Faculty of Engineering and Technology
Electrical and Computer Engineering Department
Probability and Statistical Engineering, ENEE2307
Dr. Mohammad K. Jubran
Quiz #1

Date: Time: 25 minutes
Name: أسماء عبدالرحمن فارس شجاعية Student #: 1210084

Problem 1 (10 pts):
Suppose we roll two dice; a red dice and a blue dice. The blue dice is fair. However, the probability of observing a number in the red dice is proportional to its value. Let A = "The sum of the two numbers observed on both dice is divisible by 5". Compute P(A).

red dice sample space

S = {1, 2, 3, 4, 5, 6}

P(S) = K + 2K + 3K + 4K + 5K + 6K

1 = 21K

K = 1/21

total Sample space

S = {11, 12, 13, 14, 15, 16,
21, 22, 23, 24, 25, 26,
31, 32, 33, 34, 35, 36,
41, 42, 43, 44, 45, 46,
51, 52, 53, 54, 55, 56,
61, 62, 63, 64, 65, 66}

A = {14, 23, 32, 41, 55, 64}

P(A) = $\left(\frac{1}{6} \cdot \frac{1}{21}\right) \times 6 = \frac{1}{21}$

Problem 2 (5+5 pts):

A factory has ~~three~~ two production lines; A and B. The probability that production line A fails is 20%. The probability that production line B fails is 10%. However, if production line A has failed, the probability that production line B fail double. What is the probability that:

a) both production lines fail?

$P(A\cap B) = P(A) \cdot P(B|A)$

$= 0.2 \times 0.2$

$= 0.04$

5)

$P(A\cap B) = P(A) + P(B) - P(A\cup B)$

b) at least one of the production lines will stay working?

$P(A\cup B) = P(A) + P(B)$

$= 0.2 + 0.2$

$= 0.4$

2)

$P(\bar{A}\cup\bar{B}) = 1 - P(A\cap B)$

$= 1 - 0.04$

$= 0.96$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

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12.5/15

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Faculty of Engineering and Technology
Electrical and Computer Engineering Department
Probability and Statistical Engineering, ENEE2307
Dr. Mohammad K. Jubran
Quiz #2

Date: Wednesday, 11 May 2023 — Time: 15 minutes
Name: أسماء عبد الرحمن فارس شجاعية — Student #: 1210084

Problem 1 (15 pts):

Let X be Random variables with the following cumulative distribution function

CDF

$$F_X(x) = \begin{cases} G & x < 0 \\ 0.5 & 0 \le x < 2 \\ 0.7 & 2 \le x < 3 \\ H & 3 \le x \end{cases}$$

a) Find the value of the constants G and H.

~~G = P(x<0) =~~

5

$G = P(x < 0) = 0$

$H = P(x \le \infty) = 1$

b) Find $P(1 \le x / x < 2.5)$

2.5

$$\frac{P(1 \le x < 2.5)}{P(x < 2.5)} = \cancel{\frac{0.5 + 0.2}{0.2}}$$

$$= \frac{P(1 \le x) \cap P(x < 2.5)}{P(x < 2.5)} = \frac{0.2}{0.7}$$

~~$\frac{0.5 + 0.2}{0.2}$~~

c) Find the Probability Mass function (PMF) of X?

$P(x=0) = P(0 \leq x) - P(0 > x) = 0.5 - 0 = 0.5$

$P(x=2) = P(2 \leq x) - P(2 > x) = 0.7 - 0.5 = 0.2$

$P(x=3) = P(3 \leq x) - P(3 > x) = 1 - 0.7 = 0.3$

PMF

$$P(X=x) = \begin{cases} 0.5 & x=0 \\ 0.2 & x=2 \\ 0.3 & x=3 \\ 0 & 0.w \end{cases}$$

$\frac{22}{30}$

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Electrical and Computer Engineering Department

Probability and Statistical Engineering, ENEE2307

Dr. Mohammad K. Jubran

Quiz #2

Date: Tuesday, 10/5/2022 | Time: 25 minutes

Name: ~~ي~~ ياسمين عال | Student #: 1201146

Problem 1 (30pts):

