_0 = GCD\left(\frac{5}{12}, \frac{3}{8}, \frac{1}{6}\right) = \frac{GCD(5,3,1)}{LCM(12,8,6)} = \frac{1}{24}$$

$T_0 = 24$

so the hormonice of wo for terms respectivly

10^{th}, 9^{th}, 4^{th}

$$x(t) = \sin\left(\frac{10\pi}{12}\right)t + \cos\left(\frac{9\pi}{12}\right)t + \sin\left(\frac{4\pi}{12}t\right)$$

↳ another way to write $x(t)$

for discrete signals :

$N = \frac{2\pi K}{w}$ → K: the smallest int. → (n) ﺩﻭﺭﻱ

$\frac{N}{k} = \frac{2\pi}{w} = \frac{int}{int}$

$x[n] = A\cos(w_0 n + \theta)$

$$x[n+N] = A\cos(w_0 N + w_0 n + \theta)$$

$$= A\cos(w_0 n + \theta + 2\pi k)$$

$$= A\cos(w_0 n + \theta)$$

* the original signal can now be written as :

$$x[n] = e^{jwn} = e^{jkw_0 n} = e^{jk\frac{2\pi}{N}n}$$

* $x[n] = \cos(\frac{\pi}{4} n)$

$w_0 = \frac{\pi}{4}$, $N = \frac{2\pi k}{w_0} = \frac{2\pi k}{\pi} \cdot 4$, $\boxed{k = 1}$

أقل عدد صحيح ممكن
يجعل N قيمة int.

$N = 8k$
$N = 8$ sampels

so periodic

$\boxed{2}$ $x_2[n] = 3\cos(\frac{3}{8}\pi n)$

$N = \frac{2\pi k}{w_0} = \frac{2\cancel{\pi} k}{3\cancel{\pi}} \cdot 8 = \frac{16}{3} k$, $\boxed{k = 3}$

$N = 16$ sampels

so periodic

$x[n] = 5\cos(3n)$

$N = \frac{2\pi k}{w_0} = \frac{2\pi k}{3}$, k can't be an integer number

so it's aperiodic.

$x[n] = 4\cos(0.2\pi n) - 3\cos(0.8\pi n) + 2\sin(0.3\pi n)$

* it's periodic
* the sum of any descrete periodic is periodic.

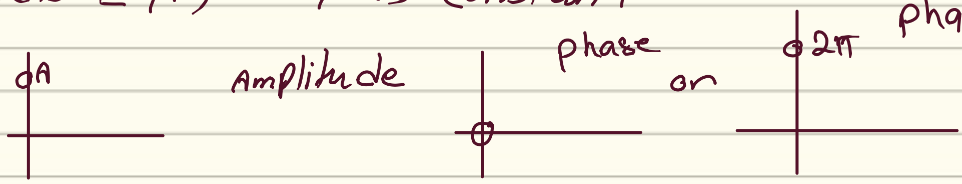
Remark:- $f, w \rightarrow$ GCD

$N, T \rightarrow$ LCM

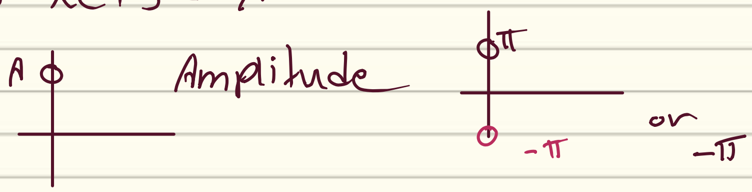
phase signals and spectra

to plot spectra :

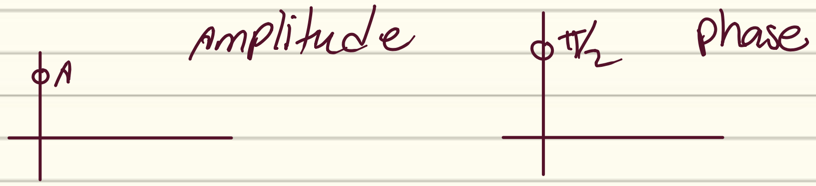
1. $\sin(\,) \longrightarrow \cos(\,)$ by subtracting $\frac{\pi}{2}$
   - $\sin(\alpha) = \cos(\alpha - \frac{\pi}{2})$
   - $\sin(\alpha) = \cos(\frac{\pi}{2} - \alpha)$
2. $-\cos(\,) \xrightarrow{\mp\pi} \cos(\,)$ * مسموح أحطو عشان الب
3. $x(t) = A$, A is constant



4. $x(t) = -A$



5. $x(t) = jA = Ae^{j\pi/2}$



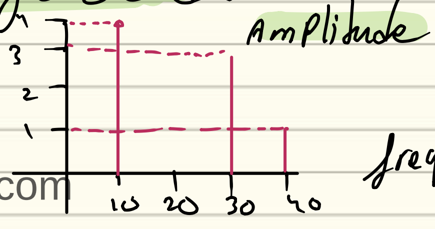
$\frac{\pi}{3} \times 2 = \frac{\pi}{2} \times 3 = \frac{3\pi}{2}$

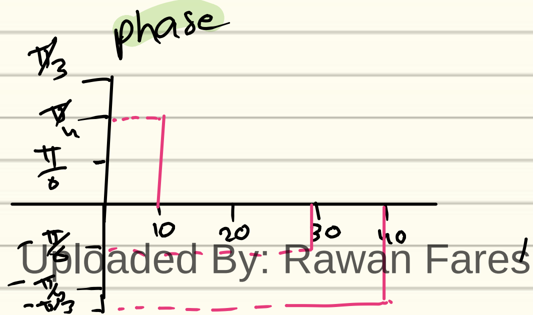
ex: $x(t) = 4\cos(20\pi t + \frac{\pi}{4}) + 3\cos(60\pi t - \frac{\pi}{6}) + \sin(80\pi - \frac{\pi}{6})$