Let X denote the actual air pressure for a car tire in pounds per square inch (psi). Suppose that X is random variable with the density function

$$f_X(x) = \begin{cases} Kx & 30 \le x \le 50 \\ 0 & otherwise \end{cases}$$

a) Determine the value of the constant K that makes $f_X(x)$ a valid probability density function.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \longrightarrow \int_{-\infty}^{30} 0 + \int_{30}^{50} Kx\,dx + \int_{50}^{\infty} 0 = 1$$

5

$$\left.\frac{Kx^2}{2}\right]_{30}^{50} = 1250K - 450K = 1$$

$$800K = 1$$

$$\boxed{K = \frac{1}{800}}$$

$$f_X(x) = \begin{cases} \frac{1}{800}x & 30 \le x \le 50 \\ 0 & o.w \end{cases}$$

b) Determine the Probability that the air pressure in a tire is between 35 psi and 45 psi.

$$P(35 \leq x \leq 45) = \int_{35}^{45} f(x)\,dx$$

$$= \int_{35}^{45} \frac{1}{800} x\,dx = \frac{1}{800}\left[\frac{x^2}{2}\right]_{35}^{45}$$

$$= \frac{1}{800}\left[\frac{45^2}{2} - \frac{35^2}{2}\right] = \frac{1}{800}\left[1012.5 - 612.5\right]$$

$$\frac{1}{800} \times 400 = \frac{1}{2}$$

c) Let $F_X(x)$ be the Cumulative Function of X. Determine $F_X(45)$.

CDF

$F_X(x) =$

$$\int_{-\infty}^{x} 0\,dx = 0 \quad \rightarrow \quad x < 30$$

$$F_X(x) = \begin{cases} 0 & x < 30 \\ & 30 \leq x \leq 50 \\ 1 & x \geq 50 \end{cases}$$

$$\int_{30}^{x} \frac{1}{800} x\,dx = \frac{1}{800}\left[\frac{x^2}{2}\right]_{30}^{x} = \frac{1}{800}\left[\frac{x^2}{2} - 450\right]$$

$$\cancel{\int_{50}^{\infty} \frac{1}{80} x\,dx}$$

$$\int_{50}^{x} \frac{1}{80} x\,dx = 1$$

$50 < x$

$$F_X(45) = \frac{1}{800}\left[1012.5\right]$$

$= .7031$

d) Determine the mean value of X.

$$\mu_x = E\{g(x)\} = \int_{-\infty}^{\infty} g(x)\, f_x$$

5

$$= \int_{30}^{50} \frac{x}{800} \times x = \int_{30}^{50} \frac{x^2}{800} = \frac{x^3}{3 \times 800} \Big|_{30}^{50}$$

$$= 52.038 - 11.25 = 40.78$$

$$f_x(x) = \begin{cases} K(1+x) & -1 \le x < 0 \\ 0 & \text{o.w} \end{cases}$$

$$f_X(x) = \begin{cases} K(1+X) & -1 \leq X < 0 \\ 0 & \text{other. wise} \end{cases}$$

① Find K

② Find the median

③ Find $P(X < -0.5 \,/\, X < -0.2)$

Dr. Mohammad K. Jubran
Quiz #2

Date: Thursday, 12/5/2022 Time: 20 minutes
Name: yasmeen Kamel Student #: 1201146

Problem 1 (30pts):
Let X be a random variable with the following probability density function

$$f_X(x) = \begin{cases} Kx^2 & 0 \le x \le 4 \\ 0 & otherwise \end{cases}$$

a) Determine the value of the constant K that makes $f_X(x)$ a valid probability density function.

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

5

$$\int_{-\infty}^{0} 0 + \int_{0}^{4} Kx^2 \, dx + \int_{4}^{\infty} 0 = 1$$

$$\frac{Kx^3}{3}\Big]_0^4 = K\left[\frac{4^3}{3} - \frac{0^3}{3}\right] = 1$$

$$21.\overline{3}K = 1$$

$$\boxed{K = .0468}$$

$$f_X(x) = \begin{cases} .0468x^2 & 0 \le x \le 4 \\ 0 & o.w \end{cases}$$

b) Determine $P(1 \le X \le 3)$

~~$P(1 \le X \le 3) = P(X \le 3) - P(X < 1)$~~

~~$= 0.468 \times 3^2 -$~~

$$\int_1^3 f(x)\,dx = \int_1^3 .0468\, x^2\, dx$$

$$= \frac{0.468\, x^3}{3}\Big]_1^3 = \frac{0.0468}{3}\left[3^3 - 1\right]$$

$$= .4056.$$

3

c) Let $F_X(x)$ be the Cumulative Function of X. Determine $F_X(3)$.