$x(t) = 4\cos(20\pi t + \frac{\pi}{4}) + 3\cos(60\pi t - \frac{\pi}{6}) + \cos(80\pi - \frac{\pi}{3})$

$\omega_1 = 2\pi f_1$	$\omega_2 = 60\pi$	$\omega_3 = 80\pi$
$f_1 = 10$	$f_2 = 30$	$f_3 = 40$
$\theta_1 = \frac{\pi}{4}$	$\theta_2 = \frac{-\pi}{6}$	$\theta_3 = \frac{-\pi}{3}$
$A = 4$	$A = 3$	$A = 1$

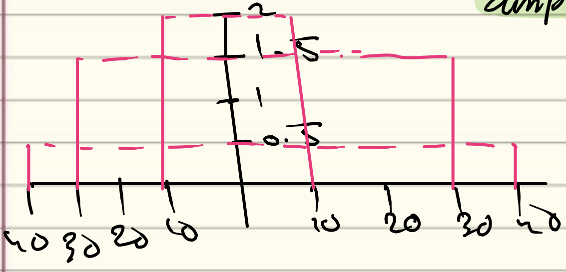
Single Sided :



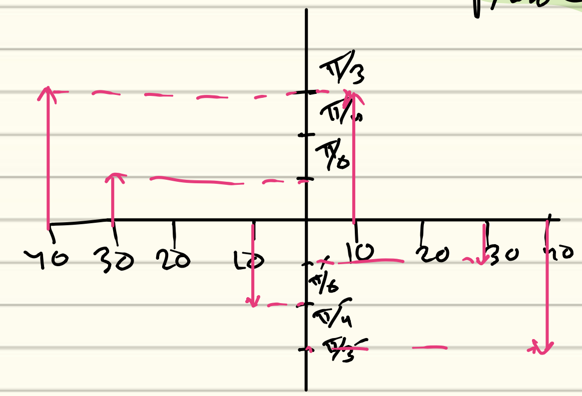


double sided

amplitude



Phase



ex:- $x(t) = 6\cos(20\pi t - \frac{\pi}{3}) + 4\sin^2(30\pi t - \frac{\pi}{6})$

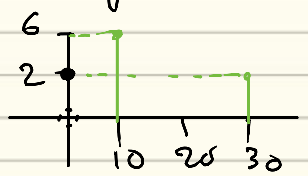
$$x(t) = 6\cos(20\pi t - \frac{\pi}{3}) + 4\left[\frac{1}{2}\left(1 - \cos\left(60\pi t - \frac{\pi}{3}\right)\right)\right]$$

$$x(t) = 6\cos(20\pi t - \frac{\pi}{3}) + 2 + 2\cos(60\pi t + \frac{2\pi}{3})$$

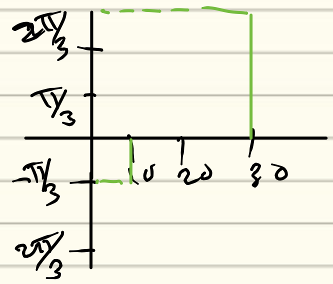
$\omega_1 = 20\pi$	$\omega_2 = 0$	$\omega_3 = 60\pi$
$f_1 = 10$	$f_2 = 0$	$f_3 = 30$
$\theta_1 = \frac{-\pi}{3}$	$\theta_2 = 0$	$\theta_3 = \frac{2\pi}{3}$
$A_1 = 6$	$A_2 = 2$	$A_3 = 2$

single sided

amplitude

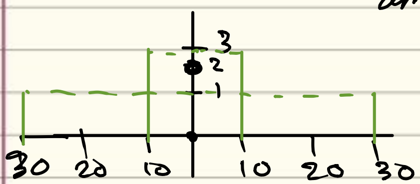


phase

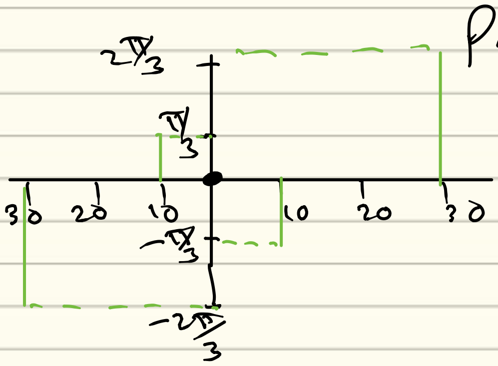


double Sided so

amplitude



Phase



ex: $x(t) = 4\cos(200\pi t)\cos(400\pi t)$ → لازم أفكر بطريقة أحلها لجمع

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \cancel{\sin\alpha\sin\beta}$

$+$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \cancel{\sin\alpha\sin\beta}$

$[\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos\alpha\cos\beta] \times 2$

$2\cos(\alpha+\beta) + 2\cos(\alpha-\beta) = 4\cos\alpha\cos\beta.$

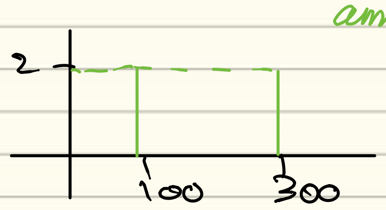
$4\cos(200\pi t)\cos(400\pi t) = 2\cos(600\pi t) + 2\cos(200\pi t)$