$F_X(3) =$

① $x < 0 \longrightarrow \int_{-\infty}^{x} 0 = 0.$

② $0 \le x \le 4 \longrightarrow \int_0^x .0468\, x^2\, dx = \frac{0.0468\, x^3}{3}\Big]_0^x$

$$= \frac{0.0468}{3}\left[x^3\right] = 0.0156\, x^3.$$

③ $x > 4 \longrightarrow 1$

$F_X(3) = 0.0156 \times (3)^3 = .4212$

5

d) Determine the mean value of X.

$\mu = E$

$$E\{g(x)\} = \int_{-\infty}^{\infty} g\,f(x)$$

$$= \int_0^4 .0468\, x^2\, dx$$

$$= \frac{.0468\, x^3}{3}\Big]_0^4$$

$$= 0.0156 \left[4^3 - 0 \right]$$

$$= .9984$$

e) Determine the Standard deviation of X.

$\sigma_x = \sqrt{\sigma_x^2}$

$\sigma_x = E\{(x - \mu_x)^2 f(x)\}$

$= \int_{-\infty}^{\infty} (x - .998)^2 \, .0468 x^2 \, dx$

4

$= \int_0^4 [x^2 - 2x .998 + .98] .0468 x^2$

$= \int_0^4 .0468 x^4 - .093 x^3 + .0458 x^2$

$\left[\frac{.0468 x^5}{5} - \frac{.093 x^4}{4} + \frac{.0458 x^3}{3} \right]_0^4$

$\sigma_x = \sqrt{[0.479 - 5.952 + .9776] - 0}$

=

f) Determine the mode of the distribution of X.

$f^*(x) = .0468\, x^2$

$f'(x) = .0468\, x = 0.$

$\boxed{X = 0}$

$f(0) = 0$

$f(4) = .0468 \times (4)^2$

$= .7488$

4

the mode = ~~.7488~~.

+4

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Electric and Computer Engineering Department

Probability and Statistical Engineering ENEE2307

Dr Alhareth Zyoud

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Quiz # 1 (B)

Date: Monday 11-10-2021 Duration: 25 minutes

Name: Rivan Jaradat Student #: 1200081

Question 1: Four light bulbs are selected at random without replacement from 16 bulbs, of which 9 are defective. Find the probability that

16 bulbs

9d $\frac{9}{16}$ 7 nond $\frac{7}{16}$

$C^7_4 = \frac{7!}{3!4!} = \frac{7\times5\times6}{3\times2\times1}$

$= 35$

a) none are defective.

~~$P = \binom{16}{4}\left(\frac{7}{16}\right)^4$~~

~~$P = C_4^7(0.43)^4$~~

~~$P = C_7^{16}\left(\frac{7}{16}\right)$~~

8/10

$C^{16}_4 = \frac{16!}{12!4!} = \frac{16\times15\times14\times13\times12!}{12!\times4\times3\times2\times1} = \frac{43680}{4\times3\times2\times1} = 1820$

$P = \frac{C^7_4}{1820} = \frac{35}{1820} = 0.0192$

b) all are defective

$C^9_4 = \frac{9!}{4!5!} = \frac{9\times8\times7\times6\times5!}{4\times3\times2\times1\times5!} = 126$

$P = \frac{126}{1820} = 0.069$

c) exactly one is defective.

$C^9_1 \times C^7_3 = \frac{9!}{8!1!} \times \frac{7!}{3!4!} = \frac{9\times8!}{8!} \times \frac{7\times6\times5\times4!}{3\times2\times1\times4!}$

$= 9 \times 35$

$= 315$

$P = \frac{315}{1820} = 0.173$

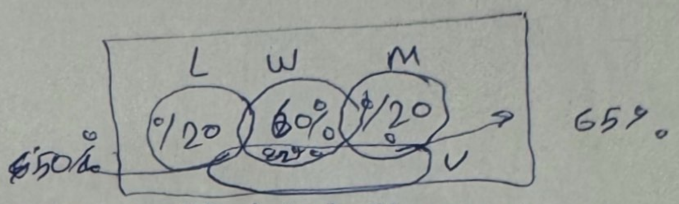
d) at least one is defective.

$C^9_4 \times C^7_3 = \frac{9!}{4!5!} \times \frac{7!}{3!4!} = \frac{9\times8\times7\times6\times5!}{4\times3\times2\times1\times5!} \times \frac{7\times6\times5\times4!}{3\times2\times1\times4!}$

$= 126 \times 35 = 4410$

$P = \frac{4410}{1820} = 2.42$

$L \quad P(L) \quad P(V|L)$
$m \quad P(m) \quad P(V|m) \rightarrow P(V)$
$W \quad P(W) \quad P(V|W)$



Question 2: Suppose that 20 percent of computer owners use a Macintosh, 60 percent use Windows, and 20 percent use Linux. Suppose that 65 percent of the Mac users have succumbed to a computer virus, 82 percent of the Windows users get the virus, and 50 percent of the Linux users get the virus.

a) We select a person at random, what is the probability that this person gets the virus?

$P(V) = P(L)P(V|L) + P(m)P(V|m) + P(W)P(V|W)$

~~P(L)~~

$= 0.2(0.5) + 0.2(0.65) + 0.6(0.82)$

$= 0.1 + 0.13 + 0.49$

$= 0.722.$

b) We select a person at random and learn that his system was infected with the virus. What is the probability that he is a Windows user?

$P(W/V) = \dfrac{P(W \cap V)}{P(V)} = \dfrac{(0.82)(0.6)}{0.722} = 1.135$??

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Probability and Statistical Engineering ENEE2307

Dr Alhareth Zyoud

Quiz # 2

30/30

Date: Thursday 5-1-2023 Duration: 25 minutes

Name: Faten Sultan Student #: 1202750

Question 1: Suppose that X is a ~~continuous~~ Discrete random variable with Cumulative Distribution Function given below

$F_X(-2)$

$$F_X(x) = \begin{cases} H & , \quad x < -2 \\ 0.2 & , \quad -2 \le x < 0 \\ G & , \quad 0 \le x < 4 \\ R & , \quad 4 \le x \end{cases}$$

$f(x)<0$

Assume P(x > 0)=0.4

1) **Determine the value of the constants H, G, R?**

→ $H = 0$

→ $R = 1$

→ $P(X>0) + P(X \le 0) = 1$

$0.4 + F_X(0) = 1$

$F_X(0) = G = 0.6$

5

$$F_X(x) = \begin{cases} 0 & X < -2 \\ 0.2 & -2 \le x \le 0 \\ 0.6 & 0 \le x < 4 \\ 1 & 4 \le x \end{cases}$$

2) **Find the P(−2 < x < 4)**

5

~~$P(x \le 4) - P(X \le -2)$~~

~~$0.6 - 0.2$~~

~~$= 0.4$~~

$P(X < 4) - P(X \le -2)$

$= 0.6 - 0.2$

$= 0.4$

3) **Find the P(0 ≤ x ≤ 4)**

$\cancel{P(X \le 4) - P(X \le 0)}$

$\cancel{f_X(4) - f_X(0)}$

$\cancel{1 - 0.6} = \cancel{0.4}$

$P(x \le 4) - P(X < 0)$

$1 - 0.2$

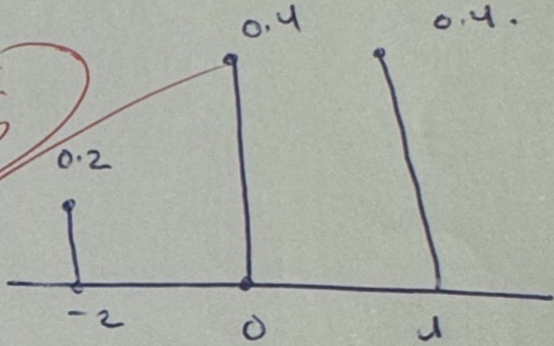
$= 0.8$

5

4) **Determine the PMF of X?**

$$P(X=x) = \begin{cases} 0.2 & x = -2 \\ 0.4 & X = 0 \\ 0.4 & X = 4 \\ 0 & \text{o.w.} \end{cases}$$

5



$\Sigma P(x) = 0.2 + 0.4 + 0.4 = 1.$

valid.