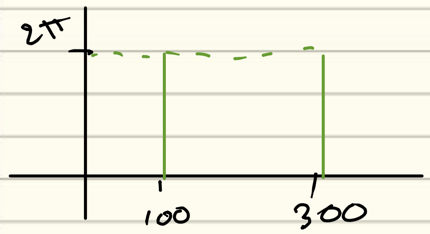
$w_1 = 600\pi$
$f_1 = 300$
$Q = 0$
$A_1 = 2$

$w_2 = 200\pi$
$f_2 = 100$
$Q_2 = 0$
$A_2 = 2.$

Single sided :

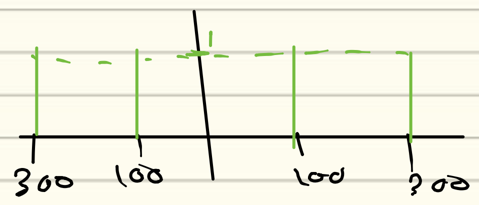
amplitude



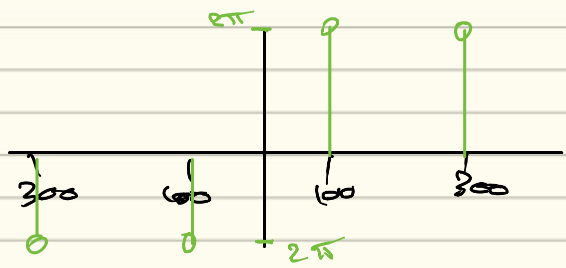


double sided :

amplitude



phase



ex: $x(t) = 2\sin(400\pi t)\sin(200\pi t)$

$\cos(\alpha-\beta) = \cos\alpha\cancel{\cos\beta} + \sin\alpha\sin\beta$

$-\cos(\alpha+\beta) = \cos\alpha\cancel{\cos\beta} - \sin\alpha\sin\beta$

$\cos(\alpha-\beta) - \cos(\alpha+\beta) = 2\sin\alpha\sin\beta.$

$2\sin(400\pi t)\sin(200\pi t) = \cos(200\pi t) - \cos(600\pi t)$

$x(t) = \cos(200\pi t) + \cos(600\pi t + \pi)$

$w_1 = 200\pi$	$w_2 = 600\pi$
$f_1 = 100$	$f_2 = 300$
$\theta_1 = 0$	$\theta_2 = \pi$
$A_1 = 1$	$A_2 = 1$

⋮ continue

ex: $x_2(t) = 6\sin(200\pi t)\cos(100\pi t).$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$+$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$

$\left[\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta\right] \times 3$

$6\sin\alpha\cos\beta = 3\sin(\alpha+\beta) + 3\sin(\alpha-\beta).$

$6\sin(200\pi t)\cos(100\pi t) = 3\sin(300\pi t) + 3\sin(100\pi t).$

$= 3\cos(300\pi t - \frac{\pi}{2}) + 3\sin(100\pi t - \frac{\pi}{2}).$

$w_1 = 300\pi$	$w_2 = 100\pi$
$f_1 = 150$	$f_2 = 50$
$\theta_1 = -\frac{\pi}{2}$	$\theta_2 = -\frac{\pi}{2}$
$A = 3$	$A = 3$

⋮ cont.

more examples :-

1. $x(t) = 4\sin(300\pi t)\sin(200\pi t)$
2. $x(t) = 3\sin^2(200\pi t) + 2\cos^2(400\pi t)$.

singularity functions

unit step function

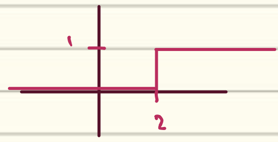
يحاكي Switch في الدارة الكهربائية من ناحية إغلاق وفتح.

* $u(t) = \begin{cases} 1, & t > 0 \\ 0, & \text{o.w} \end{cases}$ → dis Continuous at $t = 0$.

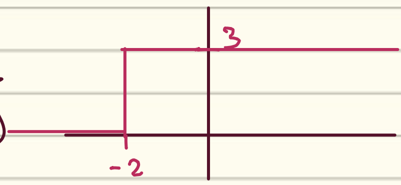
* $u(t - t_0) = \begin{cases} 1, & t > t_0 \\ 0, & \text{o.w} \end{cases}$

$\Downarrow$ Shifting to Right.

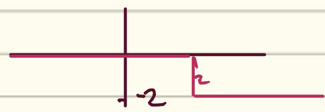
* $u(t-2) = \begin{cases} 1, & t > 2 \\ 0, & \text{o.w} \end{cases}$



* $3u(t+2) = \begin{cases} 3, & t > -2 \\ 0, & \text{o.w} \end{cases}$

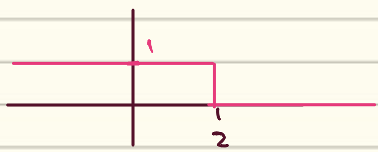


* $-2u(t-2) = \begin{cases} -2, & t > 2 \\ 0, & \text{o.w} \end{cases}$

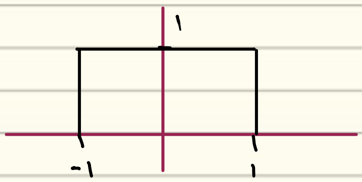


* $u(-t+2) = \begin{cases} 1, & t < 2 \\ 0, & \text{o.w} \end{cases}$

$-t+2 > 0$
$2 > t$

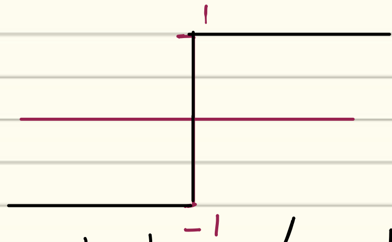


Q. express the signal $x(t)$ in terms of step function :-



Ans: $x(t) = u(t+1) - u(t-1)$

$$sgn(t) = \begin{cases} 1 , & t > 0 \\ 0 , & t = 0 \\ -1 , & t < 0 \end{cases}$$



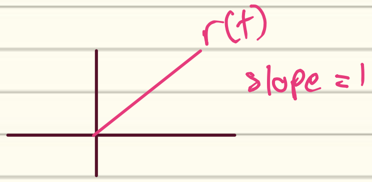
* express the sgn in terms of unit step function :-

$$sgn(t) = 2u(t) - 1$$
$$= u(t) - u(-t)$$

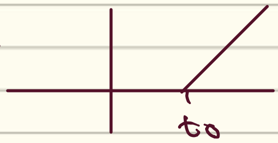
ramp function

$$r(t) = \begin{cases} t , & t > 0 \\ 0 , & o.w \end{cases}$$

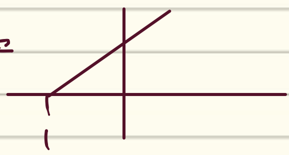
$$\frac{d\,r(t)}{dt} = u(t)$$



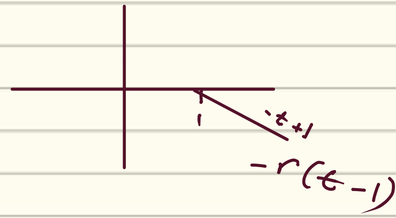
* $r(t - t_0) =$



* $r(t+1) =$

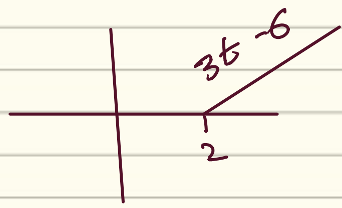


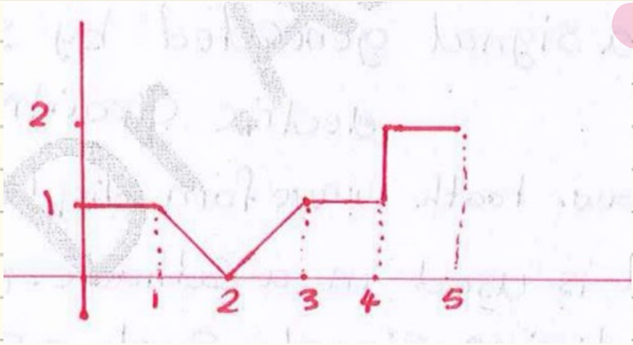
* $-r(t-1)$



* $r(3t - 6)$

$r(3(t-2))$

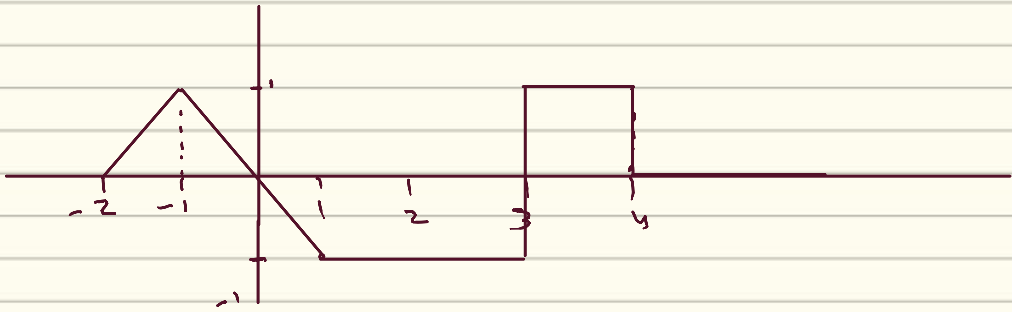




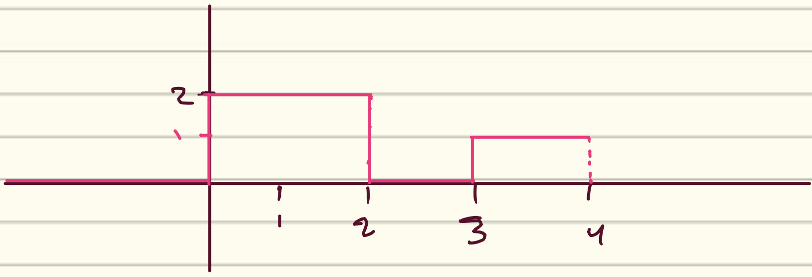
express the following Signal :-

$$x(t) = u(t) - r(t-1) + 2r(t-2) - r(t-3) + u(t-4) - 2u(t-5)$$

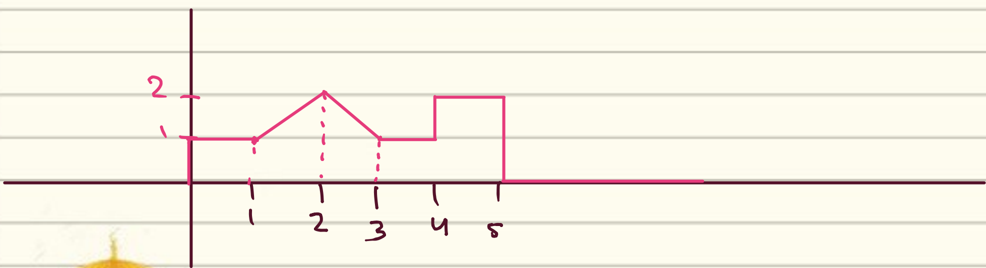
ex. $x_1(t) = r(t+2) - 2r(t+1) + r(t-1) + 2u(t-3) - u(t-4)$