5) **Determine the expected value of X?**

$$\sum_{-\infty}^{\infty} x\,P(X=x) = (-2)(0.2) + 0(0.4) + 0.4(4)$$

$$= -0.4 + 1.6$$

$$= 1.2$$

5

6) **Determine the Variance of X?**

$$\sum_{-\infty}^{\infty} (x - \mu x)^2 P(X = x).$$

$$\sum_{-\infty}^{\infty} (x - 1.2)^2 \, P(X = x).$$

$(-2 - 1.2)^2 (0.2) \quad + (0 - 1.2)^2 (0.4) + (4 - 1.2)^2 (0.4)$

$2.048 \quad + \quad 0.576 \quad + 3.136$

$= \quad 5.76.$

5

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Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Probability and Statistical Engineering, ENEE2307
Dr. Mohammad K. Jubran
Quiz#1 – Section 1

Date: Thursday, 7/4/2022 Time: 30 minutes
Name: Razan Abdelrahman. Student #: 1200531

Problem 1 (20pts):
In a game experiment a coin is flipped for two times and a dice is rolled for one time. The probability of observing a head in the coin is three times the probability of tail. Let A be the event of observing at least one head, and B is the event that two heads are observed and an even number is observed on the dice.

a) Compute $P(A)$.

+7

A = {(H,H,1), (H,T,1) (T,H,1), (H,H,2), (H,T,2), (T,H,2)
(H,H,3), (H,T,3), (T,H,3), (H,H,4), (H,T,4), (T,H,4)
(H,H,5), (H,T,5), (T,H,5), (H,H,6), (H,T,6), (T,H,6)}

$p(A) = \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6}$
$+ \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6}$
$+ \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{6}$

$= \frac{18}{96} + \frac{36}{96}$ $\boxed{\frac{54}{96}}$

b) Compute $P(B)$.

B = {(H,H,2), (H,H,4), (H,H,6)}

$p(B) = p(\{HH2\} \cup \{HH4\} \cup \{HH6\})$

$= \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{6} + \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{6}$

+4

$= \frac{9}{96} + \frac{9}{96} + \frac{9}{96}$

$= 0,28125 = \boxed{\frac{27}{96}}$

c) Compute $P(A/B)$.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P\{\{H,H,2\} \cup \{H,H,4\}, \{H,H,6\}\}}{P(B)}$$

$$= \frac{P(B)}{P(B)} = \boxed{1}$$

d) Are A and B statistically independent?

$$P(A \cap B) \stackrel{?}{=} P(A)\, P(B).$$

$$P(B) \stackrel{?}{=} P(A)\, P(B)$$

$$\frac{54}{96} \stackrel{?}{=} \frac{54}{96} \cdot \frac{27}{96}$$

No, so they are not statically independent.

Problem 2 (10pts):

In a certain assembly plant, three machines, B1, B2, and B3, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected.

1) What is the probability that it is defective?

$$P(D) = p(D/B_1)\,p(B_1) + p(D|B_2)\,p(B_2) + p(D/B_3)\,p(B_3)$$

$$= (0{,}02)(0{,}3) + (0{,}03)(0{,}45) + (0{,}02)(0{,}25).$$

$$= \boxed{0{,}0245.}$$

$0{,}3\ B_1 \xrightarrow{0{,}02} D$

$0{,}45\ B_2 \xrightarrow{0{,}03} D$

$0{,}25\ B_3 \xrightarrow{0{,}02} D$

2) If a defective item is selected, what is the probability that it was made by machine B2.

$$p(B_2/D) = \frac{p(B_2 \cap D)}{p(B_2)}$$

$$= \frac{p(D/B_2)\,p(B_2)}{p(D)}$$

$$= \frac{0{,}03(0{,}45)}{0{,}0245}$$

$$= \boxed{0{,}551}$$

10

$P(T) = k$.

$P(H) = 3k$.