2- $x_2(t) = 2u(t) - 2u(t-2) + u(t-3) - u(t-4)$



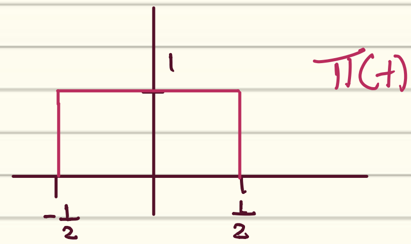
3- $x_3(t) = u(t) + r(t-1) - 2r(t-2) + r(t-3) + u(t-4) - 2u(t-5)$



unit pulse function

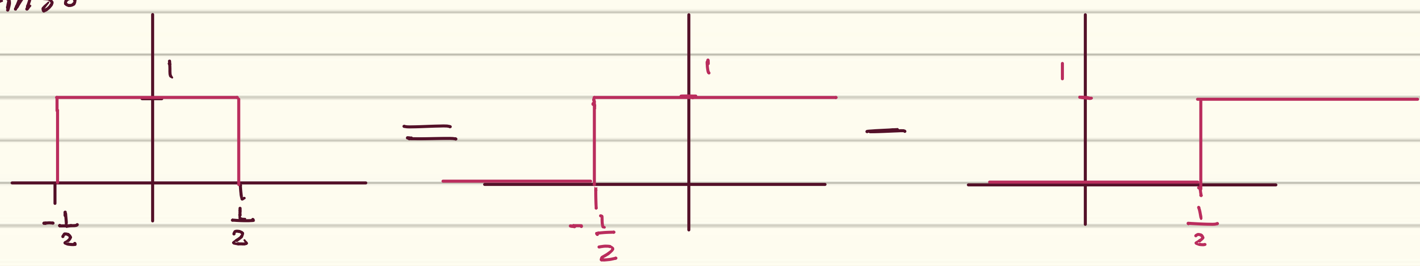
unit pulse function can be represented as :

$$\Pi(t) = \begin{cases} 1 & , -\frac{1}{2} \le t \le \frac{1}{2} \\ 0 & , \text{o.w} \end{cases}$$



ex. Express unit Pulse function in terms of unit Step function :

Ans :-



$$\Pi(t) = u\left(t+\frac{1}{2}\right) - u\left(t-\frac{1}{2}\right).$$

ex : Sketch the following Signal :

$x_1(t) = A\,\Pi(\alpha t - \beta)$, where $\beta, \alpha > 0$.

$$x_1(t) = A\,\Pi\left(\alpha\left(t - \frac{\beta}{\alpha}\right)\right).$$

Amplitude

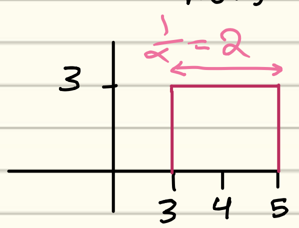
scale

Center

pulse عرض الـ $= \frac{1}{\alpha}$

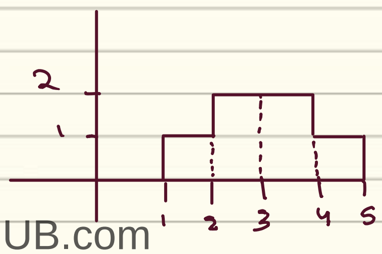
ex Sketch $x(t) = 3\Pi\left(\frac{1}{2}t - 2\right)$.

$$x(t) = 3\Pi\left(\frac{1}{2}(t-4)\right).$$



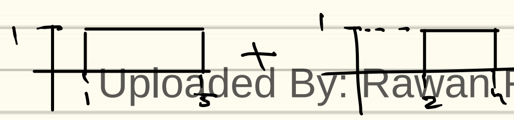
$$\frac{1}{\alpha} = 2$$

ex : express $x(t)$ in terms of Pulse function :



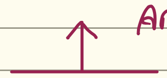
$$x(t) = \Pi\left(\frac{1}{4}(t-3)\right) + \Pi\left(\frac{1}{2}(t-3)\right)$$

$=$



Impulse functions :

* used in : 1. Network Th., Control theory and signal theory.



Area $\delta(t) = \begin{cases} 1 , & t = 0 \\ 0 , & o.w \end{cases}$

* mathmatically → goes to ∞
* engeenering → Area = 1

* it's important because of its properties and the insight it offers about the network.

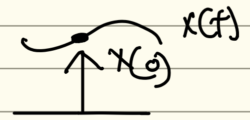
properties of $\delta(t)$ function :

1) change of variable : $\delta(at) = \frac{1}{|a|}\delta(t)$

2) Even function : $\delta(-t) = \delta(t)$

3) Sampling theorem : converting cont. signal into discrete signal.

$$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$



4) Sifting theorem :

$$\int_{t_1}^{t_2} x(t)\,\delta(t-t_0)\,dt = \begin{cases} x(t_0) , & t_1 < t_0 < t_2 \\ 0 , & o.w \end{cases}$$

substitute

5) Derivative theorem :

$$\int_{t_1}^{t_2} x(t)\,\delta^{(n)}(t-t_0)\,dt = (-1)^n x^{(n)}(t_0) = (-1)^n \left.\frac{d^n x}{dt^n}\right|_{t=t_0} \quad , t_1 < t_0 < t_2$$

* $\delta_\epsilon(t) = \frac{1}{2\epsilon}\Pi\left(\frac{t}{2\epsilon}\right) = \begin{cases} \frac{1}{2\epsilon} , & |t| < \epsilon \\ 0 , & o.w \end{cases}$