* Coin $= \{HH, HT, TH, TT\}$.

$P(S) = P(\{HH\} \cup \{H,T\} \cup \{T,H\} \cup \{TT\})$

$= P(\{HH\}) + P(\{HT\}) + P(\{TH\}) + P(\{TT\})$

$1 = 9k^2 + 3k^2 + 3k^2 + k^2$

$16k^2 - 1 = 0$.

$\frac{16}{16} k^2 = \frac{1}{16}$

$k^2 = \frac{1}{16} \longrightarrow k = \pm \frac{1}{4} \Rightarrow \boxed{k = \frac{1}{4}}$

$$\boxed{\begin{array}{l} P(k) = \frac{1}{4} \\ P(H) = \frac{3}{4} \end{array}}$$

95/10

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Probability and Statistical Engineering ENEE2307

Dr Alhareth Zyoud

Quiz # 1 Section II

Date: Thursday 24-11-2022 Duration: 25 minutes

Name: Falen Sultan Student #: 1202750

0.5

Question 1: A box contains three coins A, B and C. Coins A and B are fair coins (have a head H and a tail T), while coin C is two headed. One coin is chosen at random from the box and tossed once.

a. What is the probability that the toss results in a head H? in any order.

b. If the picked coin shows a heads H, find the probability that coin C was selected?

a) $S = \{ HHH, HTH, HHT, THH, TTH, THT, HTT \}$??

A	B	C
~~HT~~	~~HH~~	~~HT~~
HT	HT	HH

D: event that head appears

$P(A)P(H/A) + P(B)P(H/B) + P(C)\,P(H/C)$

$\frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times 1$

$\frac{1}{6} + \frac{1}{6} + \frac{1}{3} = \frac{2}{3}$ 3

b) $\frac{P(C)P(H/C)}{P(D)} \quad \frac{1/3 \times 1}{2/3} = \frac{1}{2}$ 2

Question 2: Consider a communication system. At any given time, the communication channel is in good condition with probability 0.6. An error occurs in a transmission with probability 0.2 if the channel is in good condition and with probability 0.3 if the channel is in bad condition. Let G be the event that the channel is in good condition and E be the event that there is an error in transmission.

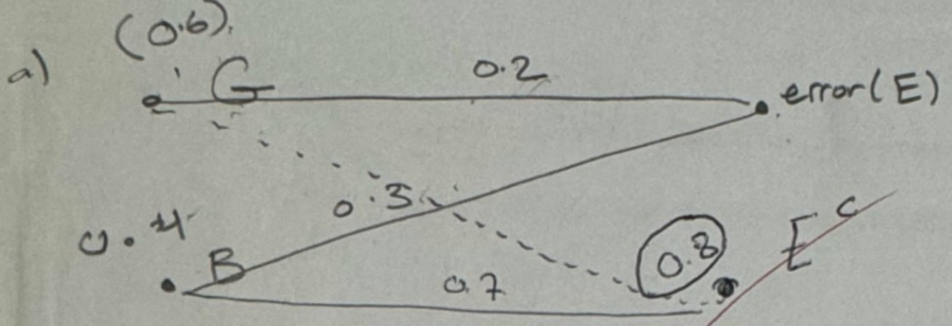
a) Find P(E)
b) Find P(G/E^C)

$0.2 = E/G$

$0.3 = E/B$

$E/G = \frac{E \cap G}{G} = \frac{G/E \cdot E}{G}$

a)



$P(E) = P(E/G) \times P(G) + P(E/B)\,P(B)$

$= 0.6 \times 0.2 + 0.3 \times 0.4$

$= 0.12 + 0.12$

$= 0.24$ → $\boxed{P(E) = 0.2}$

3

$E^c = 1 - 0.24 = 0.76$

b) $P(G/E^c) = \frac{P(G \cap E^c)}{P(E^c)} = \frac{P(G)\,P(E^c/G)}{E^c}$

1.5

$1 - P(E/G) = P(E^c/G)$

$1 - 0.6 = 0.4$

$\frac{0.6 \times 0.4}{0.76}$ (0.8)

$= 0.3157$

$P(E^c) = 1 - 0.24$

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Faculty of Engineering and Technology
Electrical and Computer Engineering Department
Probability and Statistical Engineering, ENEE2307
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Quiz #2