Q. Show that $\int_{t_1}^{t_2} x(t)\,\dot{\delta}(t-t_0)\,dt = (-1)\left.\frac{dx(t)}{dt}\right|_{t=t_0} \quad , t_1 < t_0 < t_2$

Ans : $\int_{t_1}^{t_2} x(t)\,\dot{\delta}(t-t_0)\,dt$, by using integral by parts.

let $u = x(t)$ $\qquad dv = \dot{\delta}(t-t_0)$

$du = \dot{x}(t)$ $\qquad v = \delta(t-t_0)$

$$x(t)\,\delta(t-t_0)\Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} \dot{x}(t)\,\delta(t-t_0)\,dt$$

$$= x(t_0)\,\cancel{\delta(t-t_0)}\Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} \dot{x}(t)\,\delta(t-t_0)\,dt$$

by Sampling Th. $\qquad$ by sifting Th.

$$= -\dot{x}(t_0) = (-1)\left.\frac{dx(t)}{dt}\right|_{t=t_0}$$

Q.2 Show that $\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$

Since that $\delta(t) = \dfrac{du(t)}{dt}$ $\rightarrow$ substitue.

$$= \int_{-\infty}^{\infty} x(t)\,\frac{du(t)}{\cancel{dt}}\cdot\cancel{dt} = \int_{-\infty}^{\infty} x(t)\,du(t) \rightarrow \text{integrating by parts}$$

$=$ let $u = x(t)$ $\qquad dv = du(t)$

$\qquad du = \dot{x}(t)$ $\qquad v = u(t)$

$$= x(t)\,u(t)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \dot{x}(t)\,u(t)\,dt$$

$$= x(\infty)u(\infty) - x(-\infty)u(-\infty) - \int_{0}^{\infty} \dot{x}(t)\,dt$$

$$= x(\infty)\,\overset{1}{u(\infty)} - x(-\infty)\,\overset{0}{u(-\infty)} - [x(\infty) - x(0)]$$

$$= \cancel{x(\infty)} - \cancel{x(\infty)} + x(0)$$

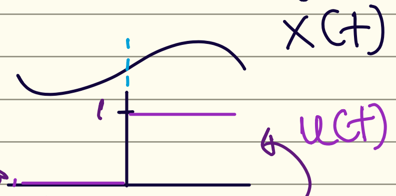
$$= x(0)$$

\# Proving sifting theorm

NOTE :-

$$\int_{-\infty}^{\infty} \dot{x}(t)\,u(t)\,dt$$

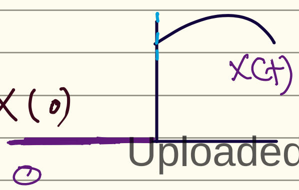
$$= \int_{0}^{\infty} \dot{x}(t)\,u(t)\,dt$$



$\therefore$, $u(-\infty) = 0$ $\quad u(\infty) = 1$

when multiplying the two function together :-

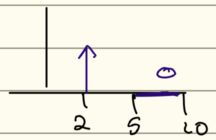
$$= \int_{0}^{\infty} \dot{x}(t)\,dt = x(\infty) - x(0)$$



EXAMPELS : Evaluate the following integrals :

a) $\int_{5}^{10} \cos(2\pi t)\,\delta(t-2)\,dt$, $2 \notin [5,10]$

$\therefore$ Ans $= 0$



b) $\int_{0}^{5} \cos(2\pi t)\,\delta(t-2)\,dt$, $2 \in [0,5]$

Ans : $= \cos(4\pi)$
$= 1$

c) $\int_{-\infty}^{\infty} [e^{-3t} + \cos(2\pi t)]\,\dot{\delta}(t)\,dt$ $\qquad 0 \in (-\infty,\infty)$

$= (-1)\left[-3e^{-3t} + -2\pi \sin(2\pi t)\right]\Big|_{t=0}$

$= 3$

d) $\int_{-\infty}^{\infty} e^{3t}\,\ddot{\delta}(t-2)\,dt$ $\qquad 2 \in (-\infty,\infty)$

$= (-1)^2 \left(3e^{3t}\right)'$

$= 9e^{3t}\Big|_{t=2} = 9e^{6}$

Ex. Find the unspecified constants, denoted as C_1, C_2 in the expression :

[1] $10\delta(t) + C_1\dot{\delta}(t) + (2+C_2)\ddot{\delta}(t) = (3+C_3)\delta(t) + 5\dot{\delta}(t) + 6\ddot{\delta}(t)$

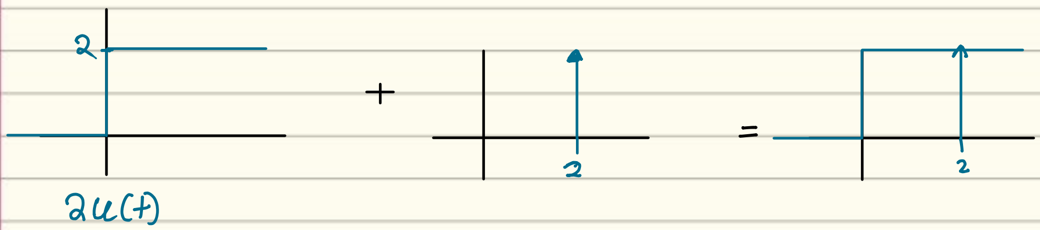
$10 = 3 + C_3 \rightarrow \boxed{C_3 = 7}$

$\boxed{C_1 = 5}$

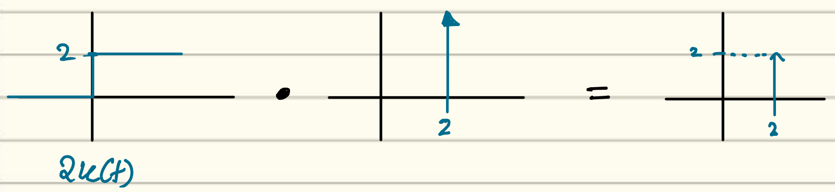
$2 + C_2 = 6 \rightarrow \boxed{C_2 = 4}$

Examples:- Sketch the following signal :-

A) $x_1(t) = 2u(t) + \delta(t-2)$.



B) $x_2(t) = 2u(t) \cdot \delta(t-2)$.



« converting Continuoes Signal ~~for~~ Discreate ».

Example :- Plot accuratly the following signal defined in terms of singularity functions :-

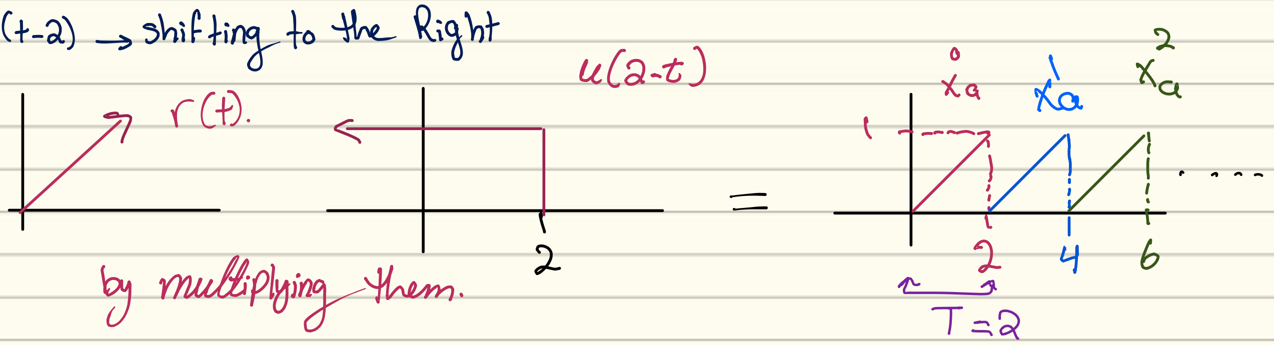
A) $x_1(t) = \sum_{n=0}^{\infty} x_a(t-2n)$, plot $0 \leq t \leq 6$, where $x_a(t) = r(t)\, u(2-t)$.

① $n=0$

$x_a(t) = r(t)\, u(2-t)$.

② $n=1$

$x_a(t-2) \rightarrow$ shifting to the Right



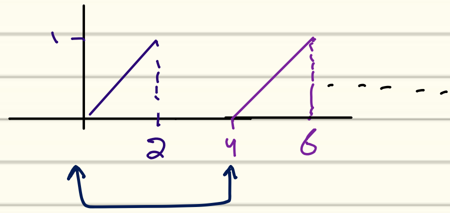
B) $X_2(t) = \sum_{n=0}^{\infty} X_a(t-4n)$; $X_a(t) = r(t)\,u(2-t)$, $0 \leq n \leq 6$

① $n=0$

$X_a(t) = r(t)\,u(2-t)$.

② $X=1$

$X_a(t-4) \longrightarrow$ Shifting $\underline{4}$ to the Right.



$T=4$

c) $X_3(t) = \sum_{n=0}^{\infty} X_0(t-3n)$, plot $0 \leq t \leq 6$, where $X_0(t) = r(t) - r(t-1) - r(t-2) + r(t-3)$.

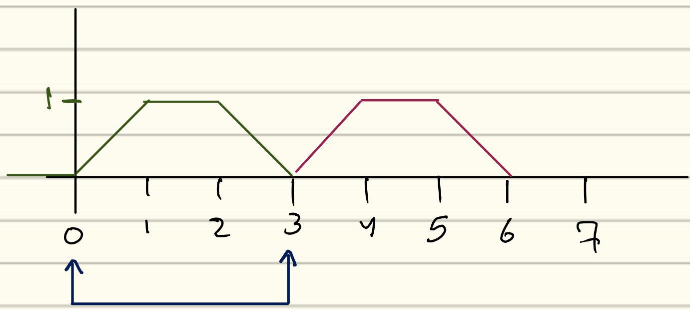
period $= 3$.

① $n=0$

$X_0(t) = r(t) - r(t-1) - r(t-2) + r(t-3)$.

② $n=1$

$X_0(t-3) \longrightarrow$ shifting 3 to the Right



$T=3$

Period $= 5$

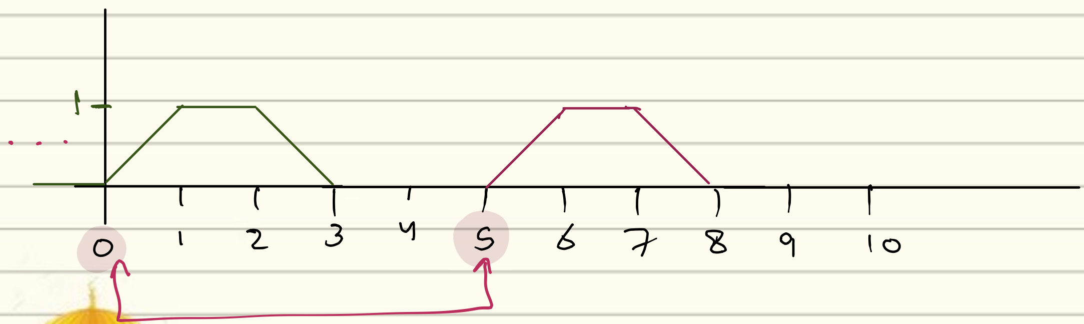
d) $X_4(t) = \sum_{n=-\infty}^{\infty} X_0(t-5n)$; $X_b(t) = r(t) - r(t-1) - r(t-2) + r(t-3)$.