Date: Thursday, 12/5/2022 Time: 20 minutes
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Problem 1 (30pts):

Let X be a random variable with the following probability density function

$$f_X(x) = \begin{cases} Kx^2 & 0 \le x \le 4 \\ 0 & otherwise \end{cases}$$

a) Determine the value of the constant K that makes $f_X(x)$ a valid probability density function.

$$\int_{-\infty}^{\infty} f_X(x)\, dx = 1$$

$$\int_{-\infty}^{0} 0\, \cancel{dx} + \int_{0}^{4} Kx^2\, dx + \int_{4}^{\infty} 0\, dx = 1$$

$$\frac{Kx^3}{3} \Big|_0^4 = 1$$

$$K\left(\frac{64}{3} - \frac{0}{3}\right) = 1$$

$$K\left(\frac{64}{3}\right) = 1$$

$$\boxed{K = \frac{3}{64}}$$

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$$f_X(x) = \begin{cases} \frac{3}{64}x^2, & 0 \le X \le 4 \\ 0, & \text{otherwise.} \end{cases}$$

b) Determine $P(1 \le X \le 3)$

$$p(1 \le x \le 3) = \int_1^3 \frac{3}{64} x^2 \, dx = \left(\frac{\cancel{3}}{64} \frac{x^3}{\cancel{3}} \right|_1^3$$

$$= \frac{x^3}{64} \Big|_1^3$$

$$= \frac{27}{64} - \frac{1}{64} = \boxed{\frac{26}{64}}$$

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c) Let $F_X(x)$ be the Cumulative Function of X. Determine $F_X(3)$.

$$F_X(3) = p(x \le 3) = \int_{-\infty}^{3} \frac{3}{64} x^2 \, dx$$

$$= \cancel{\int_{-\infty}^{0} 0 \, dx} + \int_0^3 \frac{3}{64} x^2 \, dx$$

$$= \frac{\cancel{3}}{64} \frac{x^3}{\cancel{3}} \Big|_0^3$$

$$= \frac{27}{64} - \frac{0}{64}$$

$$= \boxed{\frac{27}{64}}$$

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d) Determine the mean value of X.

$$M_X = E\{X\} = \int_{-\infty}^{\infty} x\, f_X(x)\, dx$$

$$= \int_0^4 x \, \frac{3}{64} x^2 \, dx$$

$$= \int_0^4 \frac{3}{64} x^3 \, dx$$

$$= \frac{3}{64} \frac{x^4}{4} \Big|_0^4$$

$$= \frac{3}{256} x^4 \Big|_0^4$$

$$= \frac{3}{\cancel{256}} (\cancel{256}) - \cancel{\frac{3(0)}{256}}$$

$$= \boxed{3}$$

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e) Determine the Standard deviation of X.

$$\sigma_X^2 = E\{(X-\mu_X)^2\} = \int_{-\infty}^{\infty} (x-3)^2 f_X(x)\, dx$$

$$= \cancel{\int_{-\infty}^{0} 0\, dx} + \int_{0}^{4} (x-3)^2 \left(\frac{3}{64} x^2\right) dx + \int_{4}^{\infty} 0\, dx$$

$$= \frac{3}{64}\int_0^4 (x^2 - 6x + 9)\, x^2\, dx$$

$$= \frac{3}{64}\int_0^4 (x^4 - 6x^3 + 9x^2)\, dx$$

$$= \frac{3}{64}\left(\frac{x^5}{5} - \frac{6x^4}{4} + 3x^3\right)\Bigg|_0^4$$

$$= \frac{3}{64}\left(\left(\frac{1024}{5} - \frac{1536}{4} + 192\right) - 0\right)$$

$$= \boxed{0.6}$$

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$$\sigma_X = \sqrt{\sigma_X^2}$$

$$= \sqrt{0.6}$$

$$= \boxed{0.7746}$$

f) Determine the mode of the distribution of X.

$f'_x(x)\, dx = 0$

$\frac{d}{dx}\left(\frac{3}{64}x^2\right)\Big|_{X_{mode}} = 0$

$\frac{6}{64} X = 0$

$\boxed{X_{mode} = 0}$

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