① $n=0$

$X_0(t) = r(t) - r(t-1) - r(t-2) + r(t-3)$

② $n=1$

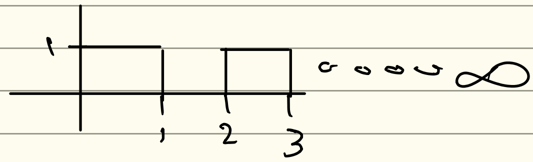
$X_0(t-5) \longrightarrow$ shifting $\underline{5}$ to the Right.



Period $= 5$.

EX. sketch the signal $y(t)=\sum_{n=0}^{\infty} u(t-2n)\,u(1+2n-t)$ $T=2$

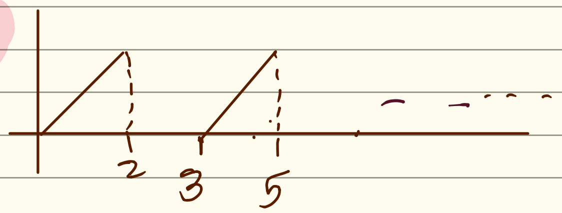
When $n=0 \rightarrow y(t)=u(t)\,u(1-t)$
$n=1 \rightarrow y(t)=u(t-2)\,u(3-t)$



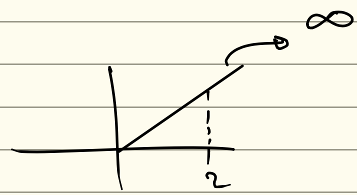
Π

EX: Express the signal shown in terms of singularity function

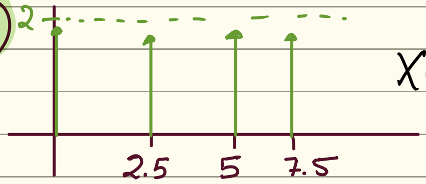
A)



$$\sum_{n=0}^{\infty} r(t-3n)\,u(2-(t-3n)).$$



B)



$$x(t)=\sum_{n=0}^{\infty} 2u(t)\,\delta(t-2.5n)$$

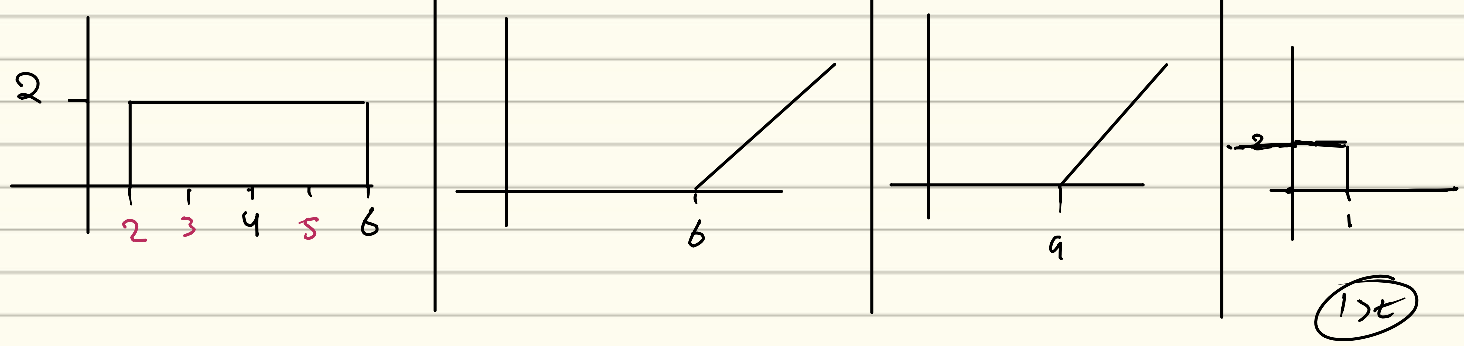
Δ

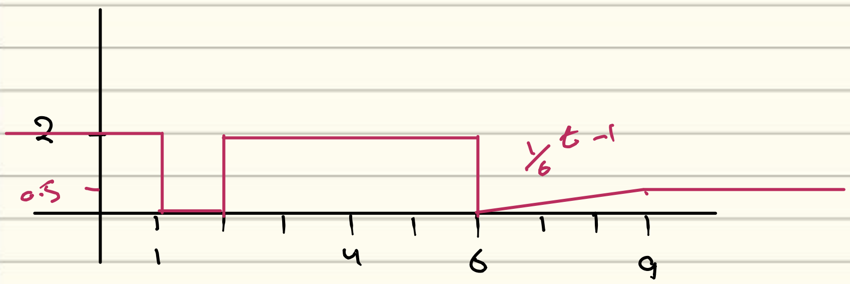
$T=2.5$,

EX: plot the following signals using elementary signal:

$$x(t) = 2\Pi\left(\frac{t-4}{4}\right) + r\left(\frac{t-6}{6}\right) - r\left(\frac{t-9}{6}\right) + 2u(1-t)$$

$2\Pi\left(\frac{1}{4}(t-4)\right)$ | $r\left(\frac{1}{6}(t-6)\right)$ | $r\left(\frac{1}{6}(t-9)\right)$ | $2u(1-t)$





" ابنا تقبل منّا
إنك أنت
السميع العليم "